UCIrvine | The Paul Merage School of Business

2022.11.24

BANA 273 Session 7

Association Rules
Market Basket Analysis

Prof. Vibs Abhishek

The Paul Merage School of Business

University of California, Irvine

Agenda

- Assignment due dates on Canvas
- Please work on your projects
  - Gather Data
  - Refer to project guidelines
- Market Basket Analysis
- Generating and identifying good Association Rules

Weka Memory

- To increase Java virtual memory for WEKA
  - On your computer go to "Start"
  - In "Search Program and Files" type: "cmd"
  - Use "cd …" to change directory to Weka folder (under Program Files)
  - On command prompt type:
  - "Java –Xmx512m –jar weka.jar"

Why mine association rules?

- The goal may be fuzzy or unstructured
- More than one class variable
- Interesting patterns (previously unknown) may emerge that can be used within a business

What can we do with this data?

A retailer (e.g. Target) has the following data sources:

1. Shopping transactions, 2. Shopper information, 3. Census data with information for each zip code

Transaction data set:

Shopper ID	Date and Time of transaction	Items included in the transaction	Store ID	Trans ID
Shopper111	09/10/2010, 12:09:01pm	Milk, egg, bread	Store123	Tran321

Shopper Information:

Shopper ID	Address	Most purchased category	Recency (days)	Frequency	Total $ (year)
Shopper111	95616	food	12	4/month	$4000

Census Data:

Zip Code	Median family Income	Median house value	Median age	Population	Population density
95616	70,000	500,000	25	60,000	5700/mile^2

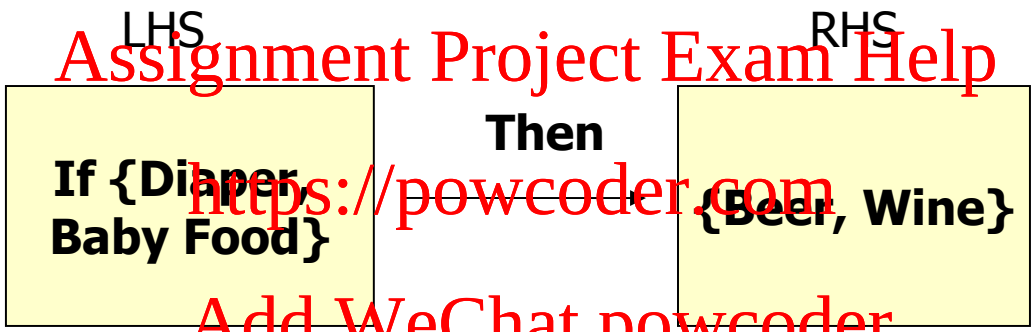
Market Basket Analysis (MBA)

- MBA in retail setting
  - Find out what items are bought together
  - Cross-selling
  - Optimize shelf layout
  - Product bundling
  - Timing promotions
  - Discount planning
  - Product selection under limited space
  - Targeted advertisement, Personalized coupons, item recommendations
- Usage beyond Market Basket
  - Medical (associated symptoms)

Association Rules

Rule format:

If {set of items} → Then {set of items}



LHS implies *RHS*

An association rule is valid if it satisfies some evaluation measures

Association Rule Discovery: Definition

- Given a set of records each of which contain some number of items from a given collection;
  - Produce dependency rules which will predict occurrence of an item based on occurrences of other items.

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Rules Discovered:

{Milk} --> {Coke}

{Diaper, Milk} --> {Beer}

Association Rule Discovery: Application 1

- Marketing and Sales Promotion:
  - Let the rule discovered be

    {Bagels, … } --> {Potato Chips}

  - Potato Chips as consequent => Can be used to determine what should be done to boost its sales.
  - Bagels in the antecedent => Can be used to see which products would be affected if the store discontinues selling bagels.
  - Bagels in antecedent *and* Potato chips in consequent => Can be used to see what products should be sold with Bagels to promote sale of Potato chips.

