Regression: Introduction & Linear Regression

Ch.4 Multivariate Data Analysis. Joseph Hair et al. 2010. Pearson
Ch.6. Learn R for Applied Statistics. Eric Hui. 2018. Apress
Ch.2 Regression Analysis. William Mendenhall and Terry Sincich. 2012. 7th edition. Pearson
Ch.7. Simple Linear Regression. David Dalpiaz. 2019

Regression in Applied Statistics

Hypothesis: **null** (H_0) and **alternative** (H_A)

Inference Test

$p < 0.05$ (alpha)

Reject

$p > 0.05$ (alpha)

Fail to Reject

Regression:

a set of statistical processes to estimate the relationships between all the variables

Descriptive Statistics

Derives dataset summary:

- **central tendency**
- **dispersion**
- **skewness**

Inferential Statistics

- **Makes inference about the population**
- **Use hypothesis testing and parameter estimation**

Model

The variable to be predicted (or modeled), y, is called the **dependent** (or **response**) variable

General Form of Probabilistic Model in Regression

$$y = E(y) + \varepsilon$$

where y = Dependent variable
$E(y)$ = Mean (or expected) value of y
ε = Unexplainable, or random, error

(Mendenhall, 2012)

- Response = Prediction + Error
- Response = Signal + Noise
- Response = Model + Unexplained
- Response = Deterministic + Random
- Response = Explainable + Unexplainable

The variables used to predict (or model) y are called **independent variables** and are denoted by the symbols x_1, x_2, x_3

$$Y = f(X) + \epsilon. \longrightarrow Y = \beta_0 + \beta_1 X + \epsilon.$$

(beta one) = Slope of the line [amount of increase (or decrease) in the mean of y for every 1-unit increase in x

(beta zero) = y-intercept of the line [the line intercepts the y-axis]

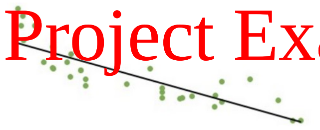
Regression Types

Independent Variables

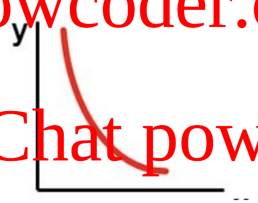
Simple	1 Independent
Multiple	> 1 Independent
Ridge	Highly correlated
Stepwise	Identification of best variables
Lasso	Ridge with variable selection

Regression Line Shape

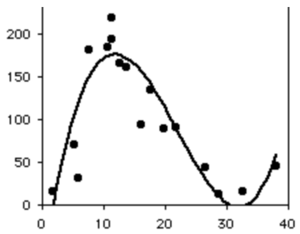
Linear



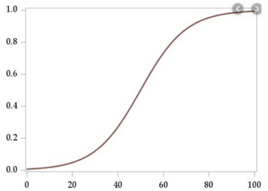
Quadratic



Curvilinear



Logistic



Dependent variable

Linear	Continuous
Logistic	Binary
Nominal	> 2 categories
Poisson	Count
Ordinal	Ordered response
Multivariate	> 1 dependent

Key Terms: Error Types

α (alpha) The level of risk we accept in making a wrong decision about a null hypothesis

Level of significance 0.05, 0.01, 0.001

When α is set to 0.05, p values < 0.05 indicate significance

	Null is true	**Null is false**
Reject null	**Type I error (False Positive)**	Right decision
Retain null	Right decision	**Type II error (False Negative)**

β (beta)

