

$$W = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 + 2x_2 = 0\}$$

prove 'W' is subspace of  $V = \mathbb{R}^2(\mathbb{R})$

$W \subseteq \mathbb{R}^2$  subset of  $V$ .

①

$$(0,0) \in W$$

$$0 + 2 \cdot 0 = 0 \Rightarrow (0,0) \in W$$

②

Let  $w_1, w_2 \in W$ , we need to prove  $w_1 + w_2 \in W$

$$\text{if } w_1 = (x_1, y_1) \in W \Rightarrow x_1 + 2y_1 = 0$$

$$\text{if } w_2 = (x_2, y_2) \in W \Rightarrow x_2 + 2y_2 = 0$$

$$\Rightarrow (x_1 + x_2) + 2(y_1 + y_2) = 0$$

$$\Rightarrow (x_1 + x_2, y_1 + y_2) \in W$$

$$\Rightarrow w_1 + w_2 \in W$$

③ Let  $c \in F$ ,  $w_1 \in W$

$$w_1 \in W \Rightarrow x_1 + 2y_1 = 0$$

$$\Rightarrow cx_1 + 2cy_1 = 0$$

$$\left. \begin{array}{l} \\ \Rightarrow (cx_1, cy_1) \in W \end{array} \right\}$$

$$\Rightarrow cw_1 \in W$$

$\Rightarrow W$  is subspace of  $\underline{\underline{V}}$ .

$$\langle x, y \rangle = x^T A y$$

$$\|x\| = \sqrt{\langle x, x \rangle}$$

$$\langle x, x \rangle = x^T A x$$

$$y = x$$

$$\Rightarrow \|x\| = \sqrt{x^T A x}$$

$$\|a - b\| = ?$$

$$x = a - b$$

$$\Rightarrow \|a - b\| = \sqrt{(a - b)^T A (a - b)}$$

$$\begin{cases} x_1 + 2x_2 = 0 \\ 2x_1 + 4x_2 = 0 \end{cases}$$

Null space

$$Ax=0$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

homogeneous  
system

Nullspace A = {  $x \in V \mid Ax=0$  }

































