CIS 471/571(Fall 2020): Introduction to Artificial Intelligence

Lecture 6: Adversarial Search

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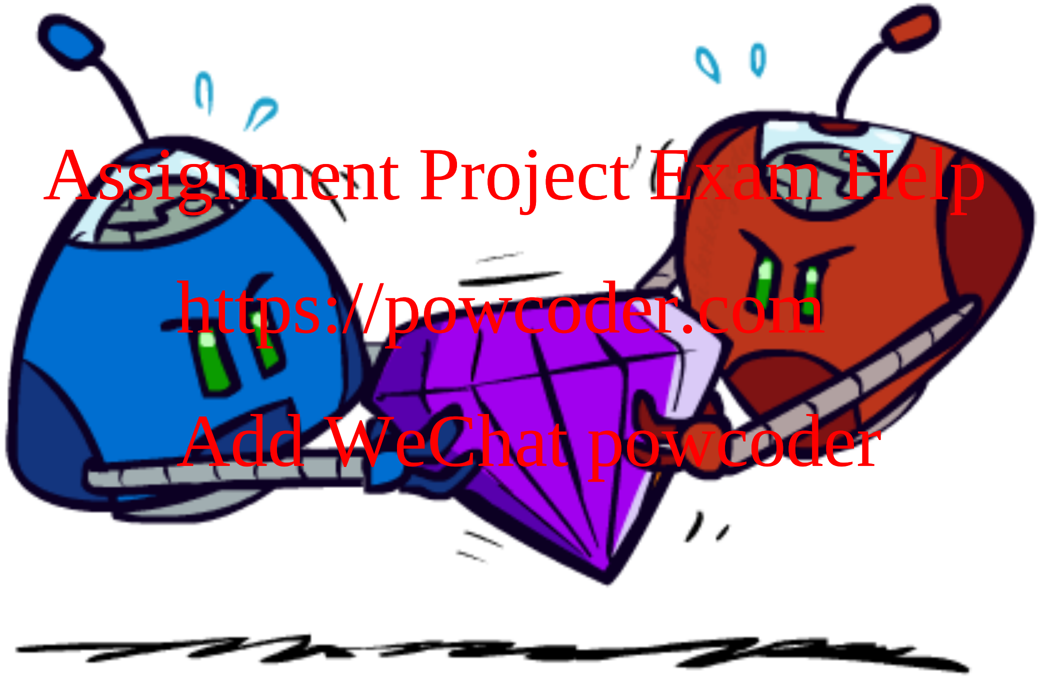
Source: http://ai.berkeley.edu/home.html

Reminders

- Project 2:
  - Deadline: Oct 27th, 2020

- Written assignment 2:
  - Deadline: Oct 24th, 2020

Adversarial Games



Types of Games

- Many different kinds of games!

- Axes:
  - Deterministic or stochastic?
  - One, two, or more players?
  - Zero sum?
  - Perfect information (can you see the state)?

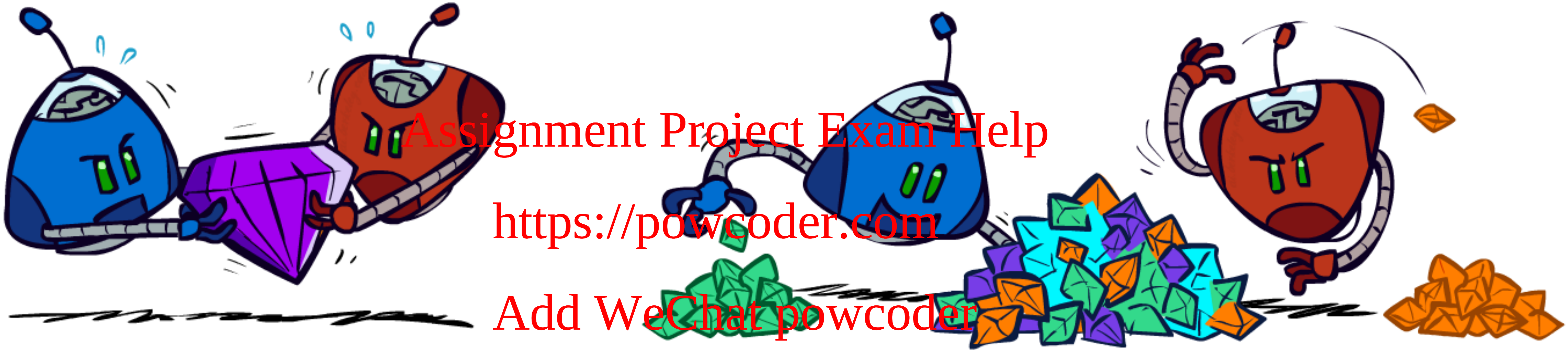
- Want algorithms for calculating a strategy (policy) which recommends a move from each state

Deterministic Games

- Many possible formalizations, one is:
  - States: S (start at s_0)
  - Players: P={1...N} (usually take turns)
  - Actions: A (may depend on player / state)
  - Transition Function: SxA → S
  - Terminal Test: S → {t,f}
  - Terminal Utilities: SxP → R

- Solution for a player is a policy: S → A

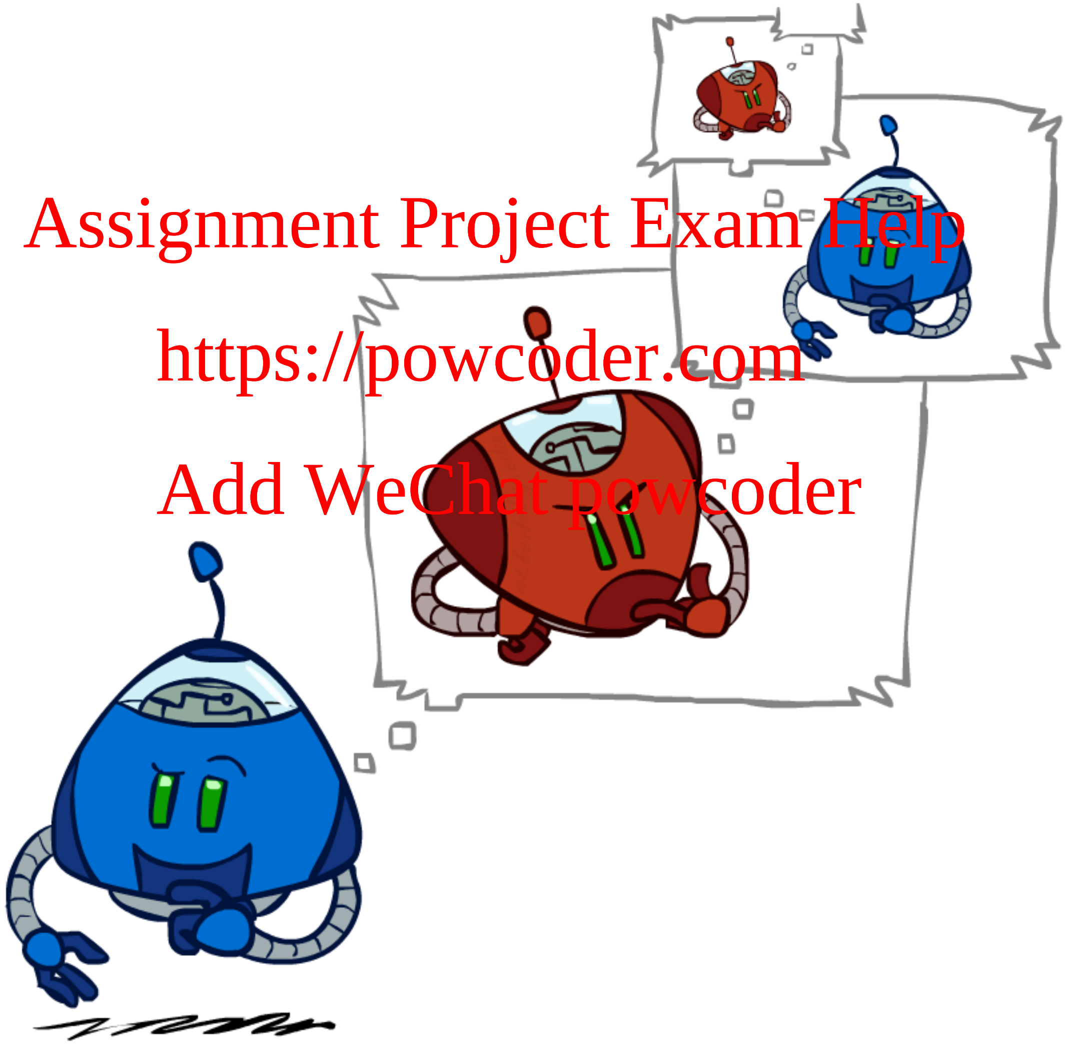
Zero-Sum Games



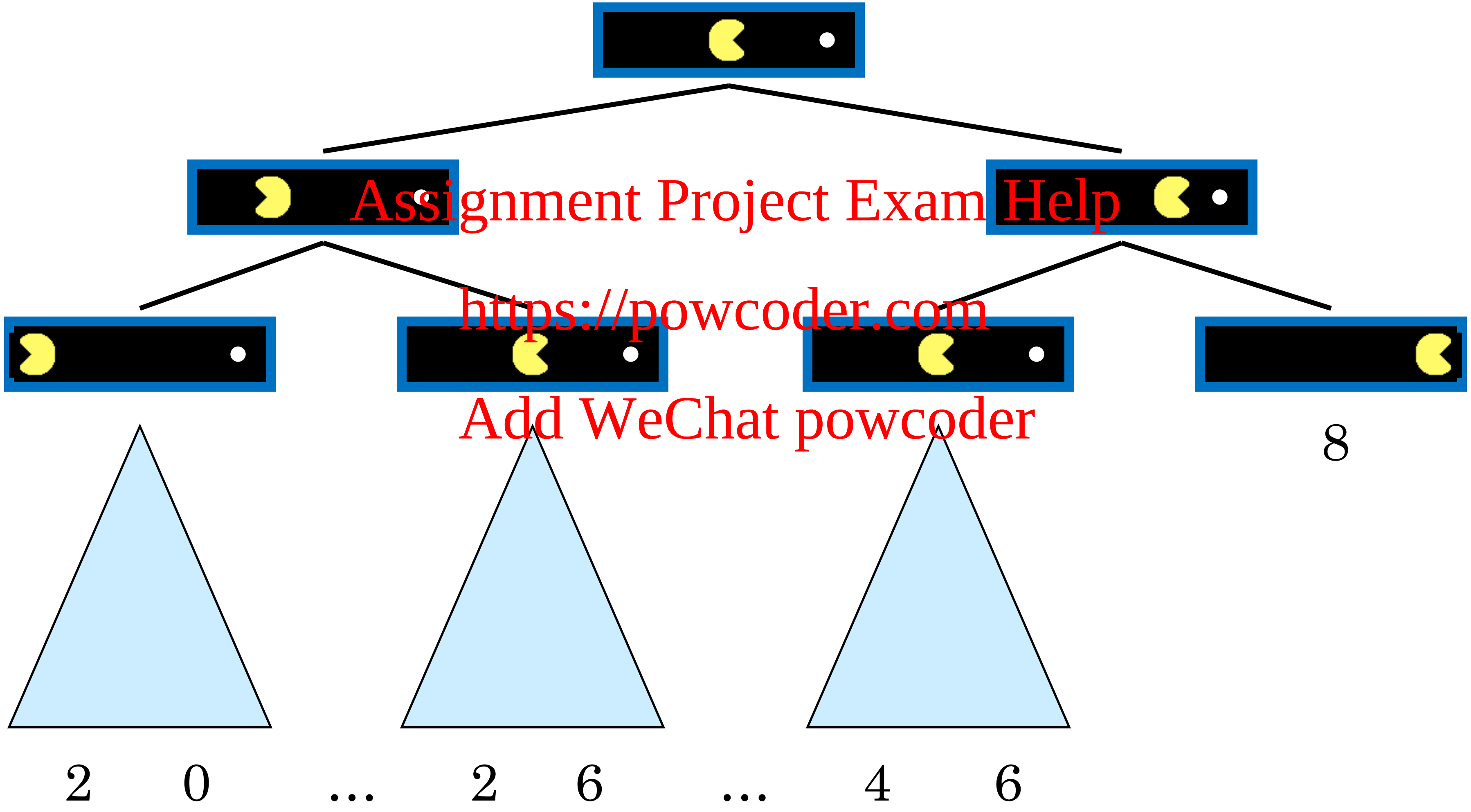
- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - Lets us think of a single value that one maximizes and the other minimizes
  - Adversarial, pure competition

- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, and more are all possible
  - More later on non-zero-sum games

Adversarial Search

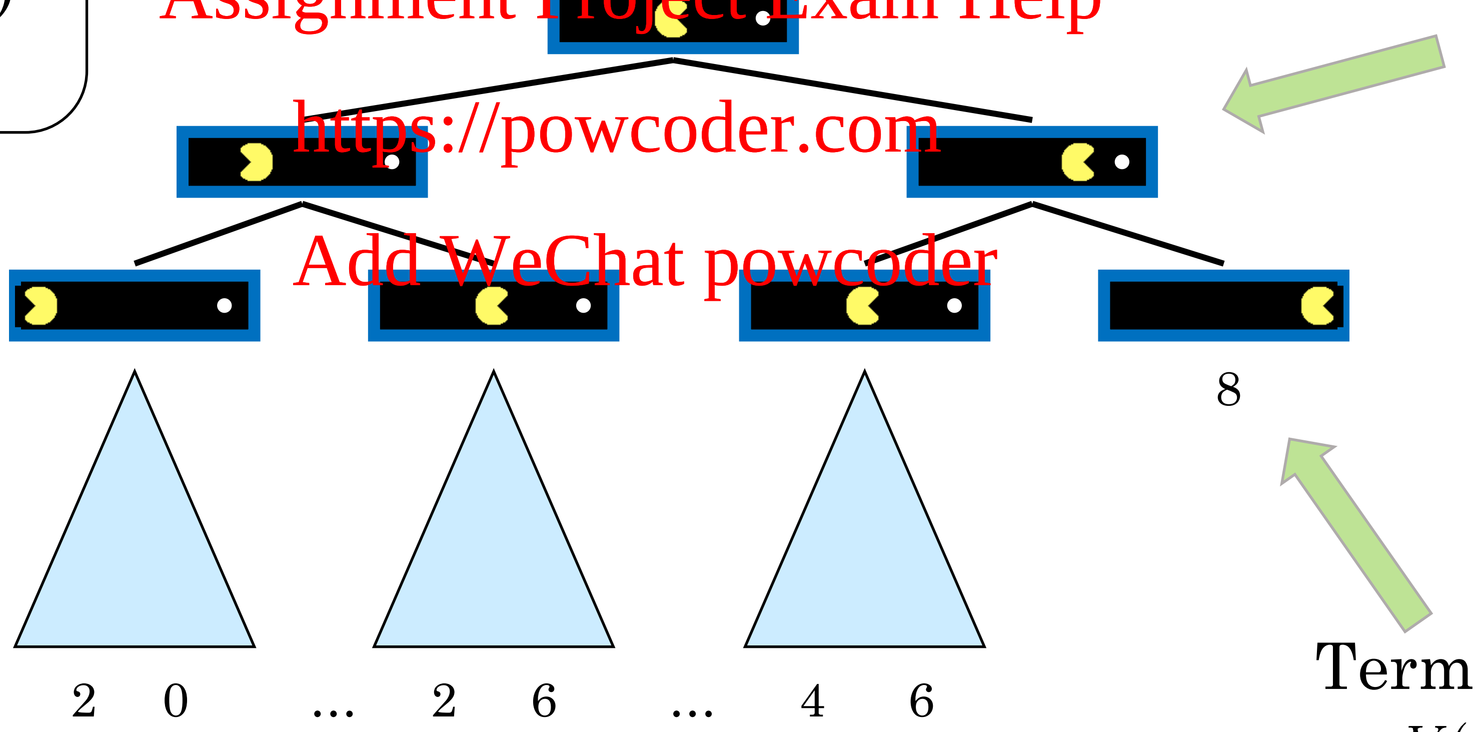


Single-Agent Trees



Value of a State

Value of a state:
The best achievable outcome (utility) from that state



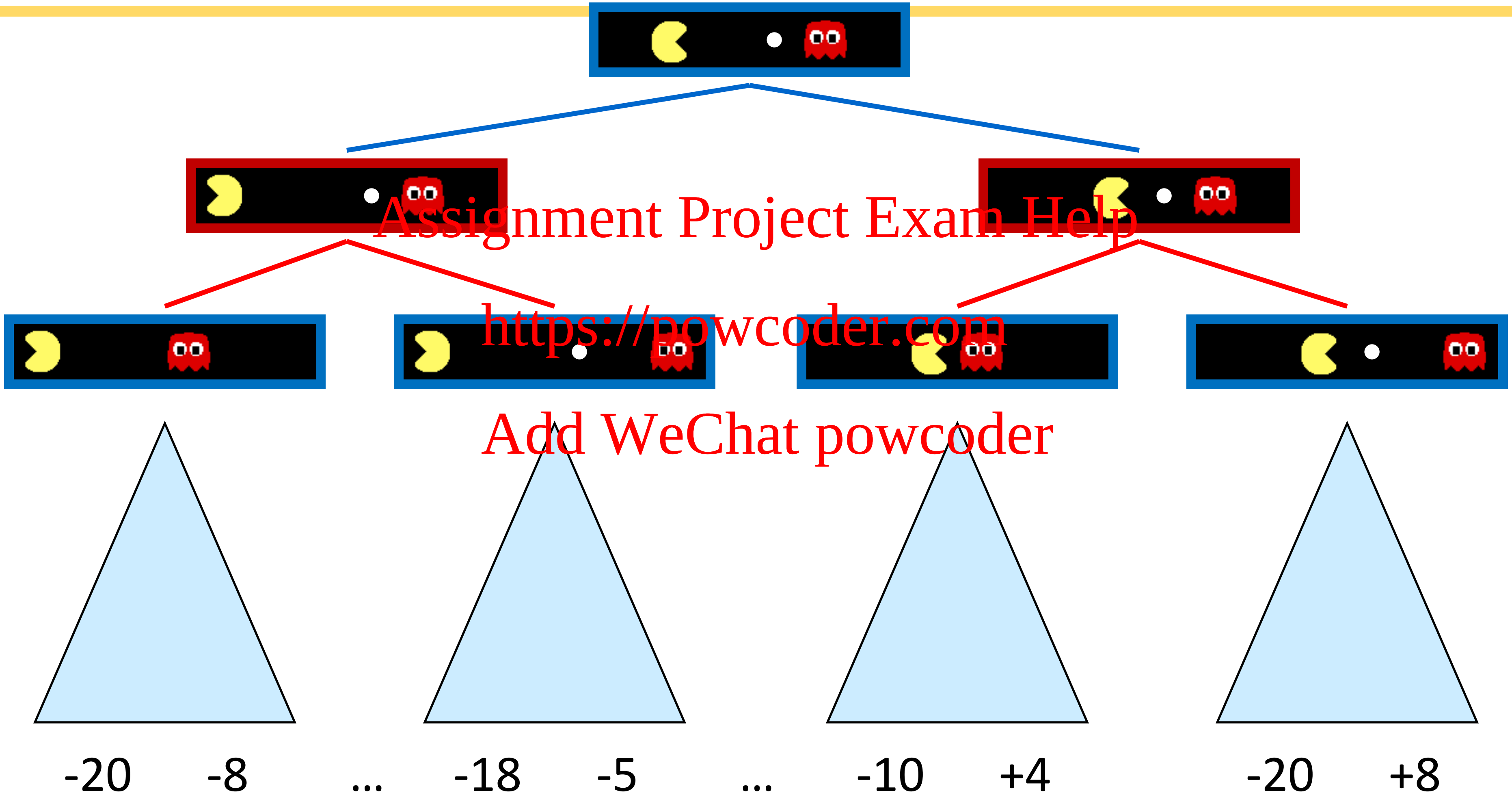
Non-Terminal States:

$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$

Terminal States:

$$V(s) = \text{known}$$

Adversarial Game Trees



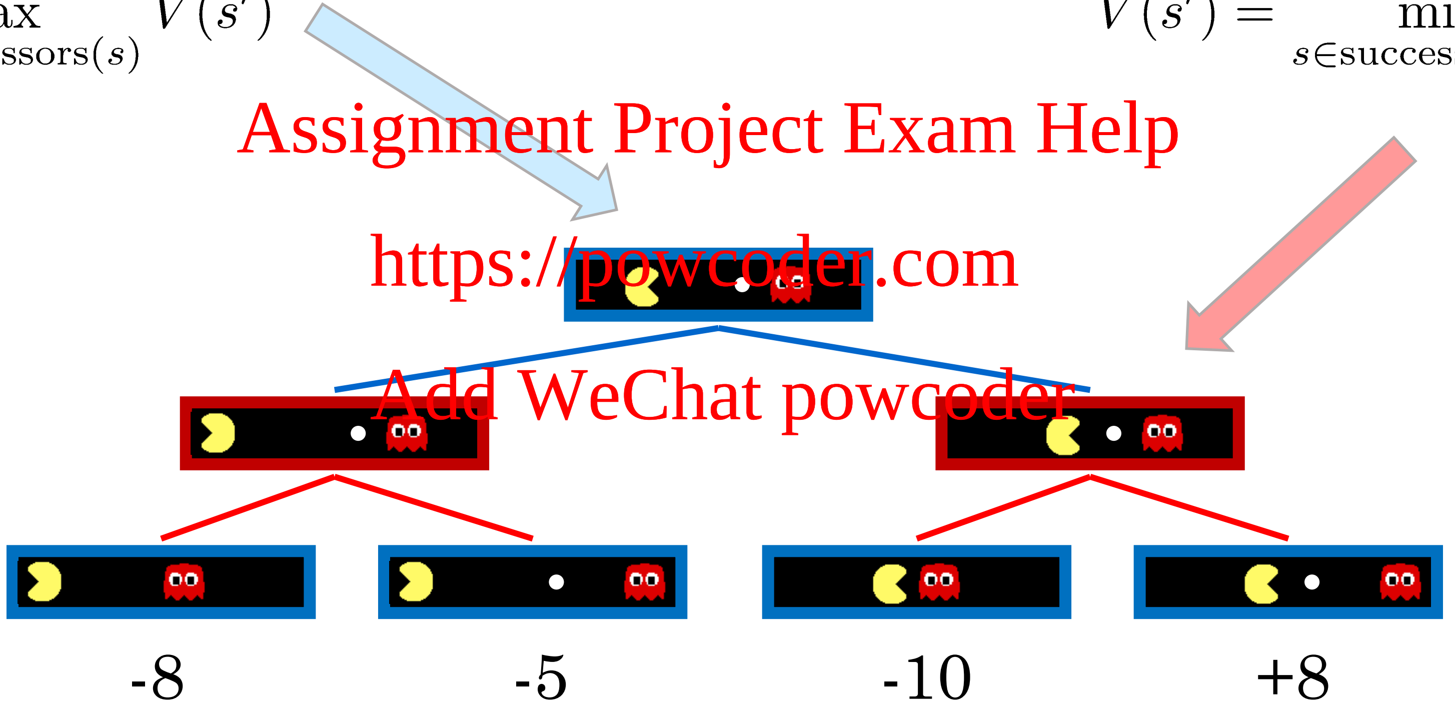
Minimax Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

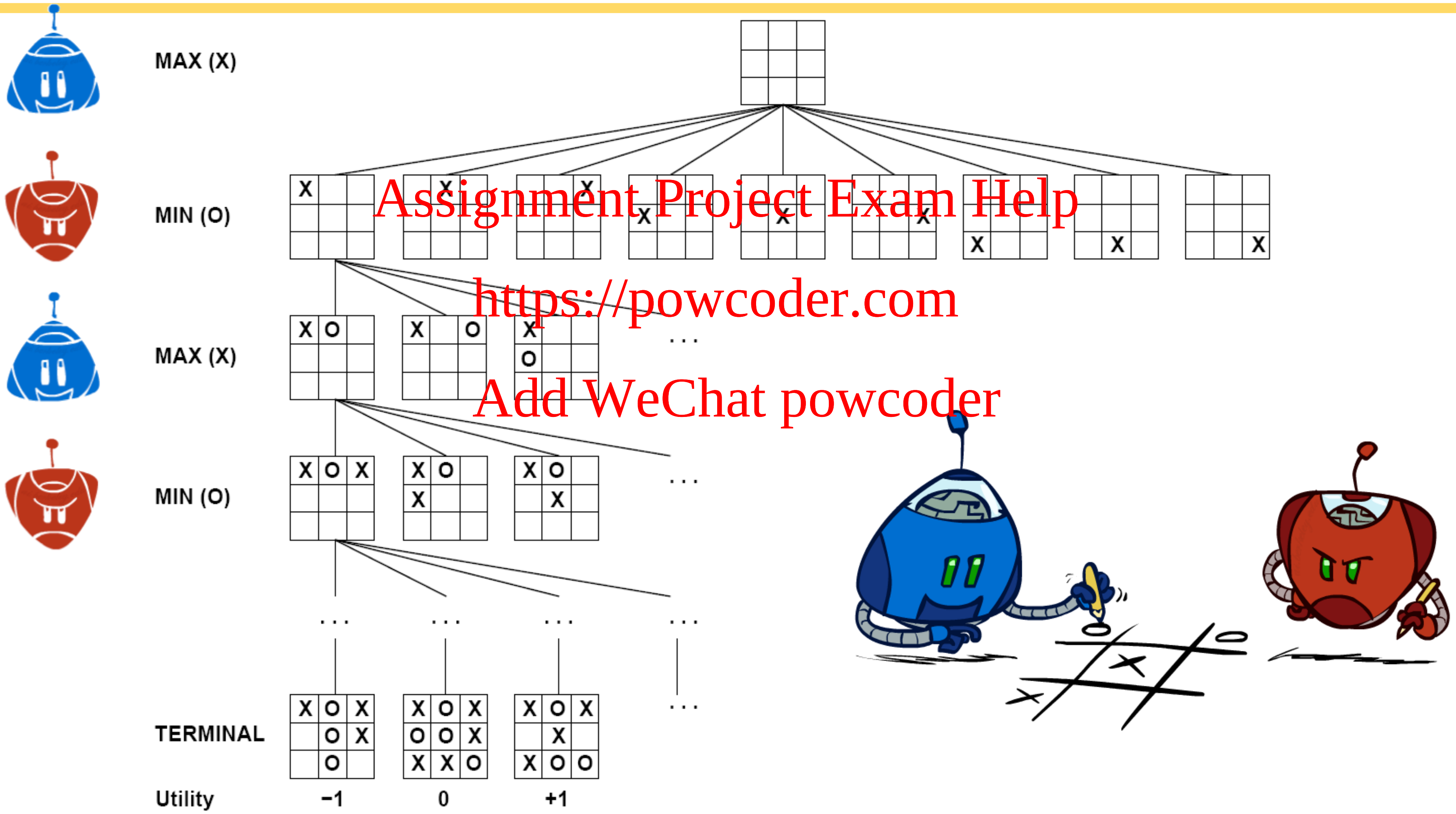
$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$



Terminal States:

$$V(s) = \text{known}$$

Tic-Tac-Toe Game Tree

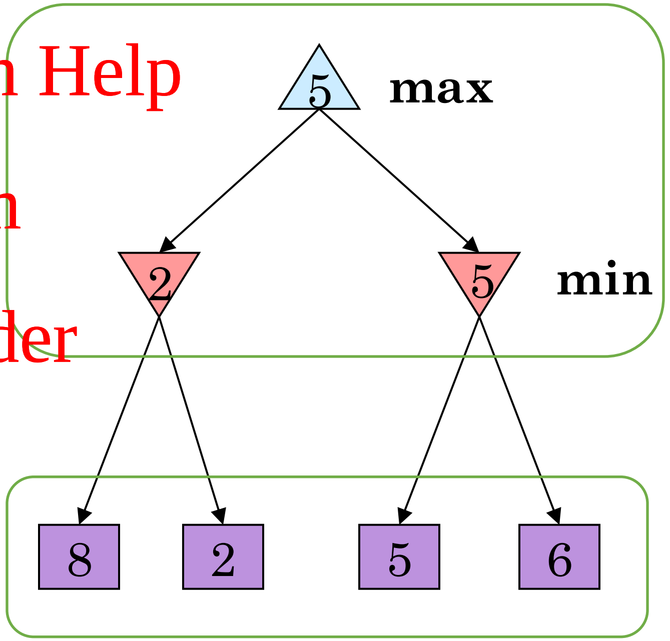


Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result

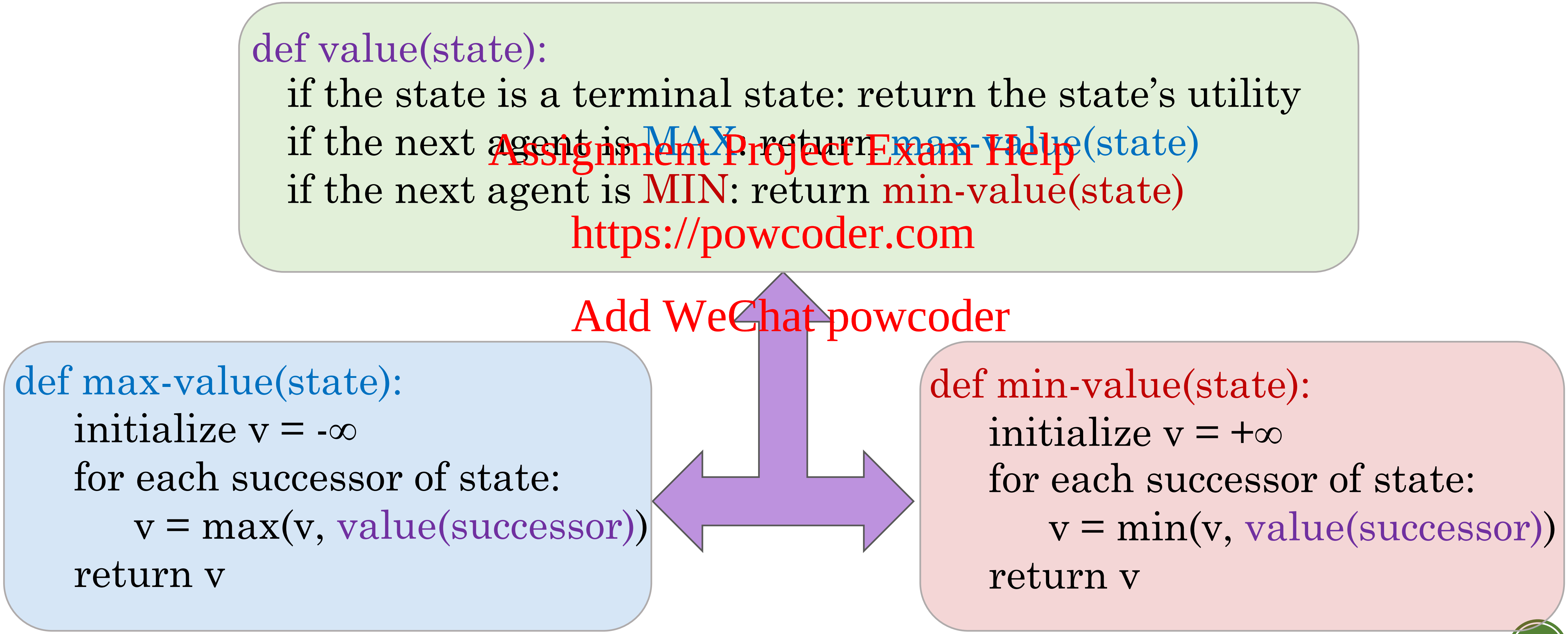
- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary

Minimax values: computed recursively



Terminal values: part of the game

Minimax Implementation

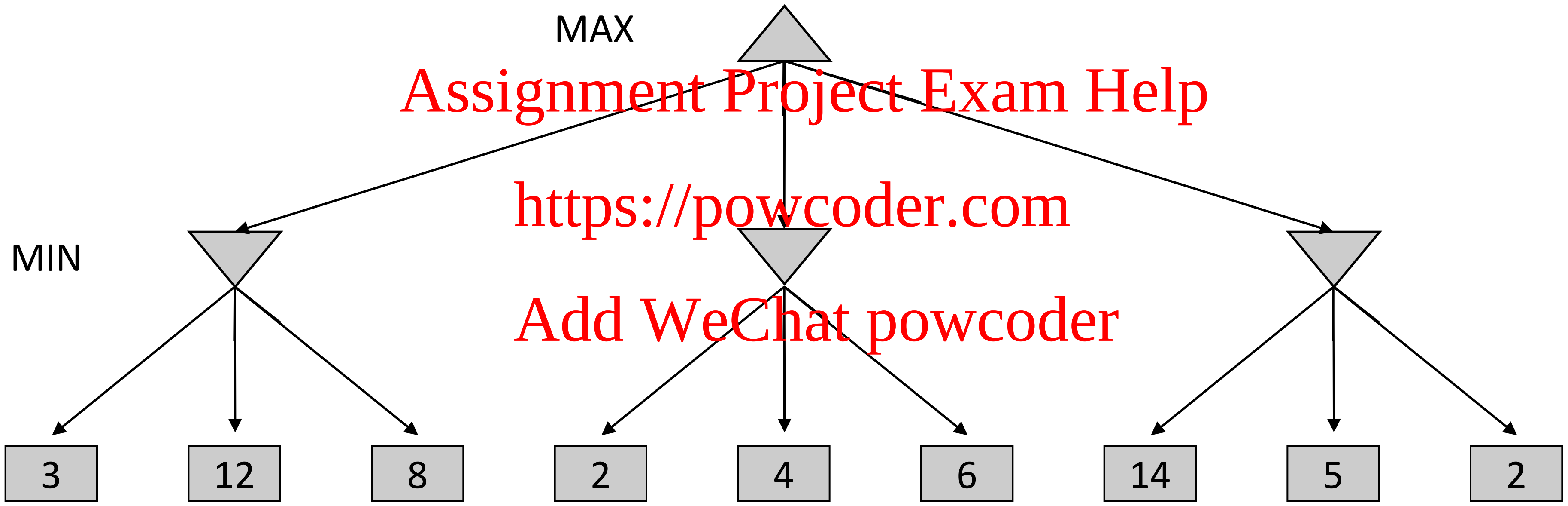


```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

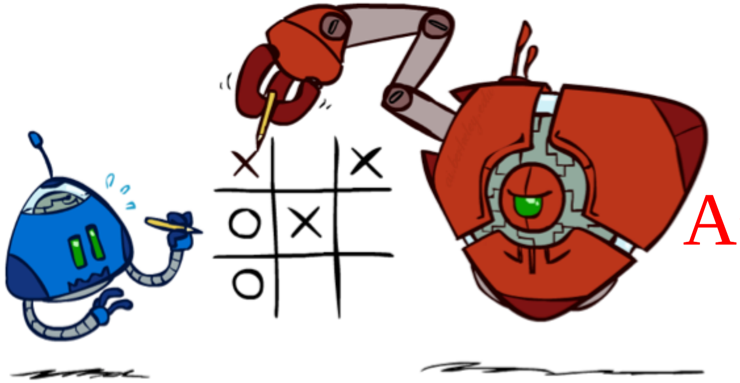
```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

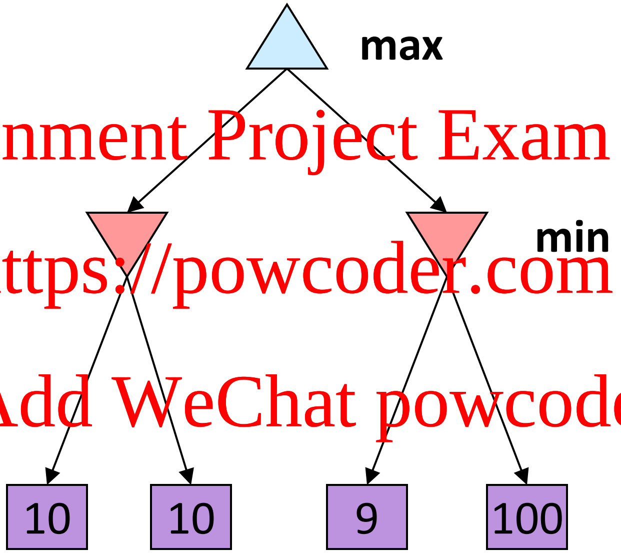
```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor))
    return v
```

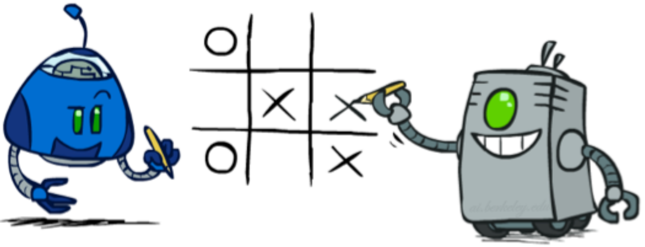
Minimax Example



Minimax Properties





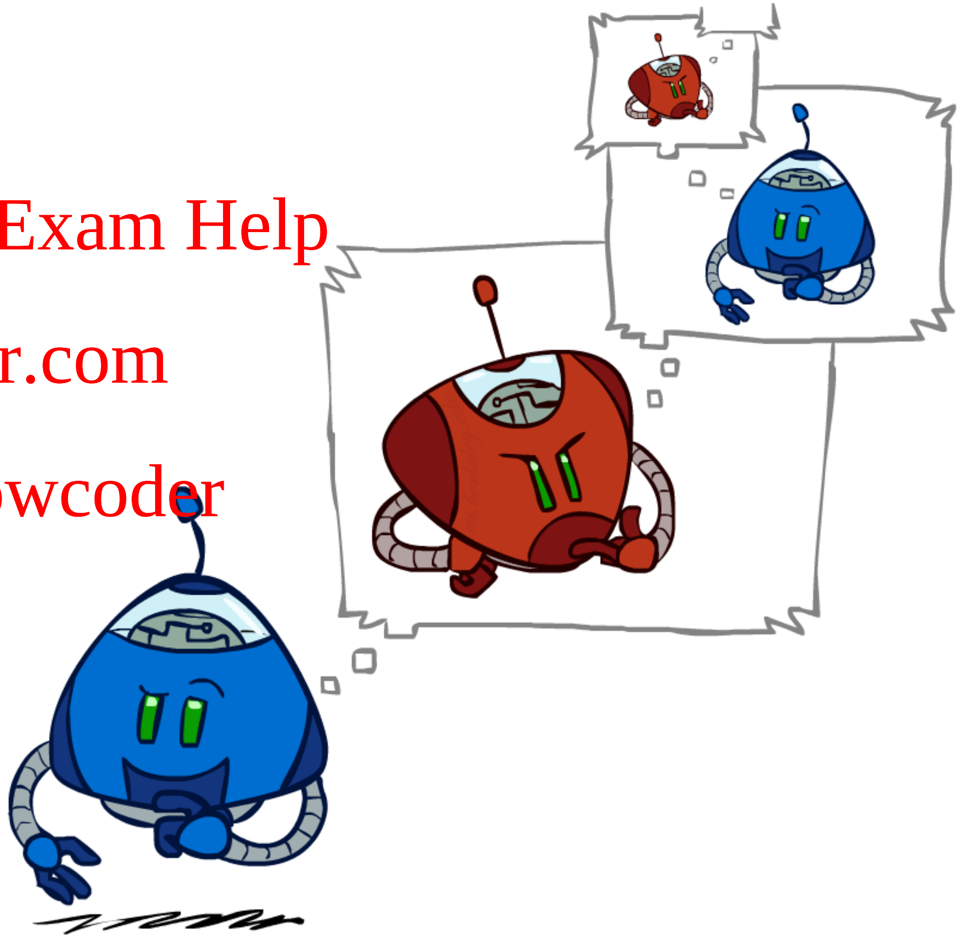


Optimal against a perfect player. Otherwise?