The probability of committing Type II error

Simple Linear Regression

$$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

Simple: y depends on only one other variable

$$\epsilon_i \sim N(0, \sigma^2).$$

Fixed known constant: x_i

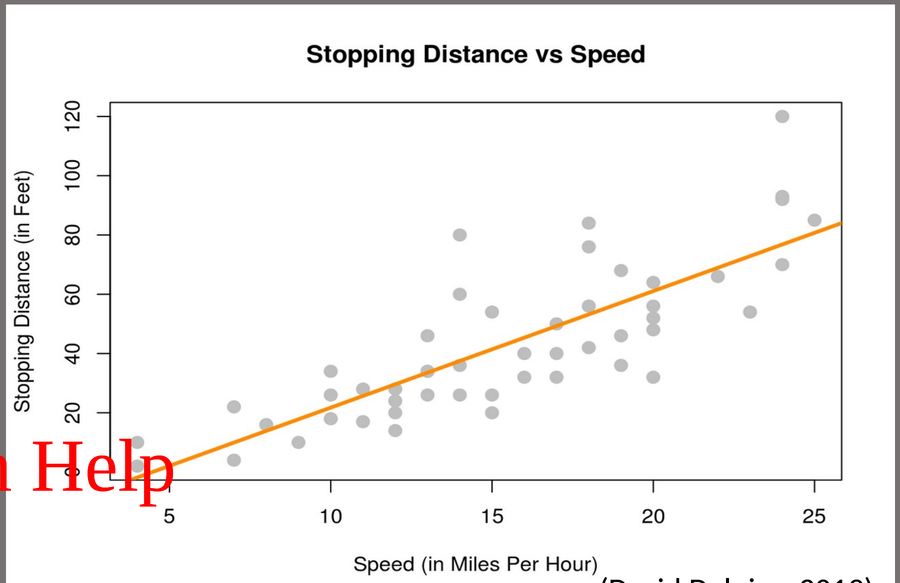
Fixed unknown parameters: β_0 β_1, and σ_2

Random unobserved variable: ϵ_i - independently and identically distributed (iid) normal random error variables

Random variable: Y_i and their possible values y_i

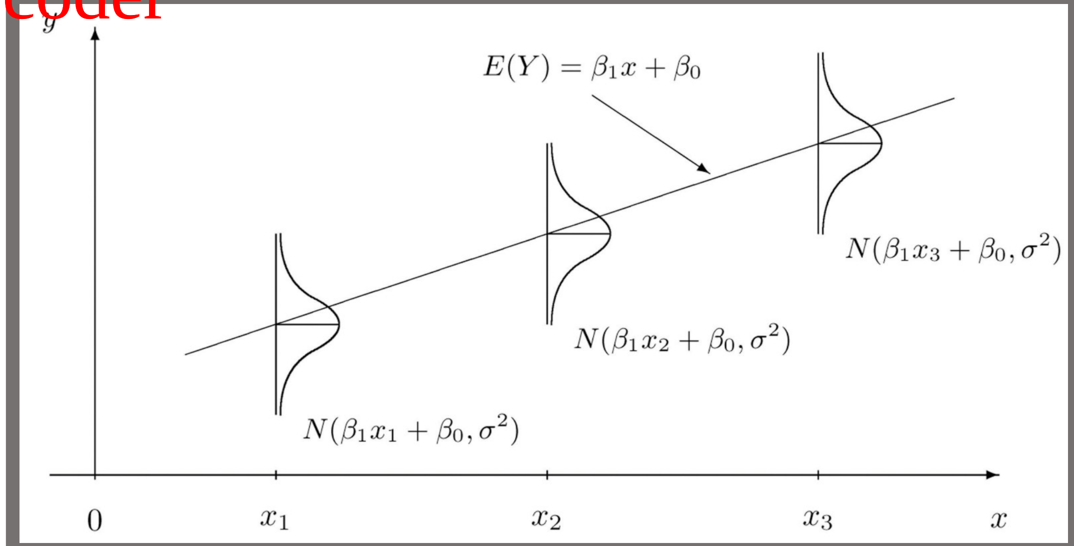
Note: for each x the y-values spread about the mean E(y) and with a standard deviation σ that is the same for every value of x.

Y - Response



(David Dalpiaz, 2019)

X - Predictor



(Shaffer and Zhang, 2019. Introductory Statistics)

Simple Linear Regression Assumptions

1. Variables Type: Continuous (Interval or Ratio)

2. Linear: The relationship between Y and x is linear

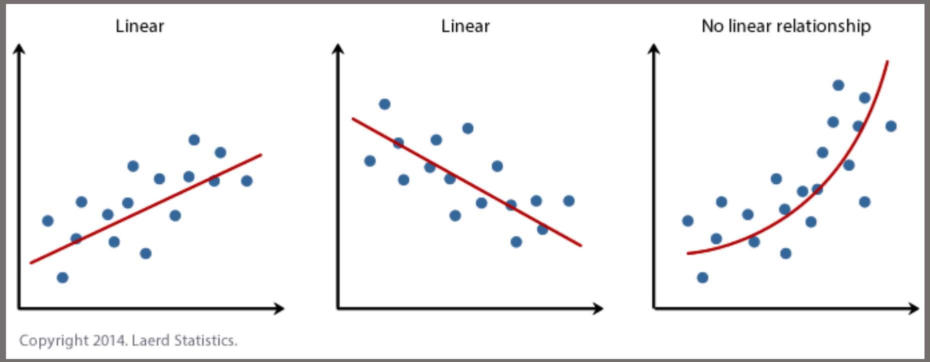
3. Outliers: There should be no significant outliers (see Ch.13 Applied Statistics in R. David Dalpiaz)

