Sparse Adaptive Filtering Techniques for Active Noise Cancellation Systems

A project report submitted in partial fulfillment of the requirements for B.Tech. Project

B.Tech.

by

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CANDIDATES DECLARATION

I hereby certify that the work, which is being presented in the report, entitled **Sparse adaptive filtering techniques for active noise cancellation systems**, in partial fulfillment of the requirement for the award of the Degree of **Bachelor of Technology** and submitted to the institution is an authentic record of my own work carried out during the period *June 2021* to *october 2021* under the supervision of **Dr. Vinal Patel**. I also cited the reference about the text(s)/figure(s)/table(s) from where they have been taken.

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ABSTRACT

In real world we encounter impulsive noise which generally are sparse in nature that they have meaningful content only for few moment. This project proposes a techniques of Sparse Adaptive Filter for acoustic impulse signal/noises. As these type of noises cannot be filtered out using normal filter and if can be filter out then they are not computationally efficient. To overcome this limitation, this paper present adaptive filter for sparse systems, which will help to filter out the impulsive noise in efficient manner. Our focus in primary focus is through this paper is to come up with new sparsity aware filter which converges much faster has much better steady state MSE. Secondly, combining the filter with other class of adaptive filter which will achieve robustness against inpulsive interference. Experiments result were shown which compares the performance of the proposed filter to the available filters in terms of both convergence rate and steady-rate.

Keywords: LMS, ZA-LMS, RZA-LMS, excess mean square error, adaptive filter, sparse system, system identification.

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ABBREVIATIONS

ANC	Active Noise Cancellation
LMS	Least Mean Square
NLMS	Normalized-LMS
PNLMS	Propotionate-NLMS
IPNLMS	Improved-PNLMS
SC-IPNLMS	Sparseness-Controlled-IPNLMS
ZA-LMS	Zero-Attracting-LMS
RZA-LMS	Reweighted-ZA-LMS
LHCAF	Logarithmic Hyperbolic Cosine Adaptive Filter
JLHCAF	Joint-LHCAF
MSE	Mean Square Error
MSD	Mean Square Deviation

NOTATIONS

w(n)	weight of the filter
μ	convergence coefficient
Δ w(n)	gradient of residual
y(n)	observed output signal
d(n)	desired output signal
x(n)	input signal
v(n)	noise signal
e(n)	residual signal
L(n)	cost function
ρ	strength of zero attraction
ϵ	shrinkage magnitude
λ	scaling factor
γ	l_1 norm panelty factor
β	attraction parameter
b	band constant

CHAPTER 1

INTRODUCTION AND LITERATURE SURVEY

This chapter discuss active noise cancellation, digital filter, adaptive filters, our objective, concept behind working and also list some of the work done in field of adaptive filter.

1.1 Introduction

This section briefly describe what active noise cancellation is and how the noises are removed from the acoustic environment. This section also contains the how the digital filters are used and on what is the working principle of it in noise cancellation system.

Active noise cancellation(ANC) system generate antinoise and introduces it into acoustic environment, this antinoise cancels the undesired sound and thus gives noise free audio. ANC system consists of different component which includes the microphone(for sensing the noise signal from environment), loudspeakers to send the antinoise into acoustic environment. It also contains different set of signal converter to change the domain of the noise signal. And most importantly it consists of the adaptive filter whose coefficient was changed as per the noise signal detected in environment. Mostly all adaptive feedforward ANC are digital.

Antinoise signal in feedforward ANC is generated using the digital filtering the reference signal and in feedback ANC using residual signal. Digital filter is the process in which the input signal is modified according to the parameter of the filter which amplify and attenuate frequency component of the input signal. Main parameter of the digital filters include:

- Filter coefficient: this defines the weight of the each component of signal in the output given by it. This parameter was determined through the adaptive algorithm.

- Length of filter: this parameter defines the how much sample of component of signal taken in calculating the output

Adaptive algorithm is the most basic yet most important part of noise cancellation systems, as they are responsible for calculating the frequency component of antinoise signal that is sent out in acoustic environment for generating noise free environment. These algorithm changes the filter coefficient based on the signal entered into the system. These algorithm updates filter coefficient values such that the error is minimised. There are different strategy used to update the weights of the coefficient, among that most used strategy was based on gradient descent. The parameter of this algorithm includes the direction of error minimization and step size in each direction. And speed of convergence of the algorithm was determined by the step-size parameter, if it is too large then algorithm fails to converge to the minimum value, on other hand if it is too small then the convergence rate of the algorithm is very low i.e. algorithm will take much time to converge to the minimum value. So how to find out the optimal value of step-size. Value of step-size depends on various factors which include the environment in which the algorithm was used. Some of the major factor on which step-size depends are signal power, characteristics of the system.