Minimax Efficiency

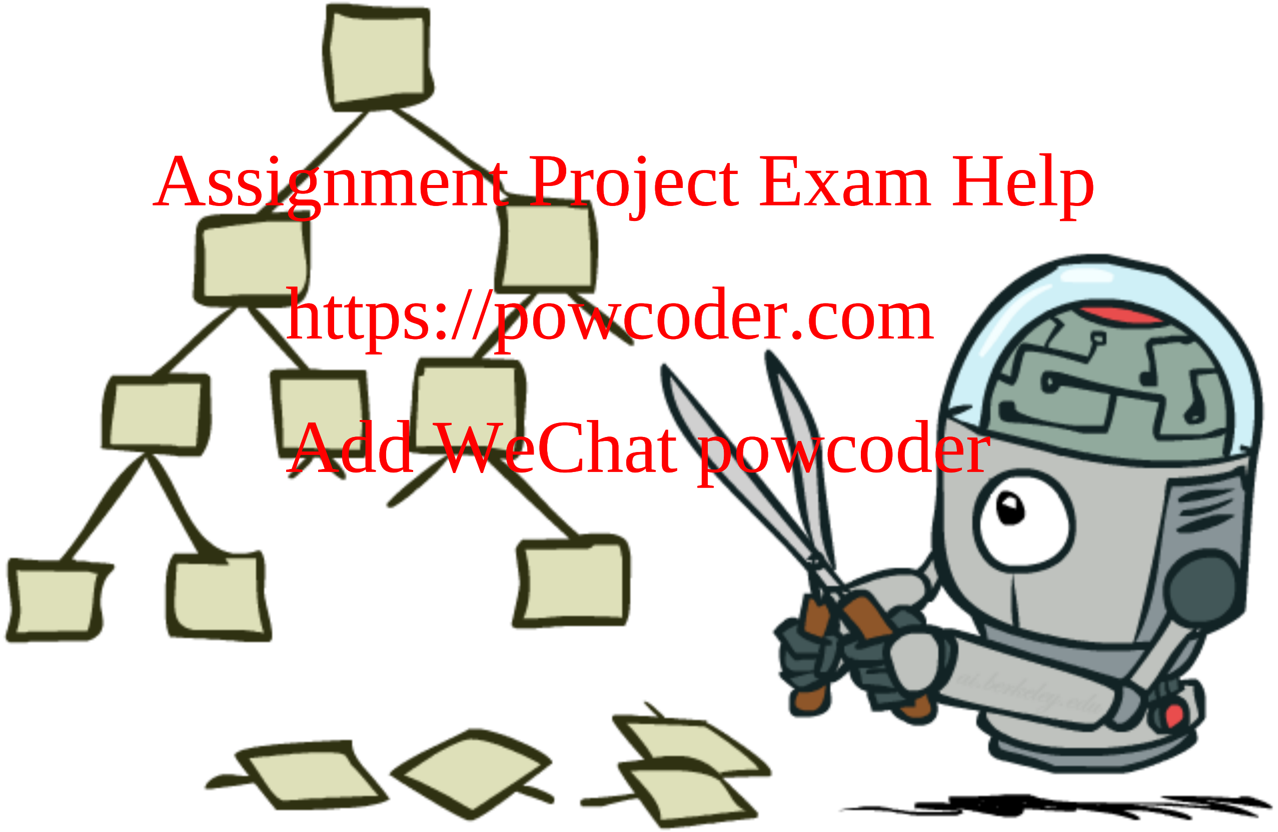
- How efficient is minimax?
  - Just like (exhaustive) DFS
  - Time: $O(b^m)$
  - Space: $O(bm)$

- Example: For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?

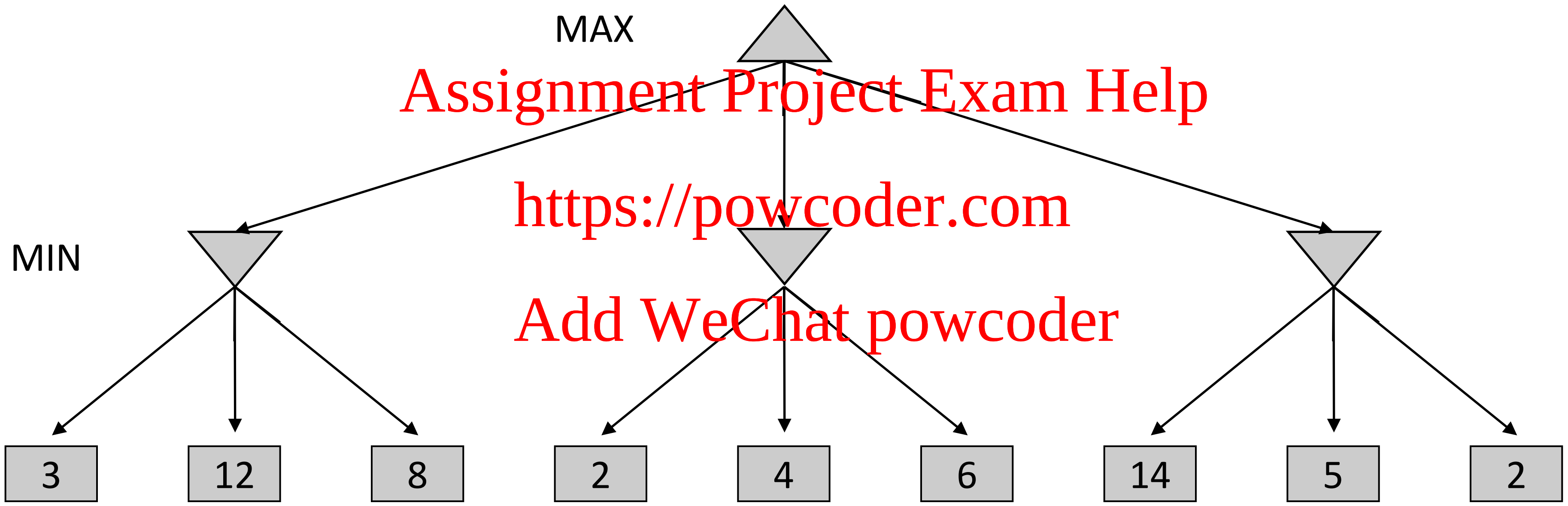


Resource Limits

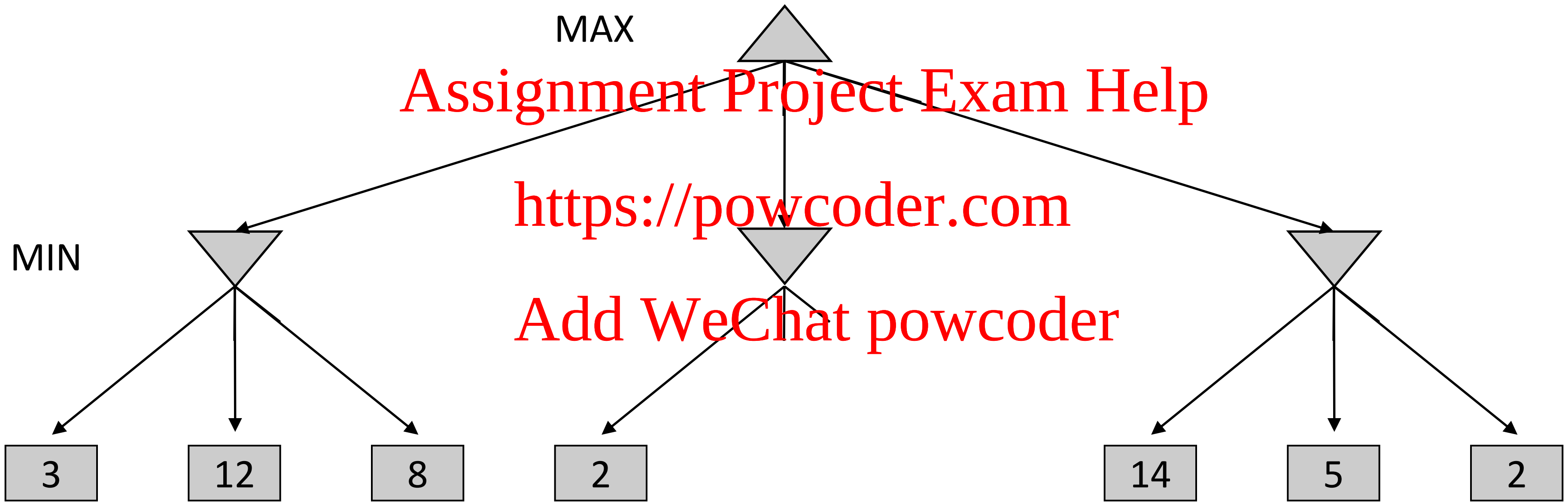
Game Tree Pruning



Minimax Example



Minimax Pruning



Alpha-Beta Pruning

- **Alpha** α: value of the best choice so far for MAX (lower bound of Max utility)
- **Beta** β: value of the best choice so far for MIN (upper bound of Min utility)
- Expanding at MAX node **n**: update α
  - If a child of **n** has value greater than β, stop expanding the MAX node **n**
  - Reason: MIN parent of **n** would not choose the action which leads to **n**

- At MIN node **n**: update β
  - If a child of **n** has value less than α, stop expanding the MIN node **n**
  - Reason: MAX parent of **n** would not choose the action which leads to **n**

Alpha-Beta Implementation

```
def value(state, $\alpha$, $\beta$):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state, $\alpha$, $\beta$)
    if the next agent is MIN: return min-value(state, $\alpha$, $\beta$)
```

```
def max-value(state, $\alpha$, $\beta$):
    initialize v = $-\infty$
    for each successor of state:
        v = max(v, value(successor, $\alpha$, $\beta$))
        if $v \geq \beta$ return v
        $\alpha$ = max($\alpha$, v)
    return v
```

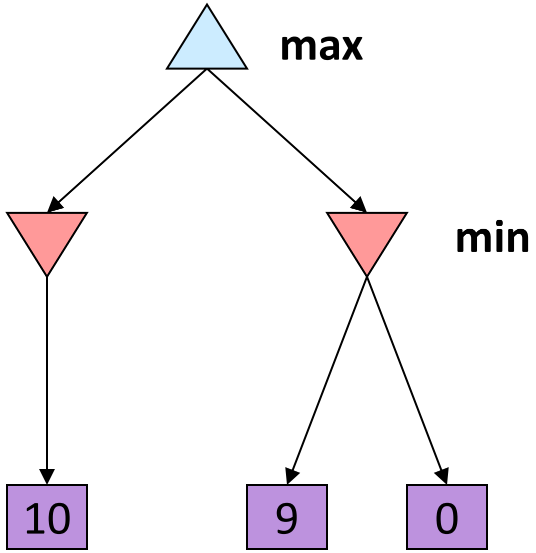
```
def min-value(state , $\alpha$, $\beta$):
    initialize v = $+\infty$
    for each successor of state:
        v = min(v, value(successor, $\alpha$, $\beta$))
        if $v \leq \alpha$ return v
        $\beta$ = min($\beta$, v)
    return v
```

Alpha-Beta Pruning Properties

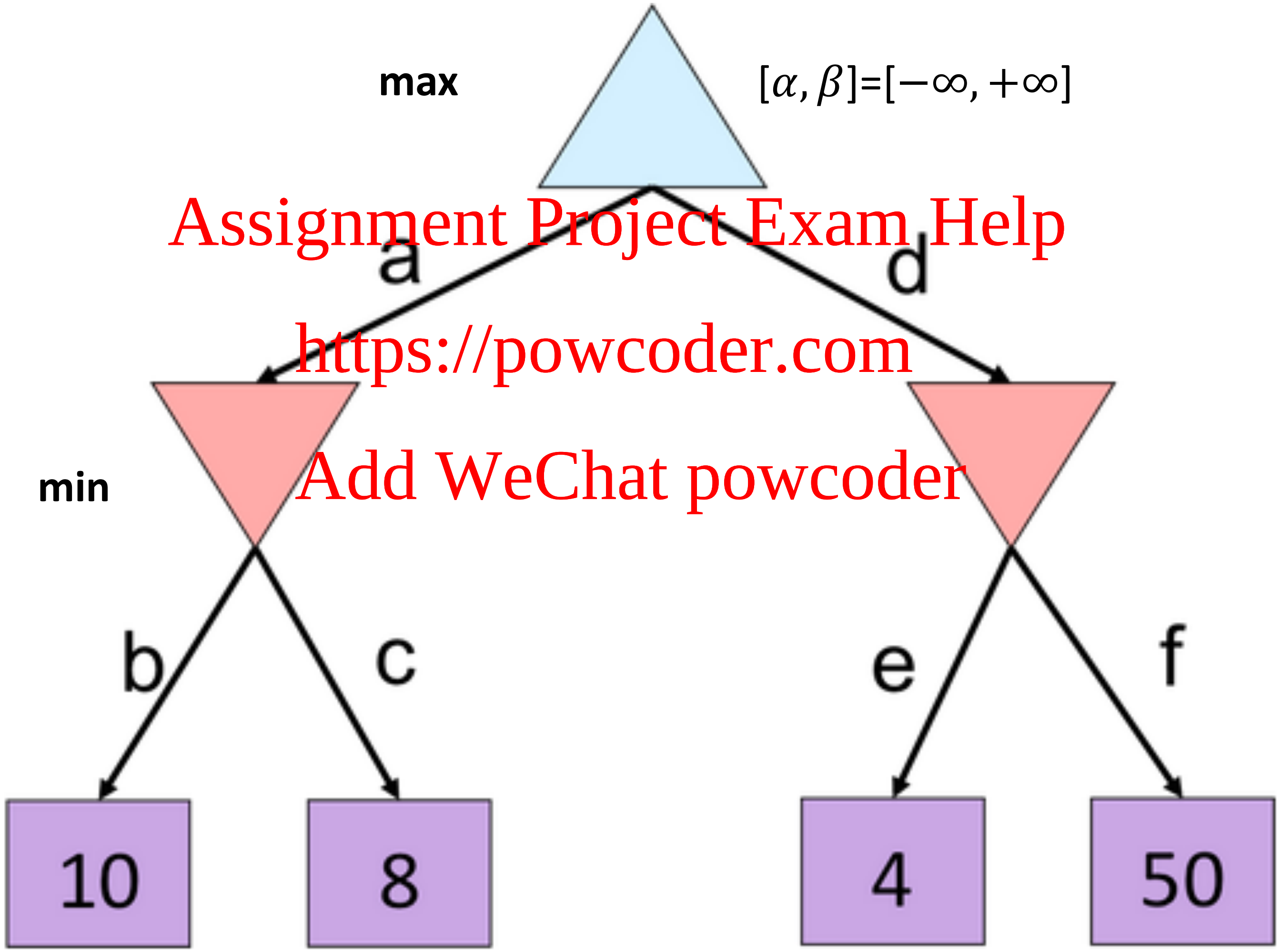
- This pruning has no effect on minimax value computed for the root!

- Values of intermediate nodes might be wrong
  - Important: children of the root may have the wrong value
  - So the most naïve version won't let you do action selection

- Good child ordering improves effectiveness of pruning



Alpha-Beta Quiz



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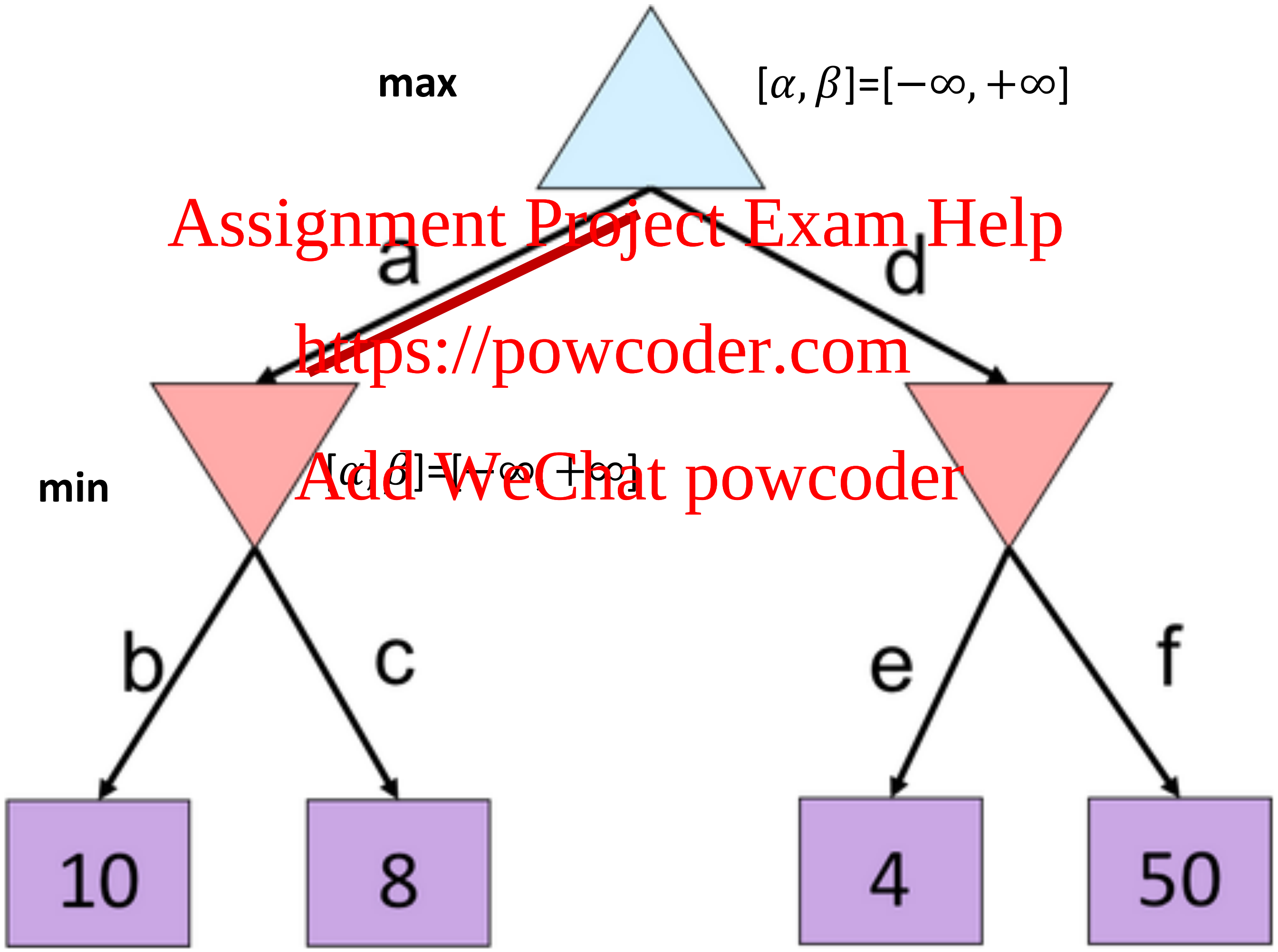
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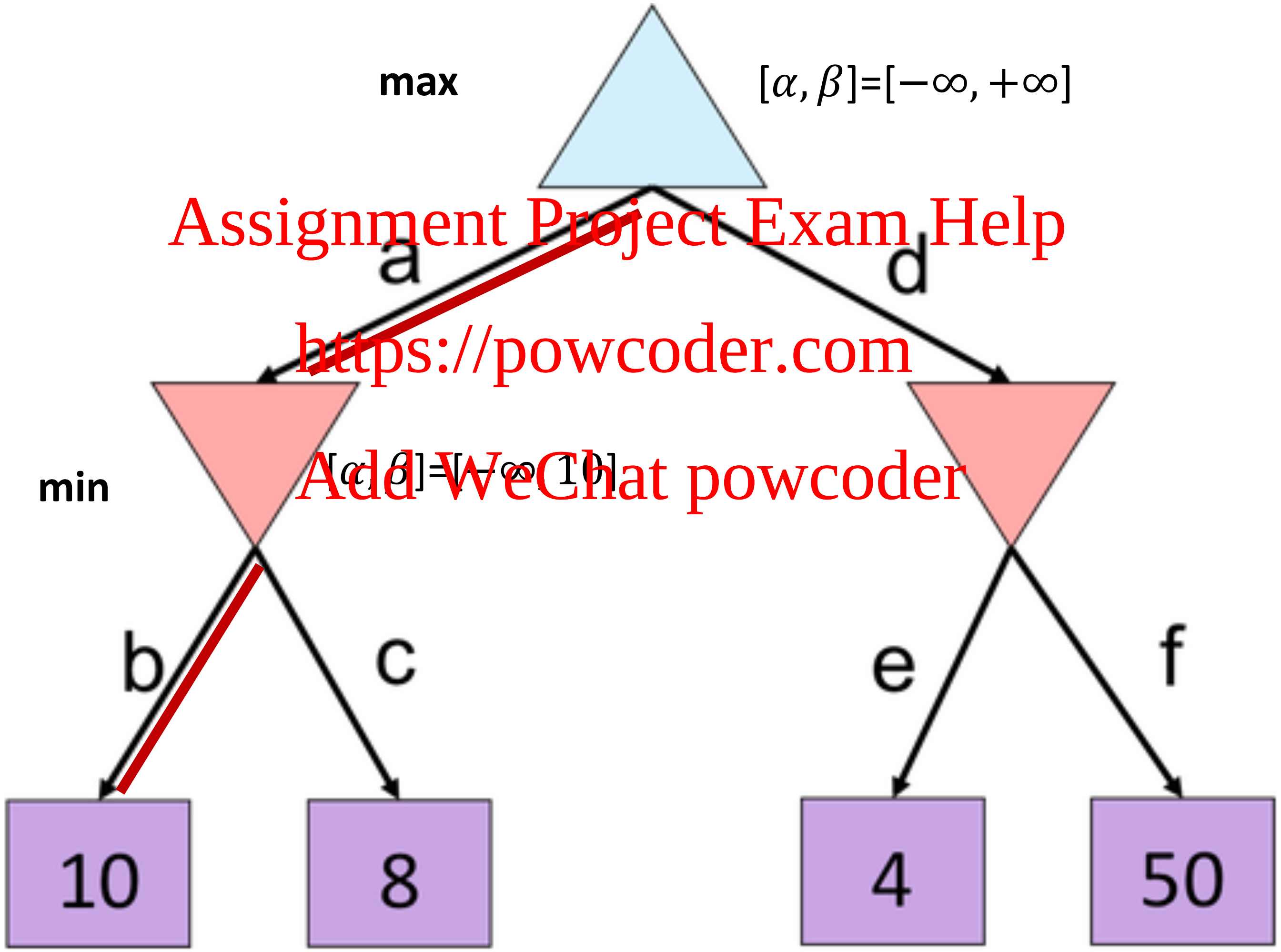
Alpha-Beta Quiz



