Introduction to AI: Tutorial 3

SAT solving using DP and DLL algorithms

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(with thanks to Krysia Broda)

This tutorial includes questions on Satisfiability for you to practice using the DP and the DLL algorithms to check whether given sets of clauses are satisfiable or not.

Question 1

Consider the following set of clauses:

$S = \{\{x_1, x_2\}, \{x_1, \neg x_2\}, \{\neg x_1, \neg x_2\}, \{x_3, \neg x_4\}, \{x_5, \neg x_3\}, \{\neg x_1, x_4, x_5\}, \{\neg x_6\}\}$

- List which literals are pure literals (if any) in S and which are unit clauses
- Use resolution to show that S is unsatisfiable.

- Apply now the improved DP algorithm, given in slide 14 of Unit 5, making use of pure literals and unit propagation where possible. Annotate each step of the derivation.

Question 2

Consider the following set S of clauses
$S = \{\{x_3, x_4, \neg x_1, x_5\}, \{\neg x_3, x_4, x_5\}, \{x_3, \neg x_4, \neg x_1\}\{x_1, x_2\}\{x_1, \neg x_2\}\{\neg x_1, \neg x_5\}, \{\neg x_3, \neg x_4, x_5\}\}$
Use the DLL (from slide 20 of Unit5) to check whether$S$ is satisfiable or not. Select branching literals in the order x_1, x_2, x_3, ...

Question 3

Consider the following problem

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I would like to organise a dinner reunion and I do not want any two of
my close friends (Tom, Sam, John and Bob) to be uninvited.
But I cannot invite both Sam and John, and
if I invite Tom I must invite John.
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- Formalise the above problem in propositional logic.
- Use the DLL algorithm to see whether the above problem is satisfiable or not. If it is provide a suggestion of people to invite.

Question 4

Consider the following set S of clauses:
$S = \{\{A, B, \neg D\}, \{D, B, A\}, \{C, E, \neg A\}, \{\neg E, F, \neg A\}, \{F, \neg C, E\}\}$

- How many pure literals are in the set S of clauses?
- If the next atom chosen for splitting is A, how many clauses are left in the A and in the $\neg A$ branches, after applying subsumption?
- Use DLL to see whether S is Satisfiable. If it is, return a model that makes it true.

Question 5

Use the DLL procedure to show the unsatisfiability of each of the following sets of clauses

1. $S = \{\{\neg A, \neg B, C\}, \{D, E\}, \{\neg C, \neg E, F\}, \{D, \neg F\}, \{A\}, \{B\}, \{\neg D\}\}$
2. $S = \{\{A, B, C\}, \{A, \neg B, C\}, \{\neg A, \neg B, \neg C\}, \{A, B, \neg C\}, \{\neg A, B, \neg C\},$
$\{\neg A, \neg B, C\}, \{A, \neg B, \neg C\}, \{\neg A, B, C\}\}$