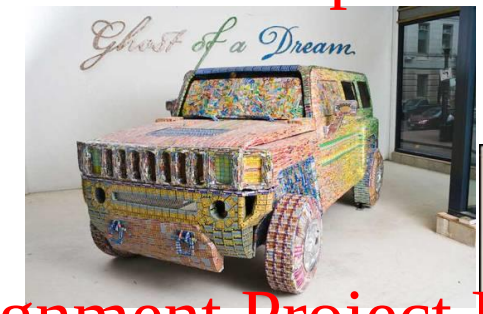
Risk Preferences and Utility

CIS 418

Source: S. Bodily, 2007

A dream of chances



Those dreams are built from losing lottery tickets, by Brooklyn-based artists Adam Eckstrom and Lauren Was and it's entitled Ghost of a Dream. The tickets were discarded by unlucky patrons. “Chance city” was built by the artist Jean Shin.

Are you risk averse?

Suppose you owned a lottery ticket that was equally likely to result in $100 loss and $125 gain.

How much would you accept for this ticket?

Expected Monetary Value (EMV)

50%x$125+50%x(-$100) = $12.5

Certainty Equivalent (CE)

the price at which you'd be willing to sell the ticket.

Risk premium (RP)

how much of the EMV you'd be willing to give up to avoid the risk of losing money.

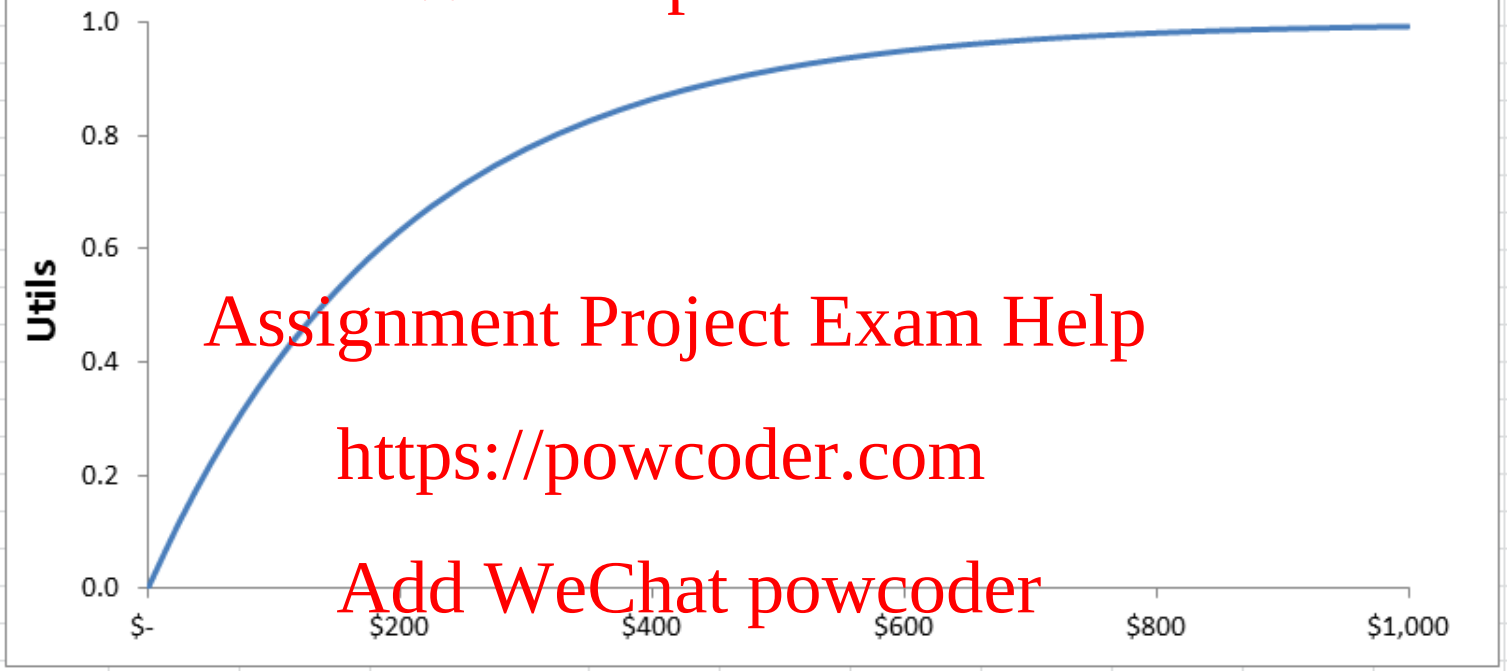
$$RP = EMV\text{-}CE$$

If RP>0 you are risk averse.

If RP<0 you are risk prone.

If RP=0 you are risk neutral.

