10. **Inference about the Box-Cox transformation**
Suppose one observes the positive values $y_1, ..., y_n$ that exhibit some right-skewness. Box and Cox (1964) suggested using the power transformation

$$w_i = \frac{y_i^\lambda - 1}{\lambda}, i, = 1, ..., n,$$

such that $w_1, ..., w_n$ represent a random sample from a normal distribution with mean μ and standard deviation σ. Suppose that the vector of parameters (λ, μ, σ) is assigned the noninformative prior proportional to $1/\sigma$. Then the posterior density of θ is given, up to a proportionality constant, by

$$g(\theta|y) \propto \frac{1}{\sigma} \prod_{i=1}^{n} \left[\phi\left(\frac{y_i^\lambda - 1}{\lambda}; \mu, \sigma \right) y_i^{\lambda - 1} \right].$$

Suppose this transformation model is fit to the following survival times (from Collett, 1994) of patients in a study on multiple myeloma.

```
 13 52  6 40 10  7 66 10 10 14 16  4
 65  5 11 10 15  5 76 56 88 24 51  4
 40  3 18  3 16 50 40  1 36  5 10 91
 18  1 18  6  1 23 15 18 12 12 17  3
```


a) Write an R function to compute the logarithm of the posterior distribution of $(\lambda, \mu, \log \sigma)$.
b) Use `laplace` to find the posterior mode of $(\lambda, \mu, \log \sigma)$ using an initial starting value of (0.1, 3, 0.5).
c) Use an MCMC algorithm such as random walk Metropolis, independent Metropolis, or Gibbs sampling to simulate 10,000 values from the posterior distribution.
d) Construct 90% interval estimates of λ, μ, and σ.
e) For these data, use the result from part (d) to decide whether a log or square root transformation is more appropriate for these data.

翻译：
关于博克斯-考克斯变换的推断

假设我们观察了正值 $y_1, \cdots, y_n$，它们表现出一些右倾性。博克斯和考克斯（1964）建议使用幂变换

$$w_i = \frac{y_i^\lambda - 1}{\lambda}, i = 1, \ldots, n,$$

其中 $w_1, \ldots, w_n$ 代表一个服从均值为 μ，方差为 σ 的正态分布随机样本。假设参数向量 (λ, μ, σ) 服从无信息先验分布正比于 $1/\sigma$。已知 θ 的后验密度

$$g(\theta|y) \propto \frac{1}{\sigma}\prod_{i=1}^{n}\left[\phi\left(\frac{y_i^{\lambda}-1}{\lambda};\mu,\sigma\right)y_i^{\lambda-1}\right].$$

假设这个转化模型适用于对多发性骨髓瘤的研究，患者的存活时间如下：

13	52	6	40	10	7	66	10	10	14	16	4
65	5	11	10	15	5	76	56	88	24	51	4
40	8	18	5	16	50	40	1	36	5	10	91
18	1	18	6	1	23	15	18	12	12	17	3

(a) 写一个 R 函数来计算(λ, μ, σ)的后验分布的对数。

(b) 使用 laplace 此命令