Association Rule Discovery: Application 2

- Supermarket shelf management.
  - Goal: To identify items that are bought together by sufficiently many customers.
  - Approach: Process the point-of-sale data collected with barcode scanners to find dependencies among items.
  - A classic rule --
    - If a customer buys diaper and milk, then he is very likely to buy beer.
    - So, don't be surprised if you find six-packs stacked next to diapers.

Association Rule Discovery: Application 3

- Inventory Management:
  - Goal: A consumer appliance repair company wants to anticipate the nature of repairs on its consumer products and keep the service vehicles equipped with right parts to reduce on number of visits to consumer households.
  - Approach: Process the data on tools and parts required in previous repairs at different consumer locations and discover the co-occurrence patterns.

Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

{Diaper} → {Beer},
{Milk, Bread} → {Eggs,Coke},
{Beer, Bread} → {Milk},

Implication means co-occurrence, not causality!

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y

$$\textbf{Support} = \frac{\text{No. of transactions containing items in LHS and RHS}}{\text{Total No. of transactions in the dataset}}$$

  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

$$\textbf{Confidence} = \frac{\text{No. of transactions containing both LHS and RHS}}{\text{No. of transactions containing LHS}}$$

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example:

$$\{\text{Milk, Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk, Diaper, Beer})}{|T|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk, Diaper, Beer})}{\sigma(\text{Milk, Diaper})} = \frac{2}{3} = 0.67$$

Rule Evaluation - Lift

Transaction No.	Item 1	Item 2	Item 3	Item 4	…
100	Beer	Diaper	Chocolate		
101	Milk	Chocolate	Shampoo		
102	Beer	Milk	Vodka	Chocolate	
103	Beer	Milk	Diaper	Chocolate	
104	Milk	Diaper	Beer		

What's the support and confidence for rule {Chocolate}→{Milk}?

Support = 3/5 Confidence = 3/4

Very high support and confidence.
Is Chocolate a good predictor of Milk purchase?

No! Because Milk occurs in 4 out of 5 transactions. Chocolate is even decreasing the chance of Milk purchase
3/4 < 4/5, i.e. P(Milk|Chocolate)<P(Milk)

Lift = (3/4)/(4/5) = 0.9375 < 1

Rule Evaluation – Lift (cont.)

- Measures how much more likely is the RHS given the LHS than merely the RHS
- Lift = confidence of the rule / benchmark confidence

  Benchmark Confidence

  = Count of RHS / Number of transactions in database

Example: {Diaper} → {Beer}

- Total number of customer in database: 1000
- No. of customers buying **Diaper**: 200
- No. of customers buying **beer**: 50
- No. of customers buying **Diaper** & **beer**: 20

- Benchmark confidence of Beer = 50/1000 (5%)
- Confidence = 20/200 (10%)
- Lift = 10%/5% = 2
- Higher Lift indicates better rule

Mining Association Rules

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

{Milk,Diaper} → {Beer} (s=0.4, c=0.67)
{Milk,Beer} → {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} → {Milk} (s=0.4, c=0.67)
{Beer} → {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} → {Milk,Beer} (s=0.4, c=0.5)
{Milk} → {Diaper,Beer} (s=0.4, c=0.5)

Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive!

Exercise 1

Compute the support for subsets {a}, {b, d}, and {a,b,d} by treating each transaction ID as a market basket.

Table 6.1. Example of market basket transactions.

Customer ID	Transaction ID	Items Bought
1	0001	$\{a, d, e\}$
1	0024	$\{a, b, c, e\}$
2	0012	$\{a, b, d, e\}$
2	0031	$\{a, c, d, e\}$
3	0015	$\{b, c, e\}$
3	0022	$\{b, d, e\}$
4	0029	$\{c, d\}$
4	0040	$\{a, b, c\}$
5	0033	$\{a, d, e\}$
5	0038	$\{a, b, e\}$

Exercise 2

Use the results in the previous problem to compute the confidence for the association rules {b, d} → {a} and {a} → {b, d}. State what these values mean in plain English.

