Decision Trees

CIS 418

Business problems involve uncertainty: the NEES example

NEES is a company that produces power. It is deciding how much to bid for the salvage rights to a grounded ship, the *SS Kuniang*.
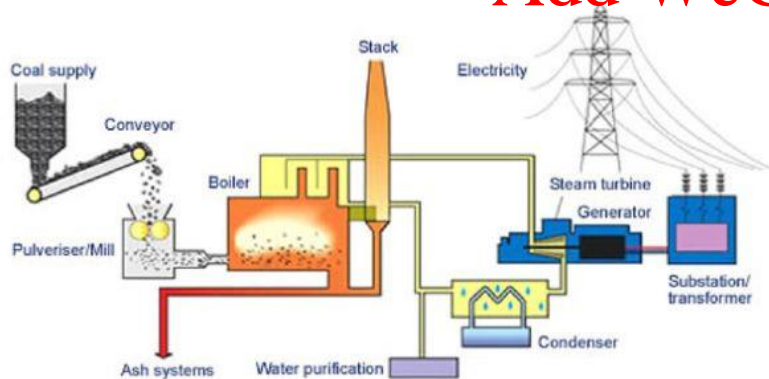
If successful, the ship could be repaired to haul coal for the company's power stations. If the bid fails, NEES could purchase a new ship or a tug/barge combination.

The higher the bid the more likely that NEES will win.

NEES decision is complicated by further uncertainty

U.S. Coast Guard (USCG) judgment about the **Marine salvage value** of the ship involves an obscure law on shipping in coastal waters. **Marine salvage** is the process of recovering a ship and its cargo after a shipwreck or other maritime casualty. USCG's judgment will not be known until after the winning bid.
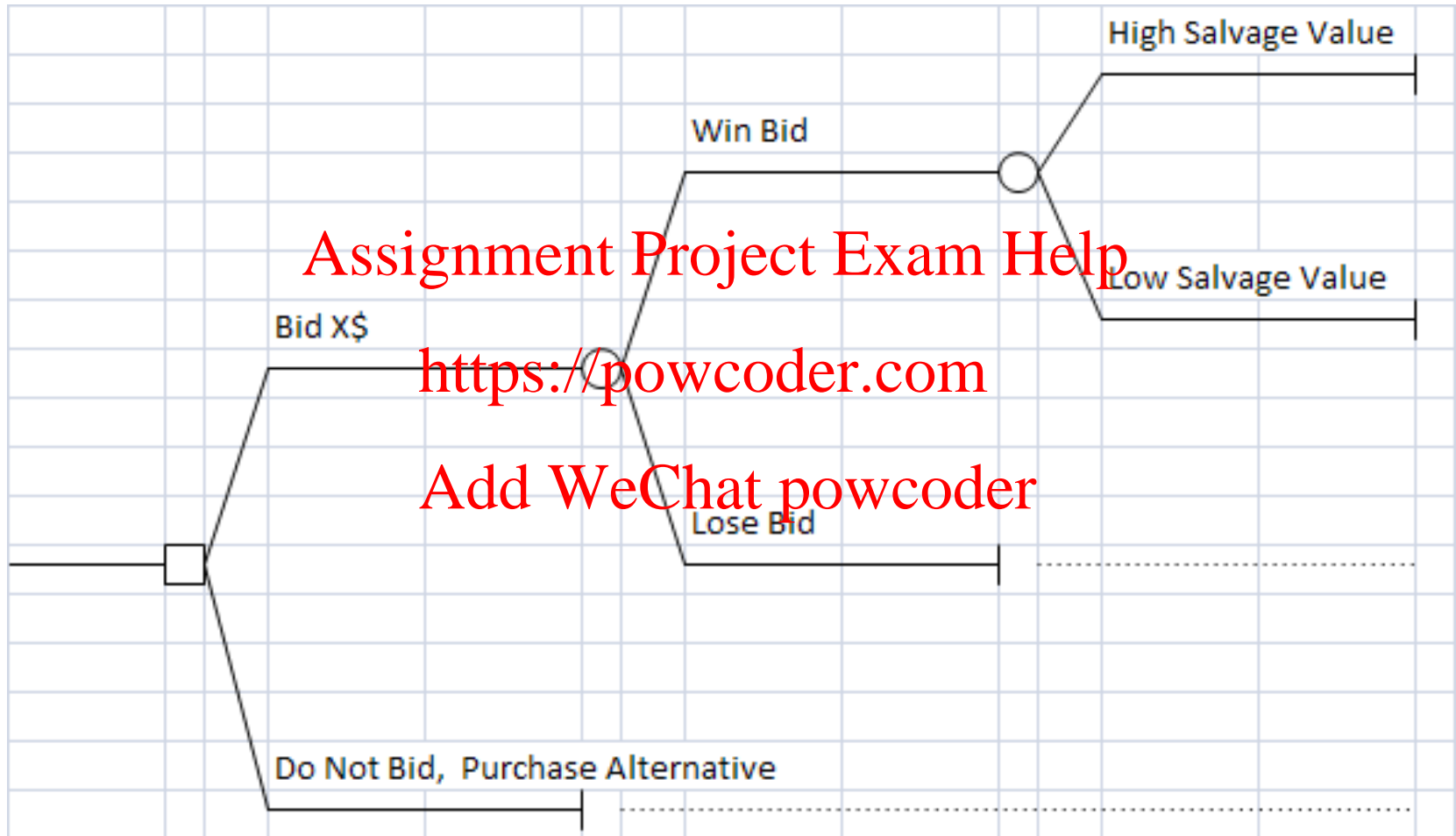
- If the judgment indicates a *low salvage value, then NEES* could use the ship for its shipping needs.

- *High salvage value means that the ship is considered* ineligible of use in domestic shipping unless expensive equipment is installed, i.e. greater expenses for NEES.

How much should NEES bid?

Decision tree diagrams timing of decisions and revelations of relevant uncertainties

Bid X$

Win Bid

High Salvage Value

Low Salvage Value

Lose Bid

Do Not Bid, Purchase Alternative

To complete the tree need to assign $ values (or utilities) to decisions and outcomes

High Salvage Value

Win Bid

$ 11.50

($X)

Low Salvage Value

Bid $X

$ 15.50

The value of the ship after realizing the marine salvage value

Lose Bid, Purchase Alternative

$ 3.20

Do Not Bid, Purchase Alternative

$ 3.20
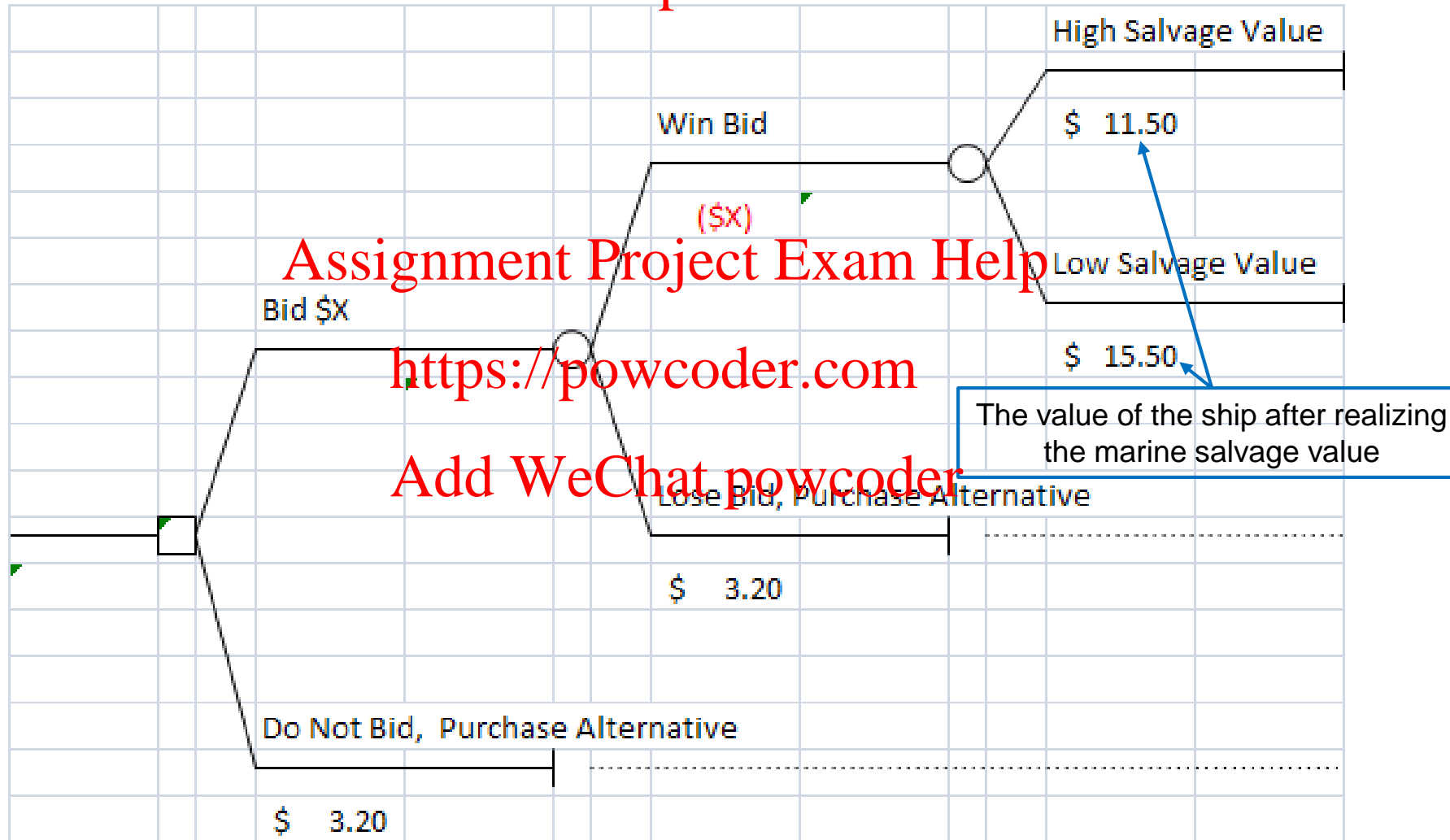
When all the cash flows are given we can assign terminal values to each branch by adding up cash flows along the branch

Label	Value
High Salvage Value	$11.5-$X
Win Bid — $ 11.50	
(\$X)	
Low Salvage Value	$15.5-$X
Bid $X — $ 15.50	
Lose Bid, Purchase Alternative	$ 3.20
$ 3.20	
Do Not Bid, Purchase Alternative	$ 3.20
$ 3.20	