4. Independence: You should have independence of observations

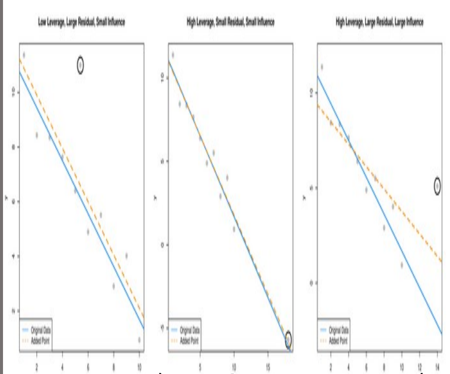
5. Equal Variance: The variances along the line of best fit remain similar.

Normal: The errors ϵ are normally distributed

Note: the values of x are fixed. We do not make a distributional assumption about the predictor variable.

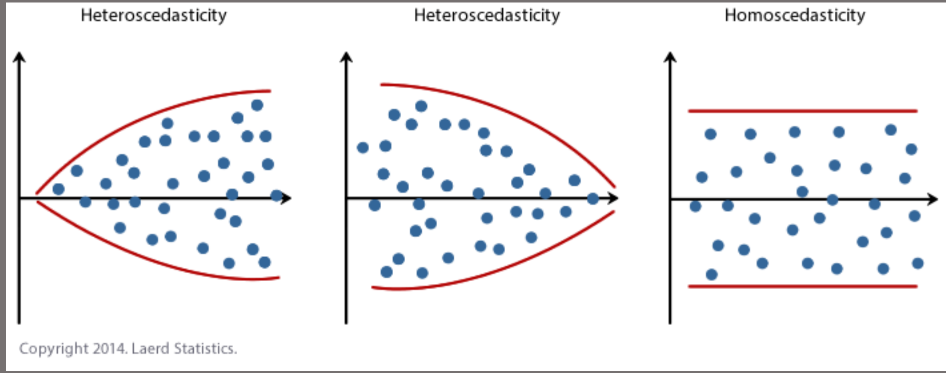


Inspect your Y and X relationship in scatterplot



(David Dalpiaz, 2019)

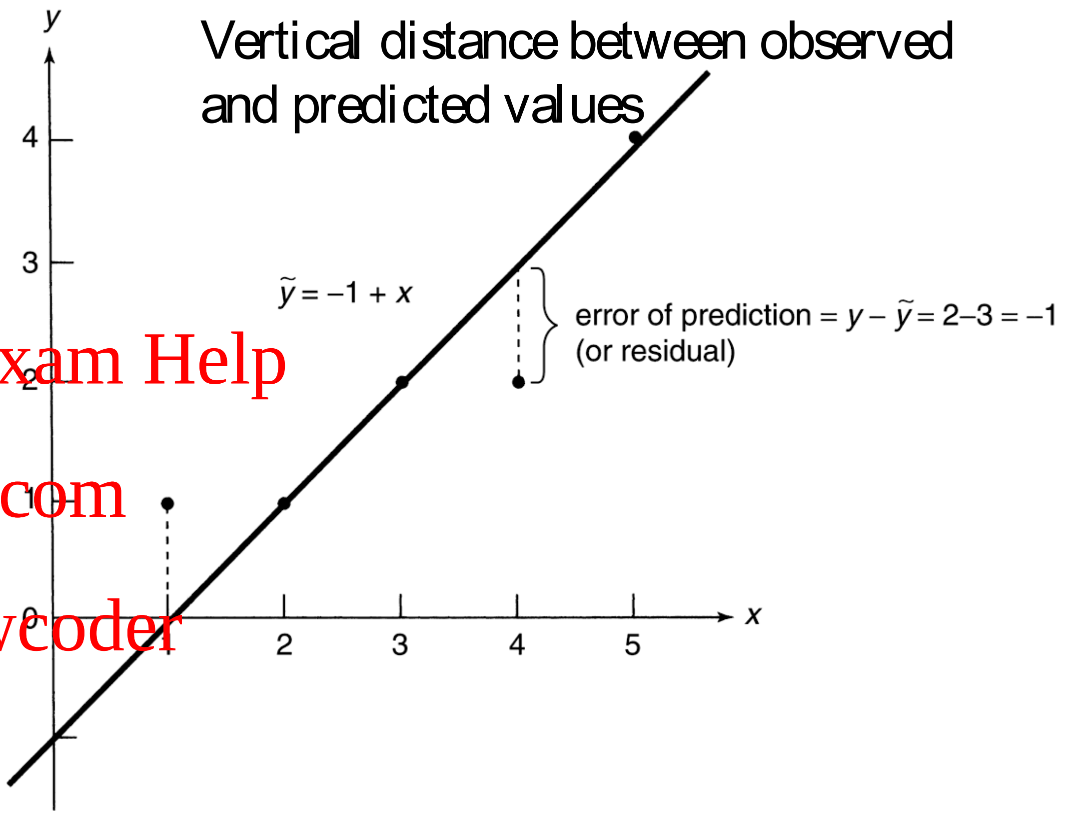
High leverage, Large residuals, Large Influence