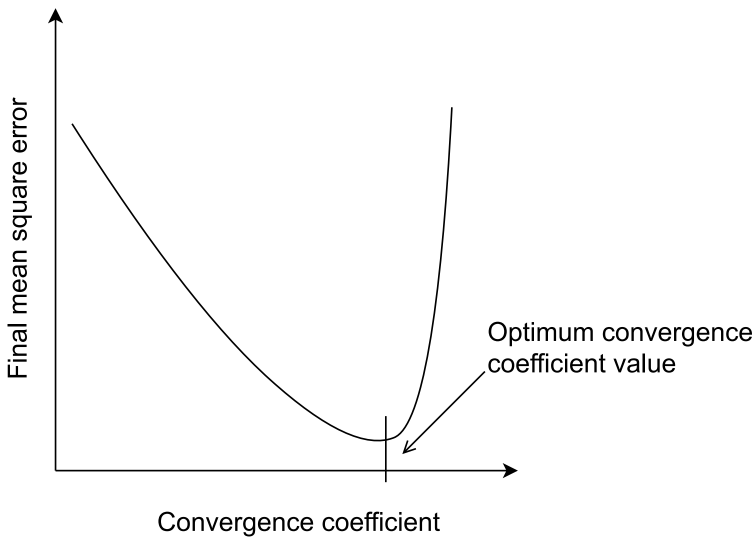

Figure 1.1: MSE v/s convergence coefficient

But how these terms are used in ANCs and how it produces the noise cancellation effect? Main component that is used in the ANCs is its controller which can also be referred as heart of an noise cancellation system. This controller is responsible for producing the noise cancellation effect, it is the main unit which performs digital filtering of the signal utilising adaptive algorithm to generate antinoise signal. Below is the typical flow diagram of the ANC system

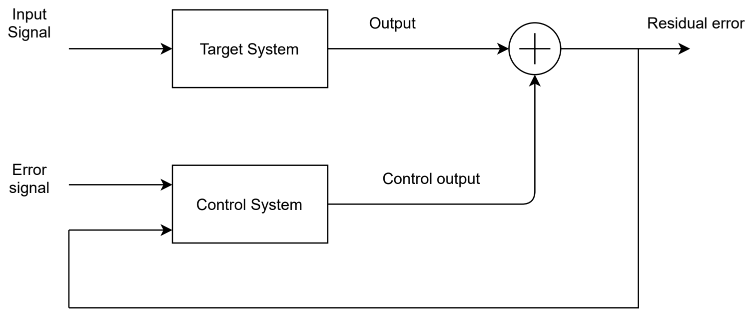

Figure 1.2: Adaptive system arrangement

Control system consist of digital filter, adaptive algorithm for modelling and transfer function. All of these components for feedforward ANC were arranged as shown in Figure 1.3

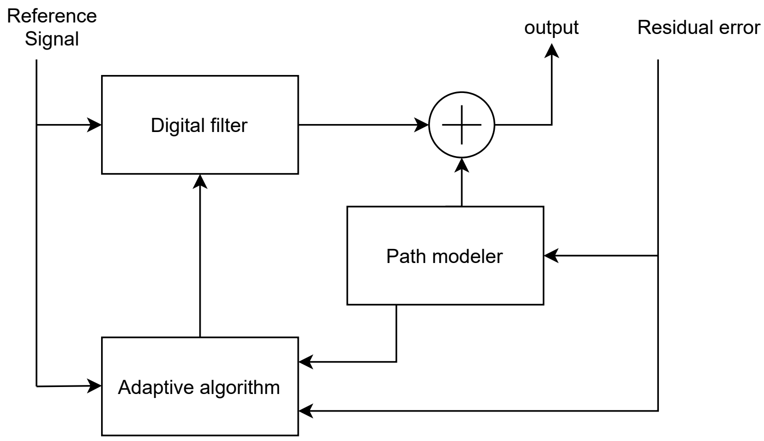

Figure 1.3: Inside flow of control system

Digital filter is nothing but set of mathematical operation that is performed on to the input signal. This acts as the transfer function of system which is the functional relation between the signal going in and coming out of the system. These transfer function determines the signal output from the system. Function output was largely dependent on the length of the digital filter we choose. Filter length is nothing but the number of weight parameter in the filter. Cancellation path modeler is the transfer function between the signal from control unit and the residual of the desired and the observed signal.

