Monte Carlo Simulation

CIS 418

Monte Carlo Simulation

- ***Simulation*** is any analytical method meant to imitate a real-life system, especially when other analyses are too mathematically complex or too difficult to reproduce.

- ***Monte Carlo simulation:*** pseudo-random numbers are generated according to probabilities associated with a source of uncertainty, e.g. stock returns, exchange rates, etc. Outcomes associated with these random drawings are then analyzed to determine the likely results and the associated risk.

Example: New product profitability

Spinners company is planning to sell a new spinner for $6. The company carried out a market survey that showed the following:

- The expected number of spinners that would be sold is 900,000 units.
- The expected fixed cost is $700,000.
- The expected unit variable cost is $3.
- The expected selling expense is $940,000.

Our goal: to create a predictive model to calculate the product profitability.

Calculate given what we know

Spinners profitability	
Data	
	Expected
Revenue per spinner	$ 6.00
# of units	900,000
Fixed cost	$ 700,000.00
Variable cost	$ 3.00
Selling expenses	$ 940,000.00
Expected profit	$ 1,060,000.00

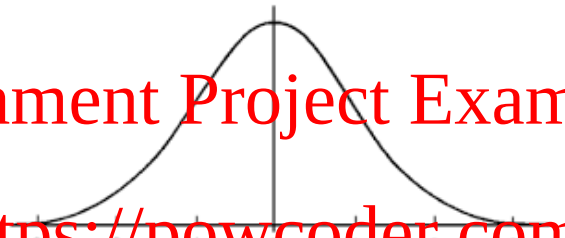
(Revenue–Variable cost)X(# of units) – Fixed cost – Selling expenses

Is that the expected profit?

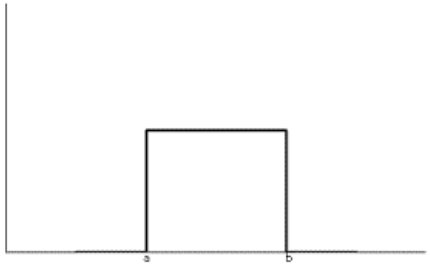
Model the profitability using probability distributions

Suppose we knew that the number of sold units, the variable costs and the selling expenses were random, and could be modeled with some probability distributions

Number of sold units is normally distributed with mean of 900,000 and standard deviation of 300,000.

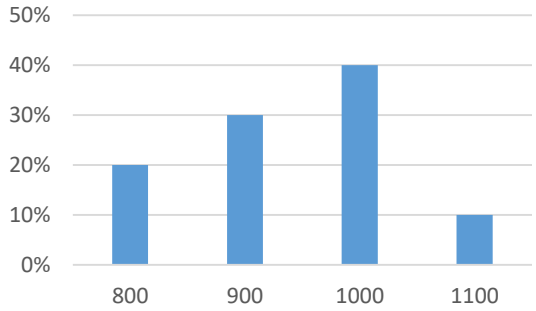


Variable costs are uniformaly distributed between $2.5 and $3.5.

