Application of Matlab for Finance: Individual Coursework

Release Date: Monday, 11th September, 2017

Due Date: 17:30 PM Friday, 3rd November , 2017

Please upload your soft copy together with the code files to the HUB by the given deadline.

This coursework is designed to test your ability to apply MATLAB to two real world financial applications: portfolio optimization and options pricing. To complete the coursework successfully, you will need to be comfortable with: linear/matrix algebra, csv.file import/export, matrix indexing & colon operator, for/while loop, simulation, user-defined function, datetime object manipulation, and help function.

The output of this coursework should be a **1-3 page report** (including figures and tables) with a brief discussion of the results and a `.m` file of **your matlab program**.

NOTE: your matlab file should be divided into subsections using %% identifier, and you MUST write clear comments in your matlab program to explain what your code is trying to accomplish.

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Question 1: Portfolio Optimization and Performance Backtesting

In this question, we are going to find out how to construct a optimal portfolio in a universe of 30 US stocks and backtest the performance of several portfolio strategies using historical data.

The dataset `equity_dataset_30.csv` can be downloaded from the HUB. It contains the daily closing prices (adjusted for stock splits and cash/stock dividends) for 30 blue-chip stocks over past 10 years. The dataset has 31 columns in total, with the first column being the date index in ISO format (yyyy-mm-dd) and the rest 30 columns containing price data for 30 stocks respectively.

(1) Import the csv data file `equity_dataset_30.csv` into Matlab, and extract the numeric price data into a variable named `px_mat`. `px_mat` should be a T-by-N matrix where $T = 2641$ and $N = 30$.

(2) Calculate the log return series for all 30 stocks according to formula

$$R_{i,t} = \ln(p_{i,t}) - \ln(p_{i,t-1}) \quad \forall i \in [1, 30], t \in [2, 2641].$$

Save the resulting stock return matrix as `ret_mat`.

(3) Split the whole sample period into In-Sample (training dataset, from 2005-01-01 to 2012-12-31) and Out-of-Sample (testing dataset, from 2013-01-01 to 2015-06-30) periods. Use two variables `ret_mat_is` and `ret_mat_oos` to store the stock return matrix for In-Sample and Out-of-Sample periods respectively. (**hints**: you may want to use `datenum` and `datestr` to convert between serial date number and the string representation of date. To find the index of the

cut-off date, try to use the function `find(.)`. **Note**: the return matrix is one-observation short than the price matrix or date vector).

(4) Calculate the historical average daily return for each stock and the historical covariance matrix by **using only the In-Sample dataset**. (hints: check the help pages for `mean(A,dim)` and `cov`)

(5) Consider the following 4 portfolios:

– Benchmark 1/N portfolio: allocate capital equally to each of the 30 stocks.

$$w_i = \frac{1}{30} \quad \forall i \in [1, 30]$$

– Portfolio 1: Maximize Sharpe ratio (short-selling is allowed).

$$\max_{w} \quad \frac{E[r_p] - r_f}{\sigma_p}$$

$$s.t. \quad \sum_{i}^{N} w_i = 1$$

– Portfolio 2: Maximize Sharpe ratio (no short-selling)

$$\begin{aligned} \max_{w} \quad & \frac{E[r_p] - r_f}{\sigma_p} \\ s.t. \quad & \sum_{i}^{N} w_i = 1 \quad \text{and} \quad w_i > 0 \quad \forall i \in [1, 30] \end{aligned}$$

– Portfolio 3: Minimize portfolio variance (short-selling is allowed).

$$\min_{w} \quad \sigma_p^2$$
$$s.t. \quad \sum_{i}^{N} w_i = 1$$

Use the historical mean return vector and covariance matrix calculated in part (4) for the in-sample period, find the optimal weights for portfolio 1, 2 and 3 respectively.

(6) Assume that we can buy/sell any fraction of shares and ignore the transaction cost associated with rebalancing portfolio daily. Backtest the performances of benchmark 1/N portfolio and optimized portfolio 1, 2 and 3 based on the weights using the In-sample dataset. Construct and plot the cumulative return for the four trategy strategies. Calculate annualized portfolio return, variance and Sharpe Ratio. Comment on which strategy perform better. (hints: You might want to consider a log 10 scale when plotting.)

(7) Next, evaluate the investment strategy based on the in-sample data with out-sample observations. Does the best performance strategy for the in-sample period still outperform in the out-of-sample period?

(8) Repeat (5) and (6) with out-of-sample data. Pick one trading strategy, comment on the weights difference between the weights here versus the weights from (6).

Question 2: Simulation & Option Pricing

Consider a stock pays no dividends, has an expected return 0.15 per annual with continuous compounding and an annual volatility of 0.3. Observe today's price \$100 per share with $\Delta t = \frac{1}{252}$ year. And the stock price follows a log-normal process as below.

$$\ln(S_t) = \ln(S_0) + (\mu - \frac{\sigma^2}{2})\Delta t + \sigma \epsilon_t \sqrt{\Delta t} \tag{1}$$

$$S_t = S_0 e^{(\mu - \frac{\sigma^2}{2})\Delta t + \sigma \epsilon_t \sqrt{\Delta t}} \tag{2}$$

(1) Simulate the path of log price of the stock over 1 year, and plot the stock price visually (hint: simulate 252 ϵ_t for $t = \frac{1}{252}, \ldots, 1$);

(2) Simulate $10,000,000$ times for the stock price, calculate today's price of an European Call and Put option on this stock with $K = 100,\ r = 0.1,\ T = 1$ year

$$C(S,T) = \max(S - K, 0) \qquad P(S,T) = \max(K - S, 0)$$

– Different from 1, now $\Delta t = 1$ year, simulate $10,000,000$ times for ϵ_T as now we want to know different possible values of S_T at the option mature date;

– The final price is the average across the simulated prices;

(3) Create a function perform the Black-Scholes Formula to determine the above

options' price, with user-defined input in define a call or put.

$$C(T) = S_0 N(d_1) - K e^{-rT} N(d_2) \qquad P(T) = K e^{-rT} N(-d_2) - S_0 N(-d_1)$$

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

(4) Compare results of the option prices based on the simulation and Black-Scholes Formula