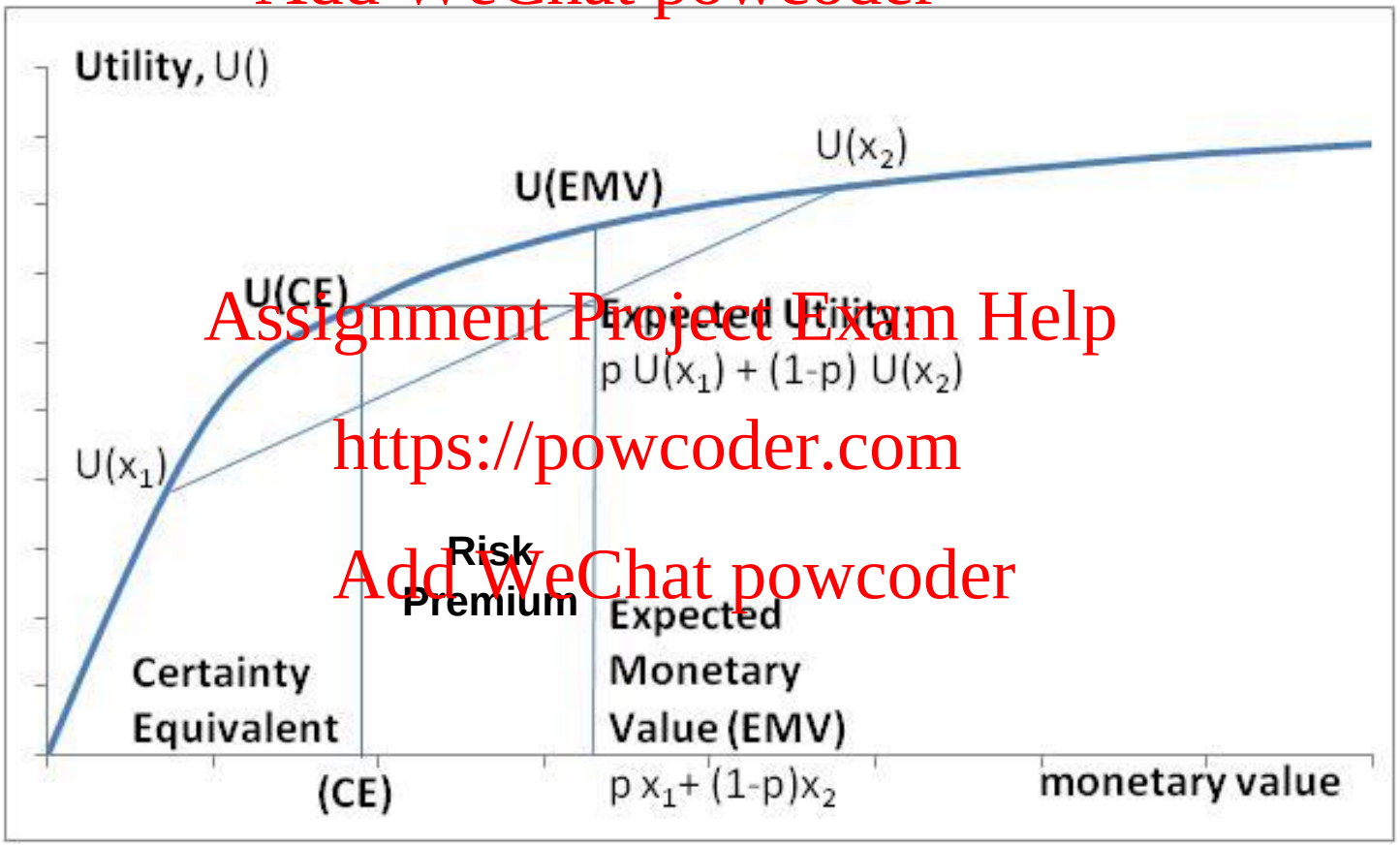
A monetary utility function translates wealth into utility



Wealth	$300	$325	$350	$375	$400	$425	$450	$475	$500	$525
Utility	0.78 utils	0.80 utils	0.83 utils	0.85 utils	0.86 utils	0.88 utils	0.89 utils	0.91 utils	0.92 utils	0.93 utils

