CISC 6525 Artificial Intelligence

Informed search algorithms:

Local, A* and Adversarial

Informed Search

- Best-first & greedy best-first search, A^* search & Heuristics – Chapter 3 (3.5-6)
- Local search algorithms – Chapter 4 (4.1)
  - Hill-climbing search
  - Simulated annealing search
  - Local beam search
  - Genetic algorithms
- Adversarial search – Chapter 5 (5.1-4)
  - Games trees & Optimality
  - α-β pruning
  - Imperfect, real-time decisions

Review: Tree search

function TREE-SEARCH(*problem*, *strategy*) **returns** a solution, or failure
  initialize the search tree using the initial state of *problem*
  loop do
    if there are no candidates for expansion **then return** failure
    choose a leaf node for expansion according to *strategy*
    if the node contains a goal state **then return** the corresponding solution
    else expand the node and add the resulting nodes to the search tree

- A search strategy is defined by picking the order of node expansion

Best-first search

- Idea: use an evaluation function *f(n)* for each node
  - estimate of "desirability"

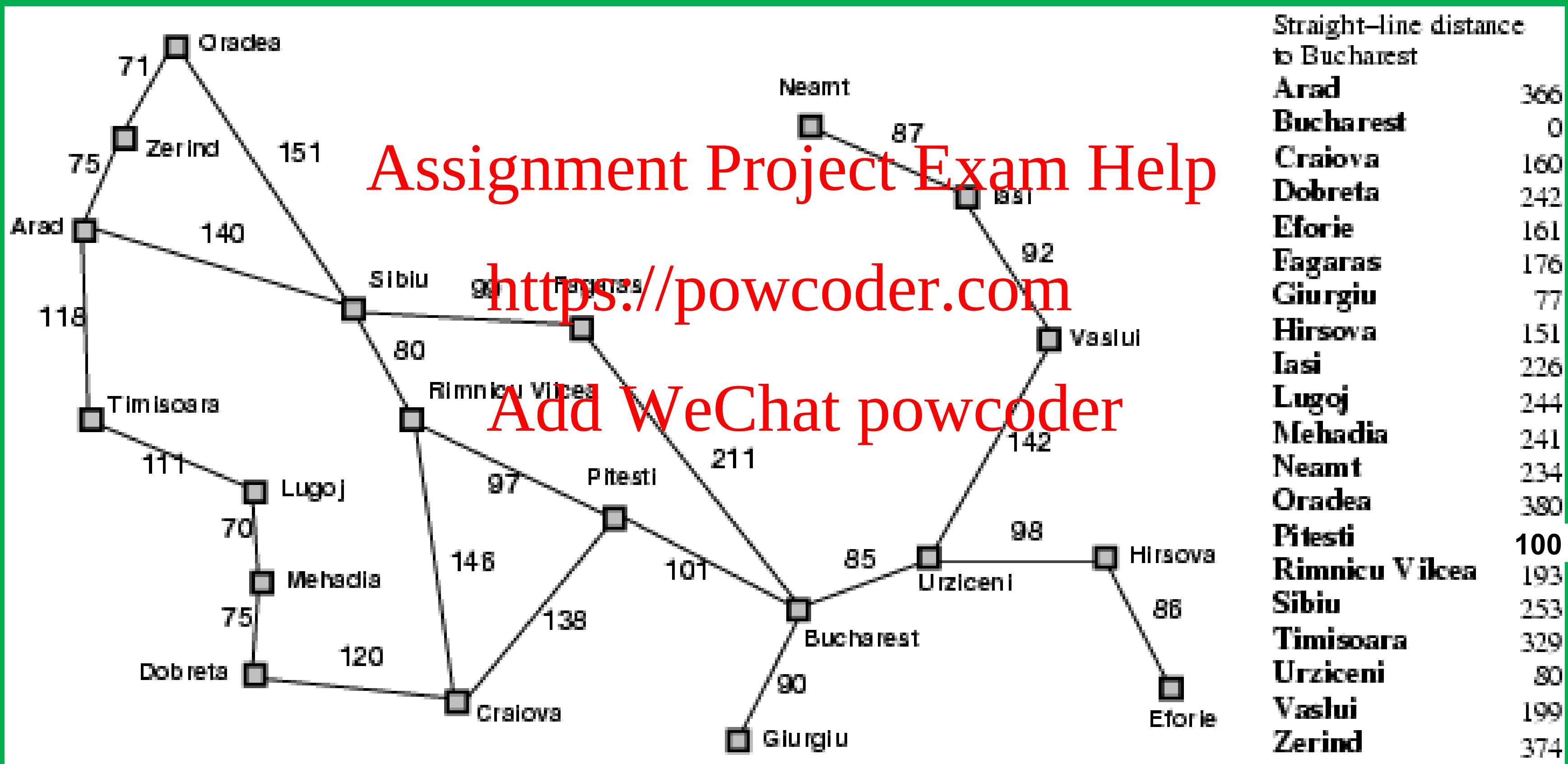
  → Expand most desirable unexpanded node