1.2 Background and motivation

Sparse impulse response was a very common type of noise which one encounter in various situation. But to filter out this one has to spend more resources. Traditional filters fail to improve their performance with sparse system and they spend same time and resources to process similar to normal systems, i.e. they don't have change in the convergence speed. There has been a lot of new adaptive filter developed for the sparse systems which utilize the sparse nature of system and was also able to handle non-sparse sys- tem as well. Proportionate normalized LMS (PNLMS) algorithm is one of the such algorithm. These algorithm update the coefficient independently. But performance of this algorithm decreases as the sparsity of the system decreases. Some improve- ment has been made and combination of filter[2] has been proposed, this model was able to handle the sparse and non sparse environment very well.

Sole motivation of this proposed project is to compare the existing methods and model to handle sparse environment and to some up with new model with different cost estimating function. Which will help researchers to easily check the performance comparison of the various model with different cost function for the sparse impulsive noises.

1.3 Literature Review

In domain of noise cancellation systems adaptive algorithm plays an important role in determining the system parameters. The classical adaptive filter like LMS [1], were very capable of modeling the system adaptively, but lacks when system was sparse in nature. Following that normalised least mean square(NLMS) algorithm were proposed but that also was not capable of utilising the sparsity of the system and end with the slow converging rate when estimating the sparse impulsive responses.

In past years there are there are many algorithms were suggested based on different schemes like partial update rule [2; 3], propotionate step-size PNLMS[4], IPNLMS[5], (SC-IPNLMS)[6] and more, in which step-size of the algorithm is depends on coefficient of the filter. Then new approach of taking norm was taken in [7] by Yilun, and two algorithm for sparse system based on *l1* norm was formed and that shows very huge improvement as compared to the previously proposed adaptive algorithm for sparse systems, first is zero-attracting LMS(ZA-LMS), which has very high improvement over previous one, furthur reweighted-zero-attracting LMS(RZA-LMS) was presented within same paper and was more better than the ZA-LMS.

Reference [10] proposed the algorithm which is combination of LMS and ZA-LMS, which was capable of handling system which has time varying sparse impulse response which works well on both type of system i.e. sparse and non-sparse. The proposed system was very good for the system whose nature is unkonwn, proposed algorithm automatically switches to the LMS algorithm when the system has very low sparsity and switches to ZA-LMS filter or switches to the solution of the combined filter for sparse case. After that many successful attemps have been made to integrate the sparsity aware filter ZA-LMS and RZA-LMS with the different filters for increasing the robustness of the algorithm. Like robust algorithm like LHCAF[9] which were not sparsity aware, were integrated with the ZA-LMS and RZA-LMS filter and new term is coined ZA-LHCAF and RZA-LHCAF for each filter respectively. This was carried furthur and to get more improved performance of hyperbolic sparse algorithm, new algorithm was suggested which uses joint hyperbolic update rule, which is termed as JHLCAF[8] is proposed.

1.4 Objective

The main objective is to do performance comparision of the existing algorithm and come up with model with different combination of cost function for sparse noises so to reduce the computational cost of identifying them in ANC systems. Following is the division of project into different phases

1. **Objective 1:** Implementation of the sparse adaptive filter using the existing model and algorithms
2. **Objective 2:** Comparing performance of the existing algorithm and simulating the MSE/MSD of algorithms.
3. **Objective 3:** Improving the existing model using different cost function and simulating the result. Report writing will be done simultaneously along with the above objective.

CHAPTER 2

DESIGN DETAILS AND IMPLEMENTATION

This chapter analyze the working of the adaptive control system arrangements and discuss the details of the classic algorithms and the new proposed algorithm.

