Data and Decision Analysis
Project work

A nonlinear mechanical resonant system is described by the discrete-time equations

$$\begin{aligned} x_1(k+1) &= x_1(k) + x_2(k) \\ x_2(k+1) &= -x_1(k) + \frac{1}{2}\,x_2(k) - \alpha\, x_2^3(k) + u(k) \end{aligned}$$

where α is a real parameter. Measurements of the second state variable are collected:

$$y(k) = x_2(k) + v(k)$$

where $v(k)$ represents measurement noise and is modeled as a white zero-mean stochastic process, with variance $\sigma_v^2 = 1$.

a) Assuming $\alpha = 0.1$, design and implement an Extended Kalman Filter for estimating the state variables $x_1(k)$ and $x_2(k)$ at each time k, based on the measurements $y(t)$ and the input $u(t)$, with $t = 1, 2, \ldots, k$.

b) Assume now that the parameter α is not know and therefore it must be estimated along with the state variables. Define an extended model in which the state vector contains a new variable $x_3(k) = \alpha$, whose dynamics can be modeled according to

$$x_3(k+1) = x_3(k) + w(k)$$

in which $w(k)$ is modeled as a white zero-mean stochastic process, with variance σ_w^2. Design and implement an Extended Kalman Filter for estimating the state variables $x_i(k)$, $i = 1, 2, 3$, using the same data employed in item a) for $y(k)$ and $u(k)$. Compare the estimation errors of x_1 e x_2, and the corresponding variances, to the values obtained in the case in which α is known.
[Suggestion: choose carefully the value of the process noise variance σ_w^2 in the equation of the state variable $x_3(k)$ and discuss the results for different values of σ_w^2].

c) In both items a) and b) evaluate:

- the consistency of the filter, by comparing the estimation errors of each state variable $x_i(k) - \hat{x}_i(k|k)$ with the corresponding confidence intervals $\pm 3\sqrt{P_{ii}(k|k)}$;
- the behavior of the filter as a function of the initial conditions $\hat{x}(0|-1)$, $P(0|-1)$.

Data available in the file `dati_silver.mat`:
`Y`: vector of measurements $y(k)$, $k = 0, \ldots, N$;
`U`: vector of inputs $u(k)$, $k = 0, \ldots, N$
`X`: matrix $N \times 2$ of true states $x_i(k)$, $i = 1, 2$, $k = 0, \ldots, N$, to be used <u>only</u> for the comparisons described in item c) (the entry k,i in the matrix corresponds to $x_i(k)$).