Decision criteria: minimize worst case

Consider bidding $10M

High Salvage Value

$ 1.50

Win Bid

$ 11.50

($10) $ 1.50

Bid $10

Low Salvage Value

$ 5.50

$ 15.50

$ 1.50

2

$ 3.20

Lose Bid, Purchase Alternative

$ 3.20

$ 3.20

Do Not Bid, Purchase Alternative

$ 3.20

$ 3.20
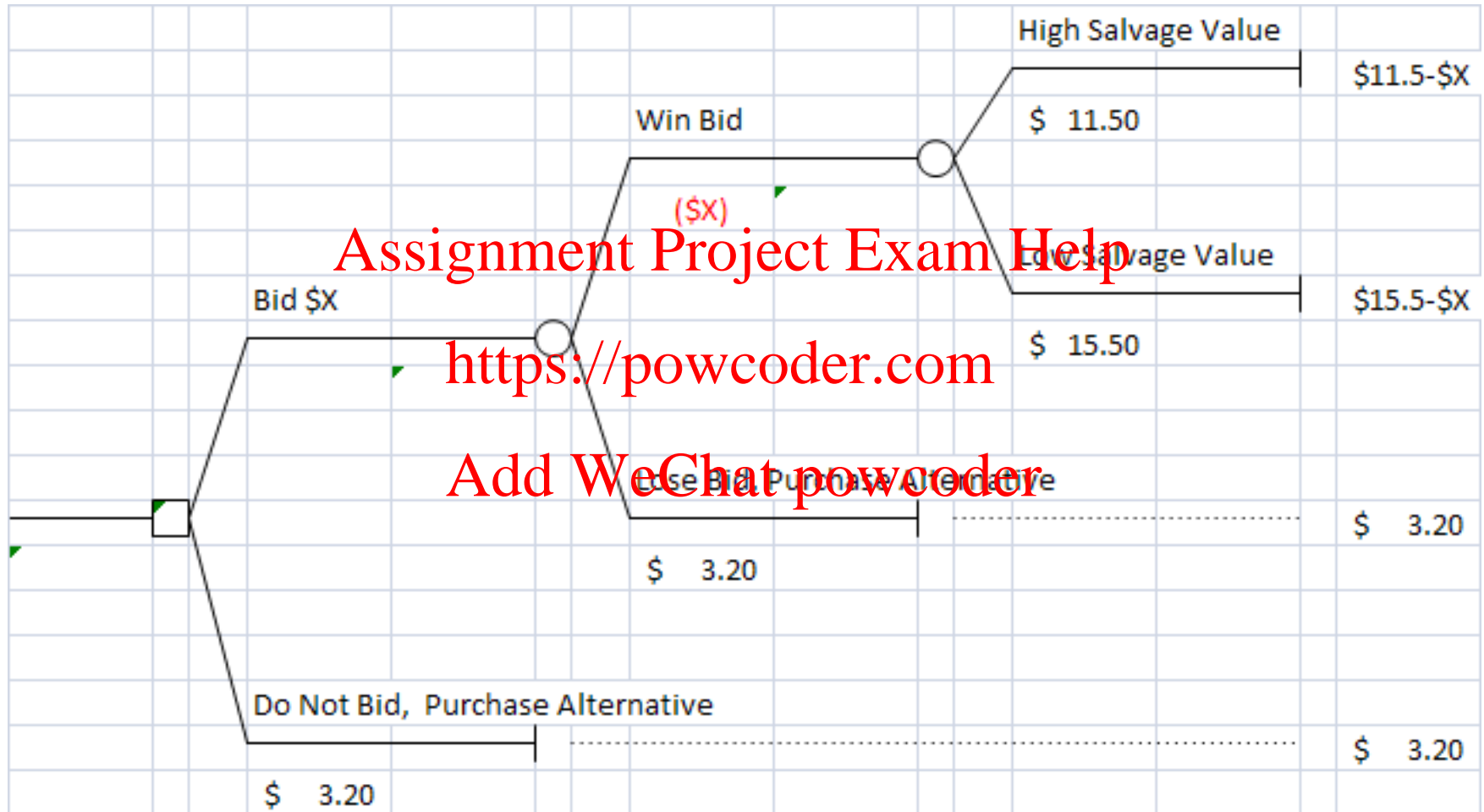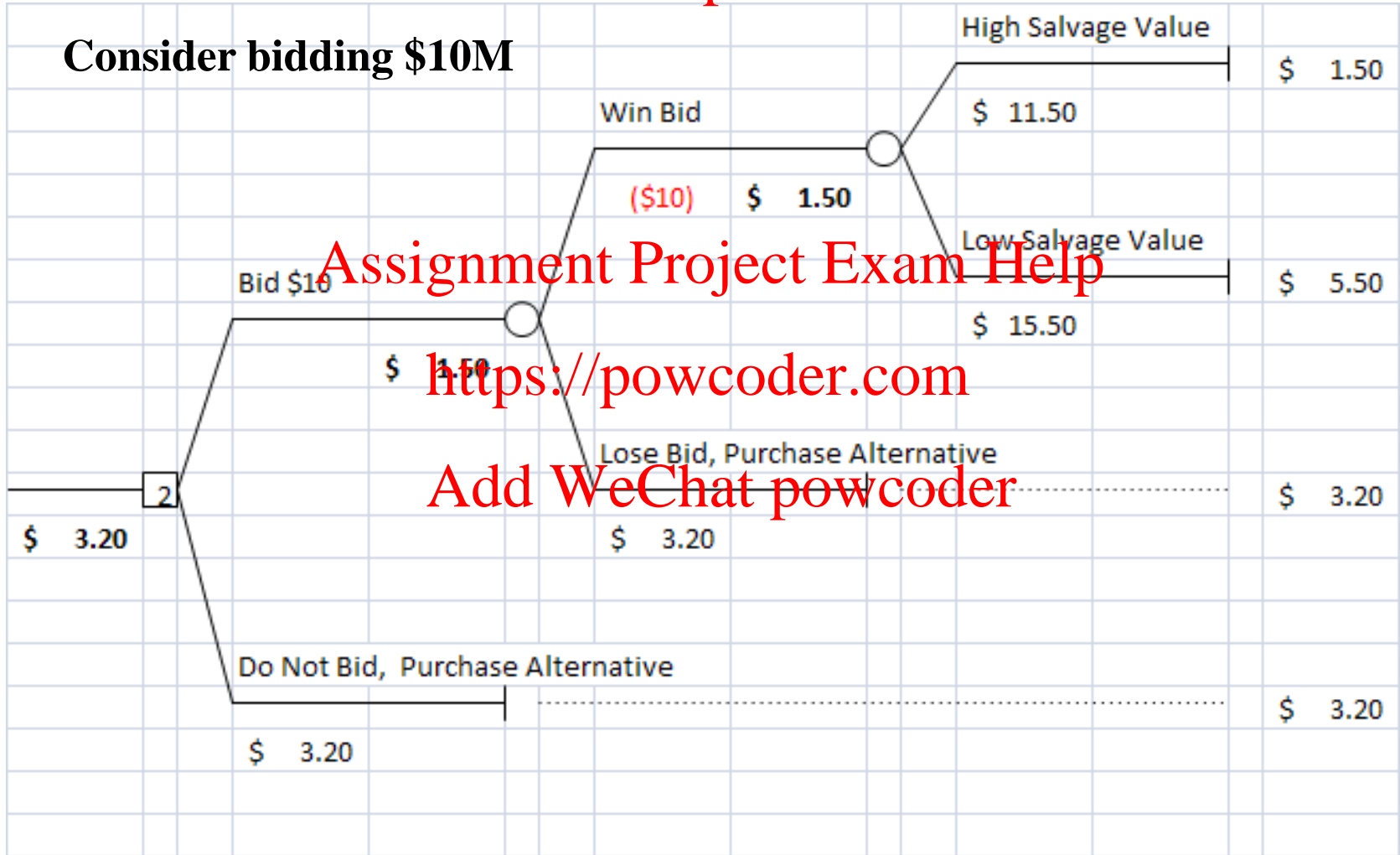
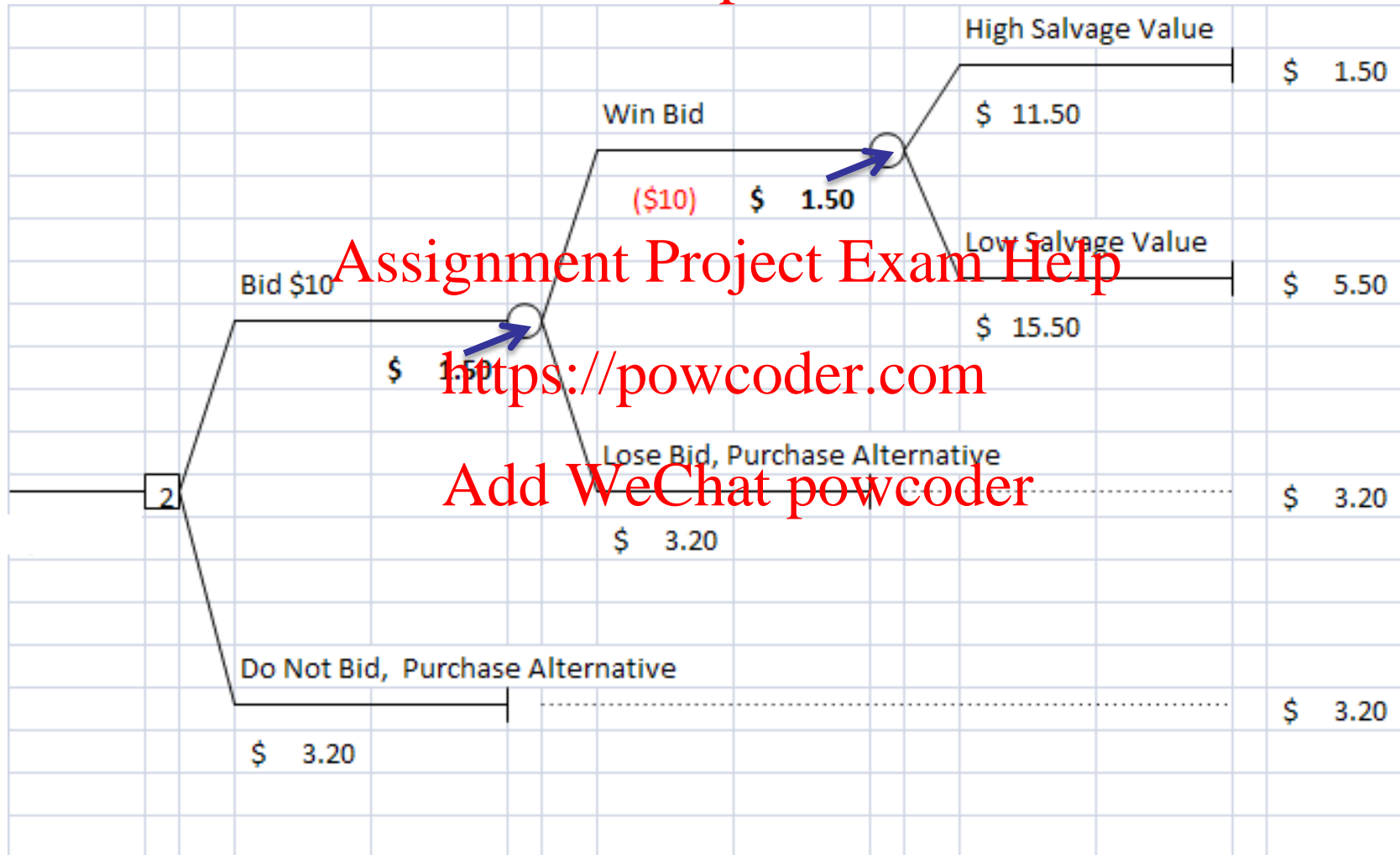# We assign values to the nodes working from the end branches toward the root of the tree
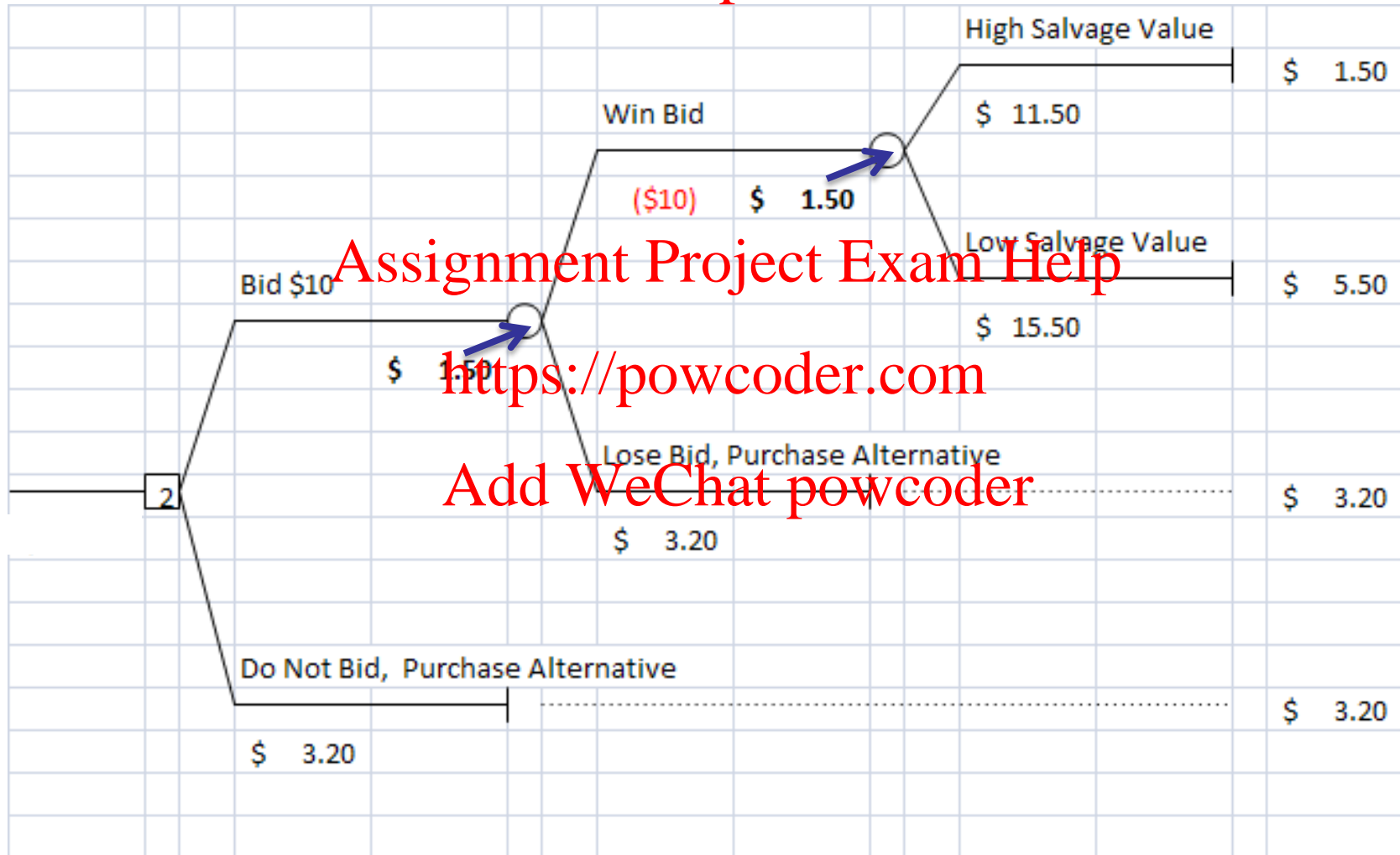
The value of an *event* node is the value corresponding to the worst-case scenario