2.1 Gradient descent Algorithm

In adaptive algorithm the weight of the filter was changes such that the residual error of the desired and observed signal was minimized. Most of the adaptive filter were based on gradient descent algorithms. These algorithm updates the weight by subtracting the gradient of the residual error, which itself is the function of the weight parameter so at end updation of weight end up on the minization of the residual error.

$$w = w - \mu \Delta w$$

where Δw is the gradient of the residual error and μ is the convergence coefficient

2.2 LMS Algorithm

Let us suppose *d(n)* denotes the desired output and *y(n)* is the observed output, *e(n)* denotes the residual signal

$$y(n) = w^T x(n) + v(n)$$

where w is vector denoting the filter coefficient

The cost function for LMS filter be $L(n) = 1/2e^2(n)$, where *e(n)* is the residual error function

$$e(n) = d(n) - y(n)$$

so the weight update rule according to the by applying the gradient descent method will be

$$w(n+1) = w(n) - \mu\Delta w(n) = w(n) - \mu\frac{\partial L(n)}{\partial w(n)} = w(n) + \mu e(n)x(n)$$

2.3 ZA-LMS Algorithm

ZA-LMS algorithm imposes penalty to the filter weight with the higher value with the help of the l_1-norm which help to converge to the most optimal coefficient much faster than traditional algorithm. But main problem with this algorithm is that algorithm is very bad when system is non-sparse.

The cost function used for the algorithm is $L_1(n) = 1/2e^2(n) + \gamma||w(n)||_1$, applying the calculating and updating the weight we get the following

$$w(n+1) = w(n) - \mu\frac{\partial L_1(n)}{\partial w(n)}$$
$$= w(n) + \mu e(n)x(n) - \rho sgn(w(n))$$

where sgn is the sign function described as follows

$$sgn(x) = \begin{cases} x/|x|, & \text{if } x \neq 0. \\ 0, & \text{otherwise.} \end{cases}$$

2.4 RZA-LMS Algorithm

To overcome the issue of ZA-LMS in case of non-sparse systems, RZA-LMS algortihm was proposed, which selectively picks the weight and updates.RZA-LMS algorithm uses logsum penalty function for implementing this.

The cost function for the algorithm is similar to that used for ZA-LMS except for the penalty part, $L_2(n) = 1/2e^2(n) + \gamma\sum_1^N log(1 + \frac{|w|}{\epsilon'})$, applying the calculating and updating the weight we get the following

$$w(n+1) = w(n) - \mu\frac{\partial L_2(n)}{\partial w(n)}$$
$$= w(n) + \mu e(n)x(n) - \rho\frac{sgn(w(n))}{1 + \epsilon|w(n)|}$$

here $\epsilon = 1/\epsilon', \rho = \mu\gamma/\epsilon$

2.5 Proposed Algorithms

Band based l0-LMS algorithm utilises has cost function similar to that of l0-LMS algorithm but in this algorithm weight which are updated not lie in single range as in case of l0-LMS, here multiple ranges of weight were modified using the update rule similar to that of the l0-LMS [11]. This help in better selection of weight to be updated. As this creates two set of weights one with extreme values of weight and other with values in between the specified ranges.

The weight update rule of the l0-LMS algorithm is

$$w(n+1) = w(n) + \mu e(n)x(n) - \rho(b^2 w(n) - bsgn(w(n)))$$

when $w(n)$ lie in between (-1/b, 1/b). In this algorithm the range of the band lie in between (-1/b , 1/b), where b is the band constant.

But in case of the proposed algorithm, the weight range is decided based on the number of bands we are using like for the n-band model the ranges for the weight will be $(-1/n^2, 1/n^2), ..., (-1/3n, 1/3n), (-1/2n, 1/2n), (-1/n, 1/n)$. Here value of b is equal to the number of bands used in algorithm.

Each weight were updated based on which band it belongs. So the value of b was halved for each range defined above and was put into weight update rule of l0-LMS, i.e. let initial value of b be b_i if the weight lie in between the range of $(-1/n^2, 1/n^2)$, value of b = b_i, when $w(n)$ lie between $(-1/(n-1)^2, 1/(n-1)^2)$, b = $b_i/2$ which goes on till b = $b_i/2^(n-1)$.

Band based PZA-LMS algorithm is very much similar to the proposed algorithm, major difference is that this algorithm imposes penalty in based on the value of the weight coefficient, which is approximated value of the l0-norm in terms of the polynomial function.