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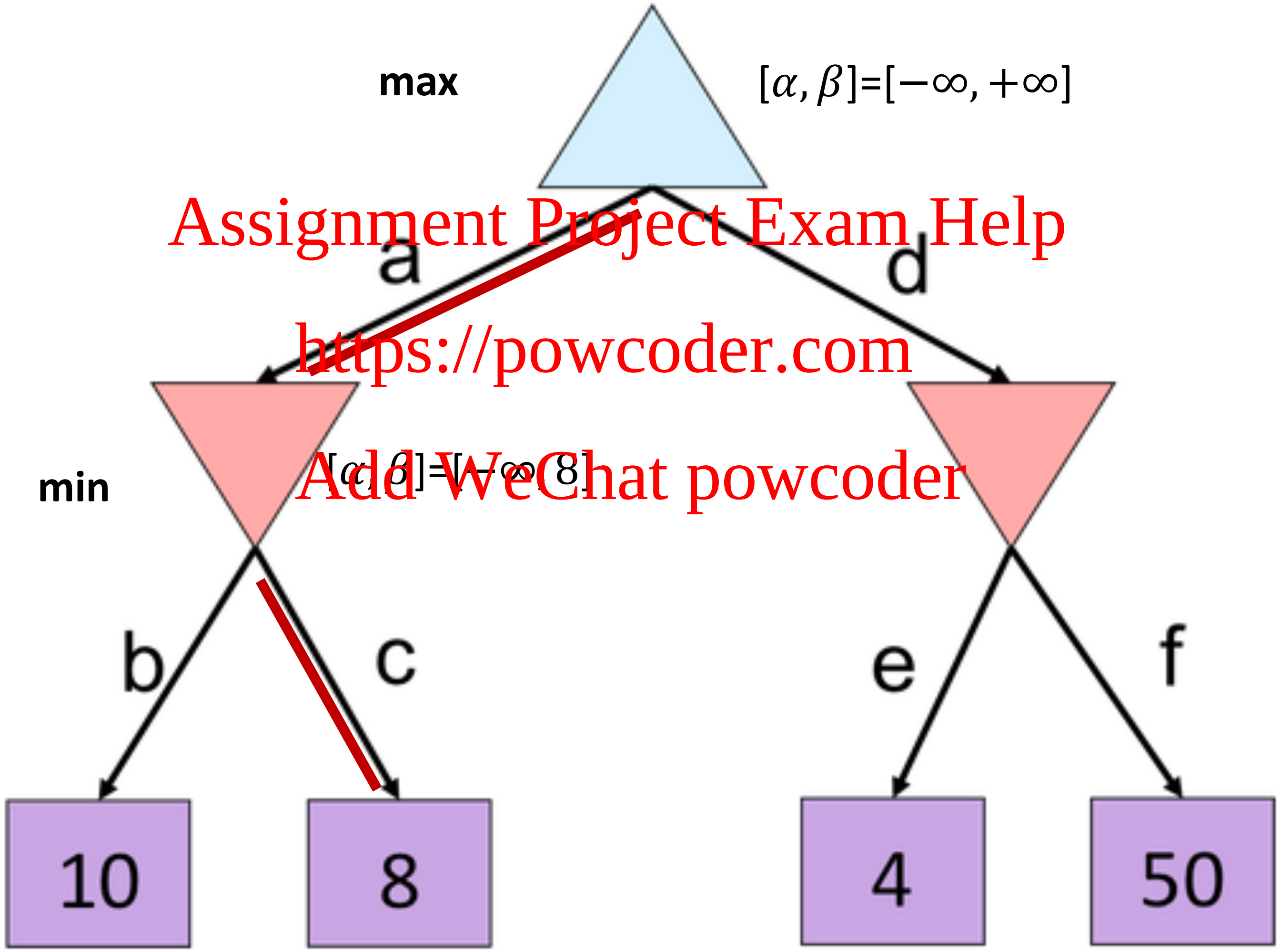
Alpha-Beta Quiz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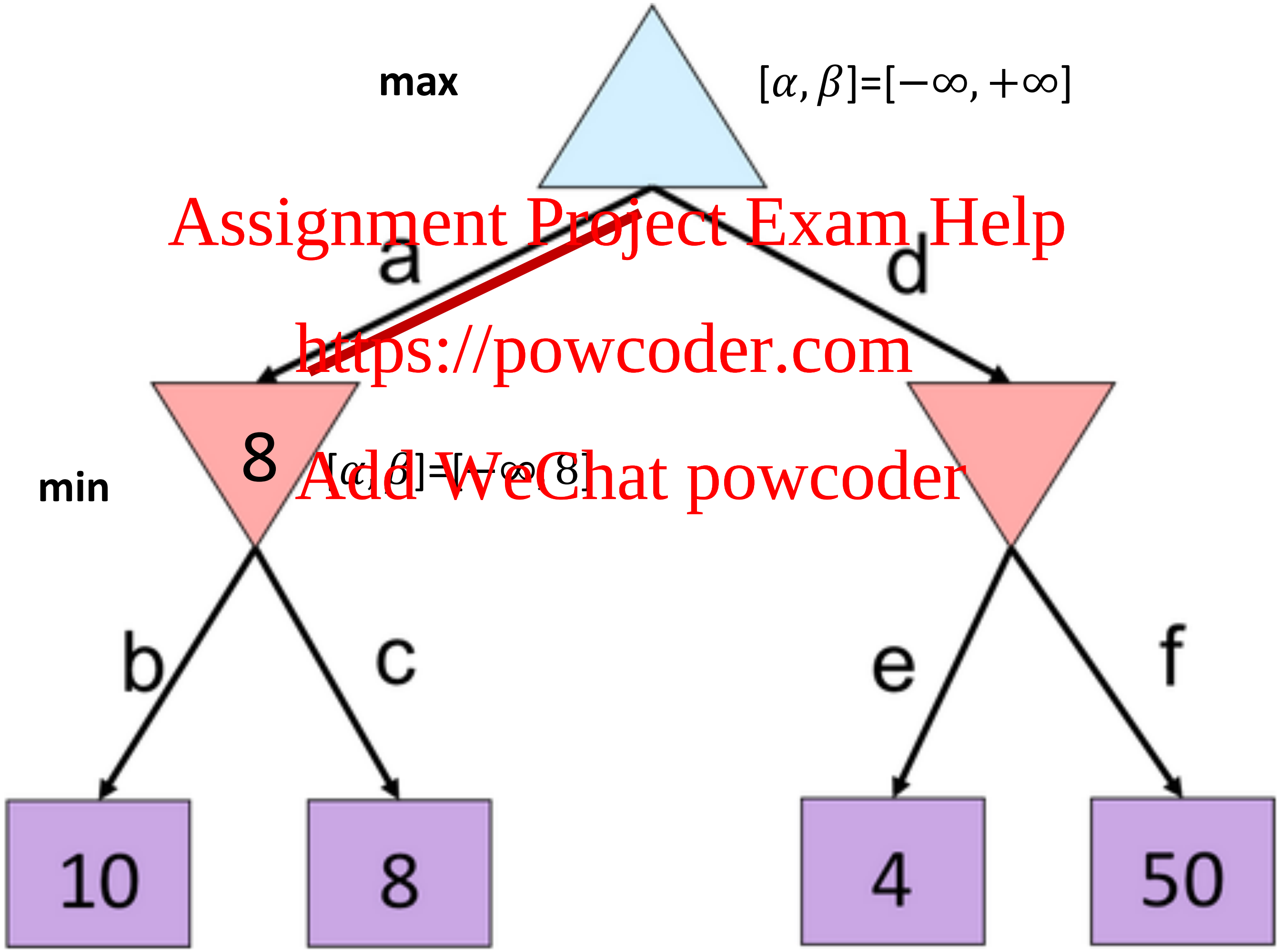
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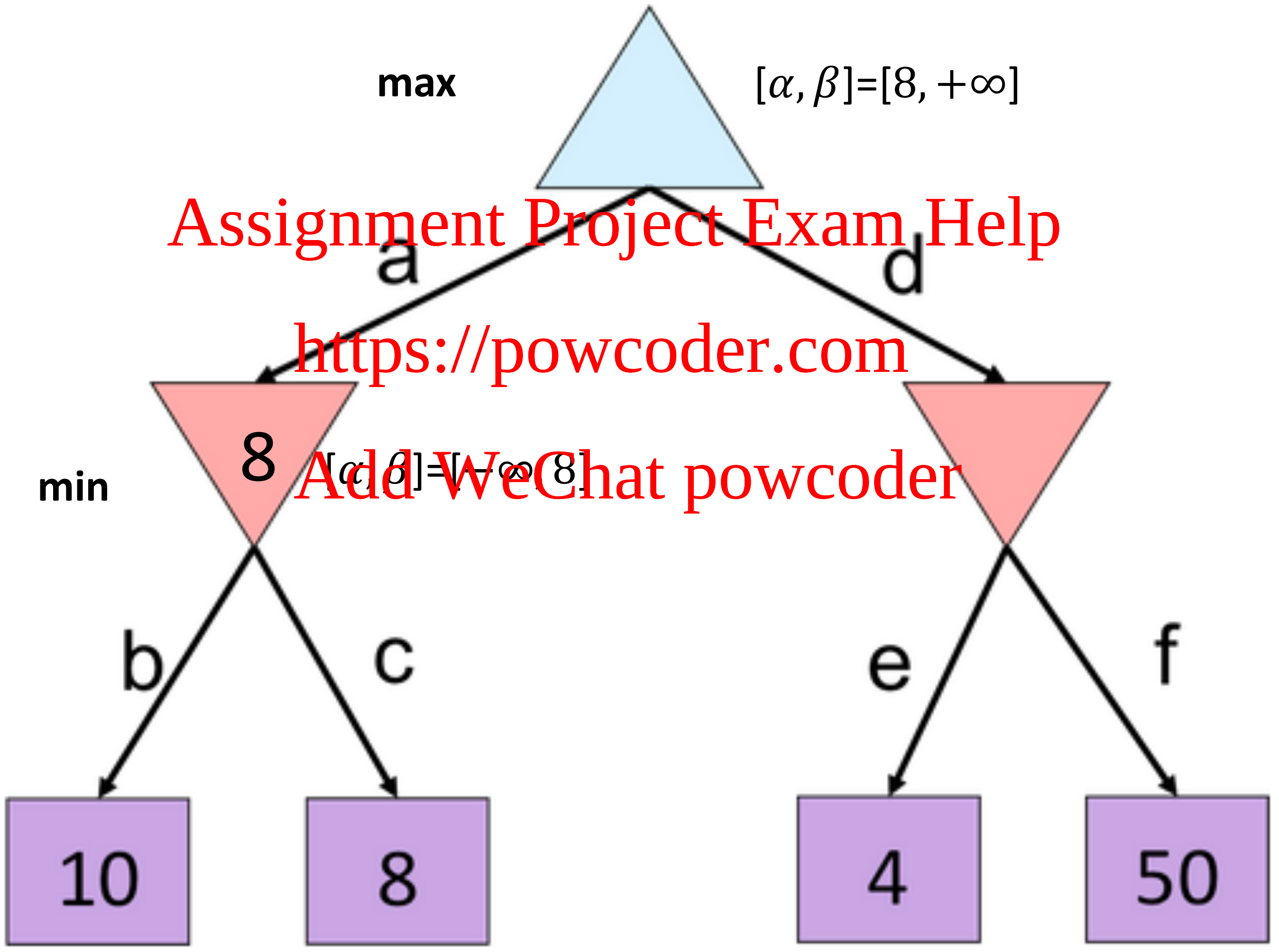

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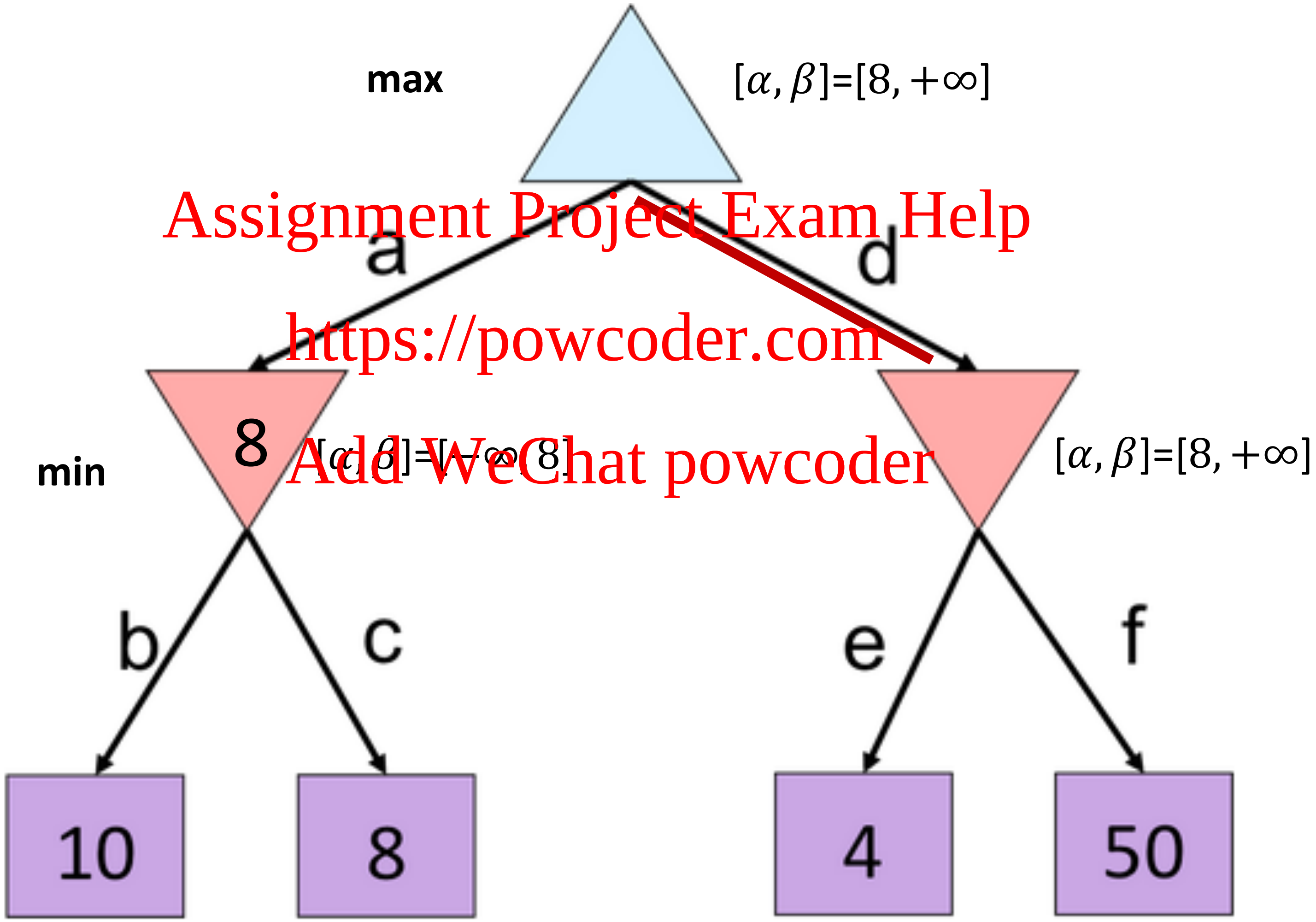
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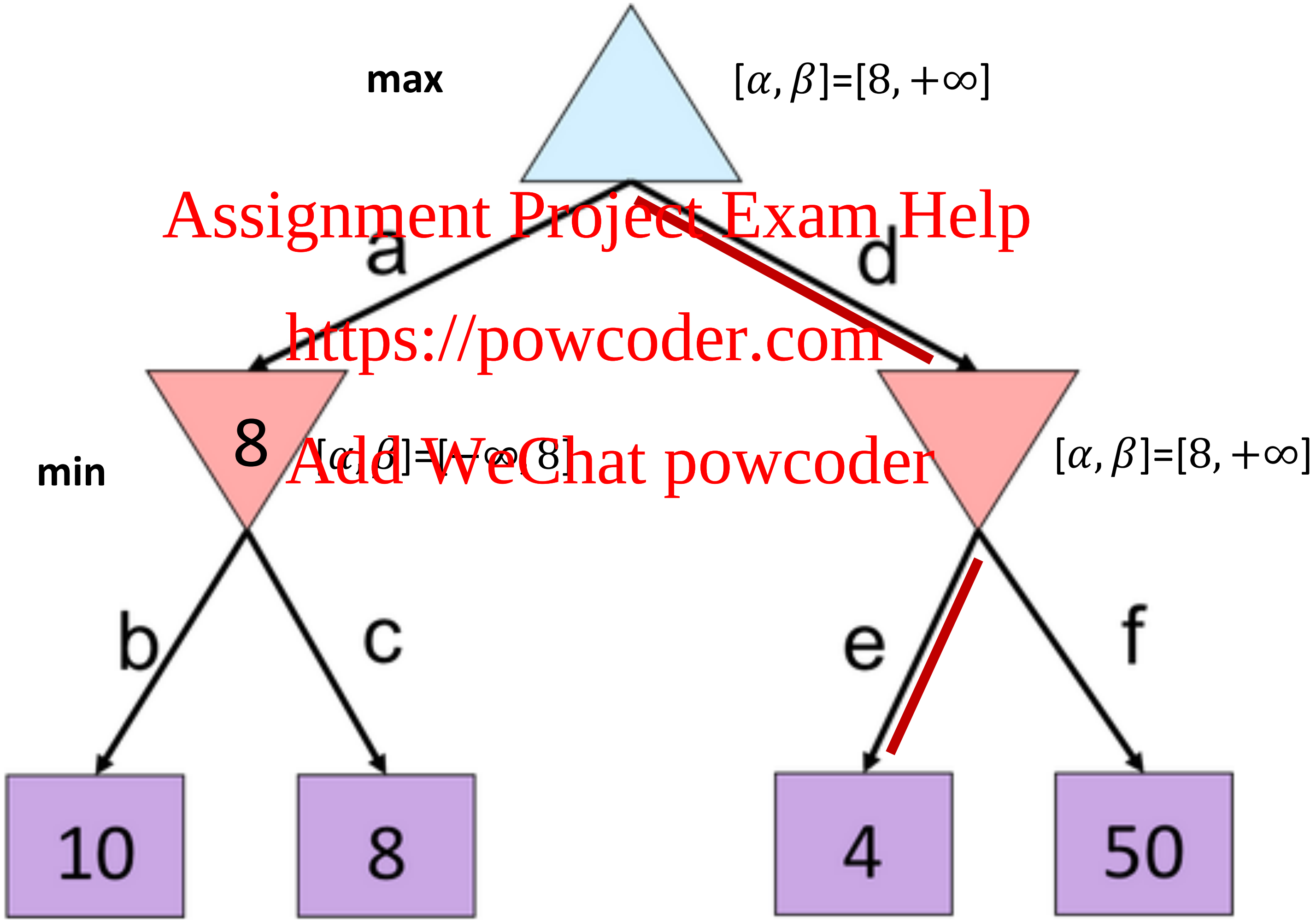
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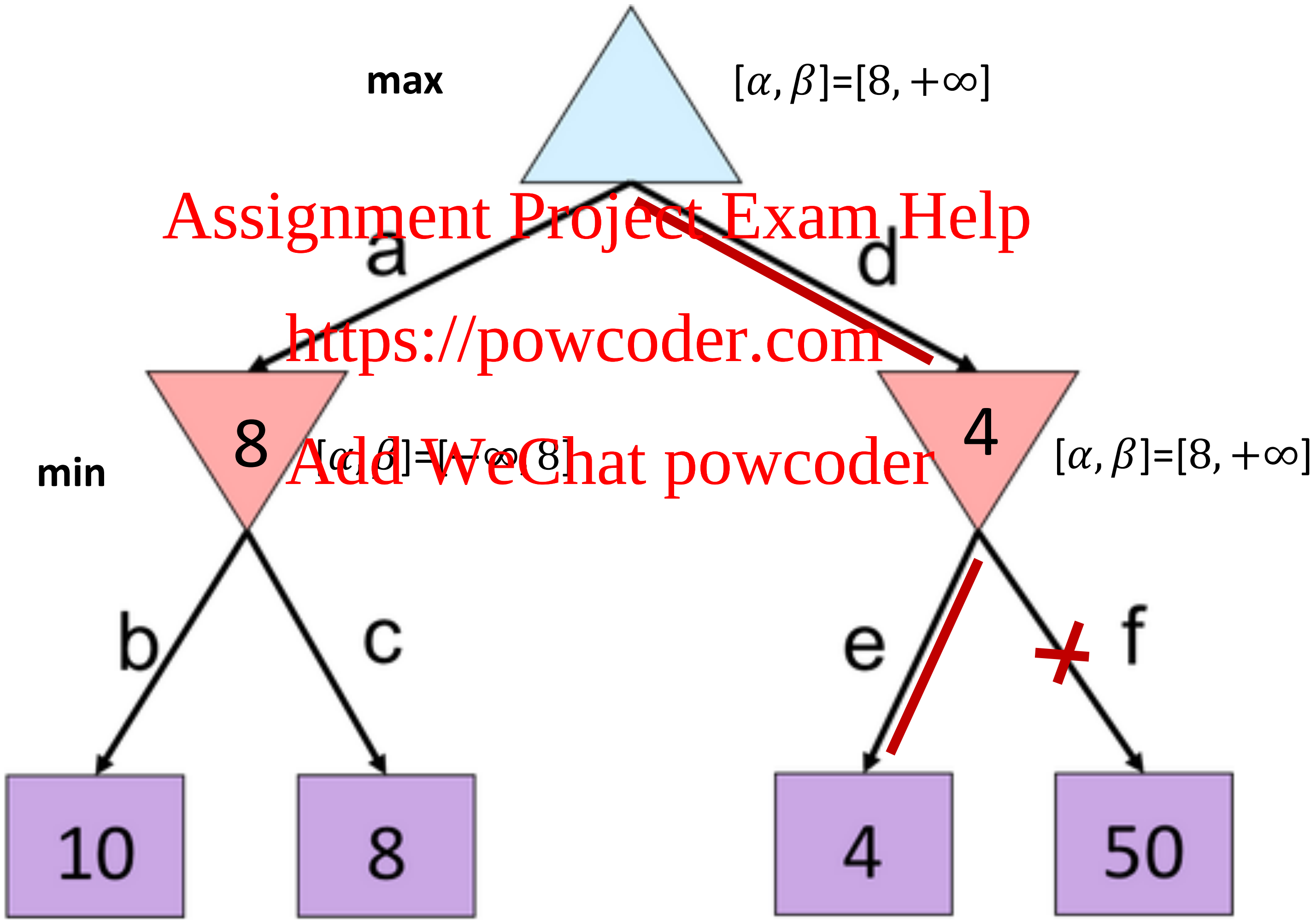
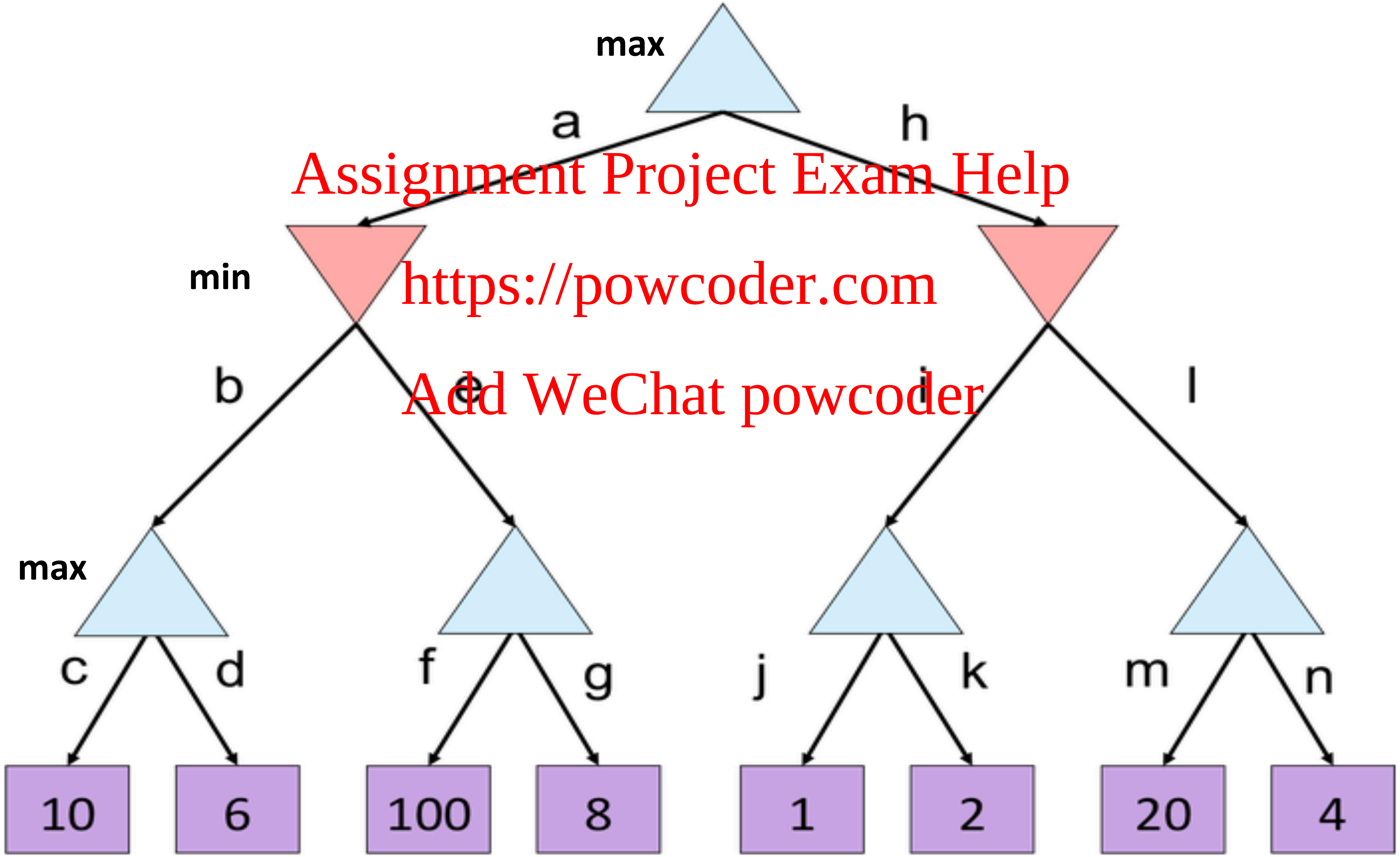