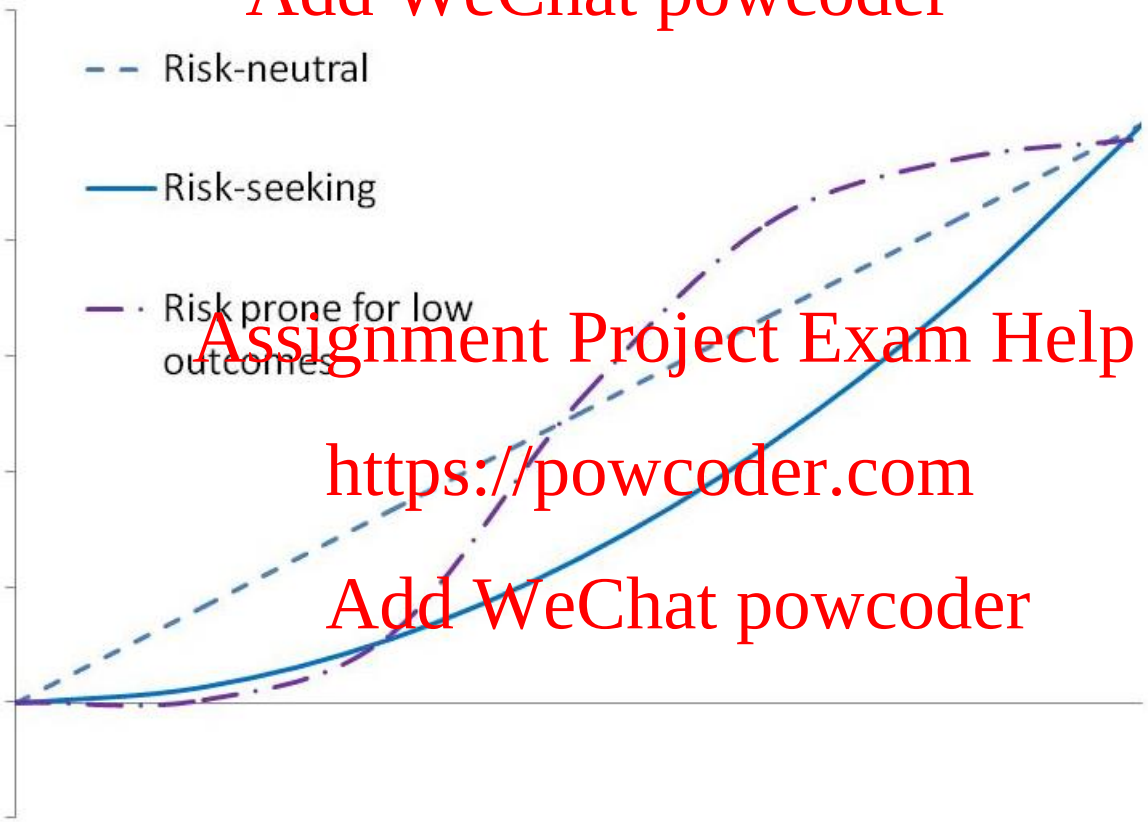
Does this utility function represents the utility of a risk-averse, risk-neutral or risk-prone decision maker?
Explain by showing an example.

Expected Utility vs. Certainty Equivalent



Under the expected utility model, decision makers make choices that maximize their expected utility. The same choices **also** maximize the certainty equivalent.

Various functions can be used to model risk



- Risk-averse function: Concave.
- Risk-neutral function: Linear.
- Risk-prone function: Convex.

Are you risk averse?

Here is an explanation by Veritasium:

https://www.youtube.com/watch?v=vBX-KulgJ1o

Example 1: A simple portfolio problem

There is an investment that for every 1$ invested returns $4.3 or $0 with equal probability.

My current wealth is $14,000. How much of $14,000 should I invest?

-- The more I invest, the higher my *expected* net wealth, but my risk goes up as well.

My optimal investment would depend on my risk preferences

- A decision maker shows constant risk aversion if she has the same positive risk premium for any two risky opportunities that have respective outcomes that differ only by a constant amount.

  Therefore, her expected utility would be modeled by a **negative exponential** function:

$$EU = 1 - e^{-CE/R}$$

- A decision maker shows decreasing risk aversion if she has decreasing risk premium for any two risky opportunities that have respective outcomes that differ only by a constant amount.

Therefore, her expected utility would be modeled by a **logarithmic** function:

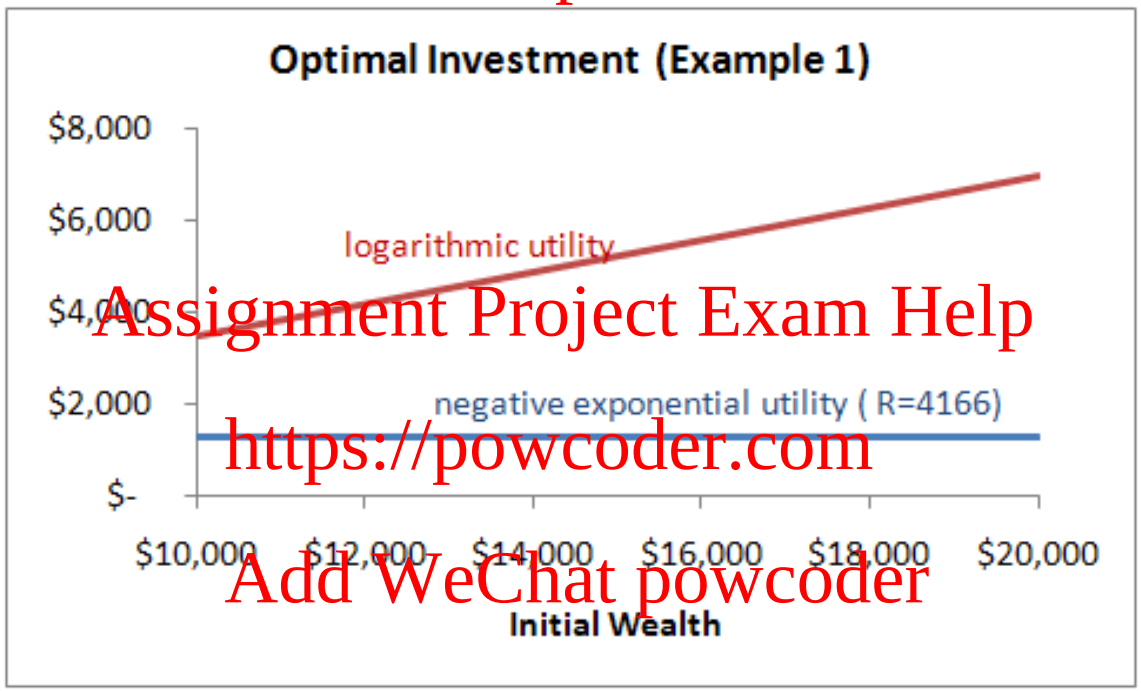
$$EU = \ln\left(CE + A\right)$$

- R, A = Risk tolerance.