The value of a *decision node* here comes from making the decision that maximizes the net profit

High Salvage Value $ 1.50

Win Bid $ 11.50

($10) $ 1.50

Low Salvage Value $ 5.50

Bid $10 $ 15.50

$ 1.50

2

$ 3.20

Lose Bid, Purchase Alternative $ 3.20

$ 3.20

Do Not Bid, Purchase Alternative $ 3.20

$ 3.20
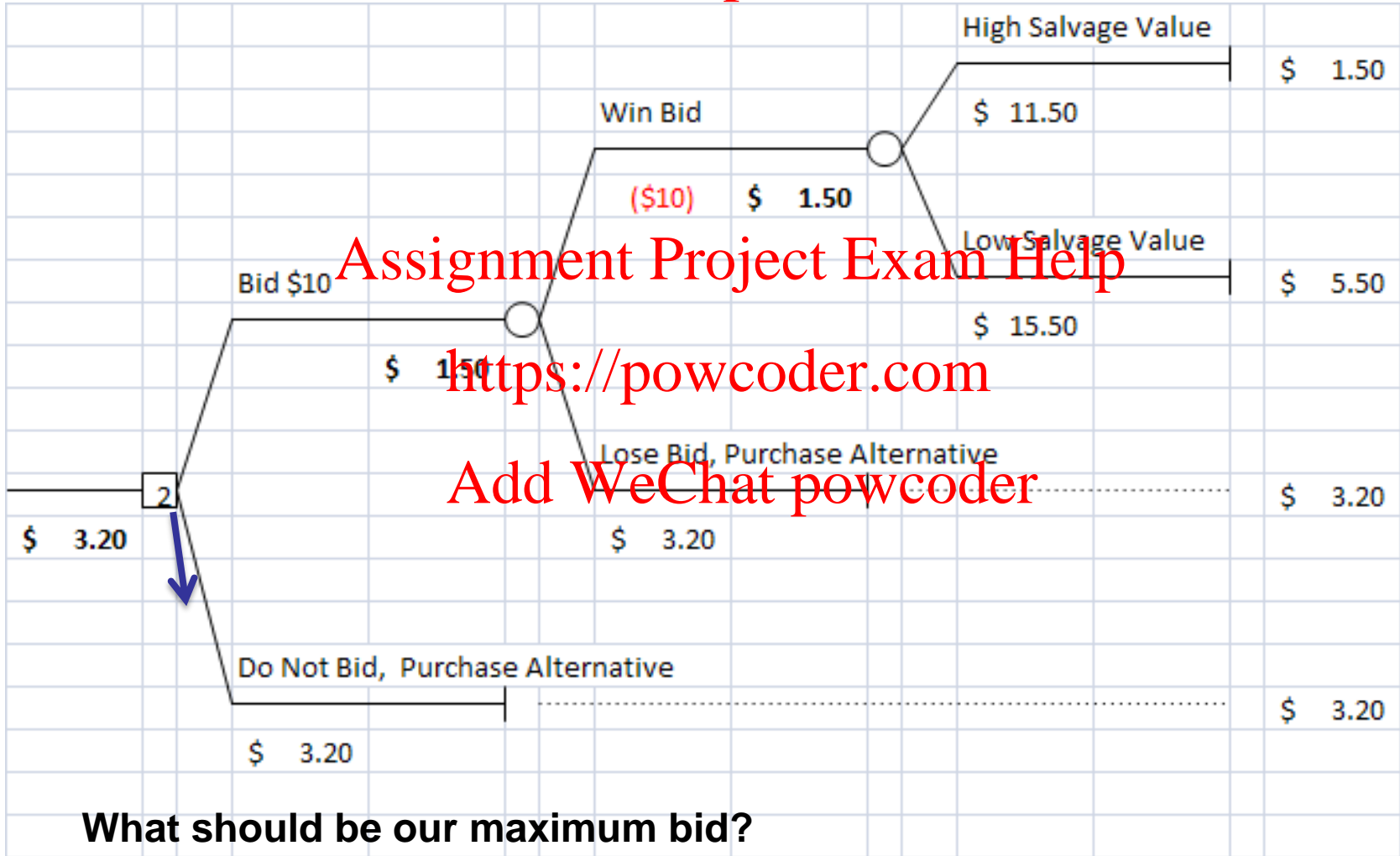
What should be our maximum bid?

If we just want to minimize the worst case it does not make sense to bid anything over ($11.50-$3.20)M = $8.3M