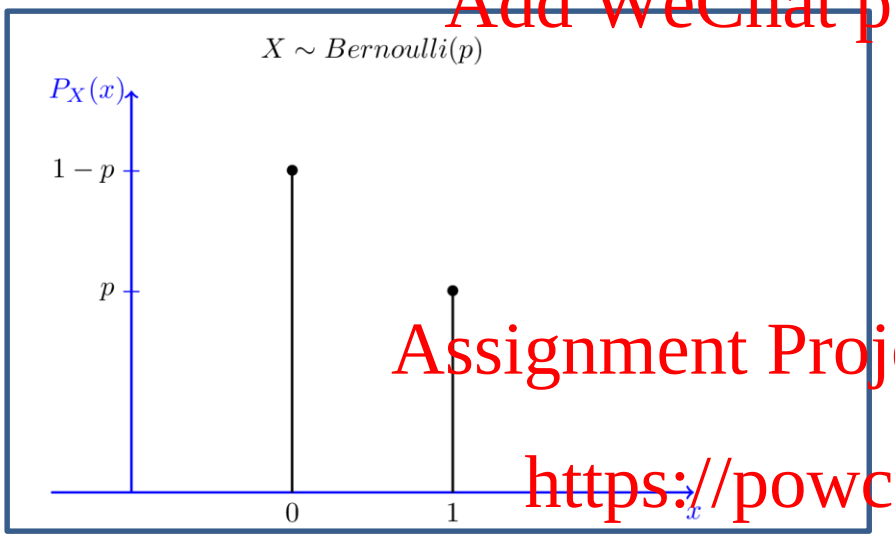


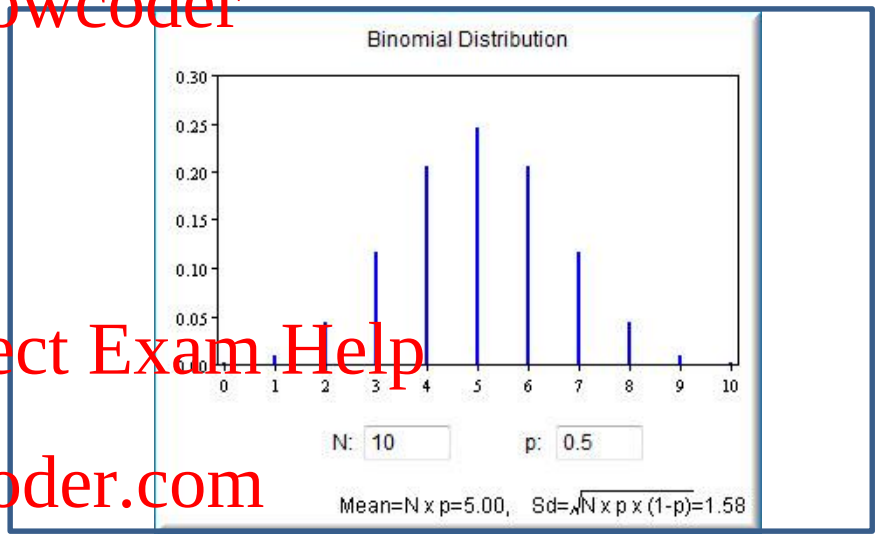
Selling expenses are discrete costume (general) distributed:

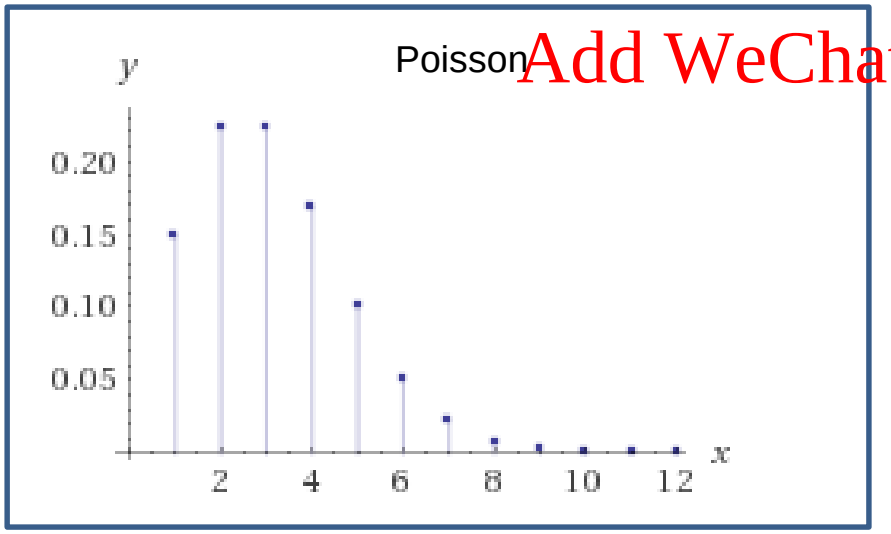
Selling expences	probability
800	20%
900	30%
1000	40%
1100	10%