Alpha-Beta Quiz 2



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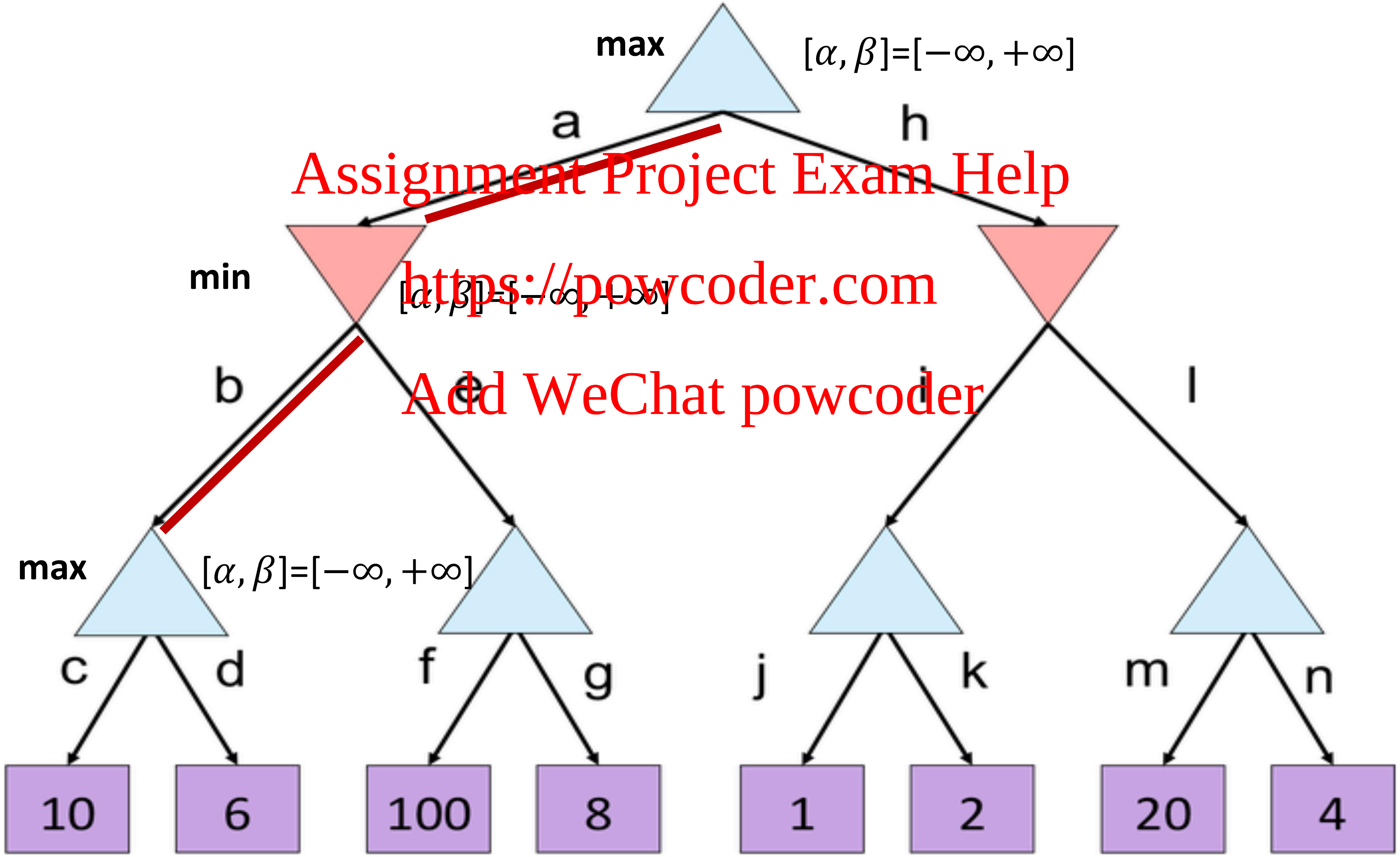

