ACS6116 Advanced Control: Assignment

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Spring 2023–24

Assignment weighting

25% of the total mark for ACS6116.

Assignment released

12:00 on Friday 15th March 2024 (Week 6).

Assignment due

23:59 on Monday 6th May 2024 (Week 11).

Penalties for late submission

Late submissions will incur the usual penalties of a 5% reduction in the mark for every working day (or part thereof) that the assignment is late and a mark of zero for submission more than 5 working days late. For more information see `https://students.sheffield.ac.uk/assessment/results/submission-if-you-submit-your-work-late`.

Feedback

This will include the overall mark, individual component marks and comments on performance on the assignment. The attached assessment criteria (at the back of this document) provide a guide to the areas on which the feedback will be provided. Note that marks may be subject to change as a result of unfair means.

Unfair means

The assignment must be completed individually. You must not work together to complete the assignment—it must be wholly your own work. References must be provided to any other work that is used as part of this assignment. Any suspicions of the use of unfair means will be investigated and may lead to penalties. See `https://www.sheffield.ac.uk/new-students/unfair-means` for more information.

Extenuating circumstances

If you have extenuating circumstances that cause you to be unable to submit this assignment on time or that may have affected your performance, please complete and submit an extenuating circumstances form—see `https://students.sheffield.ac.uk/extenuating-circumstances` for information. The form may be found here.

Assignment briefing

This assignment will assess your fundamental understanding of model predictive control and your ability to design MPC controllers and simulate and analyse MPC-controlled systems. The assignment comprises an open-ended design, simulation and analysis exercise:

Produce a report (limit: 4 pages in the provided template) containing your answer.

In order to create a level playing field between candidates' submissions, you are asked to prepare your submission using the document templates supplied on Blackboard. This is a 10pt, two-column format, which allows ample space for this assignment even with the 4-page limit. (Please note that no appendices are necessary.)

It is up to you how you tackle the problem and stucture your answer. However, it is suggested that you look at (i) the help below and (ii) the attached assessment criteria (at the back of this document) for guidance on what to include.

Assessment criteria

The assessment criteria for this exercise are derived from the module learning outcomes, which are:

1. Describe and explain the principles of more than one advanced control technique.
2. Analyse practical performance specifications and convert these into functional requirements on controllers.
3. Design, implement and evaluate an advanced control system against these requirements.
4. Compare and contrast different advanced control solutions to a particular control problem or application.
5. Describe the receding-horizon principle, and hence compare and contrast LQ-optimal control and MPC.
6. Construct a constrained finite-horizon optimal control problem—including constraint, model and cost definition—re-formulate it as an optimization problem, and recall and evaluate the analytical expression for the control law in the unconstrained case.
7. Analyse, design, implement and simulate MPC controllers with guaranteed properties, including feasibility, stability and offset-free tracking.

In particular, learning outcomes 2, 3, 6 and 7 are relevant to this assessment, and the attached assessment criteria—the marksheet that will be used to assess the assignment—are derived from these. The marksheet indicates the criteria that will be used in assessing your answer, and also the expectation for each criterion in order to achieve a mark within the specified ranges. It is suggested that you study this marksheet before completing the assignment.

These assessment criteria are deliberately broad in order to accommodate the different approaches that might be taken to solving the assignment problem.

Please note that a **4-page limit**, using the supplied template, applies to your report, and you should consider carefully how you can effectively meet the assessment criteria within this limit.

Guidance

- This assignment briefing and the module lecture notes provide the main information that is required to complete this assignment. Although we will cover in lectures the basic theory and techniques needed to complete the assignment, you may need to read ahead of the lectures—if you wait entirely until we "cover" a particular topic (*e.g.*, tracking and disturbance rejection), you might not leave enough time for a comprehensive attempt at the assignment.

- You may also wish to consult the literature relevant to your problem and review it in your report.

- Basic MATLAB programming is required, including the use of functions and loops; however, in tackling the assignment you may use the MPC-specific MATLAB functions (used in the computer exercises) available on the Blackboard page for ACS6116, plus any code you have developed to answer the computer exercises. You *should not* use the MPC toolbox in MATLAB/SIMULINK.

- The non-assessed computer exercises are good preparation for this assignment. However, the computer exercises were structured, whereas this assignment is open-ended: you need to decide what is the most appropriate approach to solve this assignment, and also how to present your results.