Heteroscedasticity **Homoscedasticity**

Fitting the Model: The Method of Least Squares

Vertical distance between observed and predicted values



Find the line that minimizes **the sum of all the squared distances** from the points to the line

y-hat

fitted line

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

deviation

residual

$$(y_i - \hat{y}_i)$$

the sum of squares of residuals

$$\text{SSE} = \sum_{i=1}^{n} [y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)]^2$$

least squares estimates

We need to find β_0 and β_1 that make the SSE a minimum.

Model Summary in R: lm()

model = **lm**(dist ~ speed, data = cars)

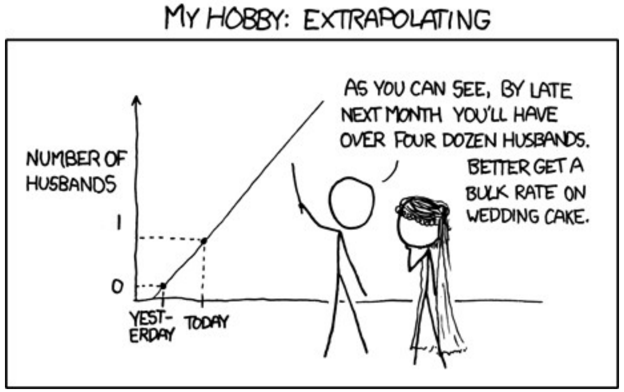
response

predictor

1 **Residuals**: 5 summary points

intercept = MEAN(distance) for x(speed) = 0

2 **slope** = for every 1 mph increase, the distance is increased by 3.9 feet



https://xkcd.com/605/

summary(model)

```
Call:
lm(formula = dist ~ speed, data = cars)

Residuals:
    Min      1Q  Median      3Q     Max
-29.069  -9.525  -2.272   9.215  43.201

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -17.5791     6.7584  -2.601   0.0123 *
speed         3.9324     0.4155   9.464 1.49e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1

Residual standard error: 15.38 on 48 degrees of freedom
Multiple R-squared:  0.6511, Adjusted R-squared:  0.6438
F-statistic: 89.57 on 1 and 48 DF,  p-value: 1.49e-12
```

Mean = 0

1

2

beta_zero

beta_one

Model Summary in R: lm()

3. **Standard Error**: The standard deviation of an estimate. Low values are ideal.
4. **t value**: coefficient/std error
5. **p value**: individual p value for each parameter
6. **Residual Standard Error**: a measure of the quality of a linear regression fit
7. **R-squared**: how well the model is fitting the actual data
8. **F-Statistic**: indicator of a relationship between predictor and response

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Felipe Rego, 2015. Quick Guide: Interpreting Simple Regression.