Risk premium as a function of initial wealth

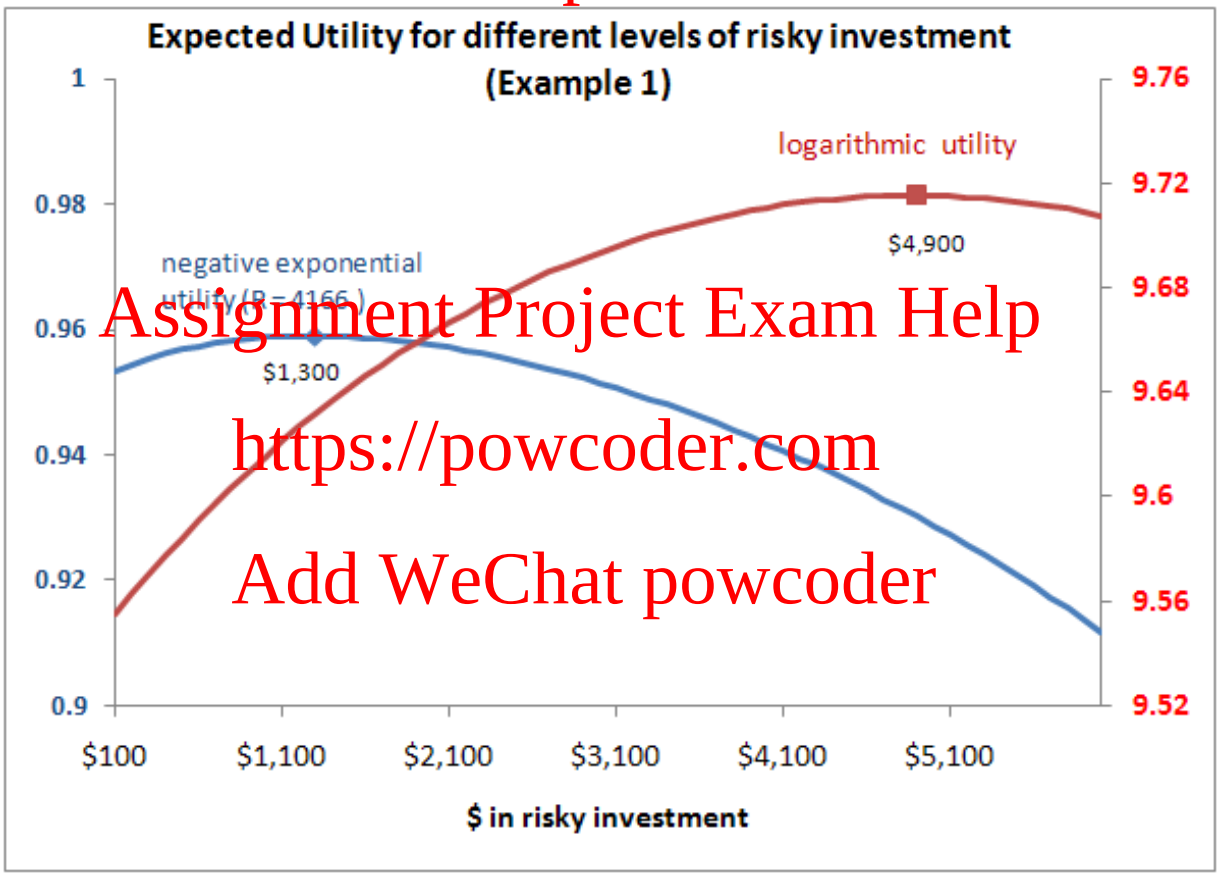
Lower risk premiums = higher risk

Risk as a function of initial wealth

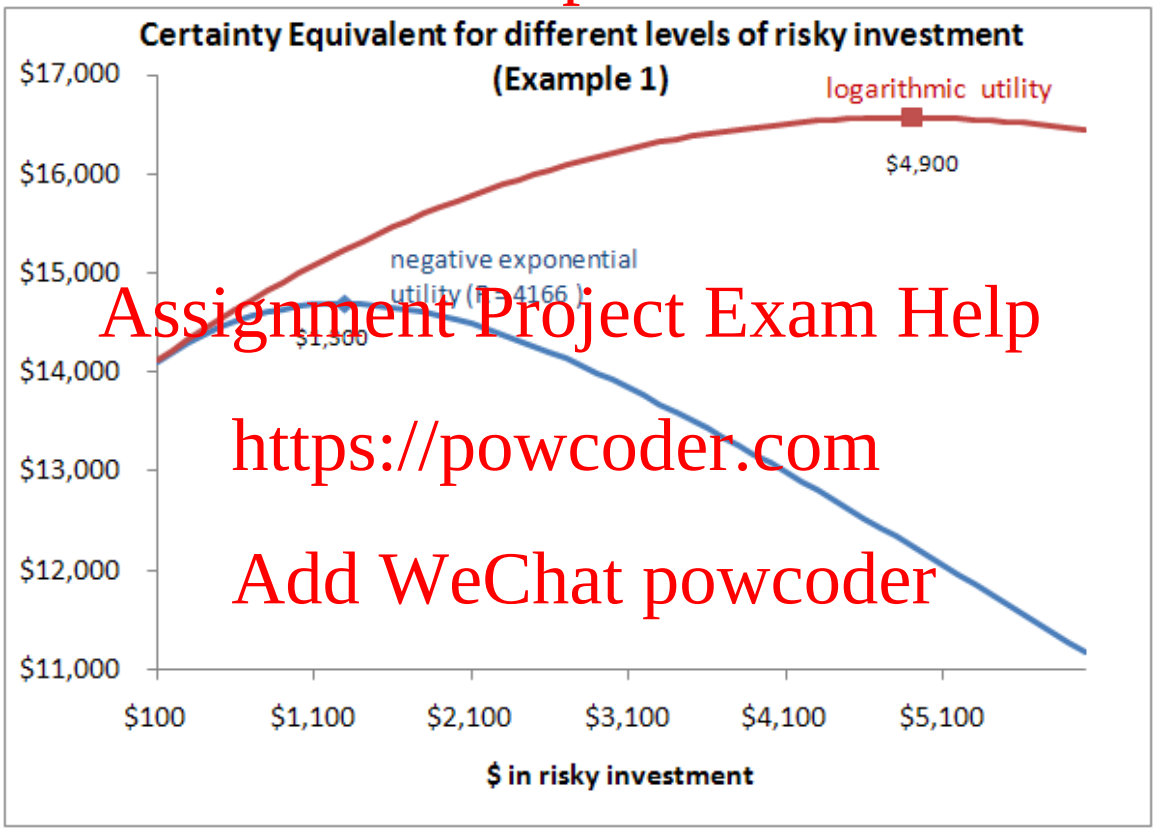


With the logarithmic utility the percentage of total wealth invested in the risky investment stays the same as the amount of wealth changes.

Optimize by maximizing EU



Optimize by maximizing CE



Calculate the Certainty Equivalent

The expected utility of the uncertain investment is equal to the utility of certainty equivalent.

Negative Exponential Utility

$$EU_{gamble} = \underbrace{1 - e^{-CE/R}}_{\text{utility of certainty equivalent}}$$

$$\Downarrow$$

$$e^{-CE/R} = 1 - EU_{gamble}$$

$$\Downarrow$$

$$-CE/R = \ln\left(1 - EU_{gamble}\right)$$

$$\Downarrow$$

$$CE = -R\ln\left(1 - EU_{gamble}\right)$$

Logarithmic Utility

$$EU_{gamble} = \ln\left(CE + A\right)$$

$$\Downarrow$$

$$\exp\left(EU_{gamble}\right) = CE + A$$

$$\Downarrow$$

$$CE = \exp\left(EU_{gamble}\right) - A$$

Calculate the Expected Utility

Investment: for every 1$ invested, returns $4.3 or $0 with equal probability.
My current wealth is $14,000.
Calculate EMV, CE and RP for $2000 investment.

Initial Wealth	$ 14,000.00
Investment	$ 2,000.00
R (risk tolerance)	$ 4,166.00

$$EU = 1 - \exp(-12{,}000 / R)$$

	Probability	Net Wealth	Utility
Lose	0.5	$ 12,000.00	0.943891
Win	0.5	$ 20,600.00	0.99288
	Expected	$ 16,300.00	0.968385
	CE	$ 14,389.93	
	RP	$ 1,910.07	

$$CE = -R \cdot \ln(1 - 0.968)$$

Example 2: Keep the investment?

The decision maker owns an investment that will result in personal wealth of either $21,000 or $11,000 in today's dollars with equal probability

Q1. The decision maker can choose to

- Keep the investment
- Sell this investment now for $14,000
- Sell half of this investment now for $7,000 and keep the other half

Which option will be preferred?

Q2. What is the minimum price that the decision maker will accept now for the entire investment?

Assume the decision maker has ***constant risk aversion*** with ***risk tolerance*** parameter $R = \$4166$.