Weight update rule for the PZA-LMS[12] algorithm is

$$w(n+1) = w(n) + \mu e(n)x(n) - \rho \frac{sgn(w(n))(1-\beta|w(n)|)}{(1+|w(n)|)^(\beta+1)}$$

where β is the attraction parameter. This was carried out further and the filter weight were divided into different bands similar to above method. And weight that lies in between these band are updates differently for each band. Like for 2-band algorithm, two bands were formed as (-1/b, 1/b) and (-1/2b, 1/2b). And for each band there are two different penalty function. Weight update rule for the band based l0-LMS algorithm is

$$c(n) = \begin{cases} ((b/2)^2 w(n) - (b/2)sgn(w(n))), & \text{if } -1/2b < w(n) < 1/2b. \\ (b^2 w(n) - bsgn(w(n))), & \text{if } -1/b < w(n) < -1/2bU1/2b < w(n) < 1/b. \\ 0, & \text{otherwise.} \end{cases}$$

$$w(n+1) = w(n) + \mu e(n)x(n) + \rho c(n)$$

Similar approach was taken for implementing the band-algorithm with PZA-LMS algorithm. Which result in the following update rule where β = b, where b is the number of band in which weight were classified.

$$c(n) = \begin{cases} \frac{sgn(w(n))(1-(b/2)|w(n)|)}{(1+|w(n)|)^5}, & \text{if } -1/2b < w(n) < 1/2b. \\ \frac{sgn(w(n))(1-b|w(n)|)}{(1+|w(n)|)^5}, & \text{if } -1/b < w(n) < -1/2bU1/2b < w(n) < 1/b. \\ 0, & \text{otherwise.} \end{cases}$$

with this the final update rule is

$$w(n+1) = w(n) + \mu e(n)x(n) - \rho c(n)$$

2.6 Implementation

Two experiment was performed for performance comparison of the algorithm described above. All simulation was done on 1500 iteration and were performed for 200 times for each filter and they are independent of each trials. This experiment simulates the adaptive filter with 16 coefficient time varying system which range from, sparse system to non-sparse system. Initially the sparsity ration was kept at 1/16 i.e. only one out of 16 coefficient was non-zero, which changes to sparsity ratio of 1/2 as the 500 iteration were completed. Which again change to non-sparse system with no zero coefficient after 1000 iteration were completed. The parameter for the algorithm was set as: $\mu = 0.05$, $\rho = 0.0007$ and $\epsilon = 200$.

Second experiment was done to adaptive filter with 16 coefficient, having sparsity ratio same as above. Simulation was done for 500 sample for 200 times for each filter and each are independent trials. Values of parameter used for experiment are $\mu = 0.05$, $\rho = 0.0007$, $\epsilon = 200$ and band constant $b = 4$. MSD vs number of iteration was plotted to compare the convergence rate of each algorithm only for the sparse system.

CHAPTER 3

RESULTS AND DISCUSSIONS

3.1 Result

3.1.1 Performance comparison in time varying systems

Figure 3.1 is the simulation result of different filter in time varying sparsity of system, for first 500 iteration when system was sparse and we can see that band based l0-LMS(BB-l0-LMS) performs much faster convergence that the other filters. Average MSD for convergence rate are -43.61, -46.97, -47.26, 48.46, -50.61 and 51.93 for LMS, ZA-LMS, RZA-LMS, l0-LMS, PZA-LMS and the BB-l0-LMS respectively. But when the system is semi sparse(in between iteration 501-1000) ZA-LMS performs worst among the them all and remaining algorithm performs similar but better than LMS. And as the system becomes non-sparse all the algorithm has very similar performance to LMS except for the ZA-LMS.