Model Summary in Python: OLS

lm()

```
y = data.dist
x = data.speed
x = sm.add_constant(x)
```

Add Intercept (None - by default)

```
model = smf.OLS(y, x)
results = model.fit()
print(results.summary())
```

import statsmodels.formula.api as smf

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   dist   R-squared:                       0.651
Model:                            OLS   Adj. R-squared:                  0.644
Method:                 Least Squares   F-statistic:                     89.57
Date:                Sat, 21 Sep 2019   Prob (F-statistic):           1.49e-12
Time:                        00:29:54   Log-Likelihood:                -206.58
No. Observations:                  50   AIC:                             417.2
Df Residuals:                      48   BIC:                             421.0
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        -17.5791      6.758     -2.601      0.012     -31.168      -3.990
speed          3.9324      0.416      9.464      0.000       3.097       4.768
==============================================================================
Omnibus:                        8.975   Durbin-Watson:                   1.676
Prob(Omnibus):                  0.011   Jarque-Bera (JB):                8.189
Skew:                           0.885   Prob(JB):                       0.0167
Kurtosis:                       3.893   Cond. No.                         50.7
==============================================================================
```

```
Call:
lm(formula = dist ~ speed, data = cars)

Residuals:
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```
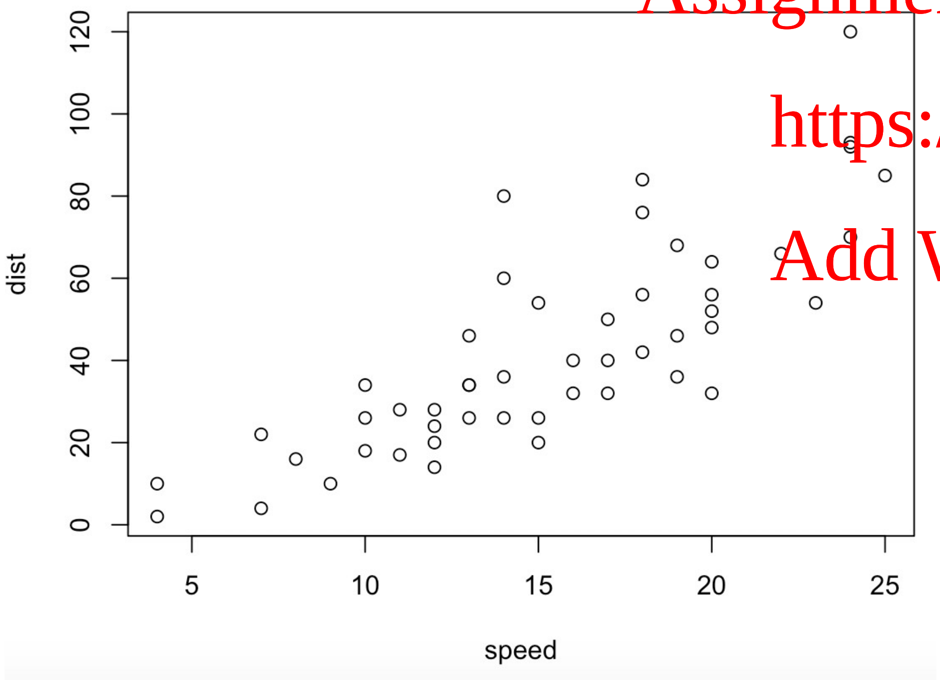

Workflow

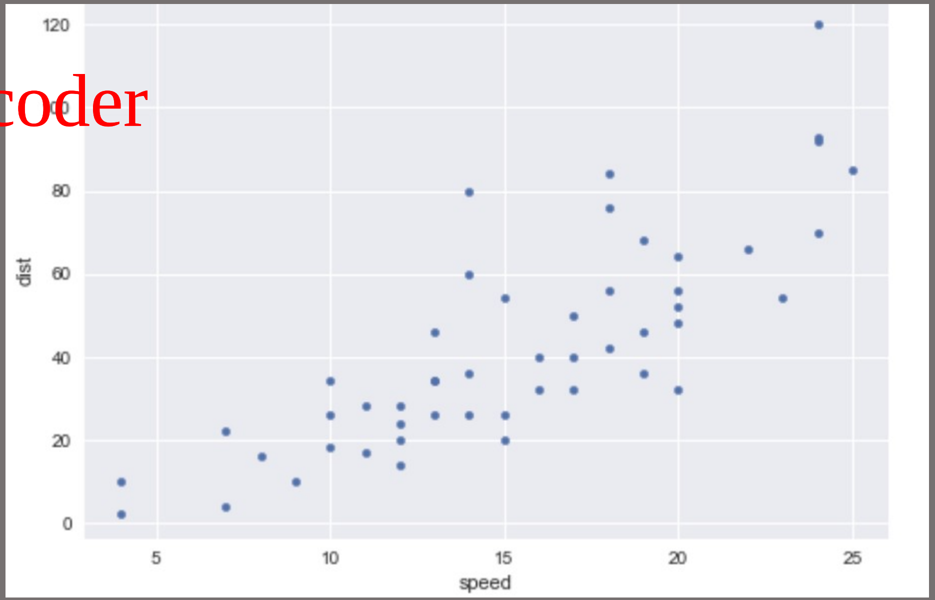
STEP 1. Confirm Linear Relationship

```
data(cars)
with(cars, plot(y=dist, x=speed))
```