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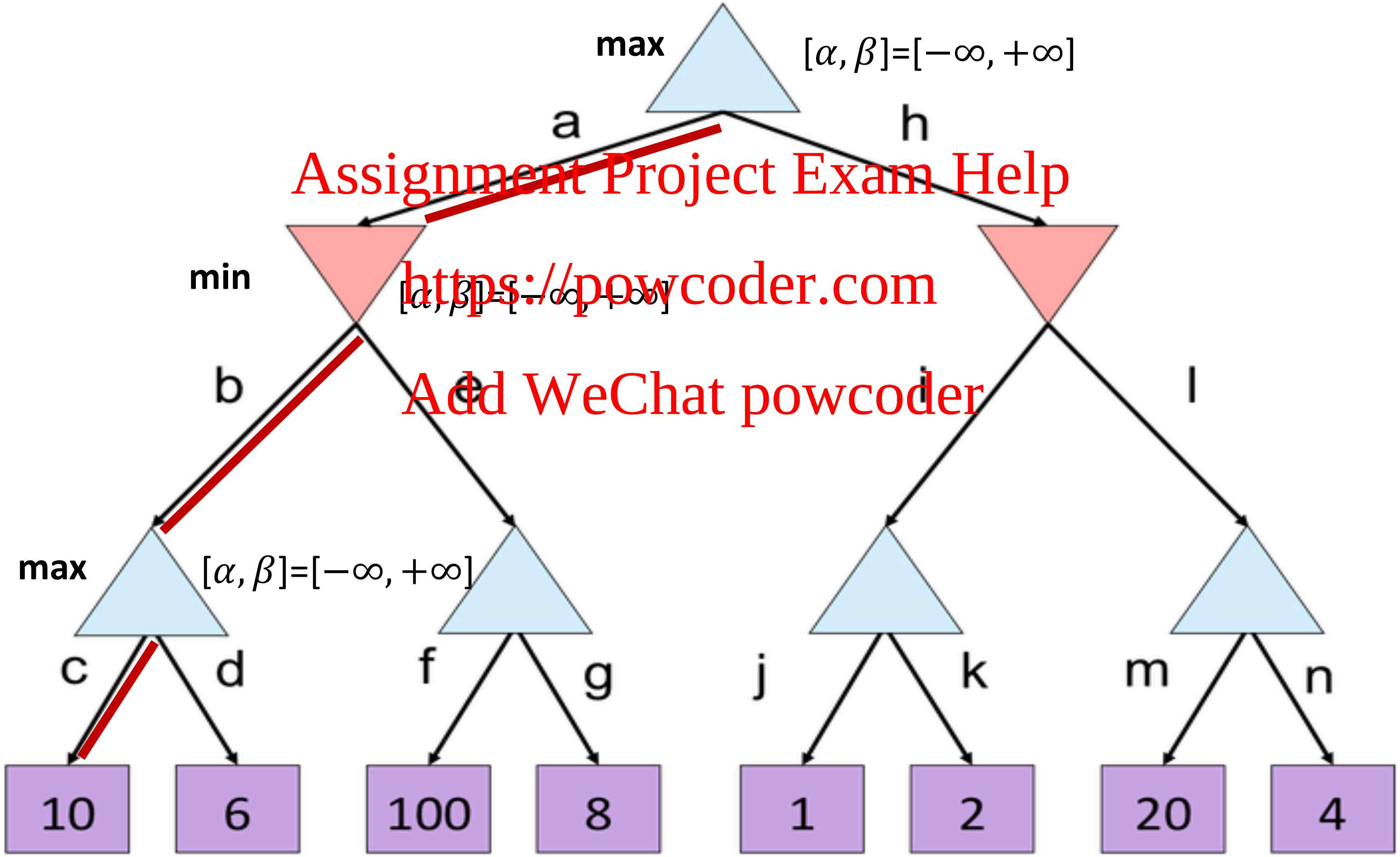
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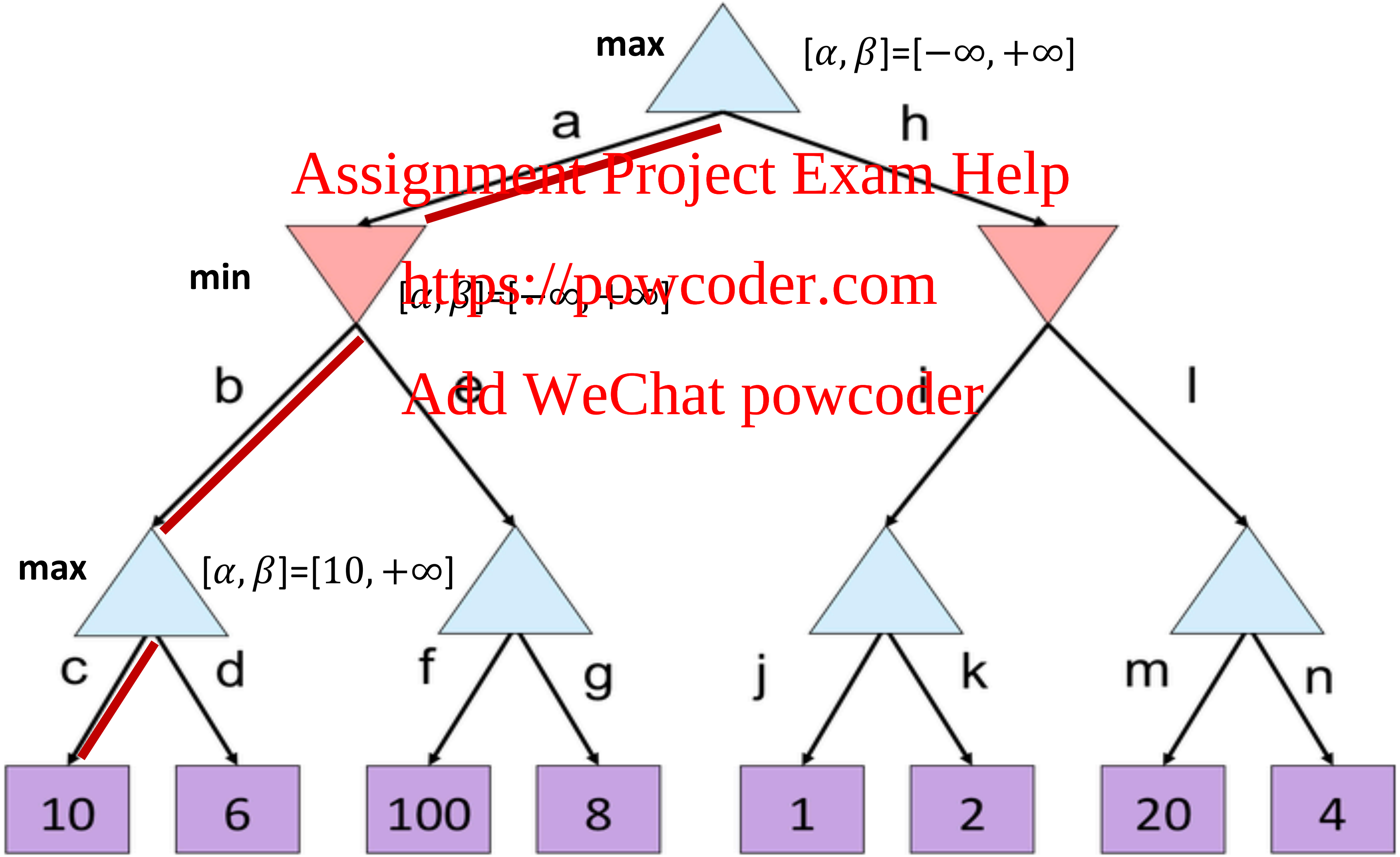
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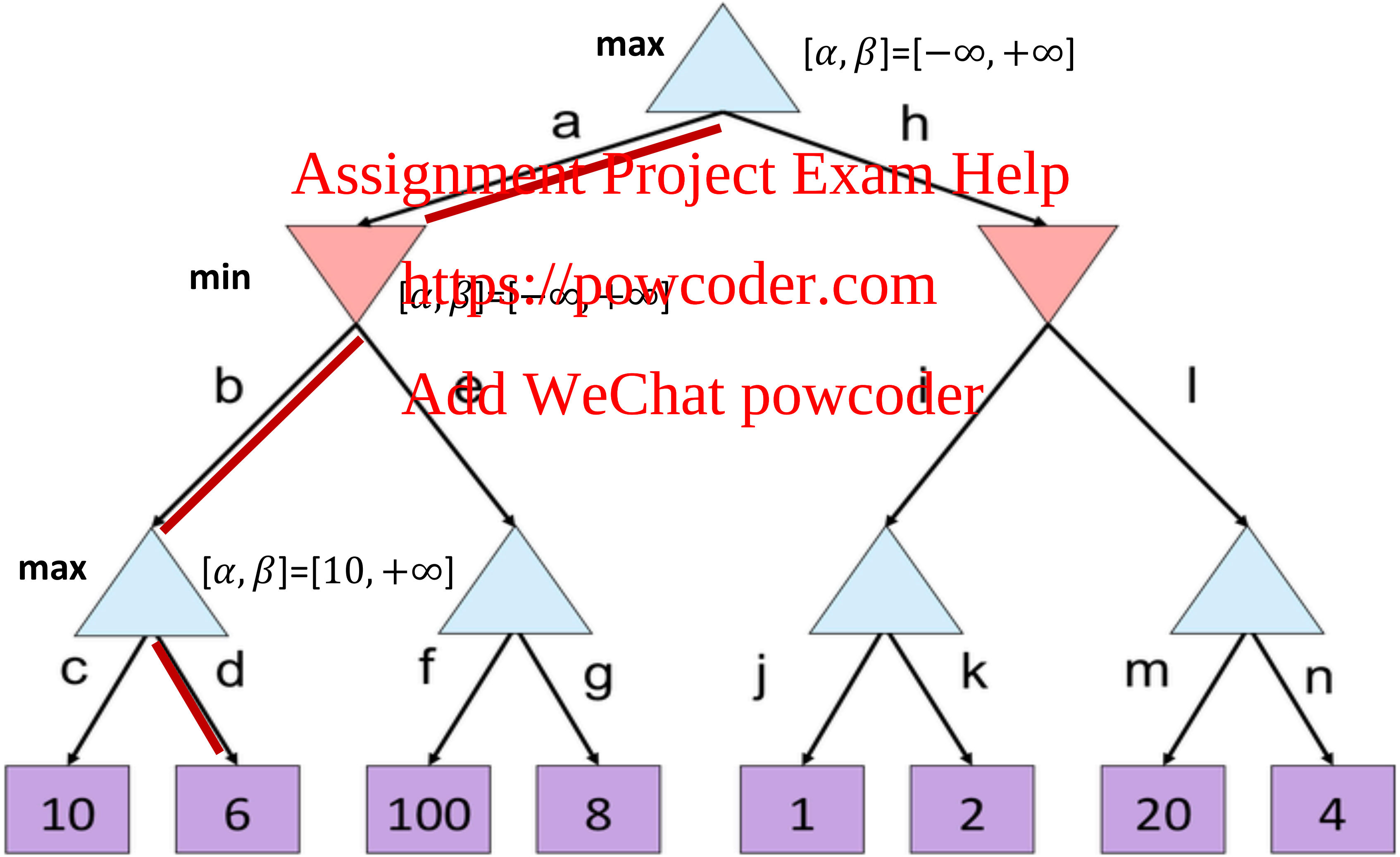
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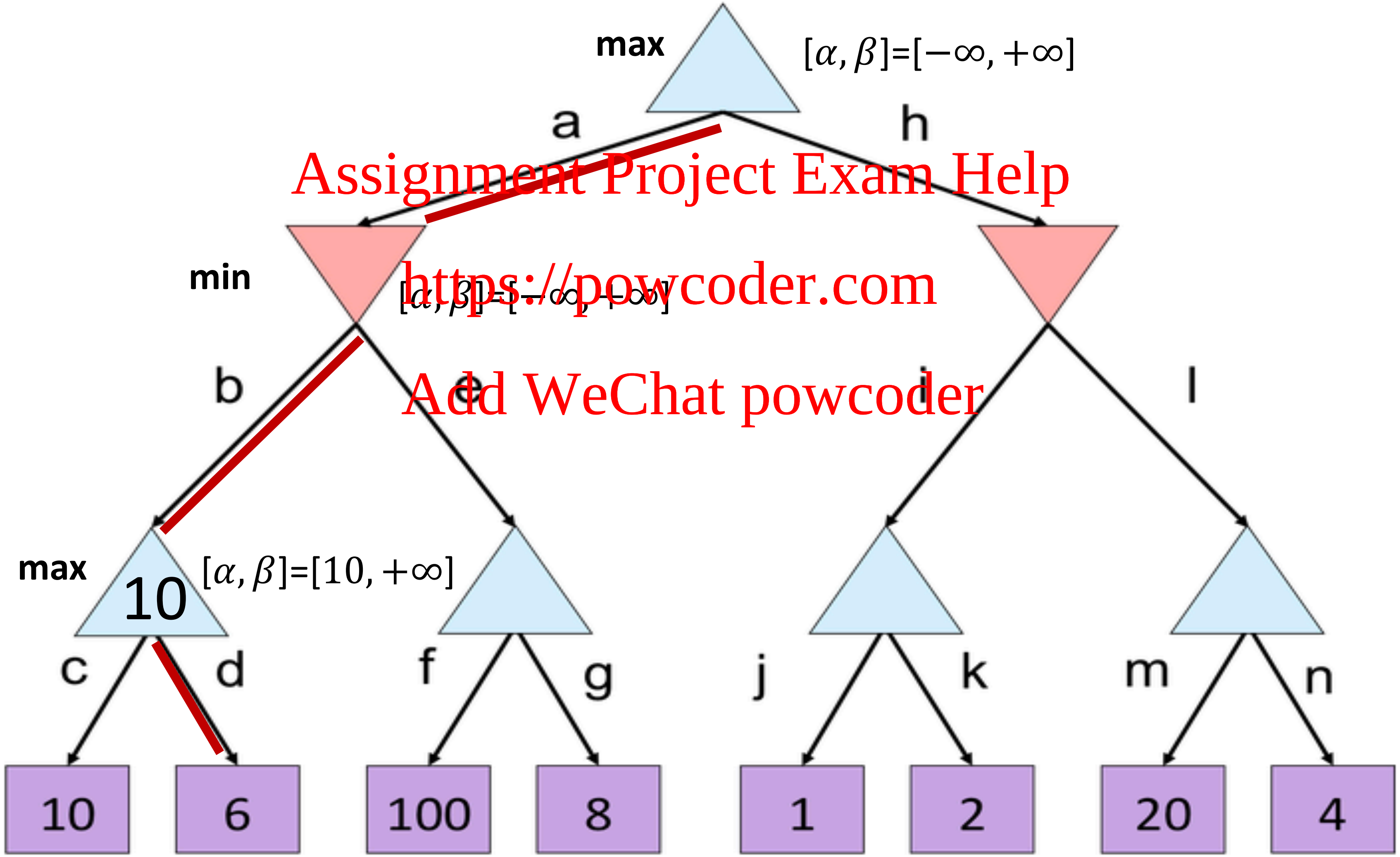
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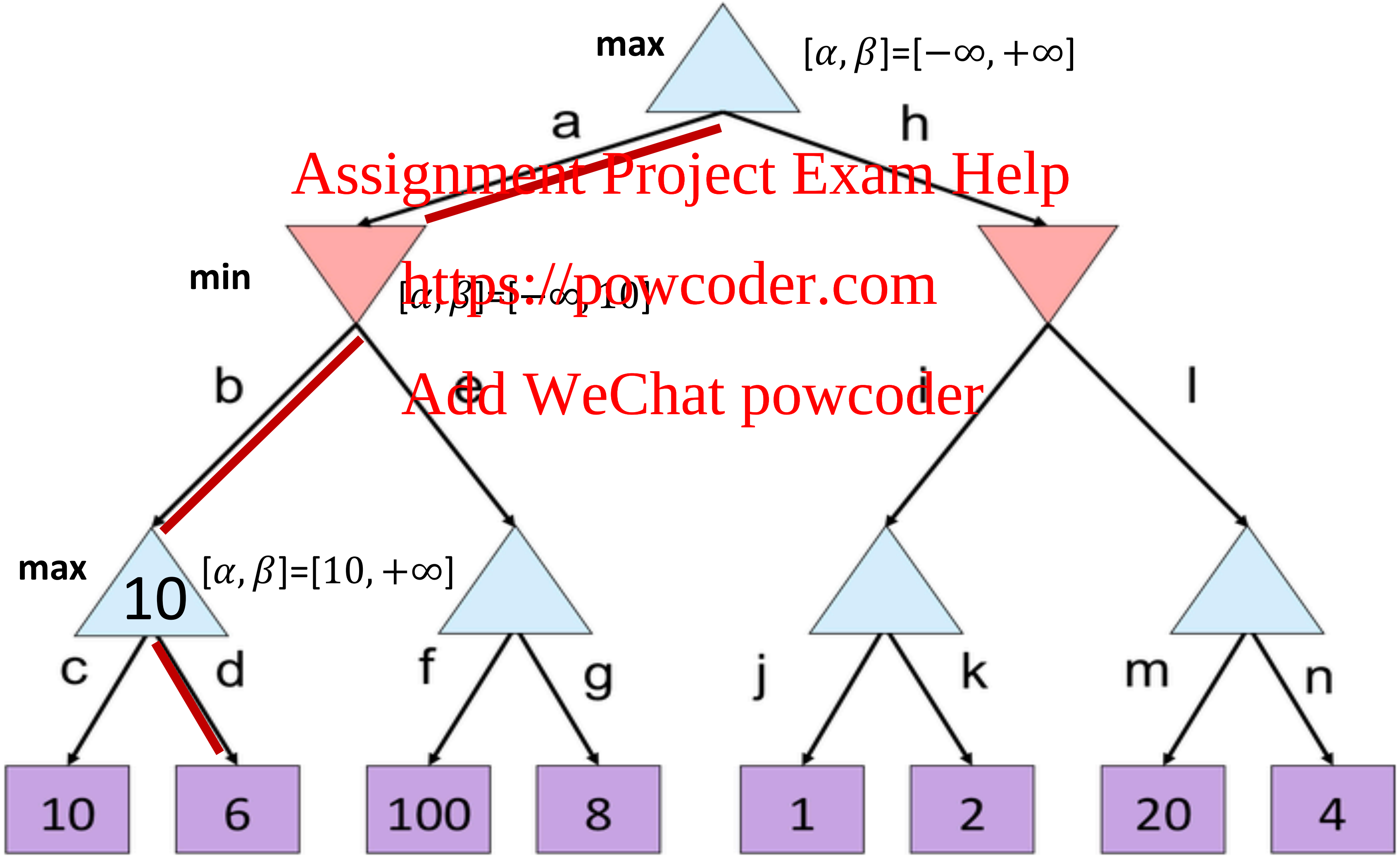
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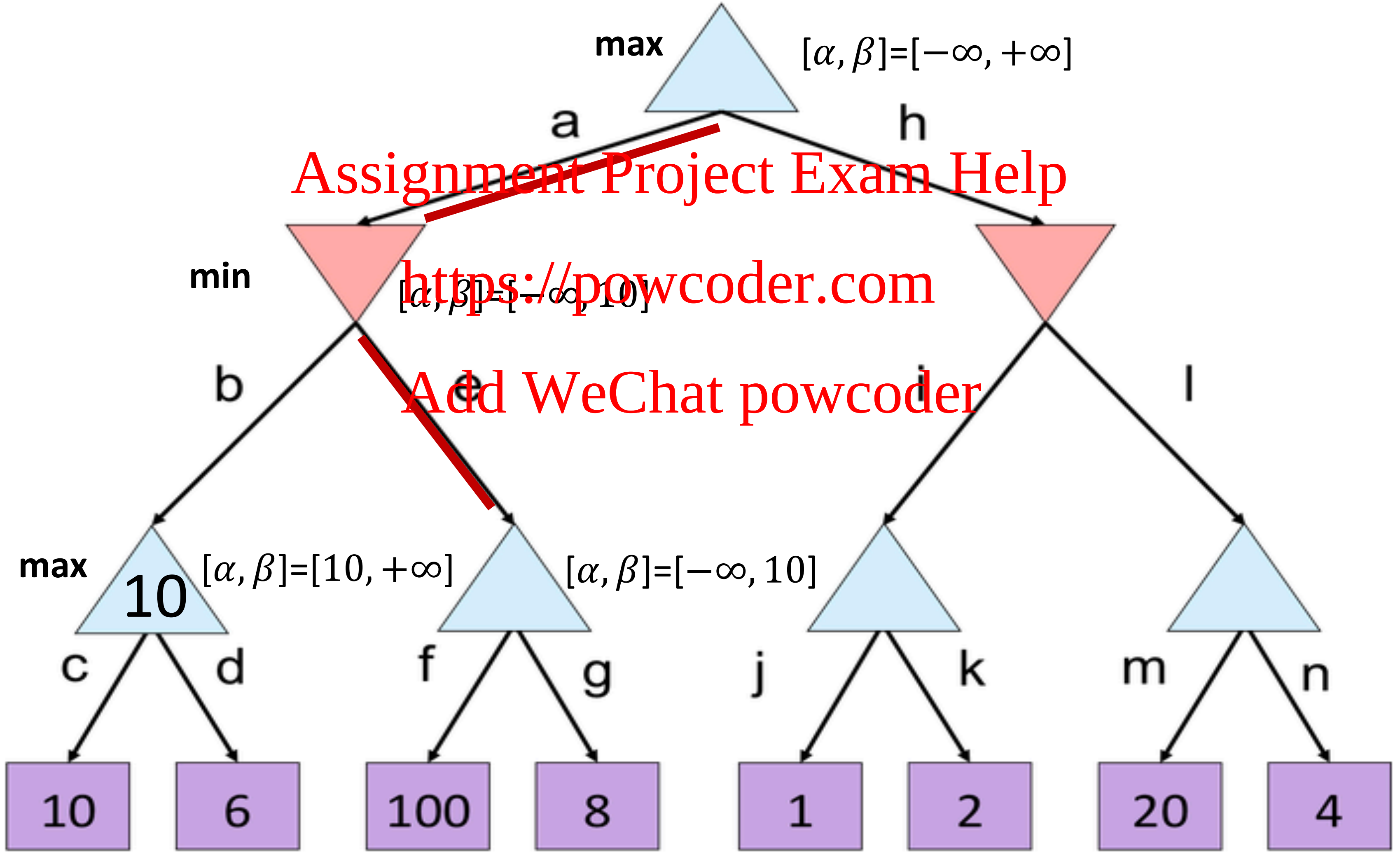
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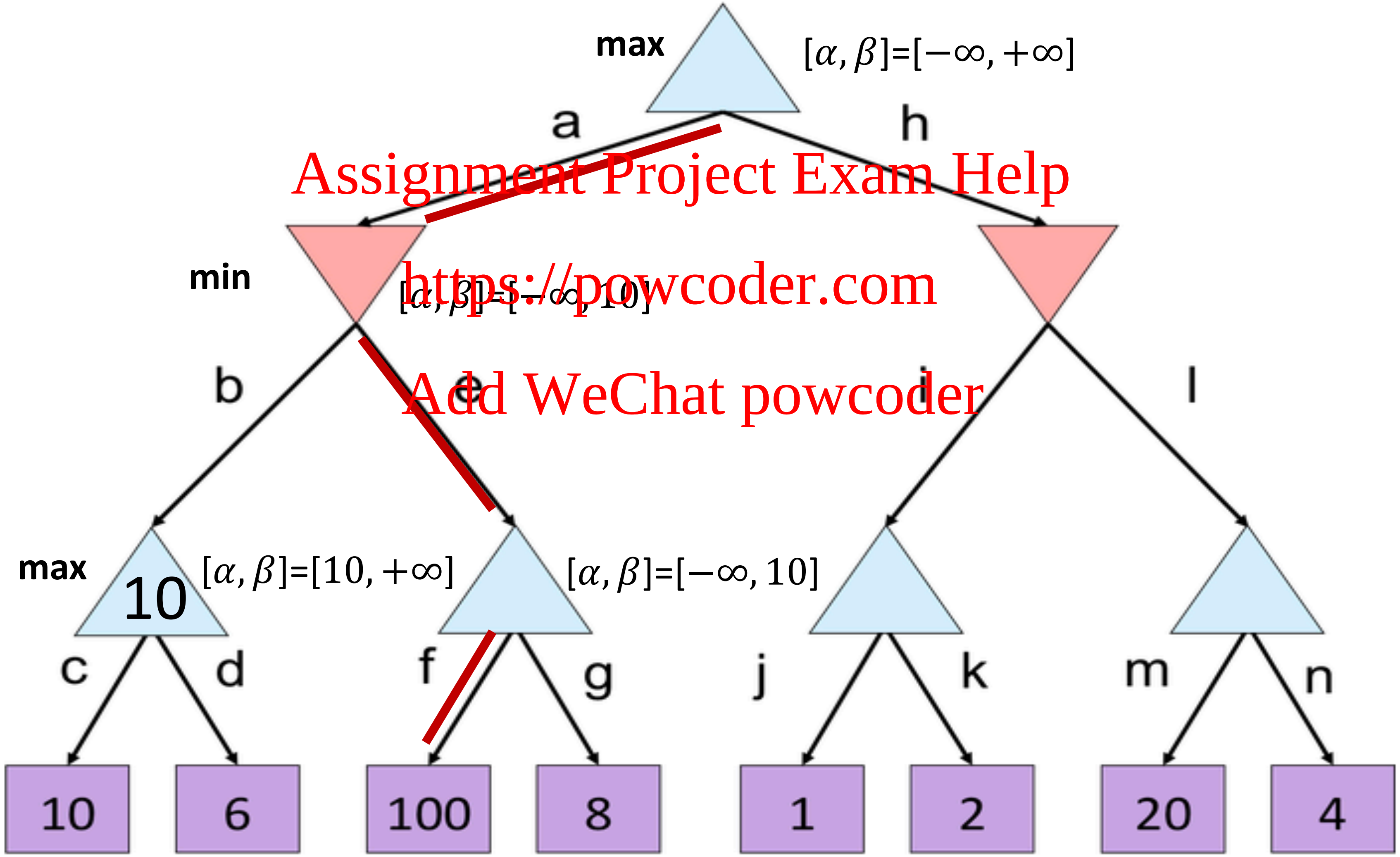
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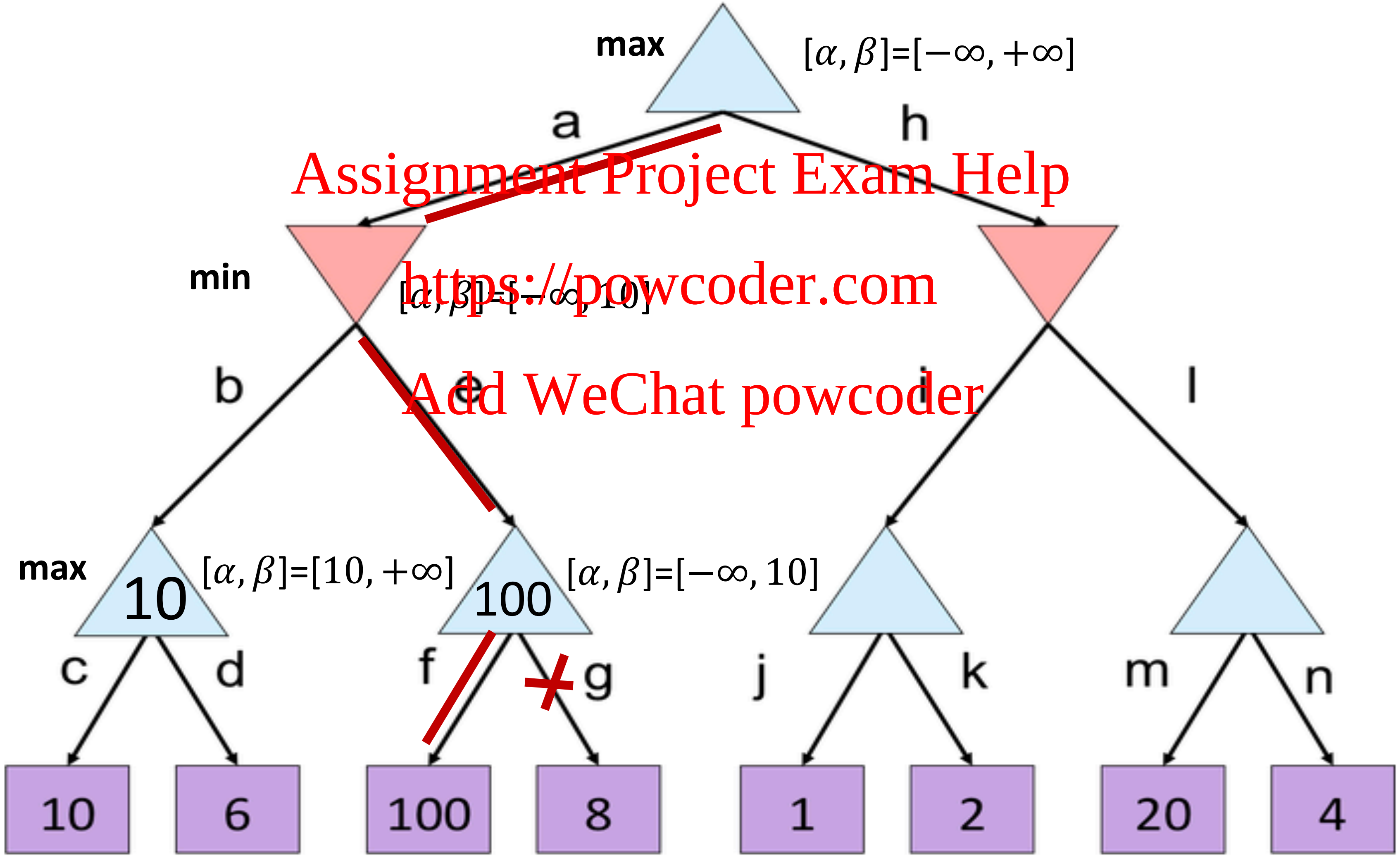
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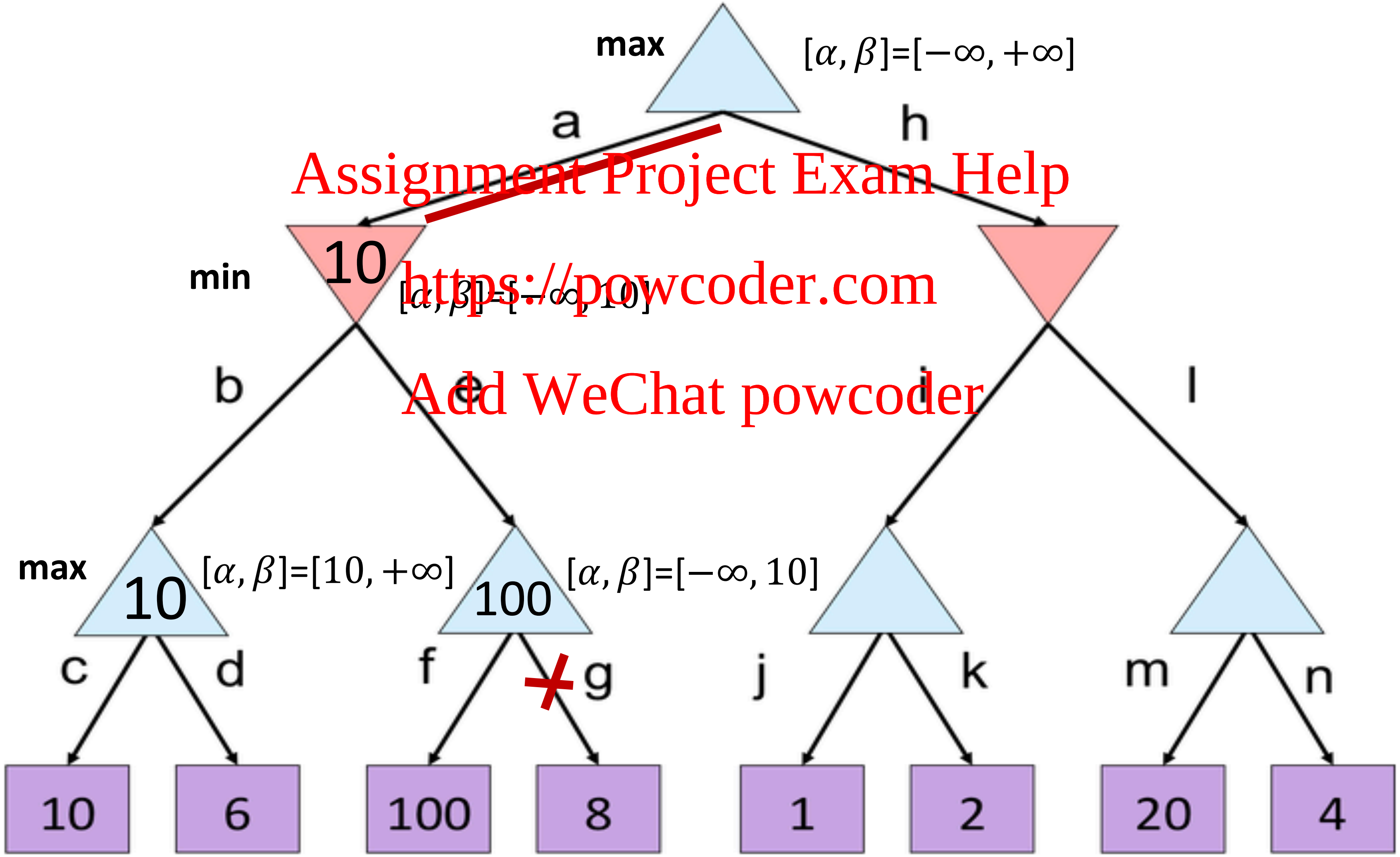