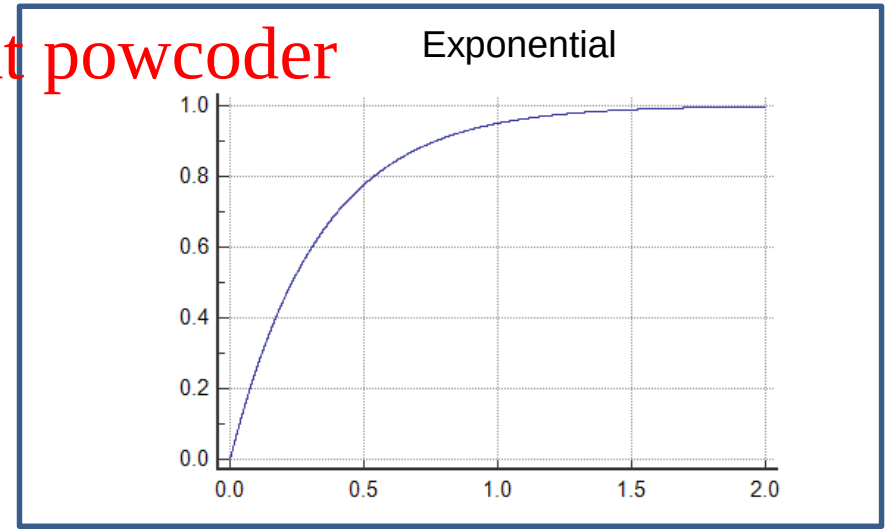


More essential distributions









Profit distribution

The profit will also be random, and can be described by a probability distribution

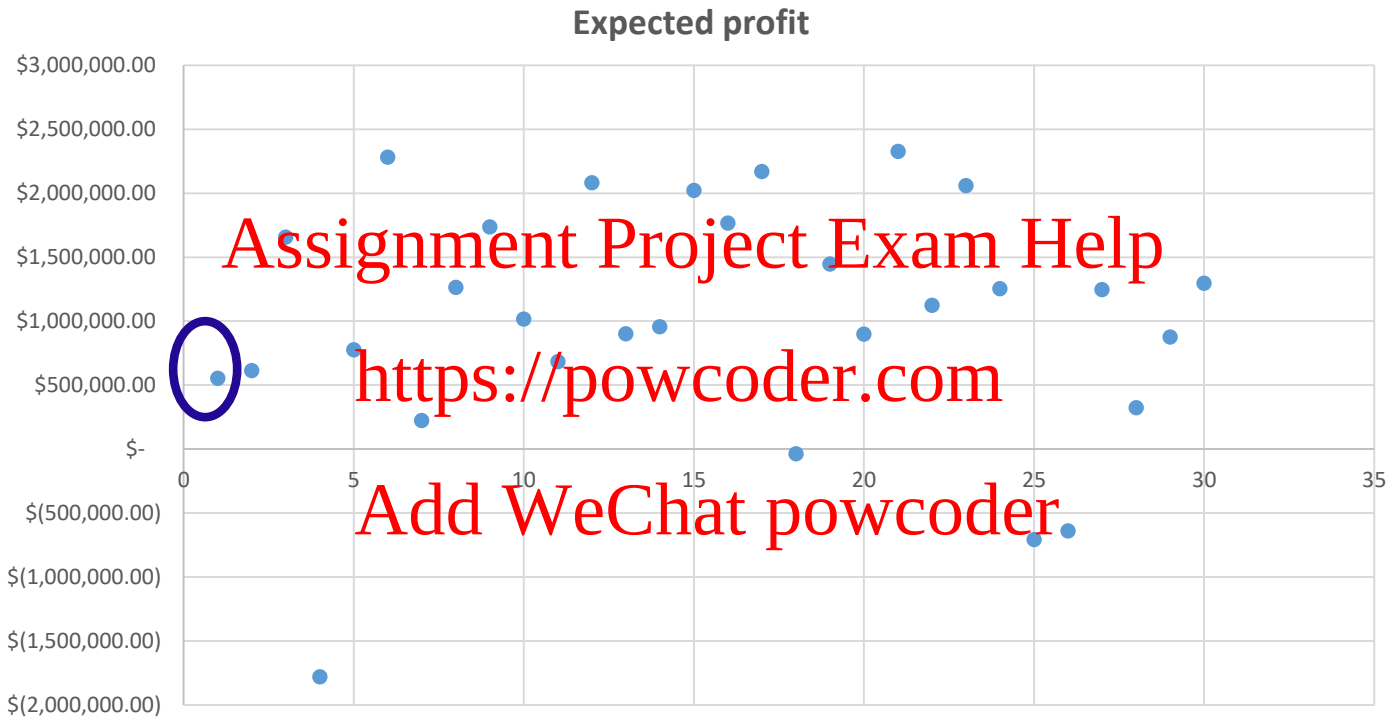
If we knew how the profit is distributed we could answer a variety of questions, e.g.