Option 1: Write VBA Functions

- To increase efficiently, we can write a VBA function to calculate the Negative Exponential Utility and the Certainty Equivalent:


```
Function NEXPEU(CE, R)
    NEXPEU = 1 - Exp(-CE / R)
End Function
```



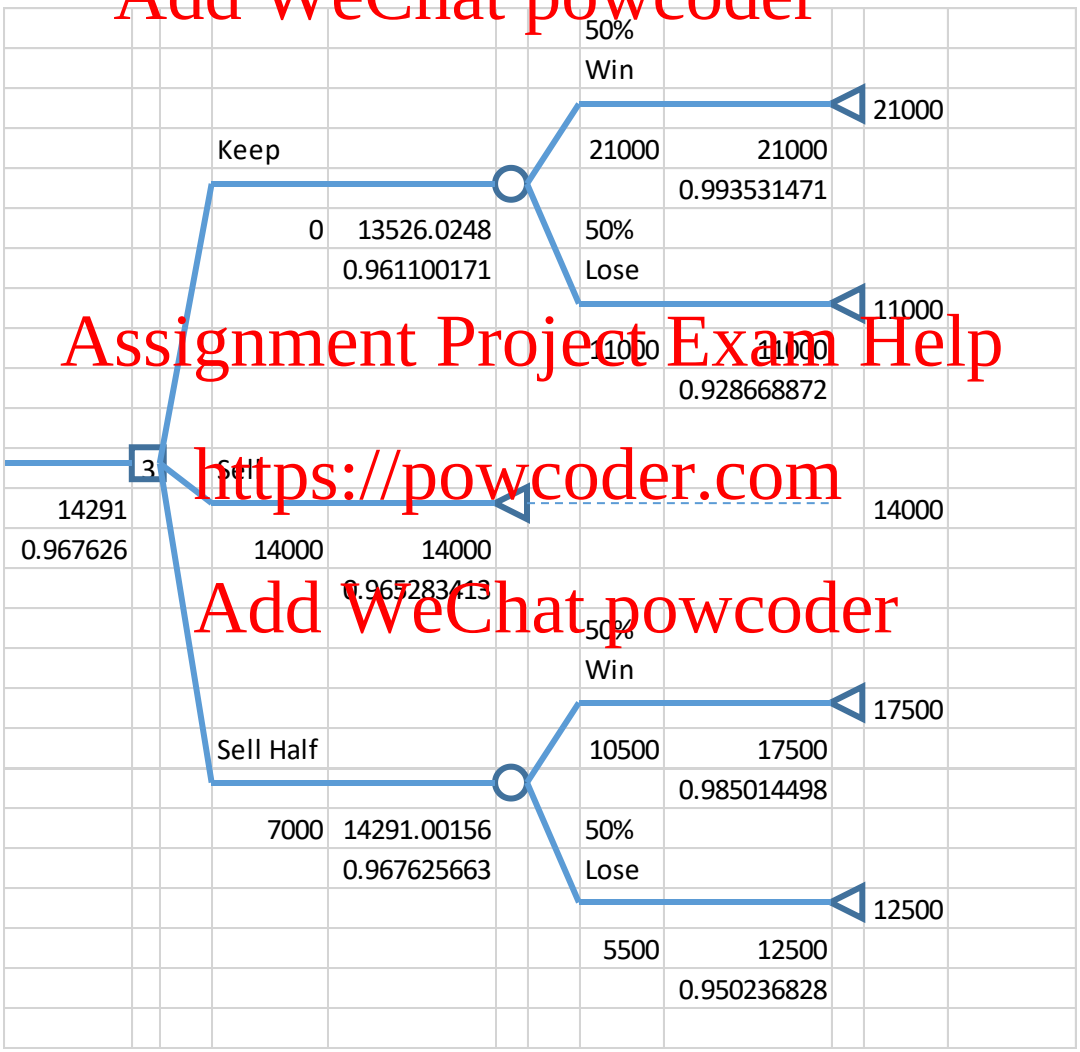
```
Function CE_NEXPEU(EU, R)
    CE_NEXPEU = -R * Log(1 - EU)
End Function
```

Option 2: Compare the expected utilities (or CEs)