Table 6.1. Example of market basket transactions.

Customer ID	Transaction ID	Items Bought
1	0001	$\{a, d, e\}$
1	0024	$\{a, b, c, e\}$
2	0012	$\{a, b, d, e\}$
2	0031	$\{a, c, d, e\}$
3	0015	$\{b, c, e\}$
3	0022	$\{b, d, e\}$
4	0029	$\{c, d\}$
4	0040	$\{a, b, c\}$
5	0033	$\{a, d, e\}$
5	0038	$\{a, b, e\}$

Exercise 3

Compute the support for itemsets {a}, {b, d}, and {a,b,d} by treating each customer ID as a market basket.

Table 6.1. Example of market basket transactions.

Customer ID	Transaction ID	Items Bought
1	0001	$\{a, d, e\}$
1	0024	$\{a, b, c, e\}$
2	0012	$\{a, b, d, e\}$
2	0031	$\{a, c, d, e\}$
3	0015	$\{b, c, e\}$
3	0022	$\{b, d, e\}$
4	0029	$\{c, d\}$
4	0040	$\{a, b, c\}$
5	0033	$\{a, d, e\}$
5	0038	$\{a, b, e\}$

Exercise 4

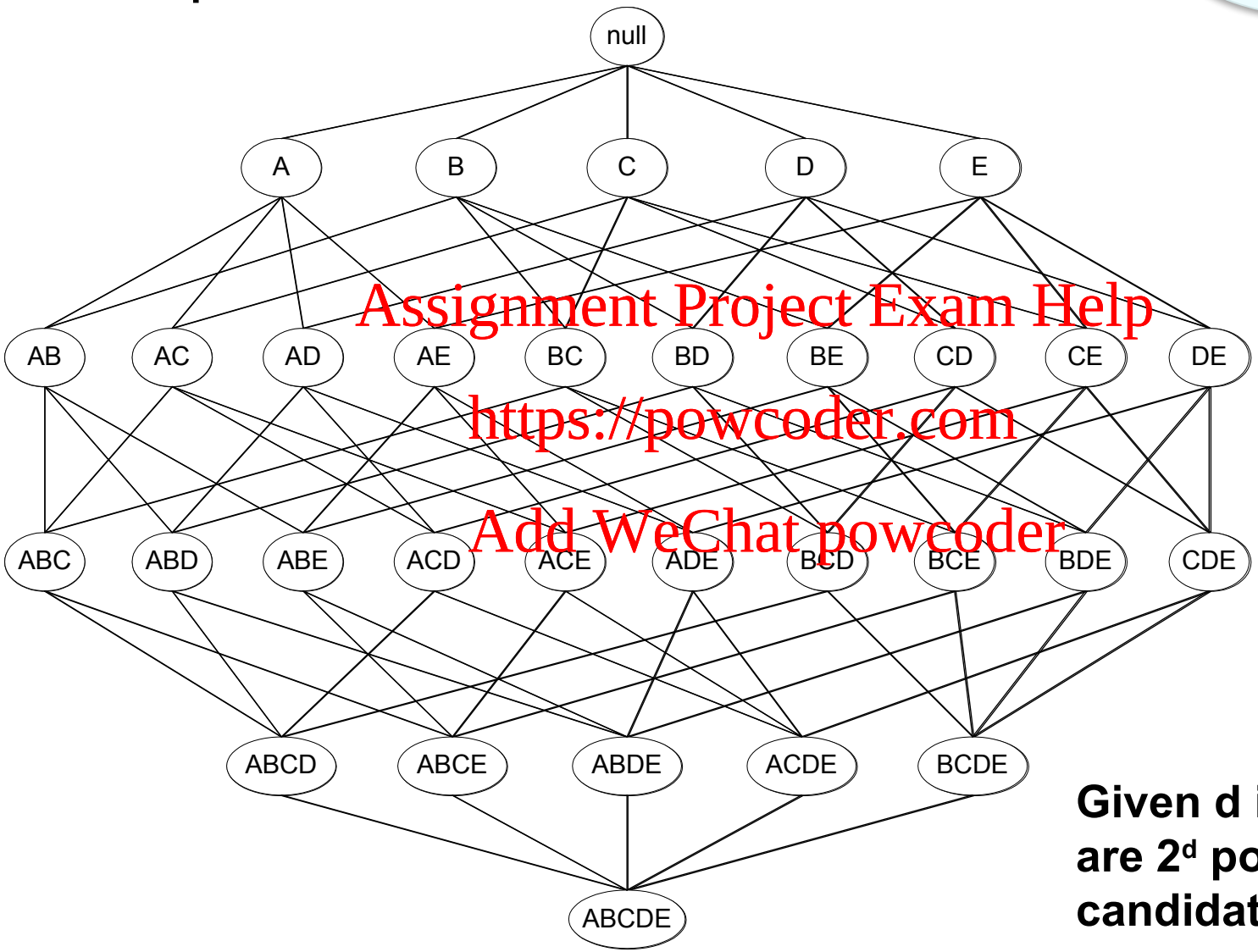
Use the results in the previous problem to compute the confidence for the association rules {b, d} → {a} and {a} → {b, d}.