The plot shows a fairly strong positive relationship

```
%matplotlib inline
import matplotlib.pyplot as plt
import pandas as pd
plt.style.use('seaborn')

df = pd.read_csv("cars.csv")

df.plot(x = 'speed', y ='dist', kind='scatter')
plt.show()
```



Workflow Example

STEP 2. Run Regression

```
model = lm(dist~speed, data=cars)
summary(model)
```

```
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```



```
import statsmodels.api as sm
y = df.dist
x = df.speed
x = sm.add_constant(x)
model = sm.OLS(y, x)
results = model.fit()
print(results.summary())
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Skew:                           0.885   Prob(JB):                       0.0167
Kurtosis:                       3.893   Cond. No.                         50.7
==============================================================================
```
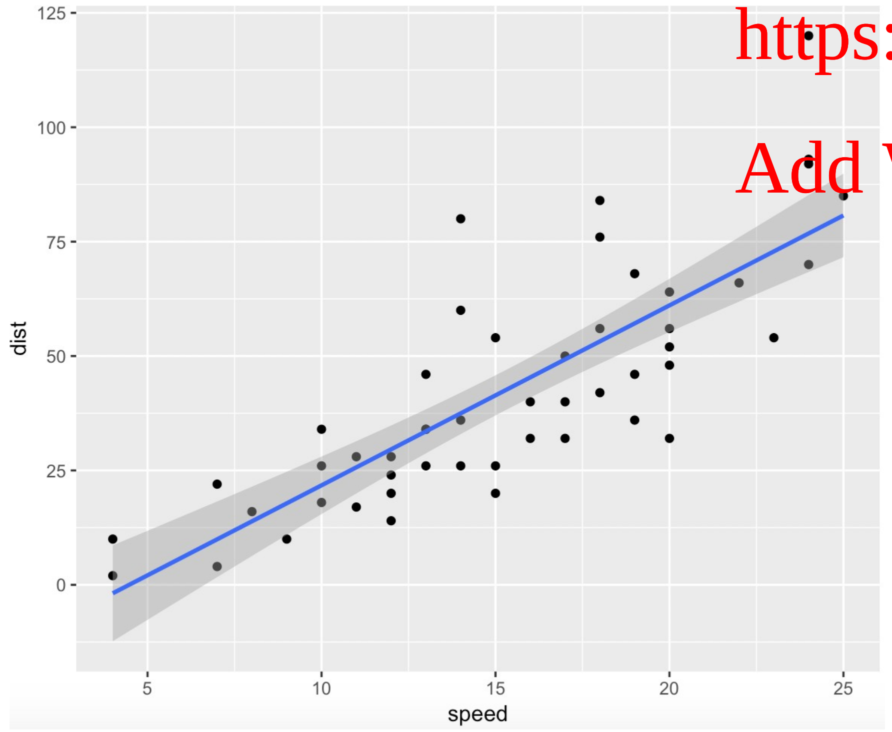

STEP 3. Interpret Summary Output

Workflow

STEP 4. Create a plot with abline

```
library(ggplot2)
ggplot(cars, aes(x=speed, y=dist))+
  geom_point()+
  geom_smooth(method=lm, se=TRUE)
```


```
import seaborn as sns
sns.set(color_codes=True)
g = sns.lmplot(x="speed", y="dist", data=df)
```