High Salvage Value

$ 1.50

Win Bid

$ 11.50

($10) $ 1.50

Bid $10

Low Salvage Value

$ 5.50

$ 15.50

$ 1.50

Lose Bid, Purchase Alternative
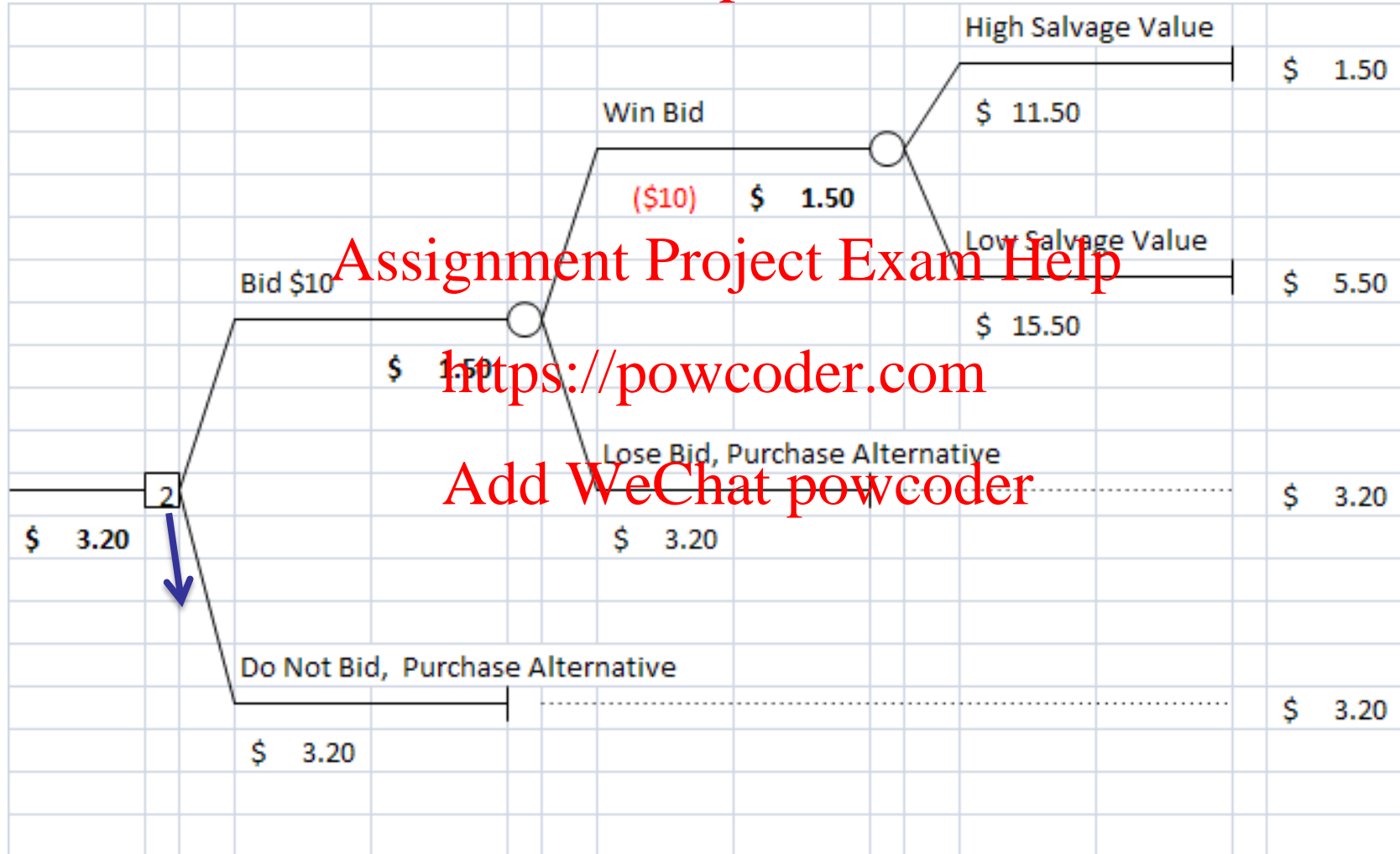
2

$ 3.20

$ 3.20

$ 3.20

Do Not Bid, Purchase Alternative

$ 3.20

$ 3.20

If two strategies result in the same "worst case" we might think about another criteria for selecting among the strategies

High Salvage Value — $ 3.50

Win Bid — $ 11.50

($8) — $ 3.50

Bid $8

$ 3.20

Low Salvage Value — $ 7.50

$ 15.50

Lose Bid, Purchase Alternative — $ 3.20

$ 3.20

$ 3.20

Do Not Bid, Purchase Alternative — $ 3.20

$ 3.20
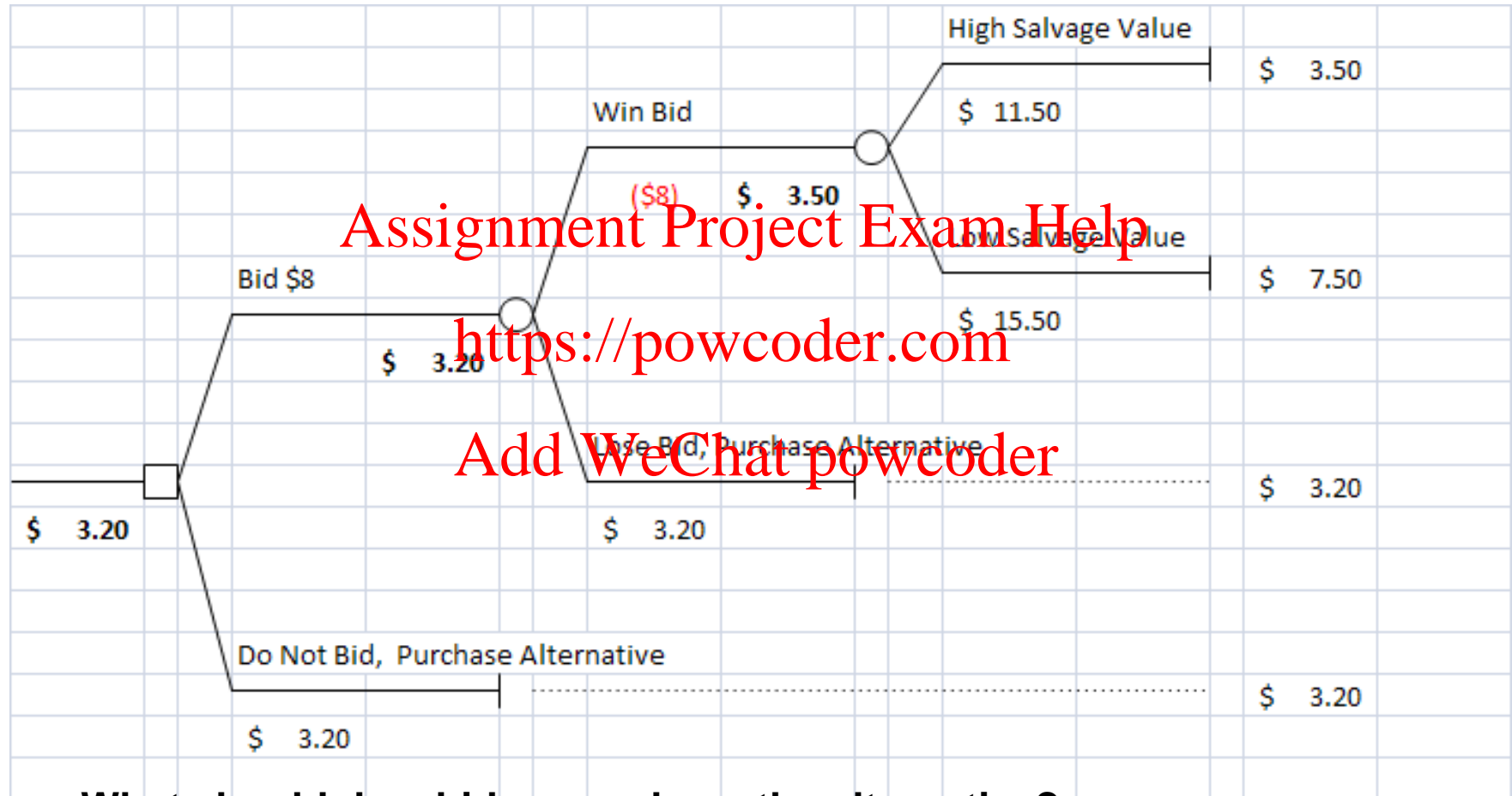
What should do – bid or purchase the alternative?

Another possible criteria: Maximizing the expected cash flow

- Expected Utility = EU = Weighted utility

- For example: you can buy a lottery ticket for 10 dollars. If you win you get 100 dollars. If you lose you get 0 dollars. The probability of winning is 8%. Would you buy the ticket?

- We answer by calculating the expected utility:

$$EU = \Pr(\text{winning}) \cdot 100 + \Pr(\text{losing}) \cdot 0 - 10$$

$$= 8\% \cdot 100 + 92\% \cdot 0 - 8 = -2 < 0$$

- Buy the ticket only if: $EU \geq 0$

Another possible criteria:
Maximizing the expected cash flow

| Bid | 8 |
| Prob Win | 60% |

70%
High Salvage Value
3.5

60%
Win Bid
11.5

-8

30%
Low Salvage Value
7.5
15.5

Bid 8

0

40%
Lose Purchase Alternative
3.2

3.2

Purchase Alternative
3.2
3.2
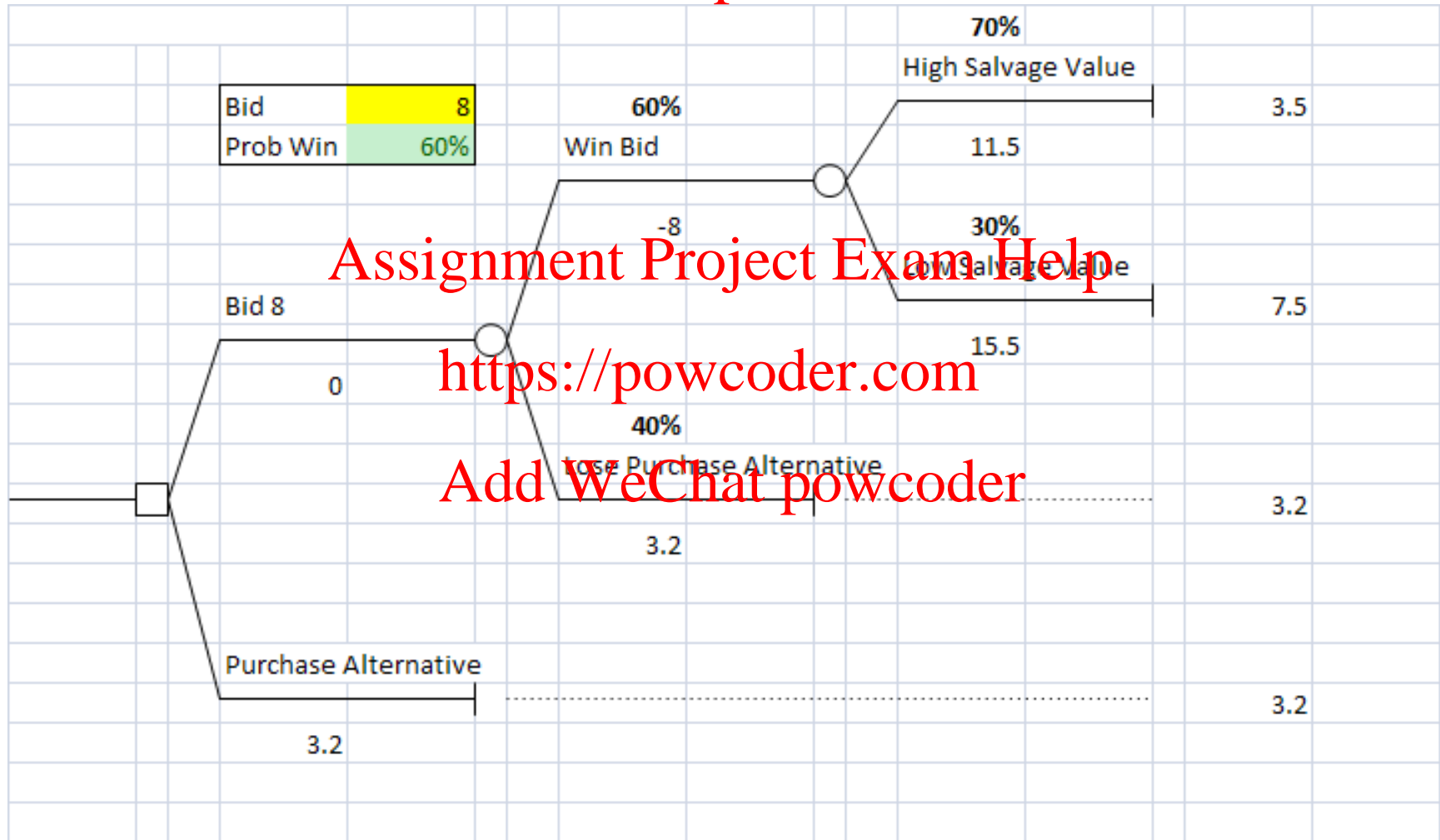
Event node values are calculated based on expected values