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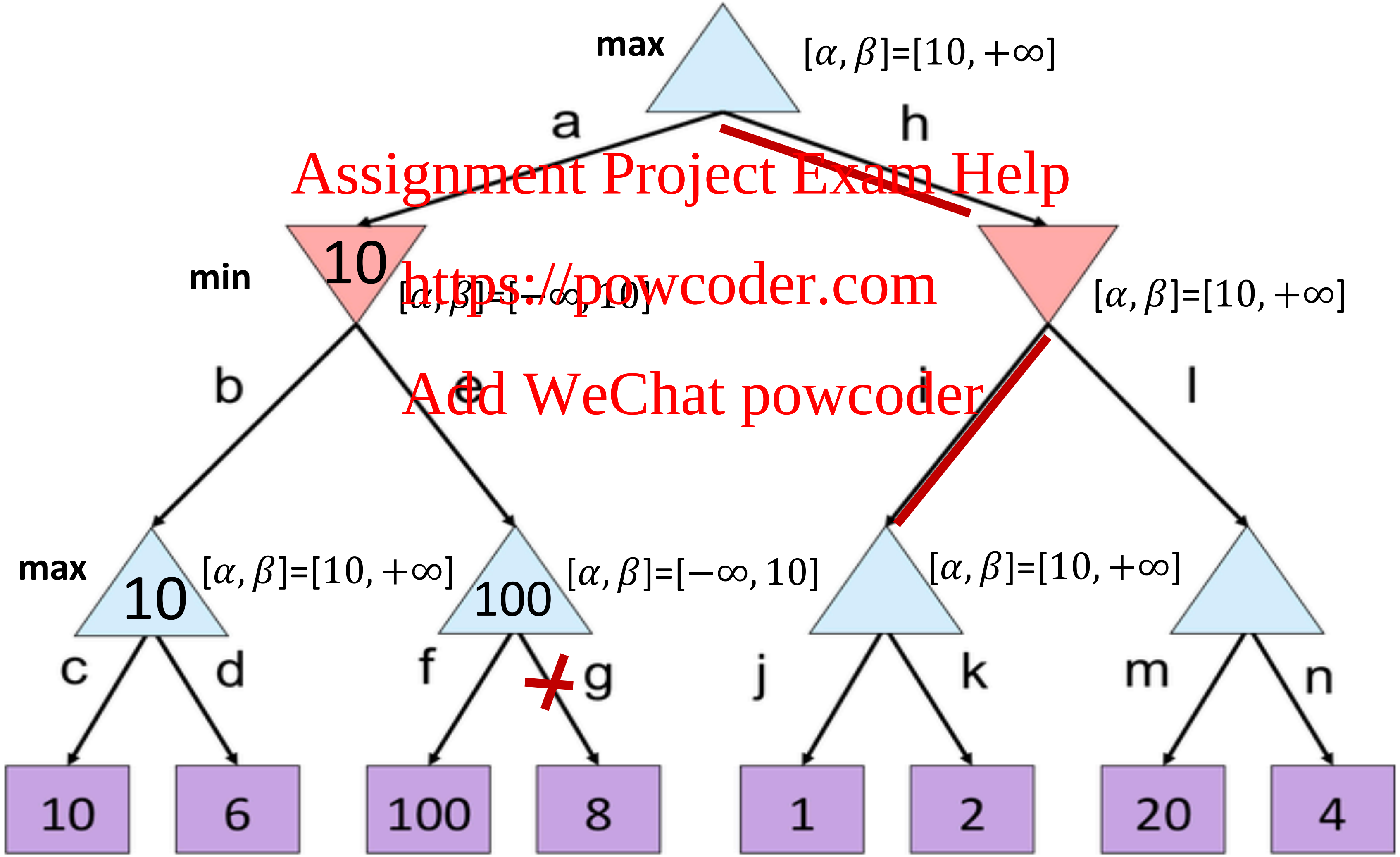

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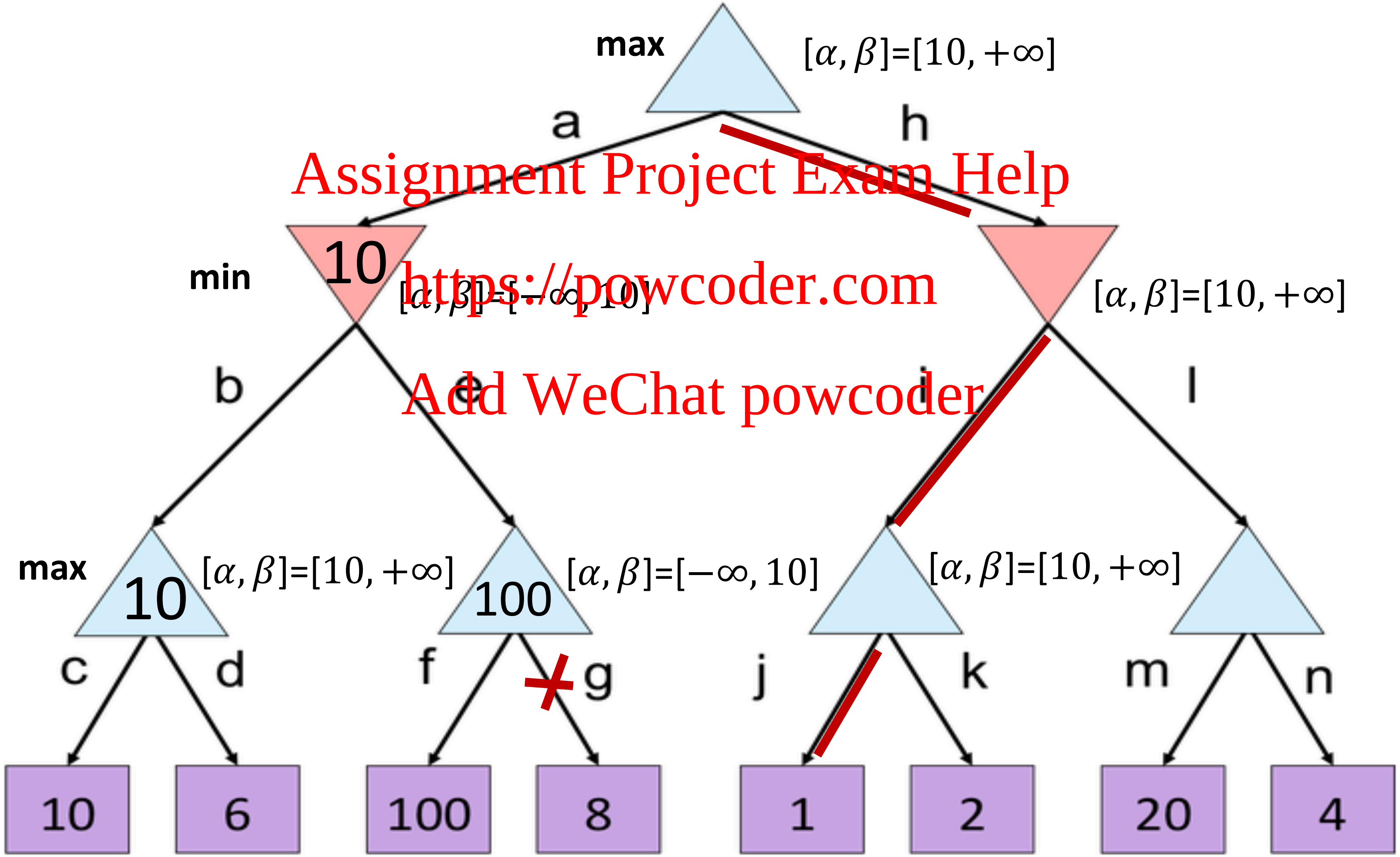

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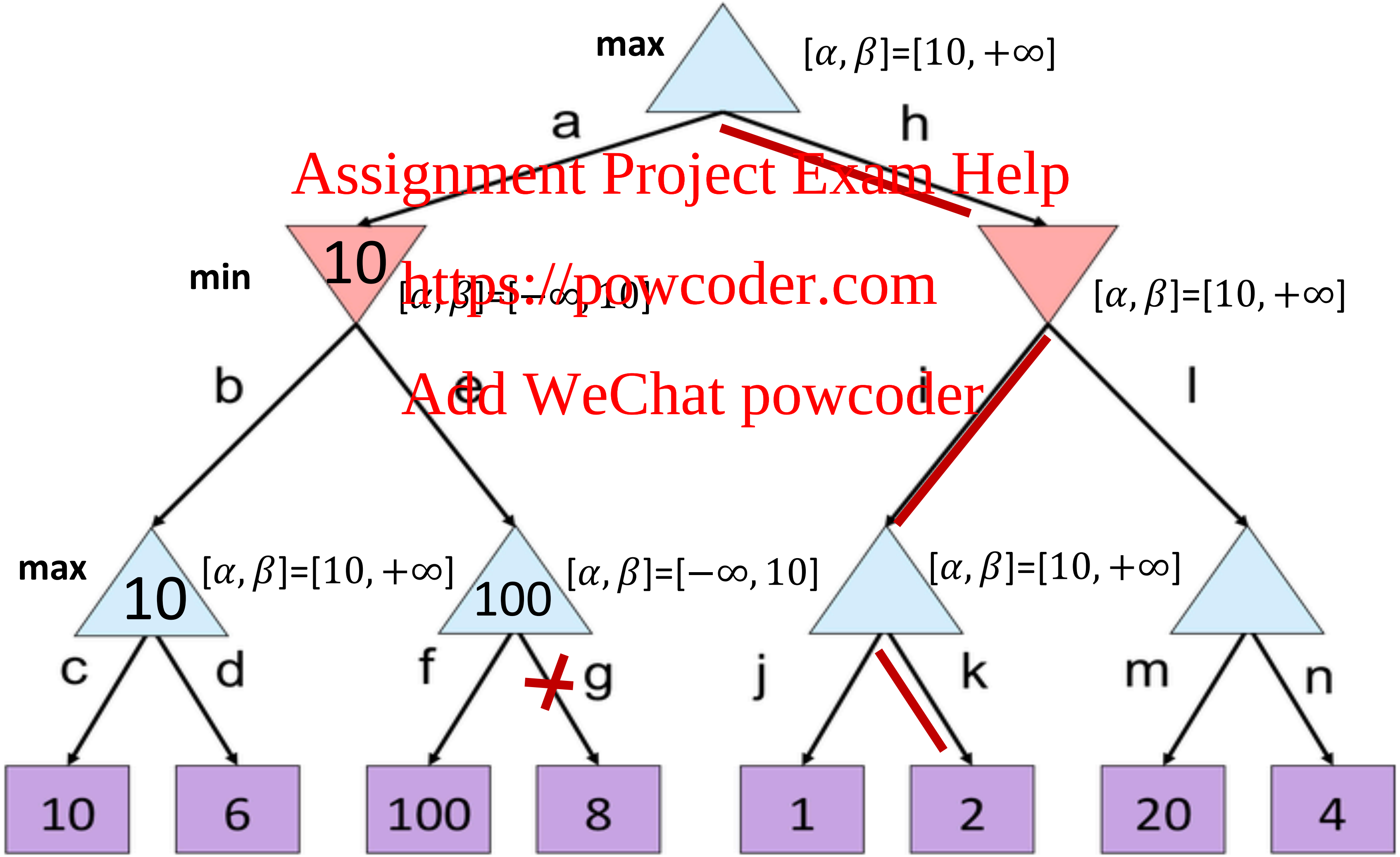

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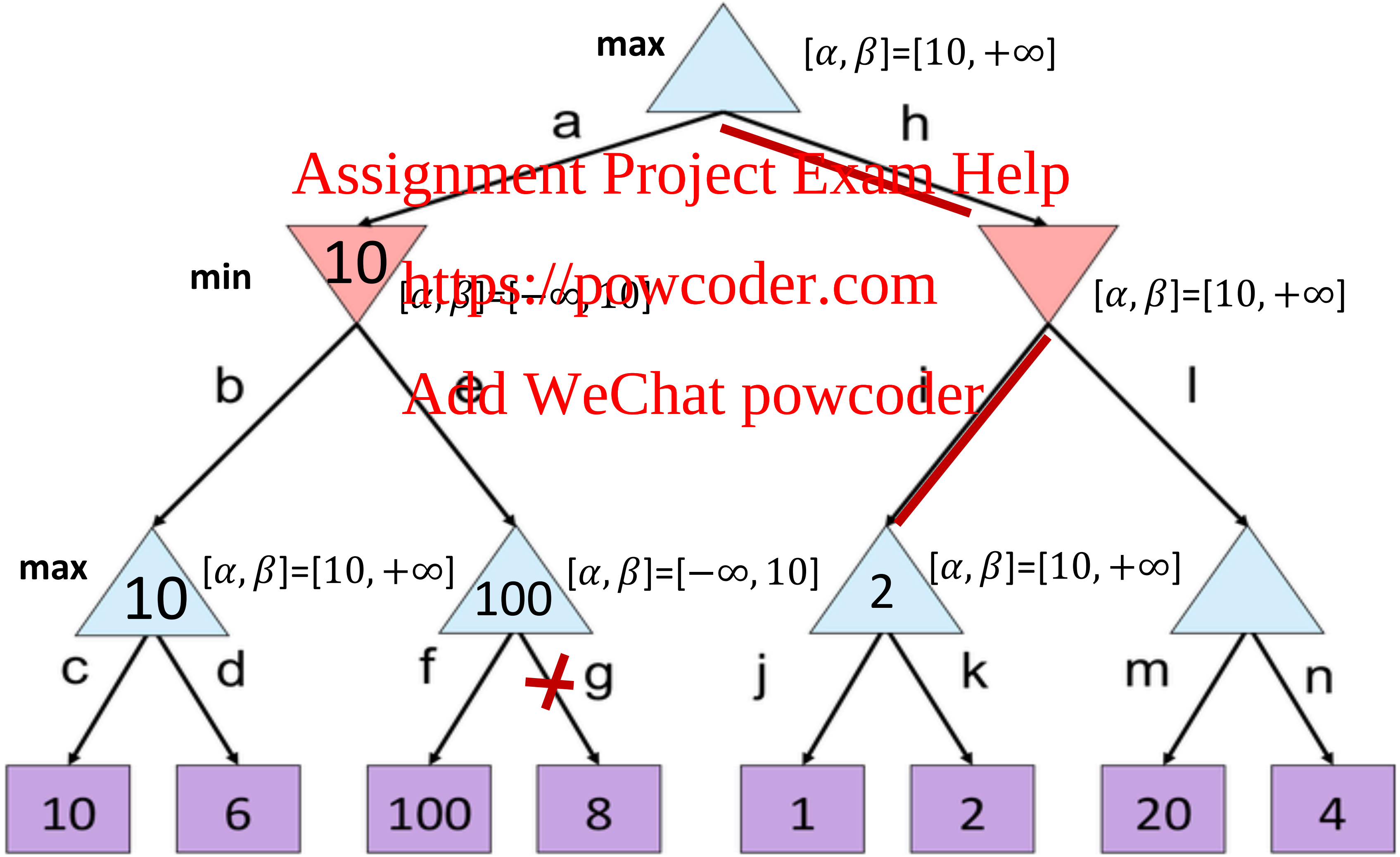