- For the report, you are recommended to consult the attached assessment criteria (at the back of this briefing) for guidance on what to include and to what level of detail. In particular, the assessment criteria suggest that your report might need to include, among other things,

  - Details of the optimal control problem / MPC formulation you used, including the correct identification and implementation of constraints.

  - A description and explanation of how the controller was designed and tuned in order to meet the specification, including the selection of all parameters, with correct explanations and justifications.

  - Clear reporting and discussion of results (including clear, *labelled* plots), and critical evaluation of the controller. (Think about more than just, for example, "Did my controller meet the spec.?" — what are the strengths and weaknesses of your solution? What could be improved?)
  - Some analysis and evaluation of your design, including its theoretical properties (if any) and its performance under different scenarios.

This is not an exhaustive list, and what you should include will vary depending on the topic you choose. However, a *suggested* outline for any report is

1. Abstract
2. Introduction
3. Problem statement
4. Design
5. Results
6. Analysis and discussion
7. Conclusion

This might *not* be the ideal structure for your report, however, and you may wish to combine or change some of these sections, depending on the approach you take and progress you make.

You do not need to include the code that you write, but you may do so (*e.g.* snippets of code) if you think it adds value to your report.

Please note that, in order to achieve the very highest marks, you will need to *go beyond* simply implementing the methods that you have learned in the lectures and practised in the computer exercises. Surprise me!

- If you do not manage to achieve a working controller or simulation, **simplify things, get something simpler working, and build systematically towards a comprehensive solution.** In fact, I advise in all cases to start tackling the problem **without constraints**, in order that you can get something working without having to wrestle with infeasibility and instability issues (which often arise from incorrect implementation of constraints). Before you consider tracking and disturbance rejection problems, *solve the regulation problem, from a non-zero initial state, first.*

- Should you need clarification or have questions on any part of the assignment then please just ask! (Talk to me in classes, post to the discussion board, or email me at `p.trodden@sheffield.ac.uk`).

Submit your report via Blackboard by 23:59 on Monday 6th May 2024

A benchmark problem in control design

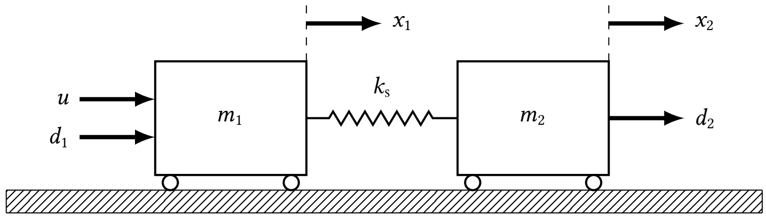

Figure 1: The Wie–Bernstein two-cart control benchmark system.

The assignment problem for 23/24 is a benchmark problem in (robust) control design. Introduced by Wie and Bernstein in the early 1990s,[1] the benchmark system has two carts connected by a spring, on a frictionless surface. Actuation may be applied to the first cart, but the performance output is the position of the second cart. The system is an abstraction of practical engineering systems that have a rigid body mode and a vibration mode.

System description and assumptions

The dynamics of the system are modelled as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_s/m_1 & k_s/m_1 & 0 & 0 \\ k_s/m_2 & -k_s/m_2 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1/m_1 \\ 0 \end{bmatrix}}_{B^c} u + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m_1 & 0 \\ 0 & 1/m_2 \end{bmatrix}}_{E^c}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix},$$

$$\text{and} \quad y = x_2,$$

where x_1 and x_2 are the respective displacements (from datums) of carts 1 and 2, x_3 and x_4 are the respective velocities, u is the control input, y is the performance output, and d_1 and d_2 are disturbances acting on carts 1 and 2 respectively.

Assume that:

1. The masses $m_1 = m_2 = 1$ and the (nominal) spring constant $k_s = 1$.
2. The full state $x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^\top$ may be measured.
3. The dynamics are discretized using zero-order hold and a sampling period of 0.1 s.[2]
4. Because zero-order hold is an *exact* discretization method, the obtained discretized model *is* the plant for simulation purposes — you do not need to simulate the true continuous-time plant in your solution.

[1] B. Wie and D. S. Bernstein, Benchmark problems for robust control design, *Journal of Guidance, Control, and Dynamics*, vol. 15, Art. no. 5, Sep. 1992.

[2] To achieve this in Matlab, consider the system input as $\begin{bmatrix} u & d_1 & d_2 \end{bmatrix}^\top$ so that the continuous-time input-to-state matrix is $\begin{bmatrix} B^c & E^c \end{bmatrix}$. Then form the continuous-time system as `sysc = ss(Ac, [Bc Ec], Cc, zeros(1,size(Bc,2)))` and discretize it with `sysd = c2d(sysc,0.1)`. Finally, recover the discrete-time B and E by extracting the appropriate columns of `sysd.b`.