Table 6.1. Example of market basket transactions.

Customer ID	Transaction ID	Items Bought
1	0001	$\{a, d, e\}$
1	0024	$\{a, b, c, e\}$
2	0012	$\{a, b, d, e\}$
2	0031	$\{a, c, d, e\}$
3	0015	$\{b, c, e\}$
3	0022	$\{b, d, e\}$
4	0029	$\{c, d\}$
4	0040	$\{a, b, c\}$
5	0033	$\{a, d, e\}$
5	0038	$\{a, b, e\}$

Frequent Itemset Generation

Optional

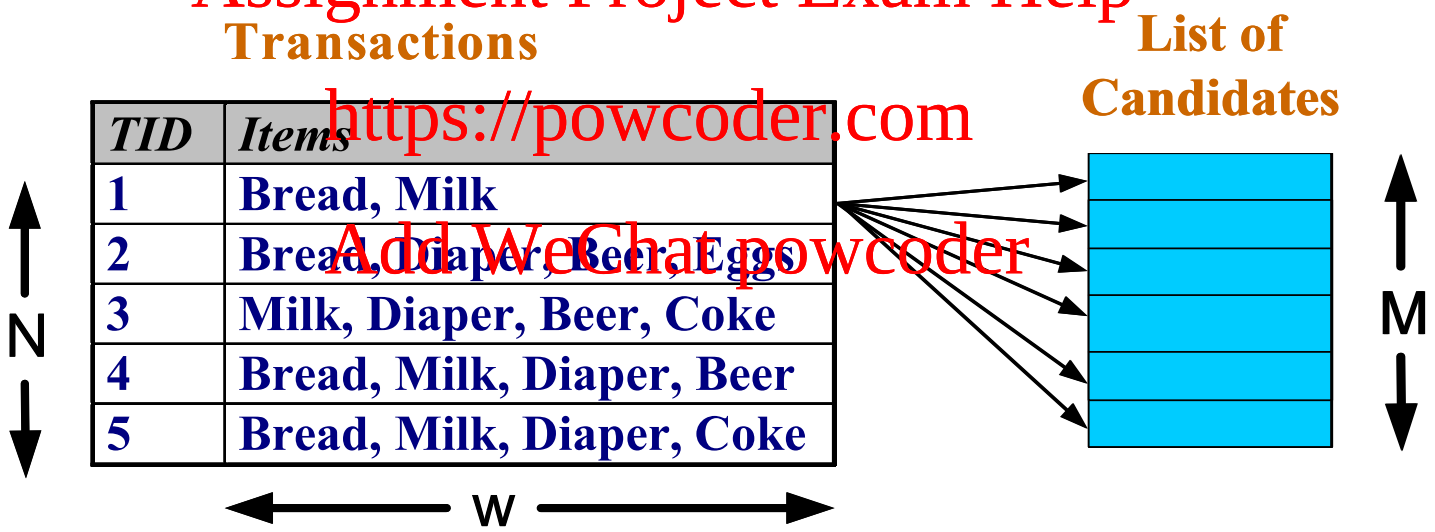


Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

Optional

- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database

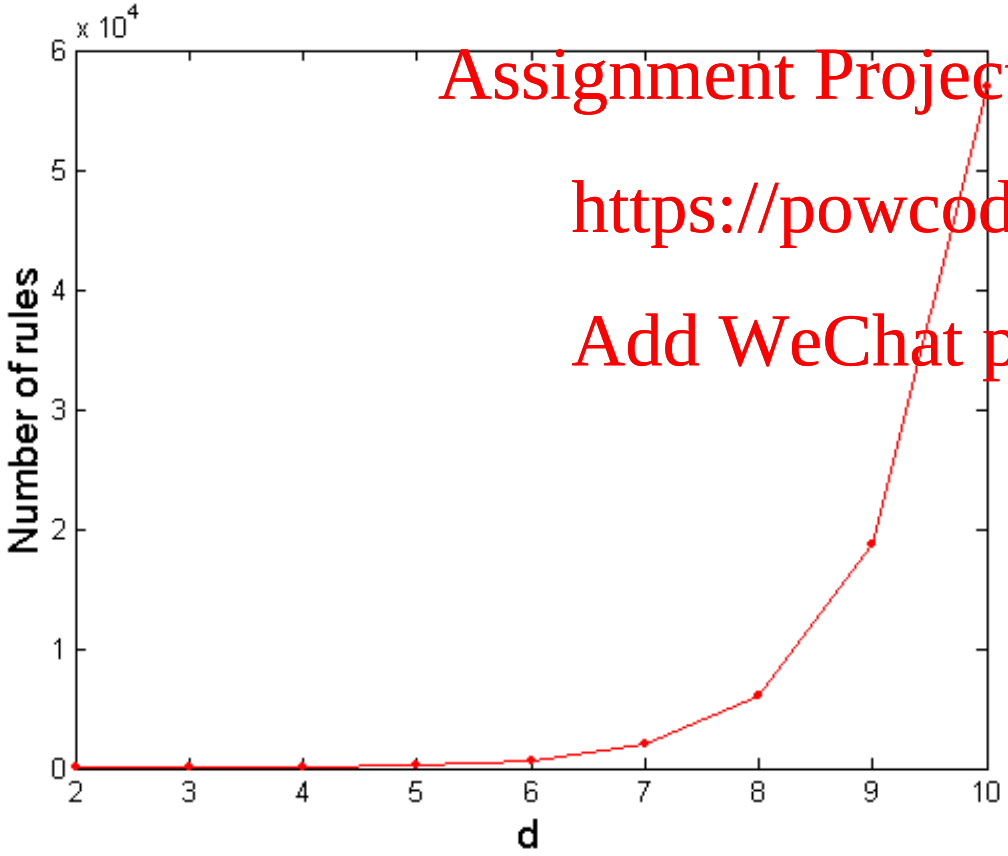


  - Match each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since $M = 2^d$!!!

Computational Complexity

Optional

- Given d unique items:
  - Total number of itemsets = 2^d
  - Total number of possible association rules:



$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k} \times \sum_{j=1}^{d-k}\binom{d-k}{j}\right]$$

$$= 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Identifying Association Rules

- Two-step approach:
  1. Frequent Itemset Generation
     - Generate all itemsets whose support ≥ minsup
  2. Rule Generation
     - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

- Frequent itemset generation is still computationally expensive

Phase 1: Finding all frequent itemsets

How to perform an efficient search of all frequent itemsets?