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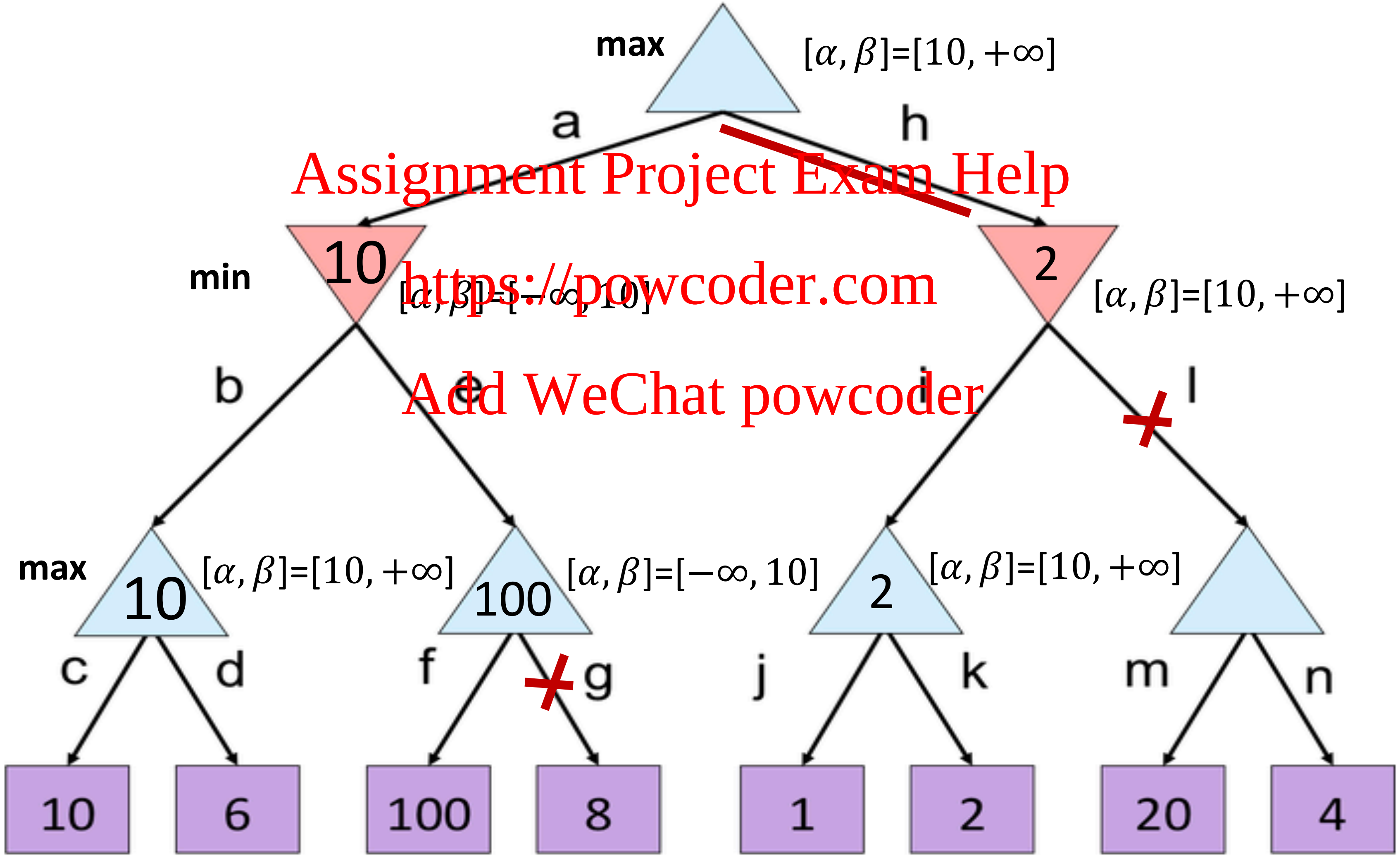
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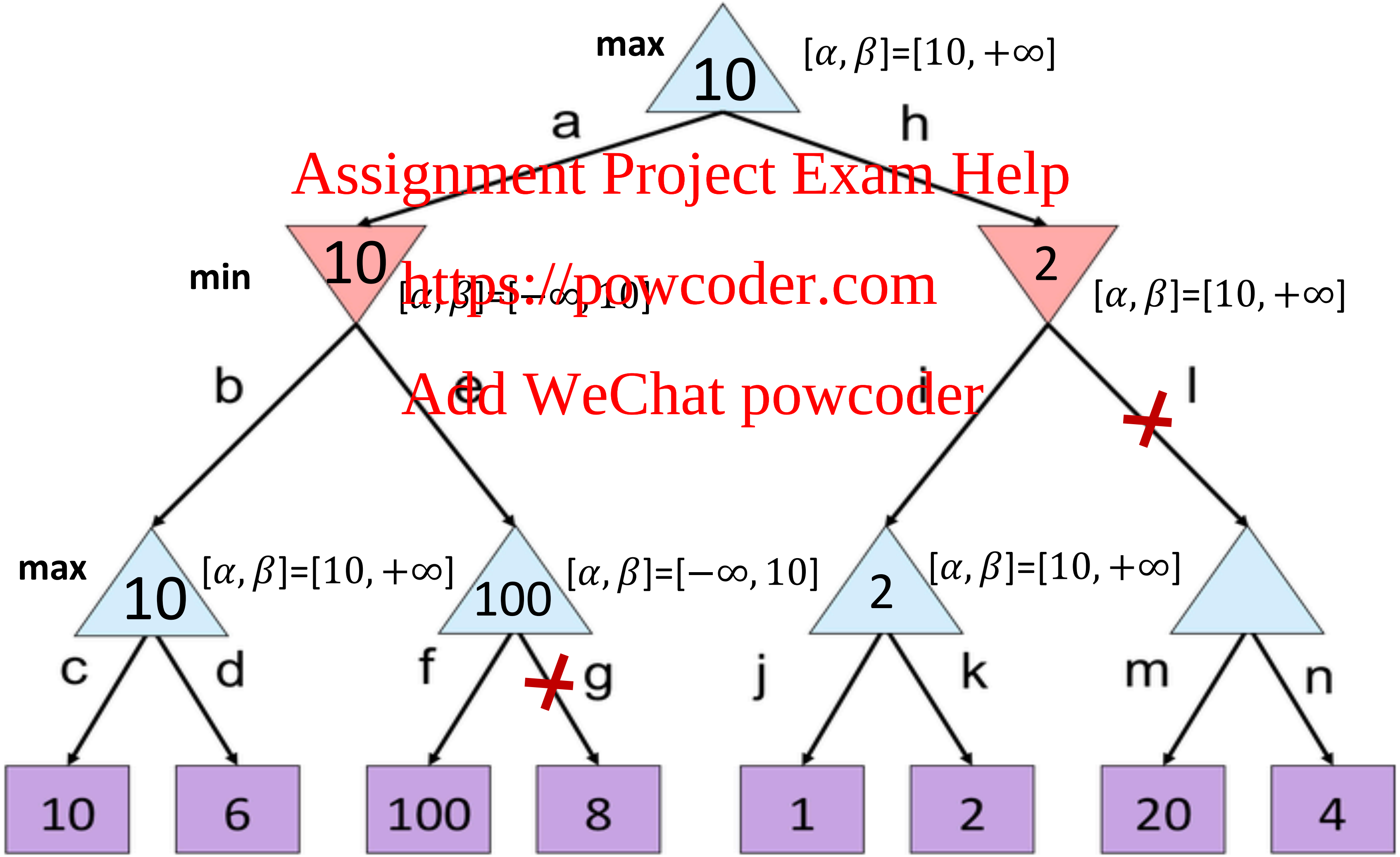
Alpha-Beta Quiz 2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[α, β]=[10, +∞]
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min
10
[α, β]=[−∞, 10]
2
[α, β]=[10, +∞]
b
e
i
l
max
10
[α, β]=[10, +∞]
100
[α, β]=[−∞, 10]
2
[α, β]=[10, +∞]
c
d
f
g
j
k
m
n
10
6
100
8
1
2
20
4

Resource Limits

Resource Limits

- Problem: In realistic games, cannot search to leaves!

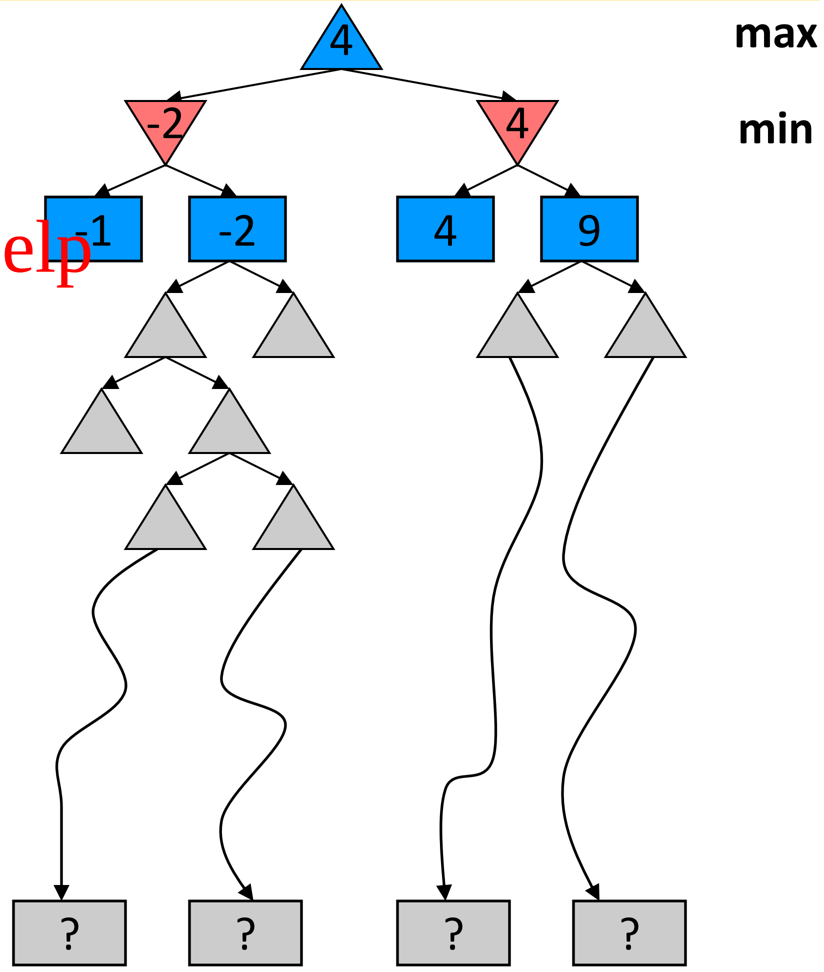
- Solution: Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions

- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - α-β reaches about depth 8 – decent chess program

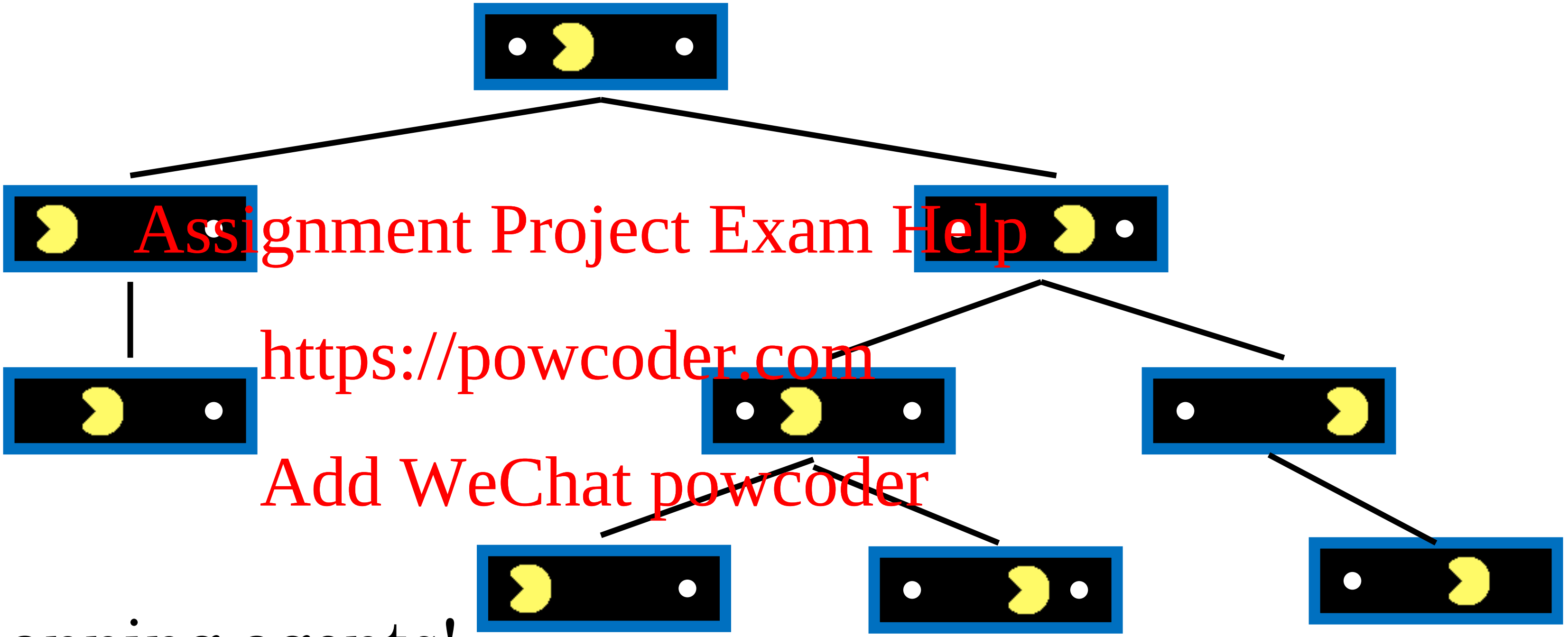
- Guarantee of optimal play is gone

- More plies makes a BIG difference

- Use iterative deepening for an anytime algorithm

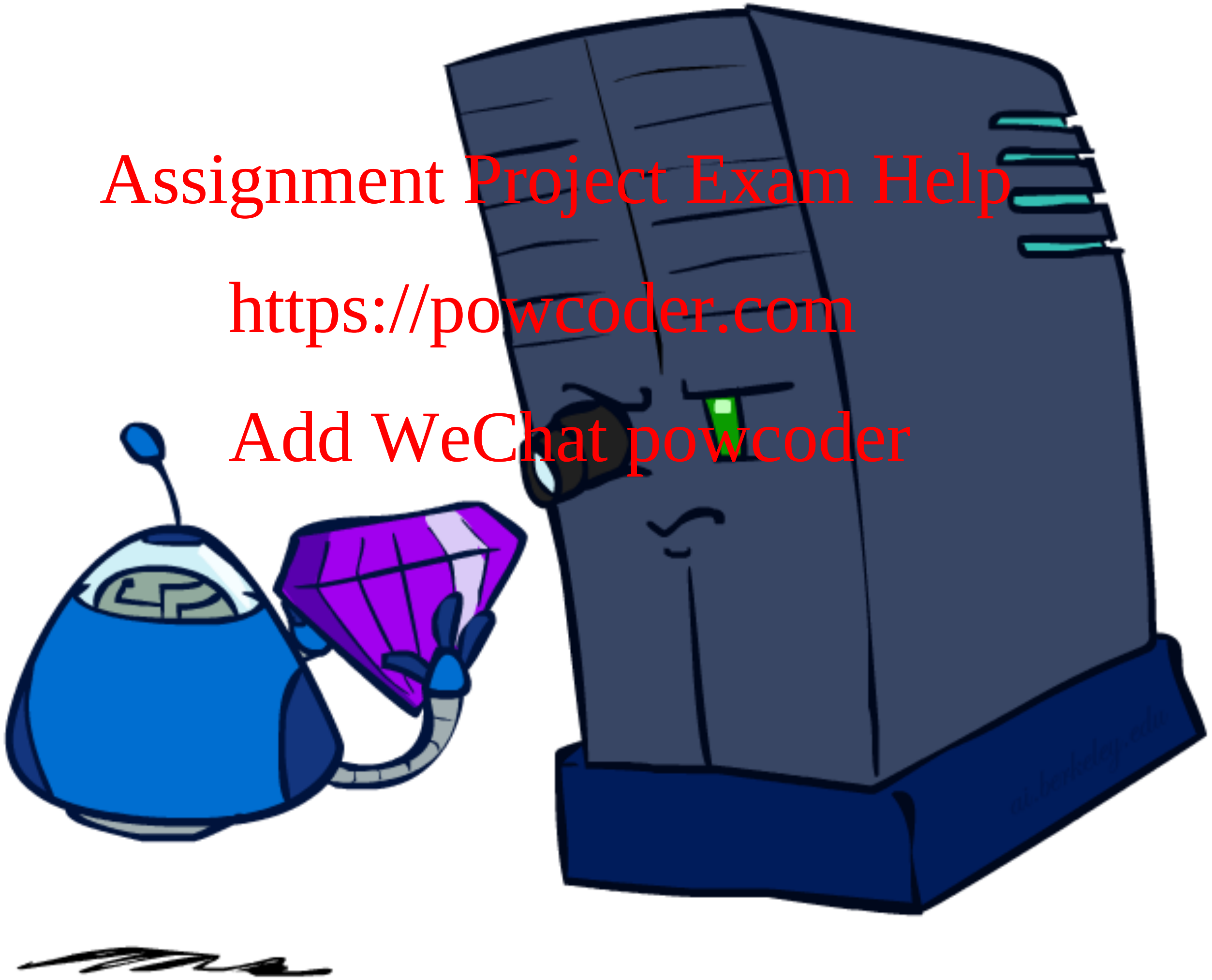


Why Pacman Starves



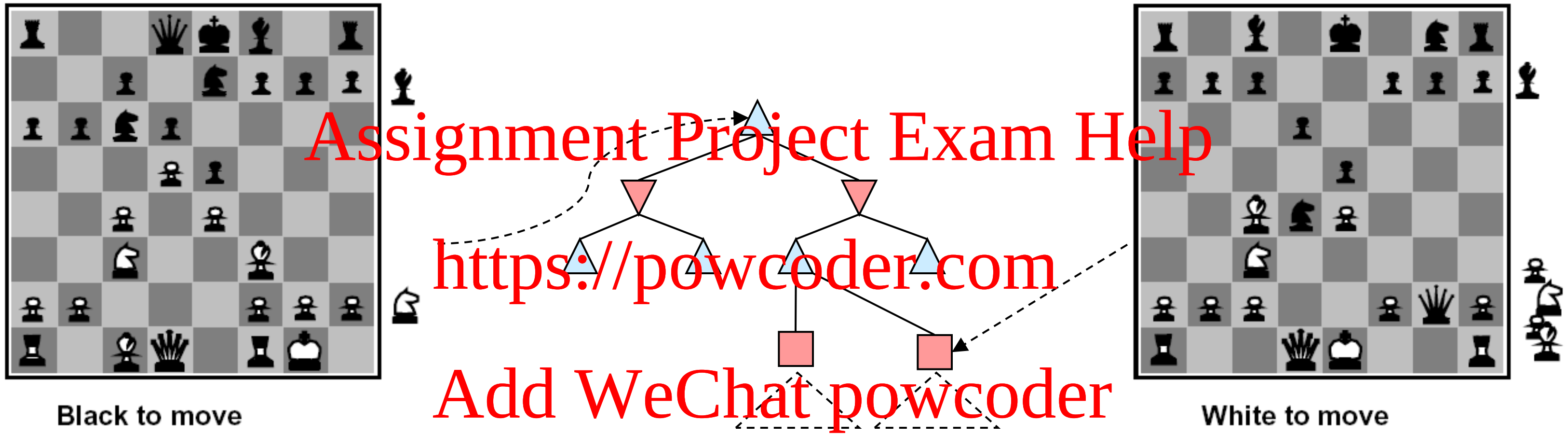
- A danger of replanning agents!
  - He knows his score will go up by eating the dot now (west, east)
  - He knows his score will go up just as much by eating the dot later (east, west)
  - There are no point-scoring opportunities after eating the dot (within the horizon, two here)
  - Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!

Evaluation Functions



Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search



Black to move

White slightly better

White to move

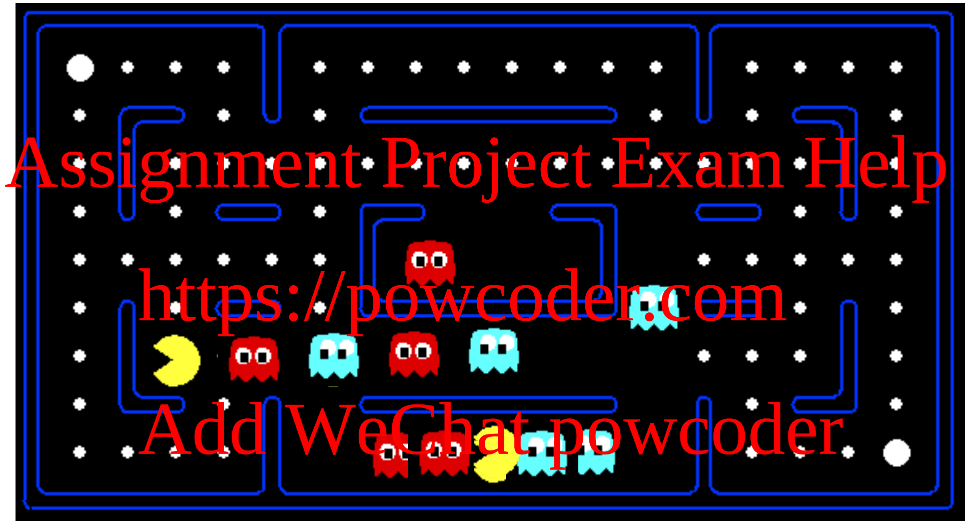
Black winning

- Ideal function: returns the actual minimax value of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

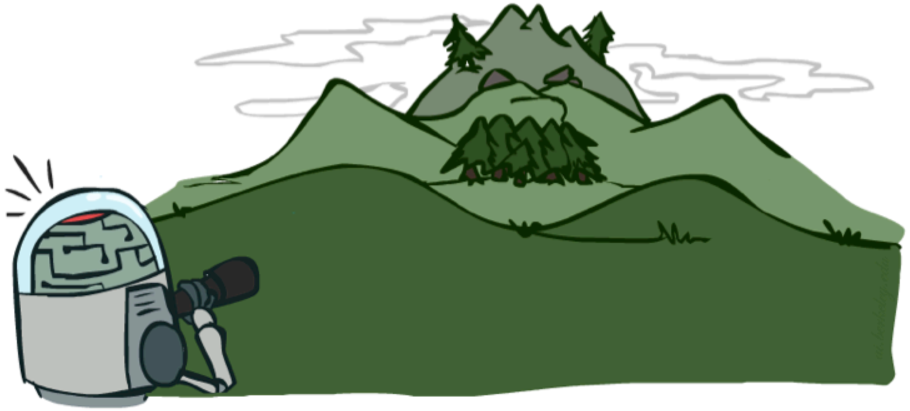
- e.g. $f_1(s)$ = (num white queens – num black queens), etc.

Evaluation for Pacman



Depth Matters

- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters
- An important example of the tradeoff between complexity of features and complexity of computation



[Demo: depth limited (L6D4, L6D5)]

Synergies between Evaluation Function and Alpha-Beta?

- Alpha-Beta: amount of pruning depends on expansion ordering
  - Evaluation function can provide guidance to expand most promising nodes first (which later makes it more likely there is already a good alternative on the path to the root)
    - (somewhat similar to role of A* heuristic, CSPs filtering)

- Alpha-Beta: (similar for roles of min-max swapped)
  - Value at a min-node will only keep going down
  - Once value of min-node lower than better option for max along path to root, can prune
  - Hence: IF evaluation function provides upper-bound on value at min-node, and upper-bound already lower than better option for max along path to root THEN can prune