- what is the expected profit (mean)?
- what is the probability that profit exceeds $2M?

We start by finding a random value for the following:

- # of sold units
- Variable cost
- Selling expenses

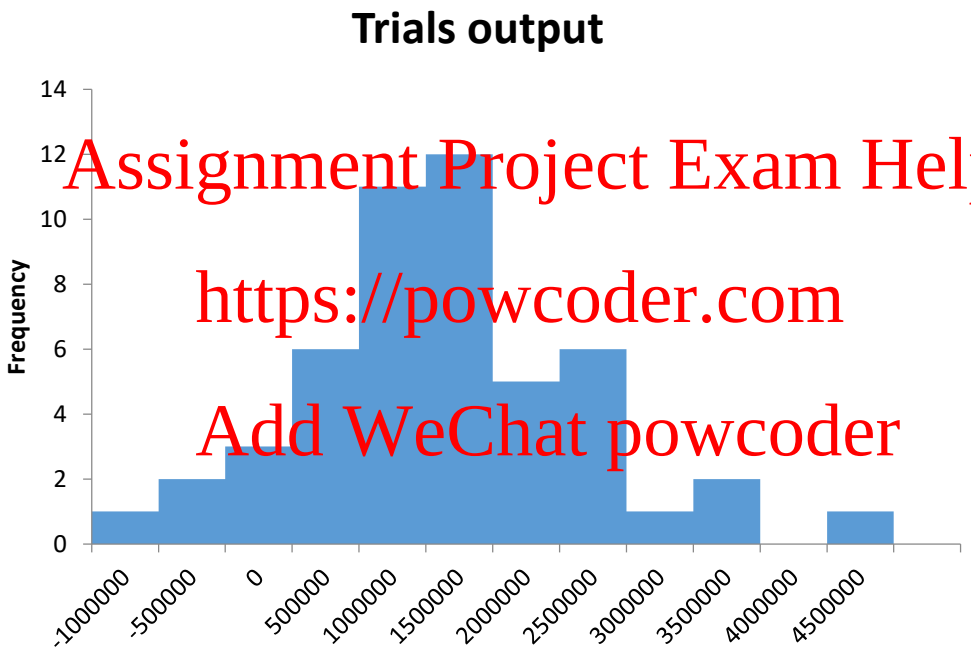
All we did so far is generate a single number from the distribution that describes the profit



We need to generate more outcomes that are possible so that we can better describe the random profit