The control task

Design, implement, simulate, analyse and evaluate a model predictive controller in order to achieve the following specifications:

1. In the absence of disturbances ($d_1 = d_2 = 0$), the controller is able to to regulate an initial state $x(0) = \begin{bmatrix} 1 & 1 & -0.2 & 0.2 \end{bmatrix}^\top$ to the origin, in a reasonable amount of time,[3] while satisfying the constraints

$$|u| \leq 1, \quad |x_3| \leq 0.5, \quad |x_4| \leq 0.5.$$

2. The same controller (*i.e.*, a controller with the same values of design parameters) should be able to perform this regulation task for values of the spring constant in the range $0.5 \leq k_s \leq 2$.

3. With the system started from rest ($x(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^\top$), the controller should be able to achieve offset-free tracking of a constant reference r with 'good' performance — *i.e.*, achieve $y = r$ in steady state, with a 'reasonable' settling time and mimimal overshoot — in the presence of constant disturbances d_1 and/or d_2 and, of course, the constraints. It should be able to achieve this for a range of different r, d_1 and d_2.

A particularly interesting scenario for task 3 is where

$$r = 1, \quad d_1 = 0.25, \quad d_2 = 0.7.$$

Design considerations

Your design should consider all, or as many, parts of this specification as possible. You should try to balance performance with computational complexity, and theoretical properties with practicality. For example, designs with guaranteed properties (*e.g.*, recursive feasibility) are generally preferable, but provided the 'cost' of these designs is reasonable.

It is strongly recommended that you build your controller in an incremental way; *i.e.*, starting with task 1 and initially in the absence of any constraints, before adding just input constraints, *etc.* (You do not, however, have to describe the entirety of this incremental design process in your report.)

For the disturbance rejection problem in task 3, you may wish to consider what assumptions are appropriate with regard to obtaining the values of d_1 and d_2: do you assume that these can be measured, or do you try to estimate them via an observer?

Finally, you should to try to provide some quantitative assessment of the performance, complexity and the limits of operation of your controller. Performance wise, consider calculating (particularly for task 3) the integral squared error (ISE) and maximum overshoot during a simulation that comprises T steps.[4]

$$\text{ISE} = \sum_{k=0}^{T} (y(k) - r)^2 \qquad \text{Overshoot} = \frac{\max_{k \in [0,T]} y(k) - r}{r}$$

A 'good' design should minimize both, where possible.

In terms of computational complexity, you might like to consider the size of the QP problem your controller has to solve, and how long this takes.

[3] You will need to make a judgement on what is 'reasonable' and therefore what your settling/convergence criterion is, *e.g.*, the value of ε in $\|x(k)\| \leq \varepsilon$.

[4] Where T must large enough to capture the full transient response of the system.

Criterion	Level				
	0–50%	50–60%	60–70%	70–80%	80–100%
Formulation, design and implentation (10 marks): Formulate and implement an MPC-based solution to the problem, including definition and/or justification of model, cost, constraints	Little or no evidence of a correct formulation/design and implementation	A formulation/design is presented and implemented, and is essentially correct but with minor errors; or, the formulation and implementation is correct, but details are unclear or not explained	A formulation/design is presented and correctly implemented, with clear explanations	A formulation/design is presented and correctly implemented, with comprehensive explanations, justifications and/or analysis	Formulation/design and implementation is to a standard that exceeds expectations; for example, using advanced techniques, further analyses, and/or insightful explanations
Investigation, results and evaluation (10 marks): Conduct a comprehensive investigation into the problem, gather and present appropriate results, and critically evaluate the findings	Little or no evidence of investigations, results and evaluations	Evidence of investigation, *e.g.*, via appropriate plots and metrics, and simple/minimal evaluation	Clear evidence of comprehensive investigation via effective plots and/or metrics; basic evaluation	Clear evidence of comprehensive investigation via well presented plots, metrics and other results; solid critical evaluation with interesting insights	Clear evidence of comprehensive and rigorous investigation that exceeds expectation; interesting and original results; detailed and insightful critical evaluation
Advanced content (5 marks):	Up to 5 marks available for engaging with advanced considerations and/or techniques from the module and especially the wider literature. This is your opportunity to propose, analyse and/or investigate something original and interesting.				