AERO9660 Advanced Propulsion

Assignment N°2 2022.

Q1

van der Waal's equation may be expressed as:

$$P = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}^2}.$$

At the critical point:

$$\left(\frac{\partial^2 P}{\partial \bar{v}^2}\right)_T = 0\,,$$

and:

$$\left(\frac{\partial P}{\partial \bar{v}}\right)_T = 0\,.$$

i) Use this fact to derive expressions for a, b in terms of $\bar{v}_c$, as well as $\bar{R}$ and T.

ii) Then insert these expressions for a and b into van der Waal's equation at the critical point to get $\bar{v}_c$, a, and b in terms of $\bar{R}$, T_c, and P_c.

For hydrogen, the critical points are:

$$T_c = 33.3K\,,$$

$$P_c = 1.30MPa\,.$$

iii) Therefore find a in $\left\{Pa\frac{(m^3)^2}{kmol^2}\right\}$ and b in $\frac{m^3}{kmol}$.

iv) For hydrogen, plot pressure (in Pa) against molar specific volume (in $m^3/kmol$) over the range:

$$0 \le P \le 2.5 \times 10^6 Pa$$

$$0 \le \bar{v} \le 1.5 m^3/kmol$$

for isotherms of $15K$, $18K$, $21K$, $24K$, $27K$, $30K$, $T_c = 33.3K$, $36K$ and $39K$.

(/35)

Q2

The Redlich-Kwong equation of state may be expressed as:

$$P = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}(\bar{v} + b)T^{(0.5)}},$$

where:

$$a = 0.42748\frac{\bar{R}^2 T_c^{2.5}}{P_c},$$

and:

$$b = 0.08664\frac{\bar{R}T_c}{P_c}.$$

i) Find a in $Pa\left(\frac{m^3}{kmol}\right)^2 K^{0.5}$ and b in $\frac{m^3}{kmol}$ given that $T_c = 33.3K$ and $P_c = 1.3MPa$.

ii) Calculate the density of hydrogen if $P = 700bar$ and $T = 400K$ using the Redlich-Kwong equation of state. This is basically the conditions you would find in a typical hydrogen tank on board an aircraft of a car.

iii) If $R = 4.124kJ/kgK$, use the ideal gas equation to find the specific volume of hydrogen if $P = 700bar$ and $T = 400K$.

iv) Predict the density of hydrogen if $Z = 1.40$.

$$\frac{Pv}{RT} = Z$$

$$v = \frac{ZRT}{P} = \frac{1}{\rho}.$$

Now you see why we need these other equations of state.

(/35)

Q3

The partition function that accounts for the vibrational component of energy storage in a molecule is:

$$Z_v = \frac{1}{1 - e^{-\frac{\theta_v}{T}}},$$

where θ_v is the characteristic vibrational temperature of the molecule and T is the absolute temperature.

Given that the component of the molar specific internal energy due to the vibrational component of energy storage is:

$$\overline{u_v} = \bar{R}T^2 \left\{\frac{\partial}{\partial T}(\ln Z_r)\right\}_V,$$

show that:

$$\overline{u_v} = \bar{R}T \frac{x}{(e^x - 1)},$$

where:

$$x = \frac{\theta_v}{T}.$$

Then as:

$$\overline{c_{v_v}} = \frac{d\overline{u_{int}}}{dT},$$

show that:

$$\overline{c_{p_v}} = \bar{R} \frac{x^2 e^x}{(e^x - 1)^2}.$$

Marks will be deducted for a lack of working.

(/30)