If we generate more trials, we improve our prediction

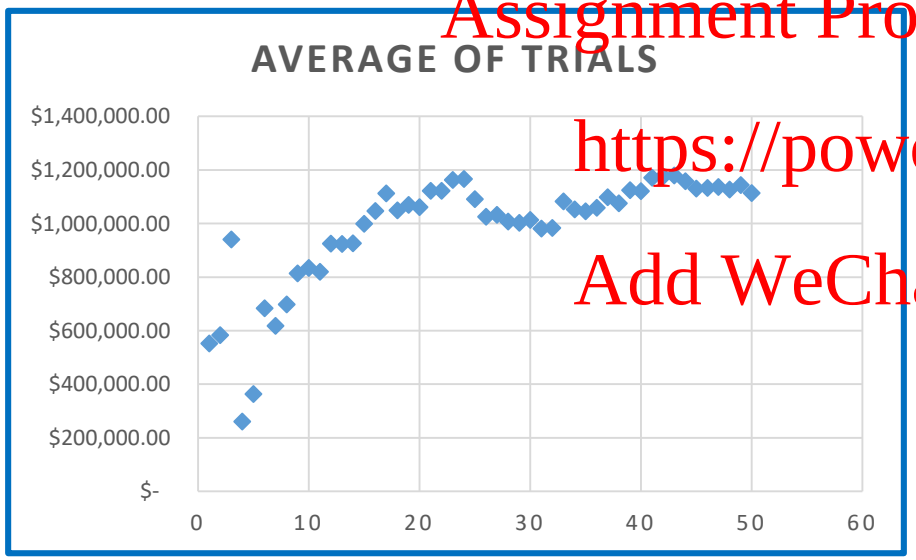
Profit is a random number from a continuous distribution, so the more trials we run the smoother the histogram

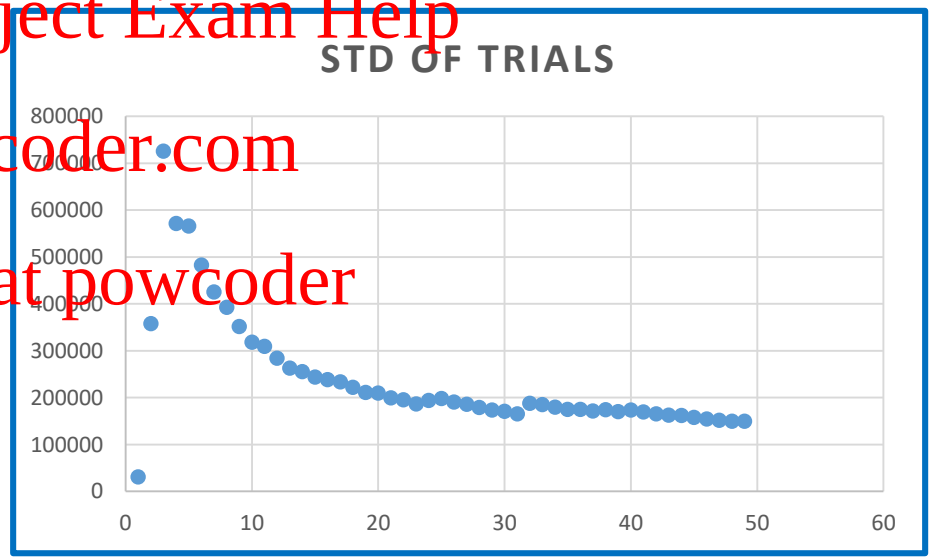


The output of Monte Carlo simulation is a distribution, which assigns different probabilities to different outcomes

More trails, better estimation

The more replications (trials, scenarios) we run the more accurate our estimates will be





The law of large numbers

- The law of large numbers is a mathematical theorem.
- By the theorem, as the number of trials of a random process increases, the percentage difference between the **expected** and **actual** values goes to zero.
- In other words: as the number of trails increases, the average of the trails outcomes converges to the real mean of the population.
- On the same time, the **average standard deviation** decreases and converges to zero.