If **{diaper, beer}** is frequent then **{diaper}** and **{beer}** are each frequent as well

This means that…

- If an itemset is not frequent (e.g., **{wine}**) then no itemset that includes wine can be frequent either, such as **{wine, beer}** .
- We therefore first find all itemsets of size 1 that are frequent.

  Then try to "expand" these by counting the frequency of all itemsets of size 2 that include frequent itemsets of size 1.

  Example:

  If **{wine}** is not frequent we need not try to find out whether **{wine, beer}** is frequent. But if both **{wine}** & **{beer}** were frequent then it is possible (though not guaranteed) that **{wine, beer}** is also frequent.
- Then take only itemsets of size 2 that are frequent, and try to expand those, etc.

Phase 2: Generating Association Rules

Assume **{Milk, Bread, Butter}** is a frequent itemset.

- Using items contained in the itemset, list all possible rules
  - {Milk} → {Bread, Butter}
  - {Bread} → {Milk, Butter}
  - {Butter} → {Milk, Bread}
  - {Milk, Bread} → {Butter}
  - {Milk, Butter} → {Bread}
  - {Bread, Butter} → {Milk}

- Calculate the confidence of each rule
- Pick the rules with confidence above the minimum confidence

Confidence of {Milk} → {Bread, Butter}:

$$\frac{\text{No. of transaction that support \{Milk, Bread, Butter\}}}{\text{No. of transaction that support \{Milk\}}} = \frac{\text{Support \{Milk, Bread, Butter\}}}{\text{Support \{Milk\}}}$$

Algorithm Apriori-Gen to Generate Frequent Itemsets

(Agrawal and Srikant 1994)

Input: two itemsets, *I and J, of size (k-1)*

Output: a supported itemset, *L of size k*

L=Null

IF (the first *(k-2) items of I and J match)*

{ copy all items of *I into L;*

copy the last item of *J into L;*

FOR (every subset *l* of *L* of size *(k-1))*

IF (*l is not supported) discard L and exit;*

Calculate, from data, support(*L);*

IF (support(*L*)< target) discard *L and exit;*

return *L;*

}

ELSE exit;

Agrawal (94)'s Apriori Algorithm—An Example

Transactions

T-ID	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

An itemset must have been purchased at least twice in order to be considered frequent or supported

Agrawal (94)'s Apriori Algorithm—An Example

Transactions

T-ID	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3rd scan

L_3

Itemset	sup
{B, C, E}	2

{A,B,C}, {A, C, E}?

Exercise 5

Transaction No.	Item 1	Item 2	Item 3	Item 4
100	Beer	Diaper	Chocolate	
101	Milk	Chocolate	Shampoo	
102	Beer	Soap	Vodka	
103	Beer	Cheese	Wine	
104	Milk	Diaper	Beer	Chocolate

Given the above list of transactions, do the following:

1) Find all the frequent itemsets (minimum support 40%)
2) Find all the association rules (minimum confidence 70%)
3) For the discovered association rules, calculate the lift

Given table below, the confidence of the rule Bread → Coke is

A: 1/4

B: 2/4

C: 3/4

D: 4/4

E: None of the above

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

What is the objective of Apriori?

A: Identify good association rules

B: Identify all frequent itemsets

C: Identify all rules from frequent itemsets

D: Determine the computational complexity of finding association rules

E: None of the above

Other Applications of Association Rules

- Recommendations: Determines which books are frequently purchased together and recommends associated books or products to people who express interest in an item.
- Healthcare: Studying the side-effects in patients with multiple prescriptions, we can discover previously unknown interactions and warn patients about them.
- Fraud detection: Finding in insurance data that a certain doctor often works with a certain lawyer may indicate potential fraudulent activity. (virtual items)
- Sequence Discovery: looks for associations between items bought over time. E.g., we may notice that people who buy chili tend to buy antacid within a month. Knowledge like this can be used to plan inventory levels.

WEKA

- Find association rules
  - Apriori
- http://facweb.cs.depaul.edu/mobasher/classes/ect584/WEKA/associate.html

Nest Week

- Clustering using K-Means