| Bid | 8 |
| Prob Win | 60% |

Bid 8

Win Bid — 60%

-8 · 4.7

70%
High Salvage Value — 11.5 → 3.5

30%
Low Salvage Value — 15.5 → 7.5

0 · 4.1

40%
Lose Purchase Alternative — 3.2 → 3.2

1 · 4.1

Purchase Alternative — 3.2 → 3.2
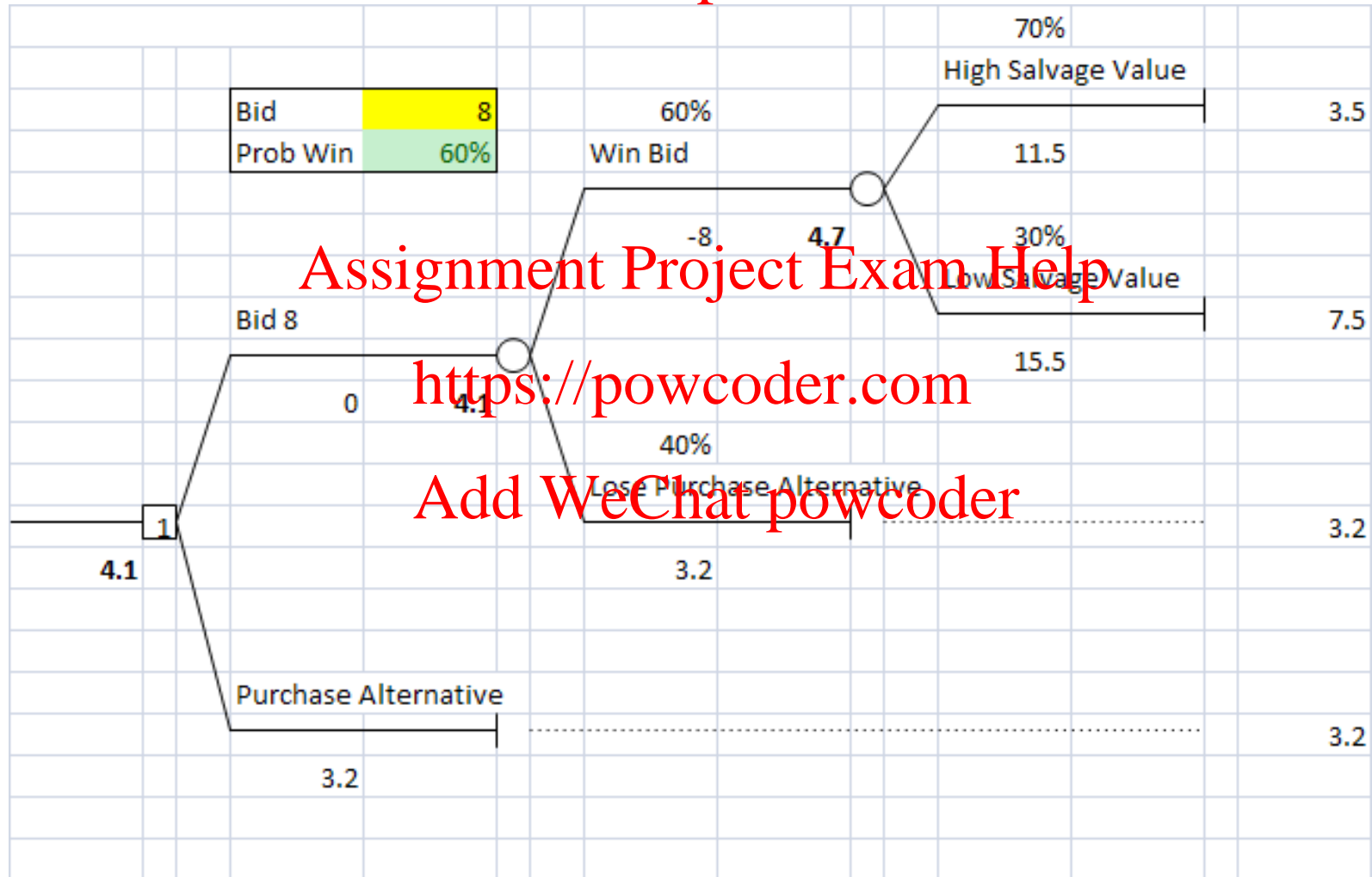
Decision node values are calculated by selecting the decision that maximizes the value of the node

| Bid | 8 |
| Prob Win | 60% |

70%
High Salvage Value

60%
Win Bid

3.5

11.5

Bid 8

-8   **4.7**

30%
Low Salvage Value

7.5

0   **4.1**

15.5

1

40%
Lose Purchase Alternative

4.1

3.2

3.2

Purchase Alternative

3.2

3.2

Bid	8
Prob Win	60%

70%
High Salvage Value
3.5

60%
Win Bid
11.5

-8   **4.7**

30%
Low Salvage Value
7.5

15.5

$$70\% \cdot 3.5 + 30\% \cdot 7.5 = 4.7$$

Bid 8

0   4.1

40%
Lose Purchase Alternative

1

4.1

3.2   3.2

$$60\% \cdot 4.7 + 40\% \cdot 3.2 = 4.1$$

Purchase Alternative

3.2

3.2

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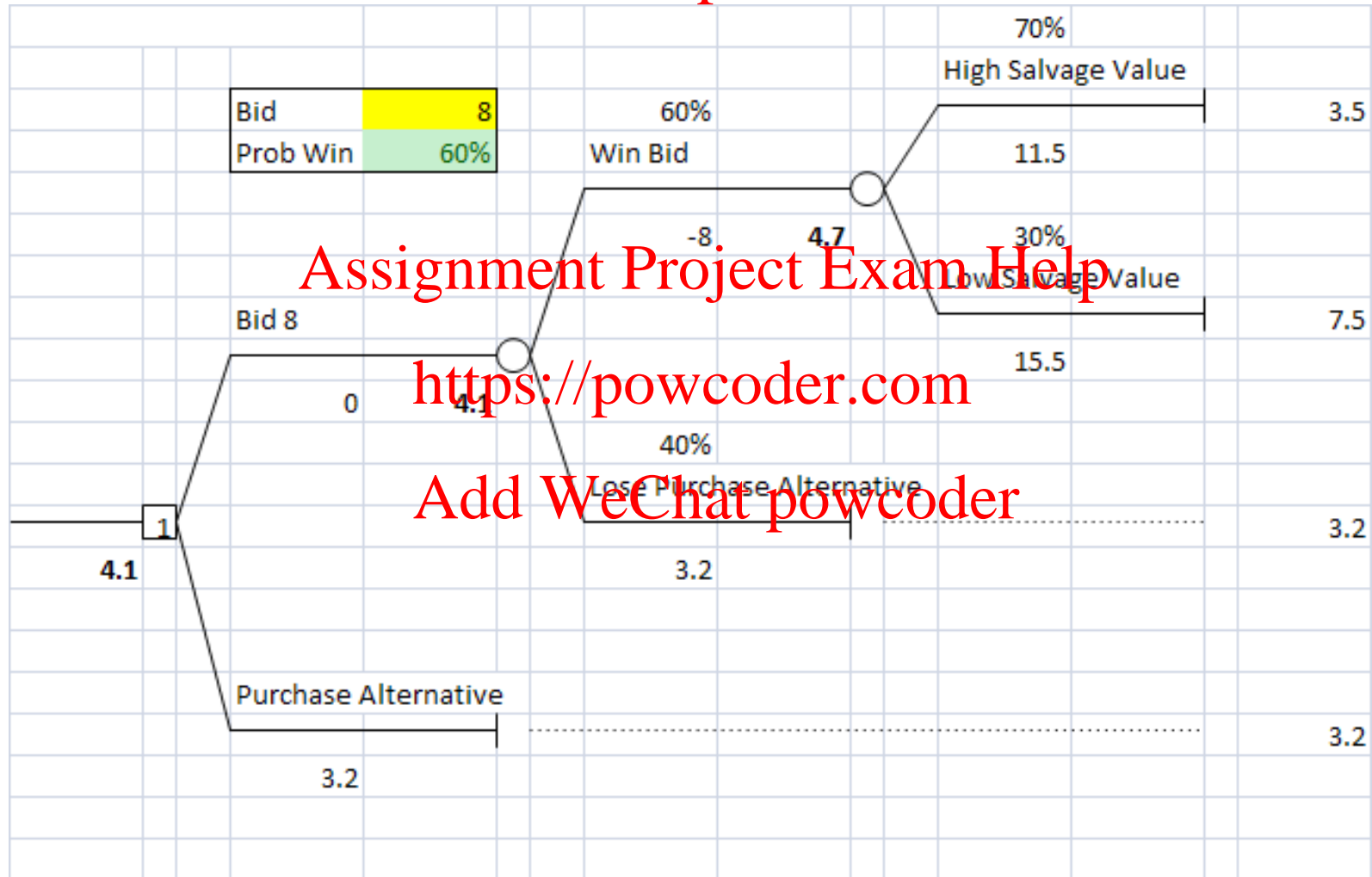
We could also assume an analytical relationship between bid size and the probability of win, and look for the optimal bid size

The problem statement reads:

The higher the bid, the more likely the company will win. They expect that a bid of

    $2M would definitely not win

    $12M would definitely win

    $8M has 60% chance of winning

We can write a formula for calculating the probability to win as a function of the bid amount:

$$Prob\_Of\_Win = (Bid - 2)/10$$

Sensitivity analysis
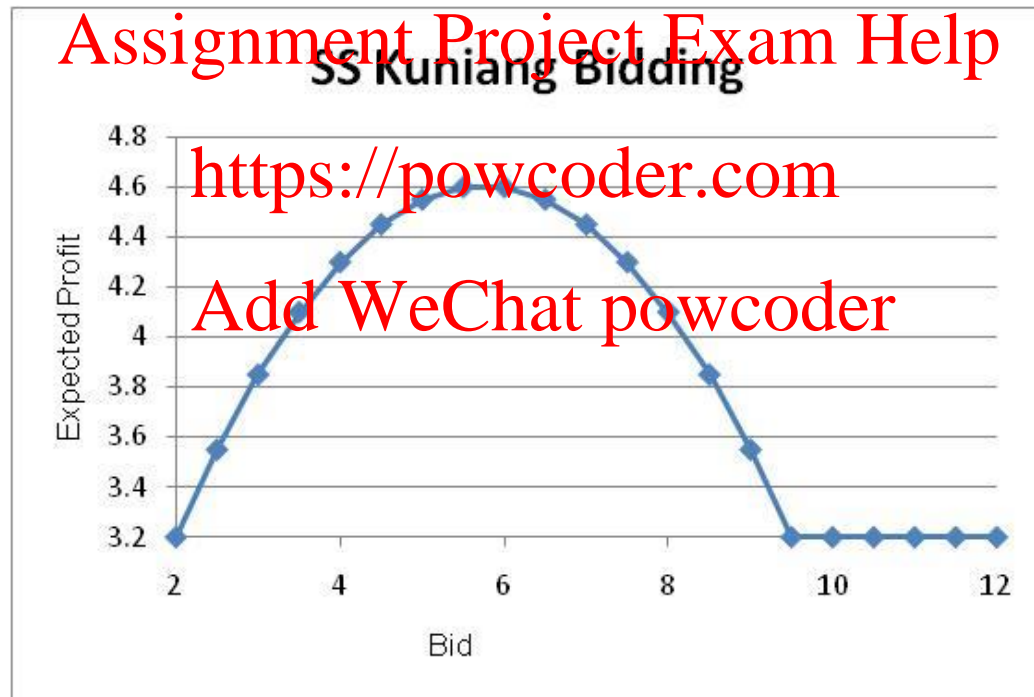
We use sensitivity analysis to calculate and compare the profit under different bids.