- We can demonstrate that by flipping an unbiased coin:
- http://www.virtualcointoss.com

- You can also watch the following demonstrations: https://www.youtube.com/watch?v=6YDHBFVIvIs
- https://www.youtube.com/watch?v=3m4bxse2JEQ

Steps to Monte Carlo Simulation

1. Generate *n*-sets of input variables from appropriate probability distributions to run *n* simulation trials
2. Collect *n* values of output, each one resulting from a separate simulation run
3. Analyze the probability distribution that describes the output

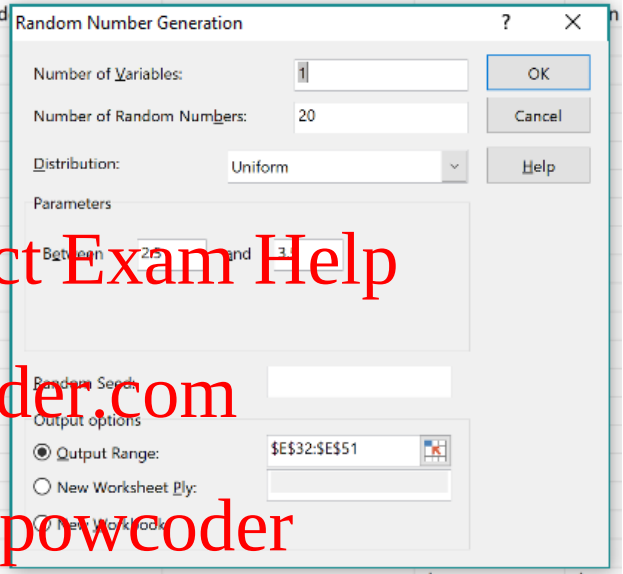
Go to the excel file "Spinners" and generate the numbers using one of two methods:

a) Using Excel's data analysis add-in

b) Using Excel's function RAND and NORMINV

Generate input random variables using Excel's Data Analysis add-in

	A	B	C	D	E	F	G
1	Spinners profitability		Scenario	Random # of units	Rand		
2			1	744202.5344			
3	Data		2	770446.1177			
4			3	1212546.49			
5		Expected	4	7501.935214			
6	Revenue per spinner	$ 6.00	5	754486.191			
7	# of units	900,000	6	1400363.058			
8	Fixed cost	$ 700,000.00	7	664215.6586			
9	Variable cost	$ 3.00	8	1082431.751			
10	Selling expenses	$ 90,000.00	9	1076640.007			
11			10	797607.4949			
12			11	826675.4003			
13	Expected profit	$1,060,000.00	12	1133012.543			
14			13	885777.4355			
15			14	985616.7466			
16	Random		15	1263202.162			
17	# of units	1156269.281	16	1203756.678			
18	Variable cost	3.221782867	17	1120702.304			
19	Selling expenses	1000000	18	566898.447			
20			19	928949.8985			
21			20	966720.9861	3.312341685	1000000	$ 898,215.70 $
22	New Expected profit	$1,512,367.13	21	1234892.775	2.820566424	900000	$2,326,259.55 $
23			22	957599.6637	3.05015717	1000000	$1,124,768.50 $
24			23	1160437.218	2.760048219	1000000	$2,059,760.63 $
25	Selling expenses distribution		24	935505.4226	2.949476608	900000	$1,253,781.18 $
26	800000	0.2	25	302445.61	3.378475295	800000	-$ 707,131.36 $
27	900000	0.3	26	294041.879	2.736060671	900000	-$ 640,265.15 $
28	1000000	0.4	27	1062693.709	3.32103946	900000	$1,246,914.51 $
29	1100000	0.1	28	679808.6042	3.318842128	800000	$ 322,674.19 $
30			29	948043.1481	3.389126255	900000	$ 875,220.96 $
31			30	832502.4191	2.52191229	900000	$1,295,516.43 $



Concluding remarks

- We use Monte Carlo simulation to simulate real-life systems when we have uncertain parameters

- We model the parameters as random variables that can be derived from a specific distribution

- We generate random numbers and calculate our model outcome

- We repeat the process many time

- The average of our outcome results from many trails would converge to the real outcome
- We can learn from the simulation the outcome distribution