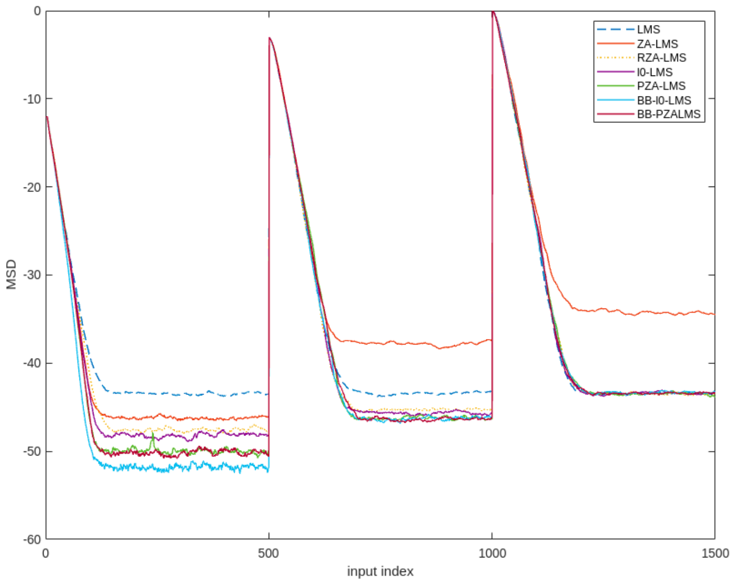

Figure 3.1: MSD(dB) for adaptive filter for different systems

3.1.2 Convergence in pure sparse system

Now looking at the convergence response of these filter only for the sparse system more carefully in Figure 3.2, we can see that from the start up to few number of iteration the behaviour of algorithms were different as compared to the behaviour when it finally converge. For example before 100th iteration the number ZA-LMS was best among the ZA-LMS, RZA-LMS and l0-LMS. But as the number of iteration passes the l0-LMS comes out to be the best among above three. But the last four lines which is nothing but the MSD of the proposed algorithm with different band values, hold their position and comes out to be the best among all. Behaviour of the effect of the bands are explained below.

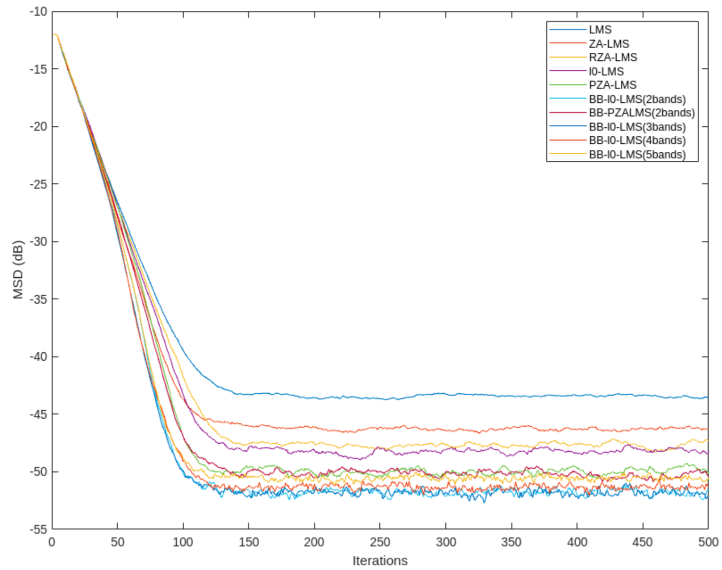

Figure 3.2: Comparison of the convergence response from algorithms

3.1.3 Effect of the number of bands on the convergence

Here in the following simulated figure we can see that as the number of bands number of bands increases performance of the algorithm also increases but after certain value the performance start to fall. In the following, algorithm with band value two performance best among all the bands, i.e first when the number of bands changes from one to two performance of the algorithm is increases but when the number of bands were increased we can see that the performance begin to fall and degrades as value increases.

3.2 Conclusion

In this paper we try to come up with algorithm having faster convergence rate, by modifying the cost function of the existing algorithm used for identification of sparse

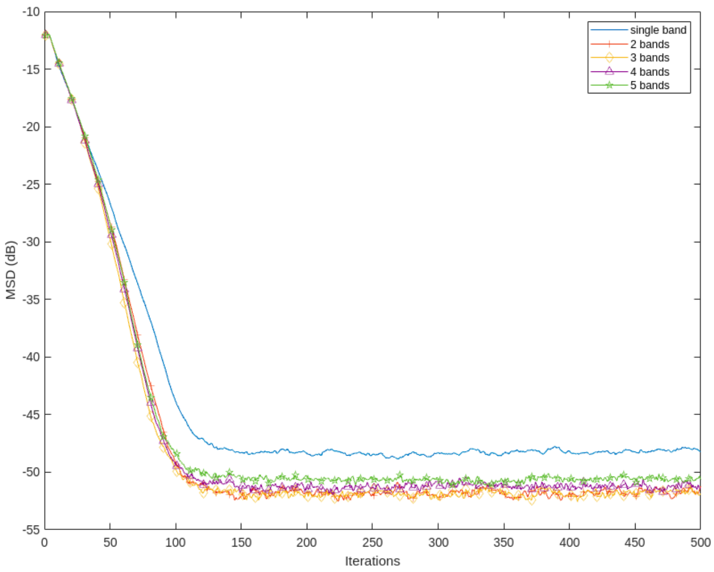

Figure 3.3: Convergence response of proposed algorithm with different band values

systems. Proposed algorithm was based on using the different polynomial update rule for different range of filter weights. Simulation result shows that proposed algorithm performs better than the other adaptive algorithms for sparse systems.

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