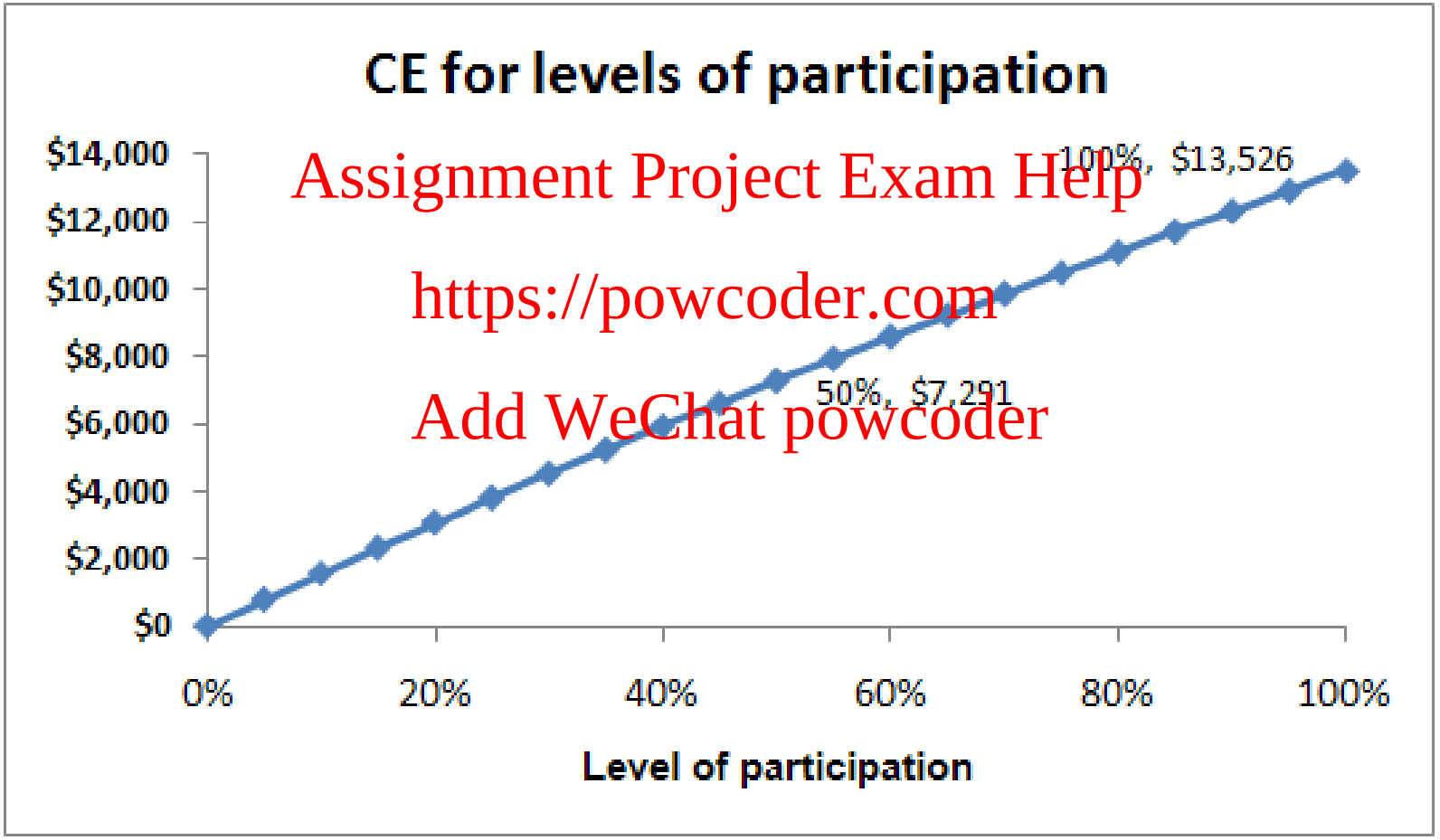
Risk Tolerance	$ 4,166				
		State			
		Good	Bad		
	Probability	0.5	0.5		
				Expected	
Option 1	Cash	$ 21,000	$ 11,000	$ 16,000	CE
	Utility	0.99353147	0.92866887	0.96110017	$ 13,526
keep					
				Expected	
Option 2	Cash	$ 14,000	$ 14,000	$ 14,000	CE
	Utility	0.96528341	0.96528341	0.96528341	$ 14,000
Sell for 14K					
				Expected	
Option 3	Cash	$ 17,500	$ 12,500	$ 15,000	CE
	Utility	0.9850145	0.95023683	0.96762566	$ 14,291

