🏠 > ib-math > analysis-and-approaches-sl > practice-exams > prediction-exams >
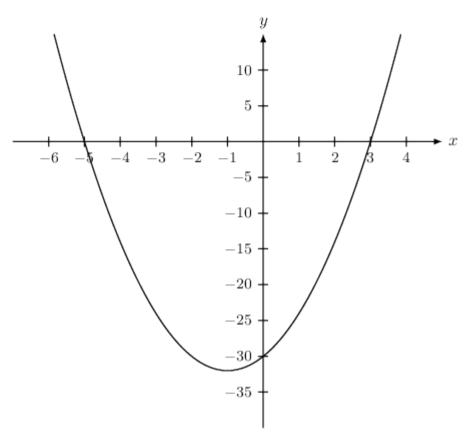may-2023-paper-1
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may 2023 paper 1

Question 0 ✕ **Non-Calculator** ⬆

[Maximum mark: 6]

The diagram below shows the graph of a quadratic function $f(x) = 2x^2 + bx + c$.
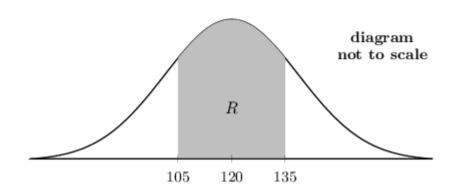
(a). Write down the value of c. [1]

(b). Find the value of b and write down $f(x)$. [3]

(c). Calculate the coordinates of the vertex of the graph of f. [2]

Question 1 ✕ Non-Calculator ⤴

[Maximum mark: 5]

The random variable X is normally distributed with a mean of 120. The following diagram shows the normal curve for X.

Let R be the shaded region under the curve between 105 and 135. The area of R is 0.4.

    (a). Write down $P(105 < X < 135)$.   [1]

    (b). Find $P(X < 135)$.   [2]

    (c). Find $P(X > 105 \mid X < 135)$.   [2]

Question 2   ✕   **Non-Calculator** ⬆

[Maximum mark: 8]

The equation of a line L_1 is $2x + 3y = -5$.

    (a). Find the gradient of L_1.   [2]

A second line, L_2, is perpendicular to L_1.

(a). State the gradient of L_2. [1] «

The point $P(4, 0)$ lies on L_2.

(a). Find the equation of L_2, giving your answer in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$. [2]

The point Q is the intersection of L_1 and L_2.

(a). Find the coordinates of Q. [3]

Question 3 ✕ Non-Calculator ⬆

[Maximum mark: 6]

Consider the curve $y = (kx - 1)\ln(2x)$ where $k \in \mathbb{R}$ and $x > 0$.

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The tangent to the curve at $x = 2$ is perpendicular to the line $y = \dfrac{2}{5 + 4\ln 4}x$.

Find the value of k.

Question 4 ✕ Non-Calculator ⬆

[Maximum mark: 7]

Consider the functions $f(x) = 3\cos(x) + \dfrac{9}{2}$ and $g(x) = 3\cos\left(x + \dfrac{\pi}{3}\right) + A$, where $x \in \mathbb{R}$ and $A < \dfrac{9}{2}$.

«

(a). Describe a sequence of two transformations that transforms the graph of f to the graph of g. [3]

The y-intercept of the graph g is at the point $\left(0, \dfrac{9}{2}\right)$
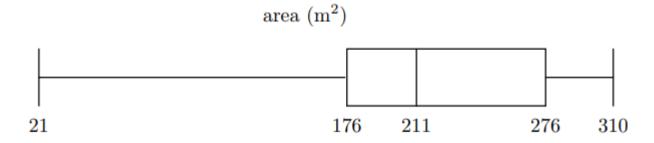
(a). Find the range of g. [4]

Question 5   ✕   Non-Calculator ⬆

[Maximum mark: 7]

A real estate company keeps a register of the monthly cost of rent, R, of their apartments and their corresponding area, A, in m^2.

The areas of the apartments registered are summarised in the following box and whisker diagram.

area (m^2)

21   176   211   276   310

«

(a). Find the smallest area A that would not be considered an outlier.   [3]

The regression line A on R is $A = \dfrac{5}{12}R - 50$.

Meanwhile, the regression line R on A is $R = \dfrac{5}{2}A + 100$.

(a). One of the apartments has a monthly rent of $\$480$. Estimate the area of the rental.   [2]
(b). Find the mean rental cost of all the real estate company's apartments.   [2]

Question 6   ✕   Non-Calculator ⬆

[Maximum mark: 12]

A particle moves along the x-axis with a velocity, $v \text{ ms}^{-1}$, at time t seconds given by the function

$$v(t) = 2 + 7t - 4t^2$$

«

For $0 \leq x \leq 3$. The particle is initially at the origin.

(a). Find the value of t when the particle reaches its maximum velocity.   [3]
(b). Sketch a graph of v against t showing any points of intersection with the axes.   [5]
(c). Find the displacement of the particle from the origin after 2 seconds.   [4]

Question 7 ✕ **Non-Calculator** ⬆

[Maximum mark: 18]

The first three terms of an infinite sequence, in order, are

$$2\ln x, \quad q\ln x, \quad \ln\sqrt{x} \quad \text{where} \quad x > 0.$$

First consider the case in which the series is geometric.

(a).

(i). Find the possible values of q.

(ii). Hence or otherwise, show that the series is convergent.   [3]

(b). Given that $q > 0$ and $S_\infty = 8\ln 3$, find the value of x.   [3]

Now suppose that the series is arithmetic.

(a).

(i). Show that $q = \dfrac{5}{4}$.

(ii). Write down the common difference in the form $m\ln x$, where $m \in \mathbb{Q}$.   [4]

(b). Given that the sum of the first n terms of the sequence is $\ln\sqrt{x^5}$, find the value of n.   [8]

Question 8 ✕ **Non-Calculator** ⬆

[Maximum mark: 11]

Carmen plays a game at a stall at a local fair. The stall owner lays n cards face down on a table. Three of the cards say "Win" on the hidden side, the others say "Try Again". Carmen draws cards from the table, one after the other, without replacement. The game ends when Carmen draws a card that says "Win".

(a). Find the probability, in terms of n, that the game will end on her 2nd draw. [2]

(b). Given that $n = 8$, find the probability that the game will end on her 6th draw. [2]

«

Carmen plays the game when $n = 8$. She pays $6 to play and can earn money back depending on the number of draws it takes to obtain a card that says "Win". She earns no money back if she obtains a card that says "Win" on her 6th draw. Let M be the amount of money that she earns back playing the game. This information is shown in the following table.

Number of draws	1	2	3	4	5	6
Money earned back (M)	10	5	4	$\dfrac{3k}{4}$	$\dfrac{k}{3}$	0

(a). Find the value of k so that this is a fair game. [7]