We set the bid price to be our sensitivity parameter (and not as optimization parameter, because we do not use optimization model to find the bid price).

SS Kuniang Bidding

Let's take a break to talk about Probability

Probability = chance.
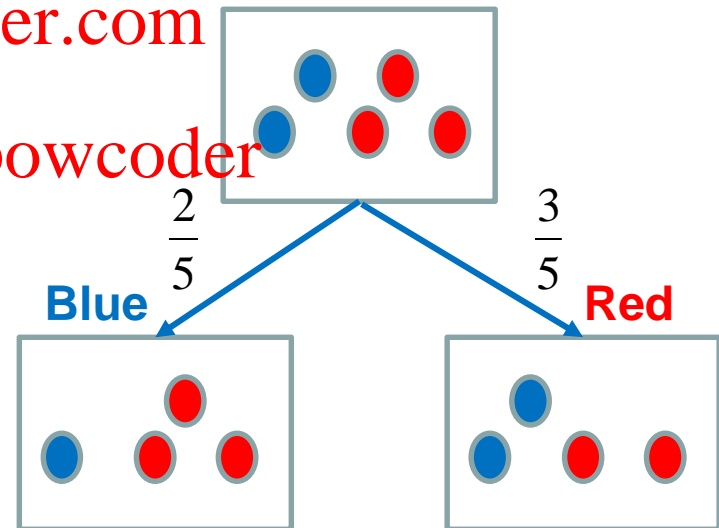We use probability to measure uncertainty.
Probability measurements: 0 to 1 or 0% to 100%.

For example: From a box that contains 2 blue and
3 red balls, we randomly select one ball (with our
eyes closed).

What are the chances of getting a blue ball?

$$\frac{2}{5}$$

$$\frac{3}{5}$$

Blue

Red

What are the chances of drawing two blues?

Assumption: no replacements.

$$\Pr(Blue) = \frac{2}{5}$$

$$\frac{3}{5} = \Pr(\mathrm{Re}\,d)$$

Blue

Red

$$\Pr(Blue\big|Blue) = \frac{1}{4}$$

$$\frac{3}{4}$$

Blue **Red** **Blue** **Red**

$$\frac{2}{4}$$

$$\frac{2}{4}$$
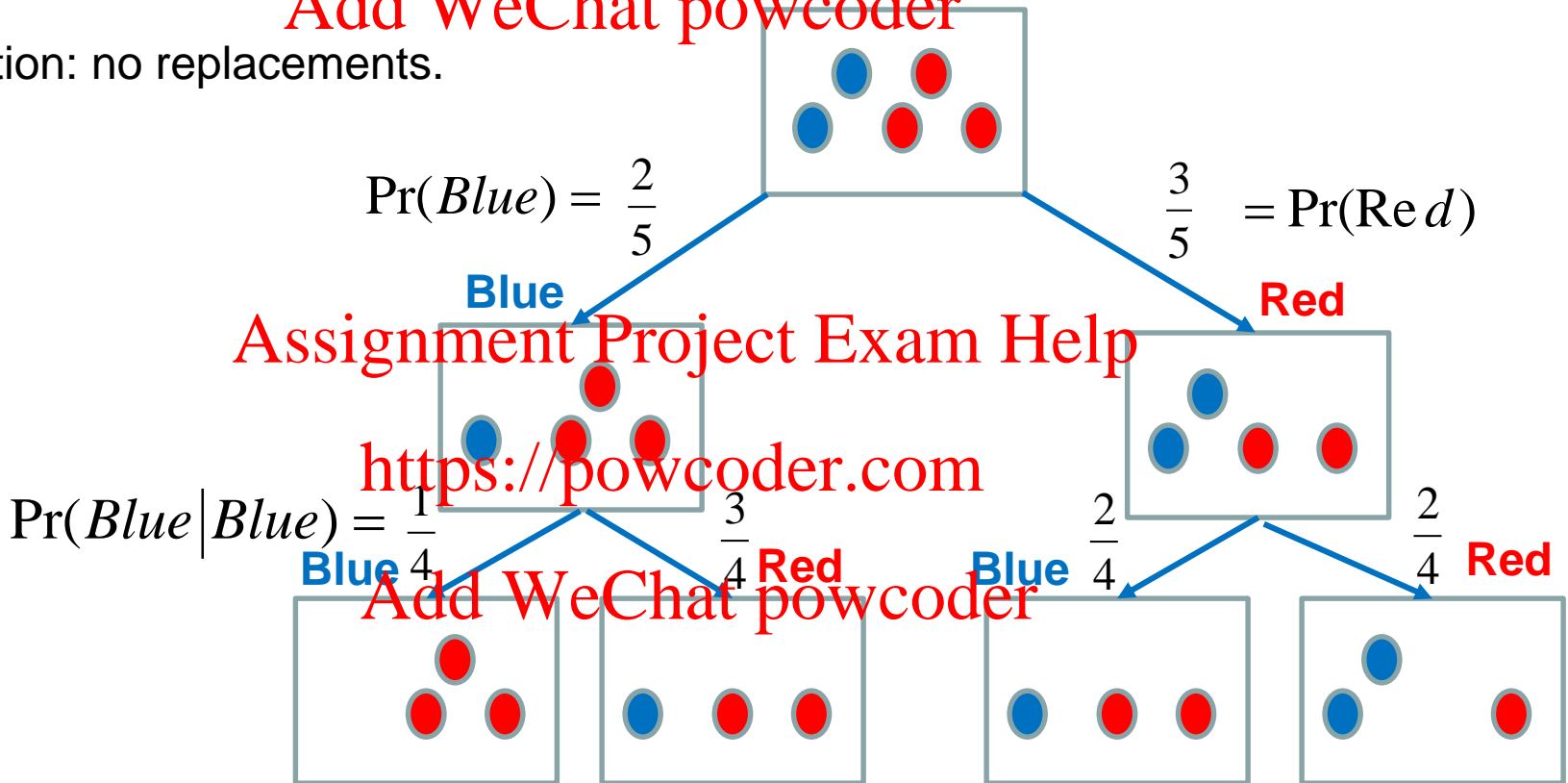
$$\Pr(Blue, Blue) = \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10}$$

$$\Pr(Blue, Blue) = \Pr(Blue) \cdot \Pr(Blue\big|Blue)$$

What are the chances of drawing red and then blue?

Assumption: no replacements.
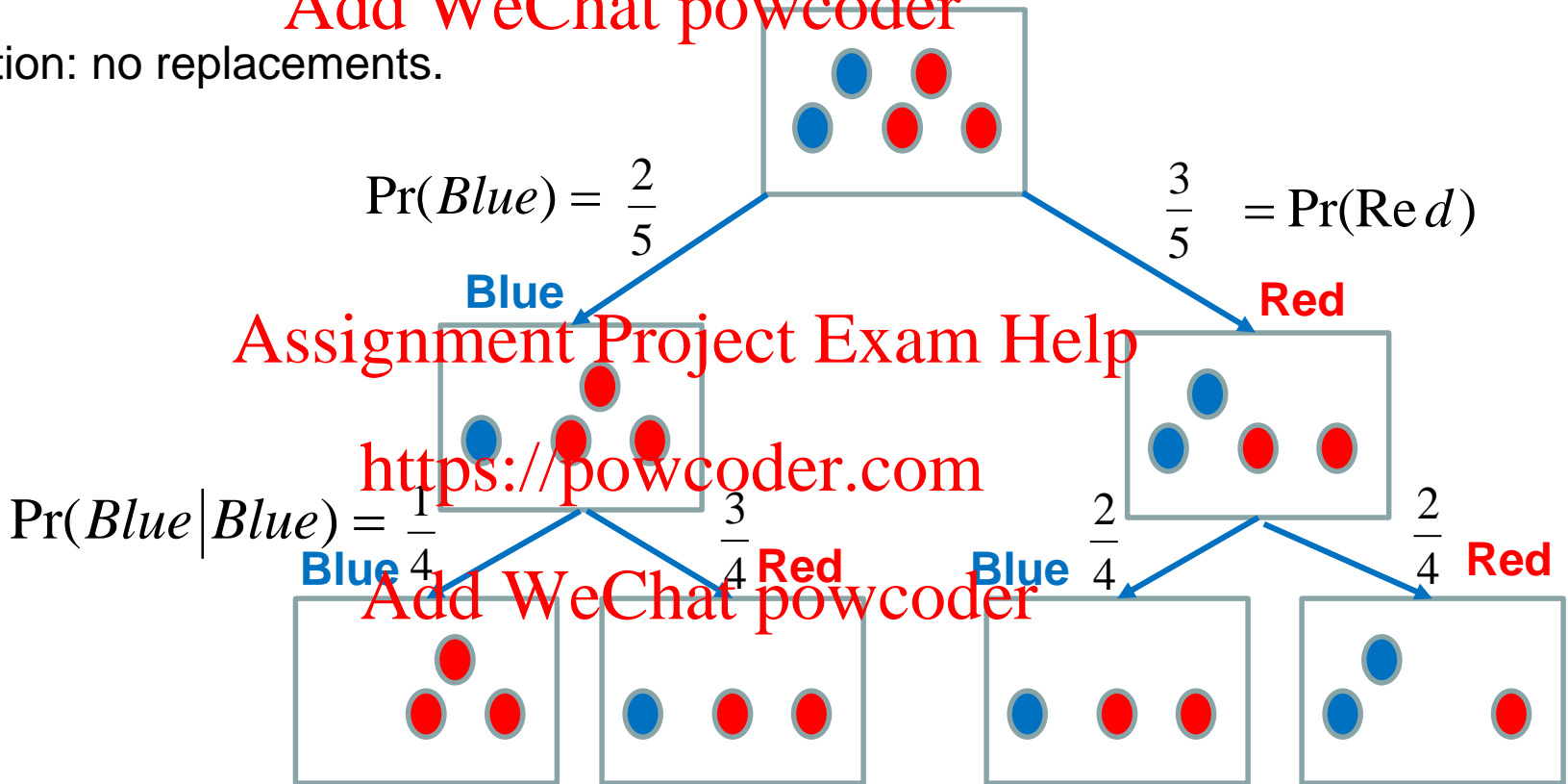
$$\Pr(Blue) = \frac{2}{5}$$

$$\frac{3}{5} = \Pr(\mathrm{Re}\,d)$$

Blue

Red

$$\Pr(Blue\,|\,Blue) = \frac{1}{4}$$

Blue $\frac{3}{4}$ **Red**

Blue $\frac{2}{4}$

$\frac{2}{4}$ **Red**

$$\Pr(red, Blue) = \frac{3}{5} \cdot \frac{2}{4} = \frac{3}{10}$$

$$\Pr(Red, Blue) = \Pr(Red) \cdot \Pr(Blue\,|\,Red)$$

Conditional probability

$\Pr(A|B) = \Pr(E\text{vent A would happen if we know that event B happened})$

$$\Pr(A \text{ and } B) = \Pr(B) \cdot \Pr(A|B) = \Pr(A) \cdot \Pr(B|A)$$

Bayes' rule:

$$\Pr(A|B) = \frac{\Pr(A \text{ and } B)}{\Pr(B)}$$

$$\Pr(B|A) = \frac{\Pr(A \text{ and } B)}{\Pr(A)}$$

Interesting video regarding Bayes' rule:
https://www.youtube.com/watch?v=R13BD8qKeTg

Total Probability Theorem

Recall that the probability that the first draw would be of a blue ball is $\dfrac{2}{5}$.