Decision: Sell ½ for 7K

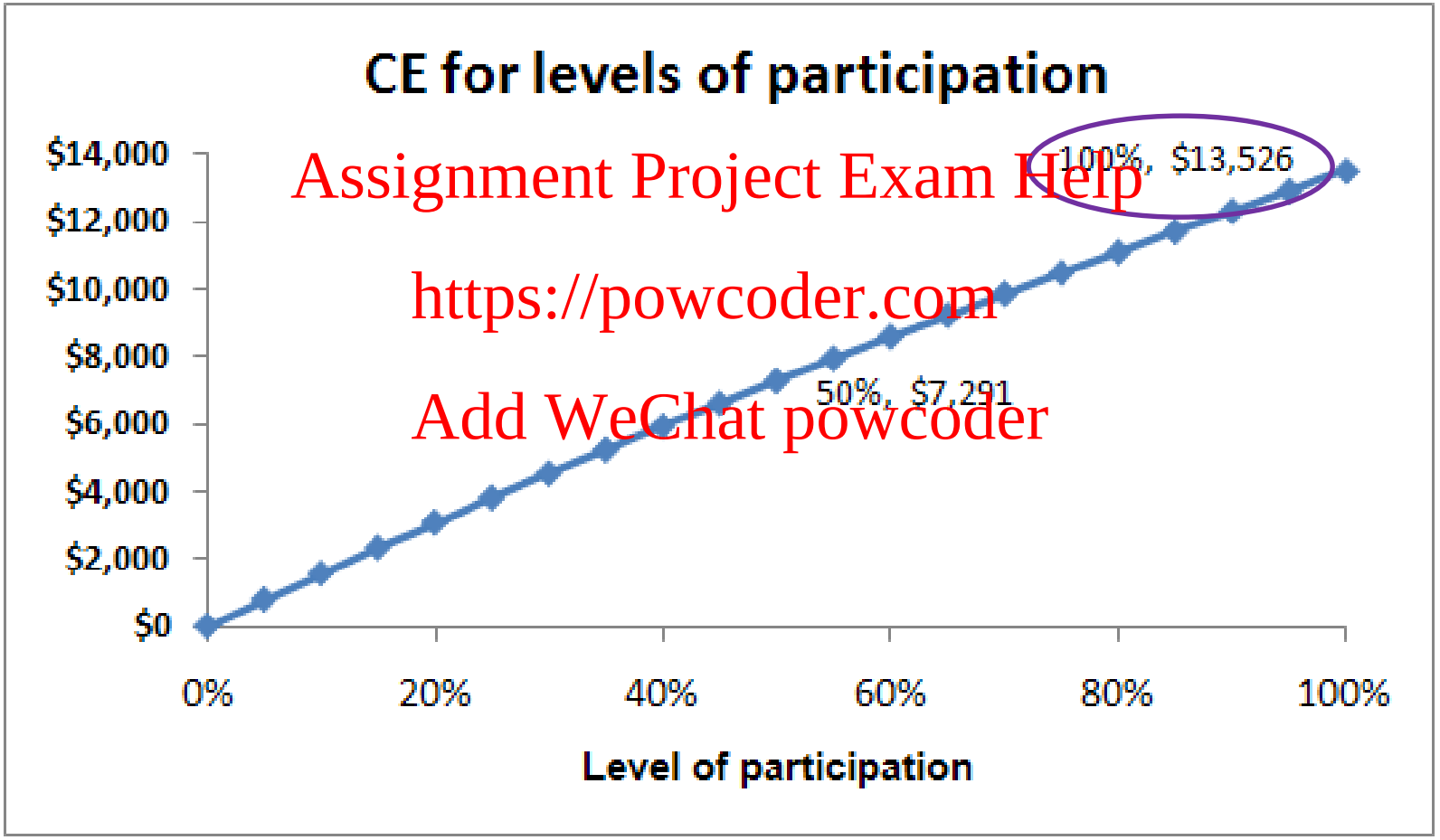
Option 3: Use a decision tree



For this investor, the second 50% of this investment are worth less than 7K in CE, but not the first 50%



Q2: The minimum this investor will accept right now for the investment is $13,256



Example 3: Bet on what horse?

I would like to bet $5000 on a horse. The odds, and my beliefs about the probability of winning are given below.

Horse #	Name	Listed odds	Payout on $1 bet	P(W) mine
1	Waste O'Time	3 to 2	$ 2.50	35%
2	Fool's Folly	3 to 1	$ 4.00	10%
3	First in Ninth	7 to 2	$ 4.50	10%
4	Save Your Money	4 to 1	$ 5.00	30%
5	Tea Biscuit	17 to 1	$ 18.00	15%
				100%

So if I place $100 bet on Tea Biscuit, and Tea Biscuit comes in first I would get $1800. That will happen with 15% probability. With 85% probability Tea Biscuit will not come in first, and I will lose my $100.

Sample spreadsheet to compare utilities of betting on different horses

Risk aversion coefficient		Bet size					
R	$ 25,000	5000					
Horse #	Name	Payout on $2 bet	P(W)	Wealth if in first	Wealth if not in first	Expected Return on Bet	Expected Utility
1	Waste O'Time	$ 2.50	35%	$ 12,500	$ -	$ 4,375	1.4E-01
2	Fool's Folly	$ 4.00	10%	$ 20,000	$ -	$ 2,000	5.5E-02
3	First in Ninth	$ 4.50	10%	$ 22,500	$ -	$ 2,250	5.9E-02
4	Save Your Money	$ 5.00	30%	$ 25,000	$ -	$ 7,500	1.9E-01
5	Tea Biscuit	$ 18.00	15%	$ 90,000	$ -	$ 13,500	1.5E-01

$$EU = 1 - e^{-CE/R}$$

What if you could split the bet among the horses?

Risk tolerance parameter							
R	$ 25,000						
	Waste O'Time	**Fool's Folly**	**First in Ninth**	**Save Your Money**	**Tea Biscuit**		
Prob of coming first	35%	10%	10%	30%	15%		
Payout on $1 bet	$ 2.50	$ 4.00	$ 4.50	$ 5.00	$ 18.00		
Bets	$ 400.00	$ 1,000.00	$ 1,500.00	$ 2,000.00	$ 100.00	**Expected**	
Winnings if first	$ 1,000.00	$ 4,000.00	$ 6,750.00	$ 10,000.00	$ 1,800.00	$4,695.00	**CE**
Utility if first	0.04	0.15	0.24	0.33	0.07	$ 0.16	$4,403.39

Built a spreadsheet that includes the above data.

- What would be your objective?
- What are the decision variables?
- What are the constraints?

Optimize using two different objectives: risk-averse vs. risk-neutral.
Compare the results.