What are the chances that the second draw would be of a blue ball?

Answer: Total probability theorem:

$$Pr(\text{2nd is blue}) = Pr(\text{2nd is blue}|\text{1st is blue})\cdot Pr(\text{1st is blue}) + Pr(\text{2nd is blue}|\text{1st is red})\cdot Pr(\text{1st is red})$$

Meaning: $Pr(\text{2nd is blue}) = \dfrac{2}{5}\cdot\dfrac{1}{4} + \dfrac{3}{5}\cdot\dfrac{2}{4} = \dfrac{2}{5}$

A Mobile Oil Company has recently acquired rights to a new potential source of natural oil in Alaska.

- The current market value of these rights is $90,000. The company could sell these rights now.

- However, if there is natural oil at the site, it is estimated to be worth $800,000; although the company would have to pay $100,000 in drilling costs to extract the oil.

- The company believes there is a 25% probability that the proposed drilling site actually would hit the natural oil reserve.

- Alternatively, the company can pay $30,000 to first carry out a seismic survey at the proposed drilling site.

- The survey is not totally accurate: there is a 20% chance that the survey is favorable when oil is not present (false positive, type I error); and a 40% chance that the survey result is unfavorable when there is oil at the site (false negative, type II error).

Model formulation

- What is the company objective?
  - Maximize profit

- What decisions does the Mobile Oil Company face?
  - Drill
  - Sell
  - Survey

- How to calculate the expected value of the survey option?

- **Go to the excel file "Mobile oil company"**

What is the probability of a favorable survey?

```
                          25%              75%

                    Oil                          No oil

            60%           40%        20%              80%

    Favorable      Unfavorable    Favorable      Unfavorable
```
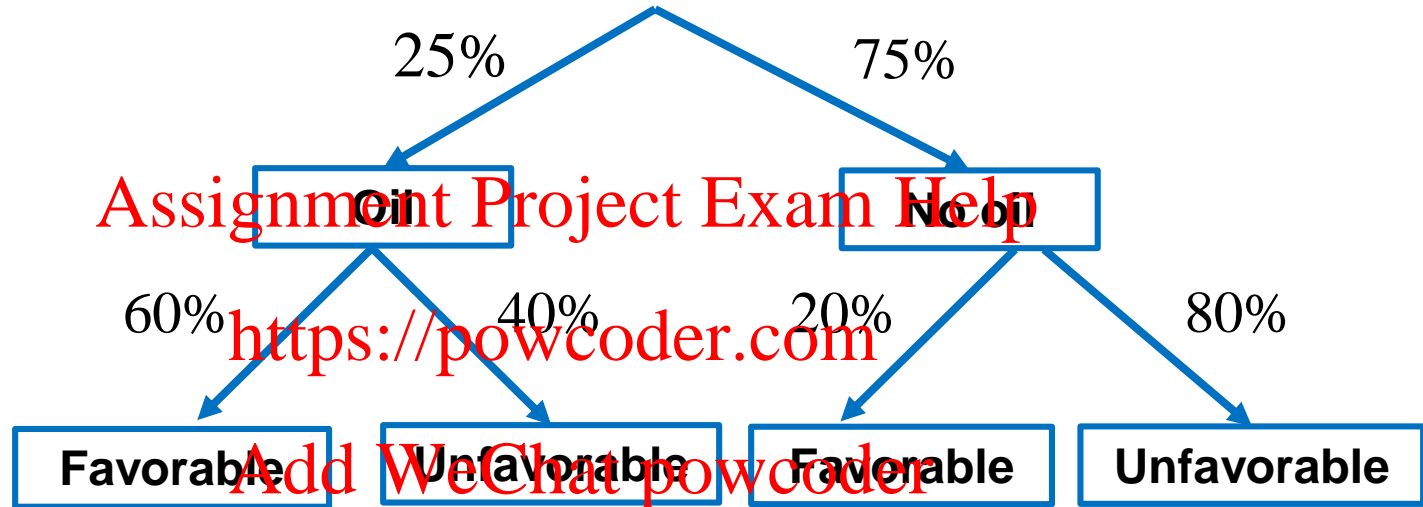
What is the probability of a favorable survey?

$Pr(Favorable) = 0.25 \cdot 0.60 + 0.75 \cdot 0.20 = 0.30 = 30\%$

$Pr(unfavorable) = 1 - Pr(Favorable) = 0.70 = 70\%$

What is the probability of finding oil given a favorable survey?

```
              25%                    75%

           Oil                    No oil

     60%        40%         20%          80%

 Favorable   Unfavorable  Favorable   Unfavorable
```
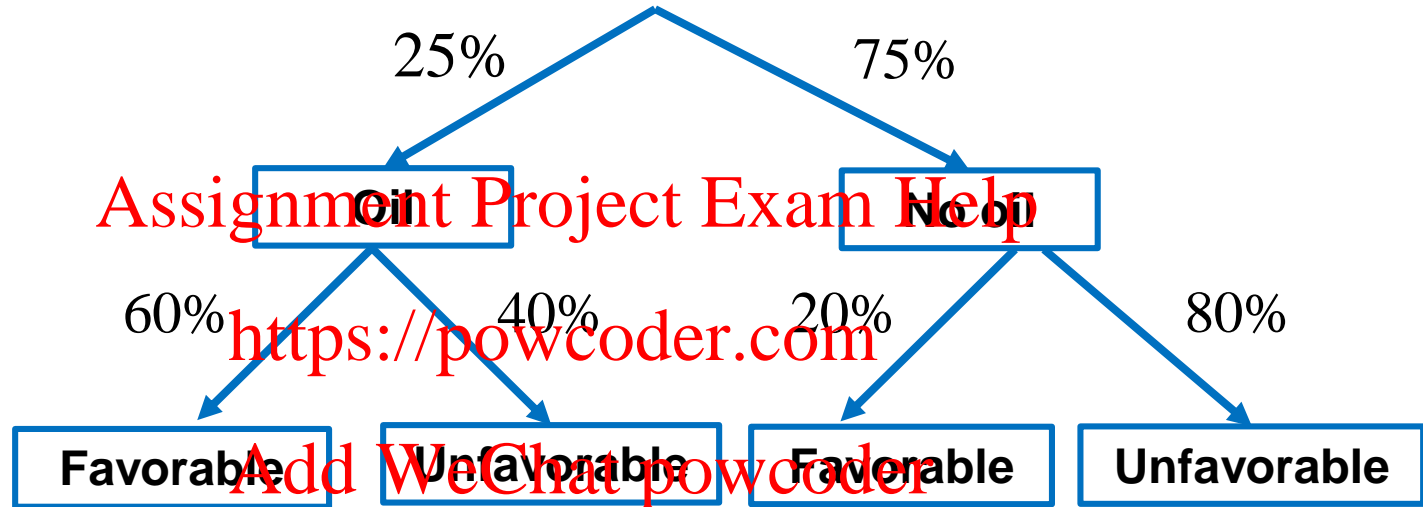
$$\Pr(\text{Oil}|\text{Favorable}) = \frac{\Pr(\text{Oil and Favorable})}{\Pr(\text{Favorable})} = \frac{0.25 \cdot 0.6}{0.3} = 0.5 \quad \Rightarrow \quad \Pr(\text{No Oil}|\text{Favorable}) = 1 - 0.5 = 0.5$$

$$\Pr(\text{Oil}|\text{Unfavorable}) = \frac{\Pr(\text{Oil and Unavorable})}{\Pr(\text{Unfavorable})} = \frac{0.25 \cdot 0.4}{0.7} = 0.143 \quad \Rightarrow \quad \Pr(\text{No Oil}|\text{Unfavorable}) = 1 - 0.143 = 0.857$$

What probabilities do we need for the decision tree?
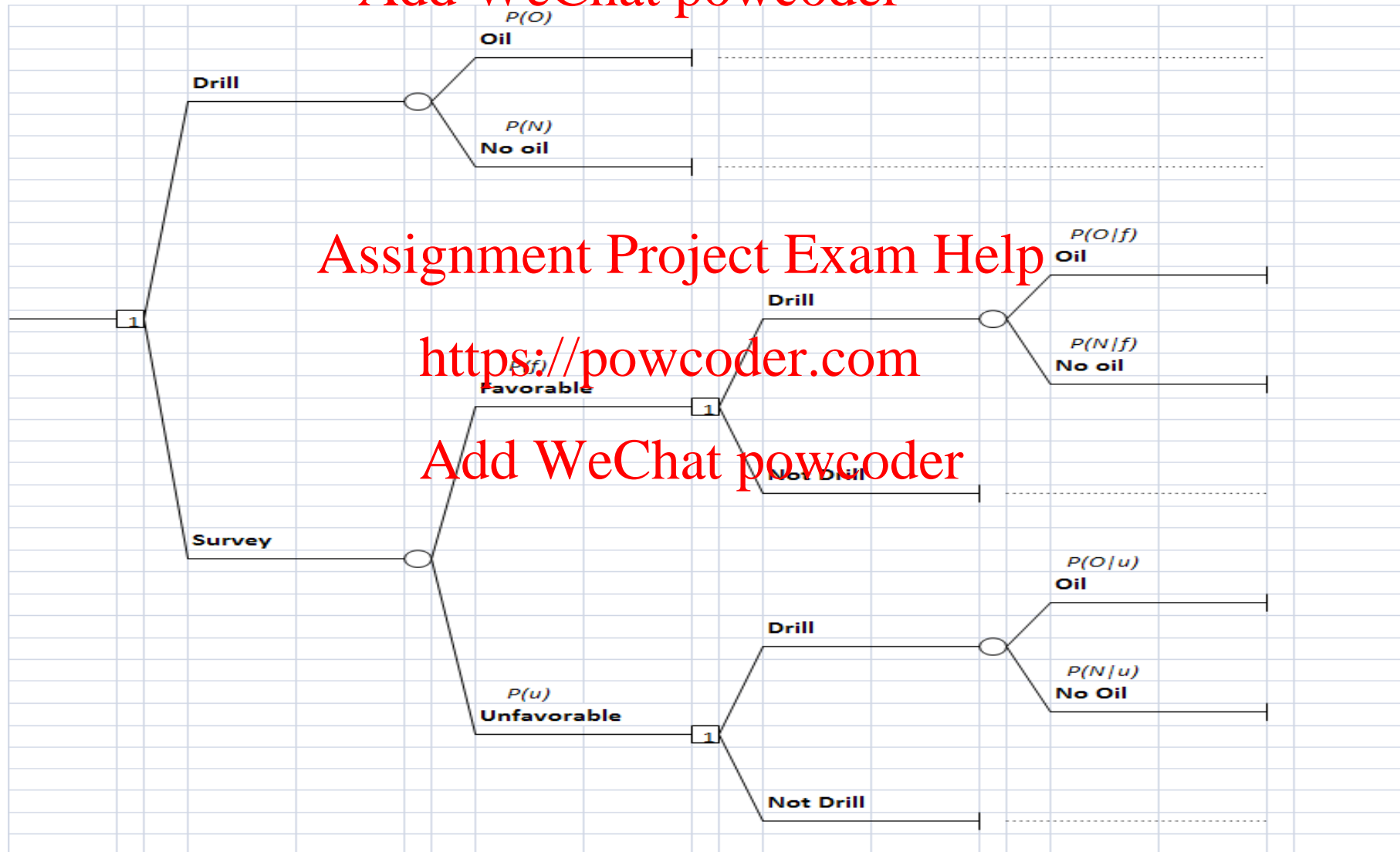
What is the probability of finding oil if the survey is favorable?

Survey
-30

Favorable
30%

Drill
-100

Oil 50%
800

No Oil 50%
0

Don't drill
0

Unfavorable
70%

Drill
-100

Oil 14.3%
800

No oil 85.7%
0

Don't drill
0
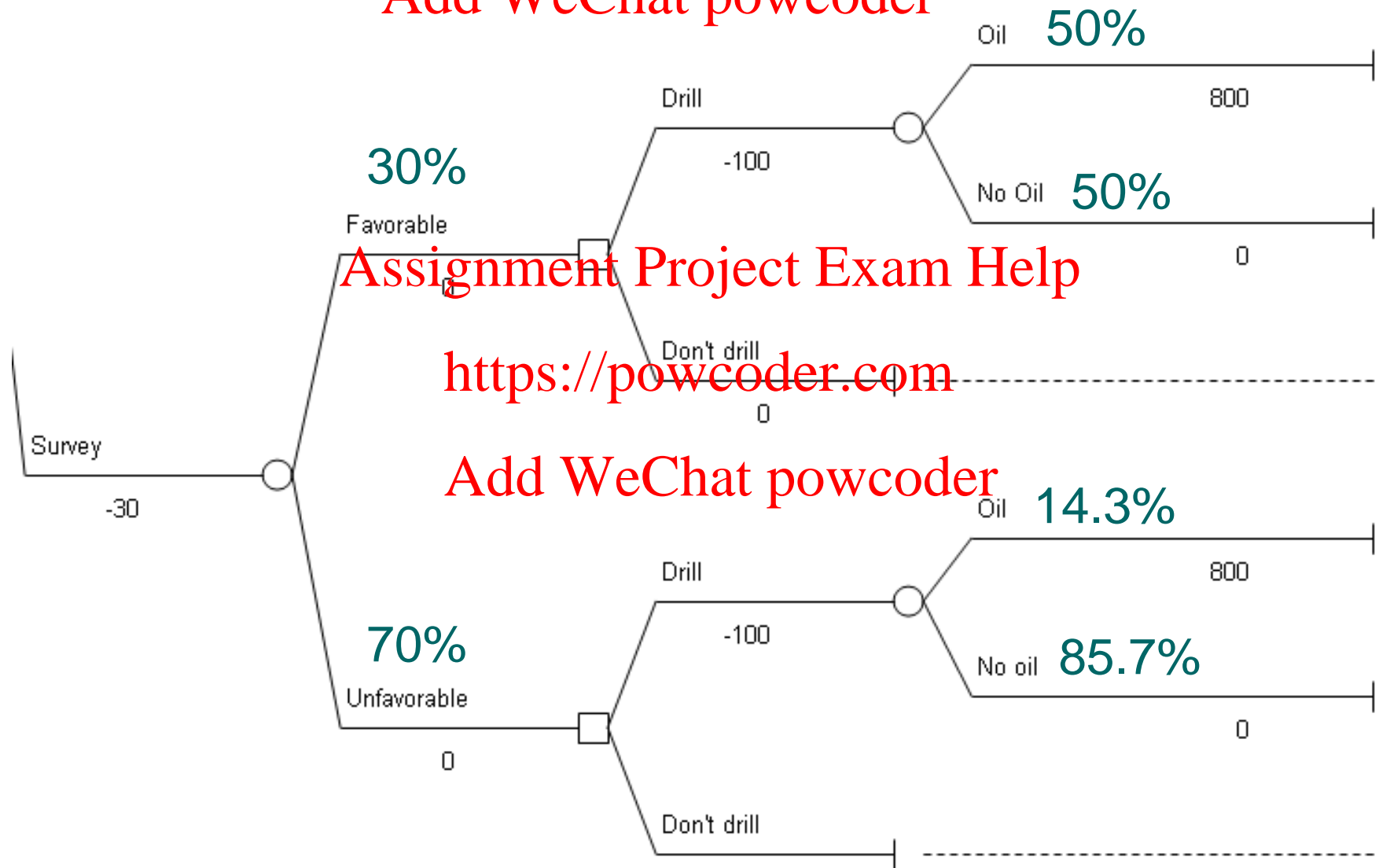
Would choose to drill, no matter what the survey results

ePlan Academic

Sell Rights
90   90   90

                    25%
                   Oil
Drill   700   25%
         -100   100   800   700
                    75%
                   No Oil
                   0   -100   -100   75%

2
100
                                        50.0%
                                       Oil
                                                            670   15.0%
                              Drill   800   670
                              -100   270
                                       50%
                                       No Oil
                   30.0%   0   -130   -130   15.0%
                   Favorable
                   0   270
                              Don't drill
                                                            -30
                              0   -30

Survey
-30   70
                                        14.29%
                                       Oil
                              Drill   670   10.0%
                                       800   670
                              -100   -15.7143
                                       85.71%
                   70.0%   No oil
                   Unfavorable
                                        0   -130   -130   60.0%
                   0   -15.7143
                   1
                              Don't drill
                                                            -30
                              0   -30
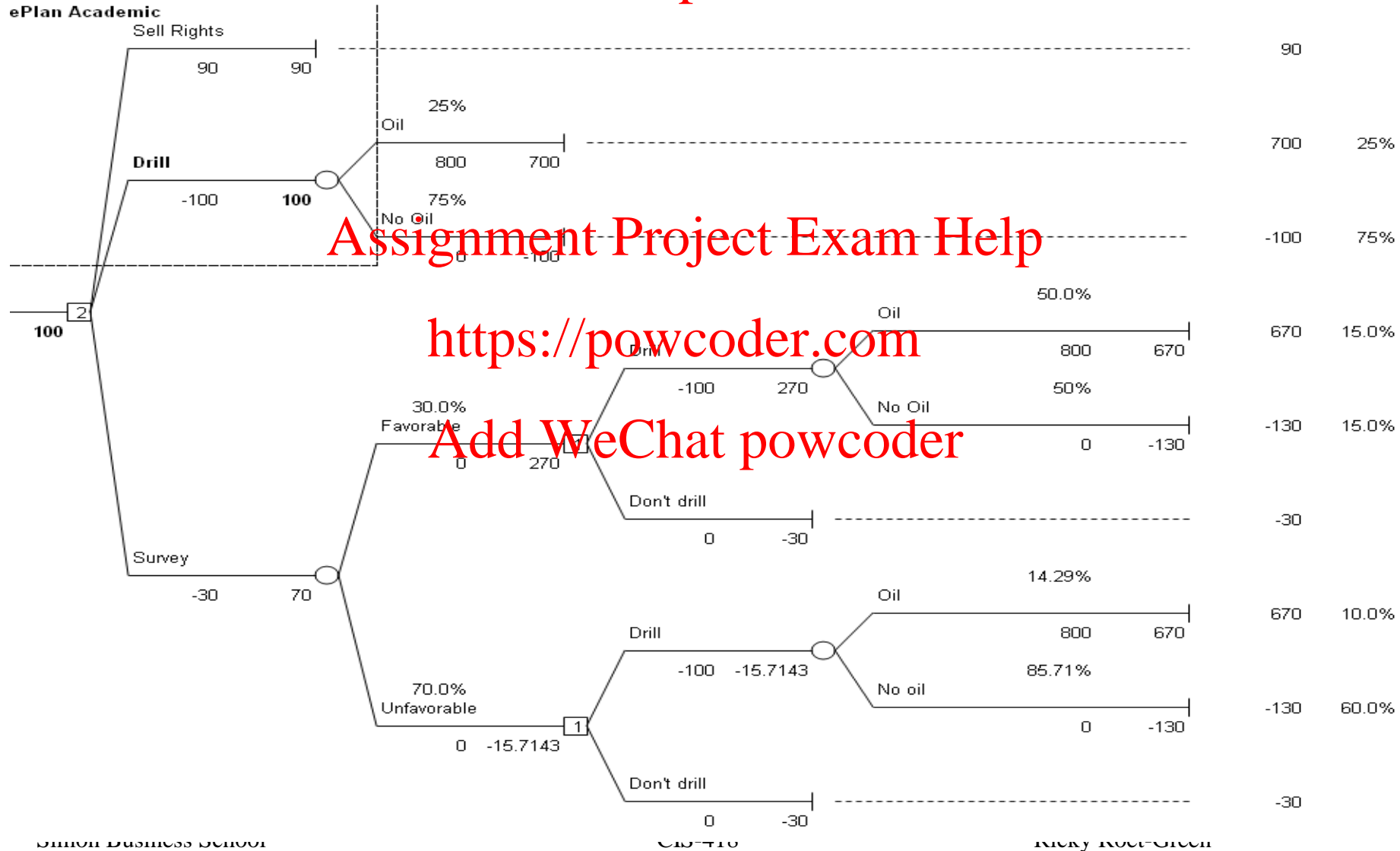
Questions

1. Would the decision be different if the survey would cost less?

2. If the accuracy of the survey was different, would that affect the decision?