- Implementation:

  Order the nodes in fringe in decreasing order of desirability

- Special cases:
  - greedy best-first search
  - A^* search

Romania with step costs in km



Greedy best-first search

- Evaluation function $f(n) = h(n)$ (heuristic)
- = estimate of cost: from n to *goal*
- e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest
- Greedy best-first search expands the node that appears to be closest to goal

Straight line distance

Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
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Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

Greedy best-first search example

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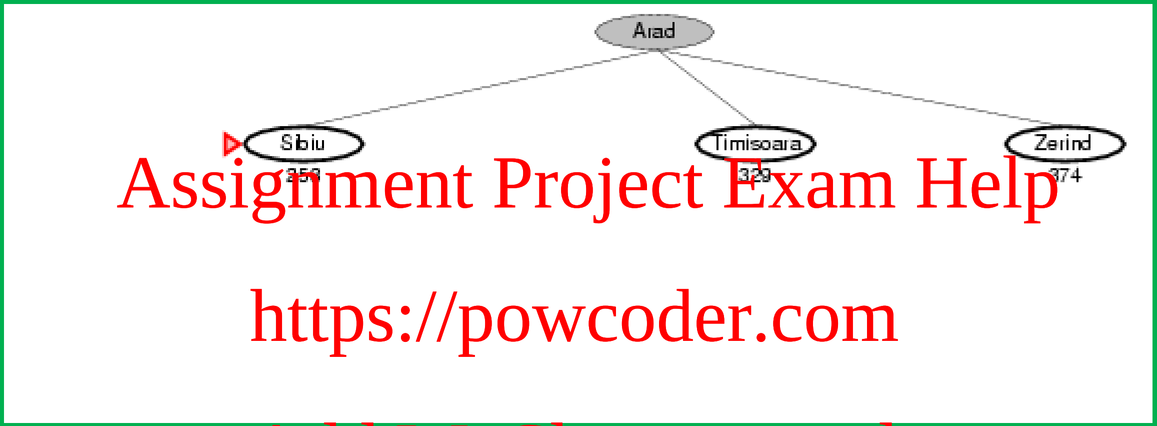

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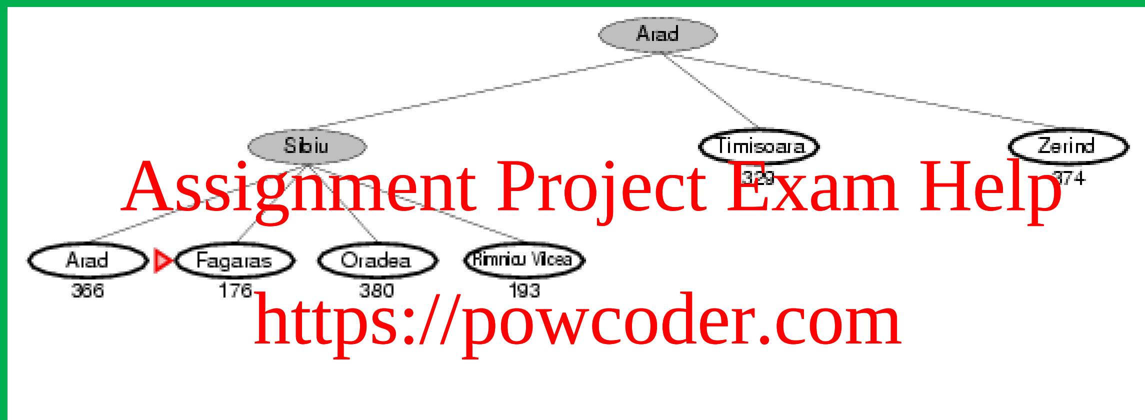

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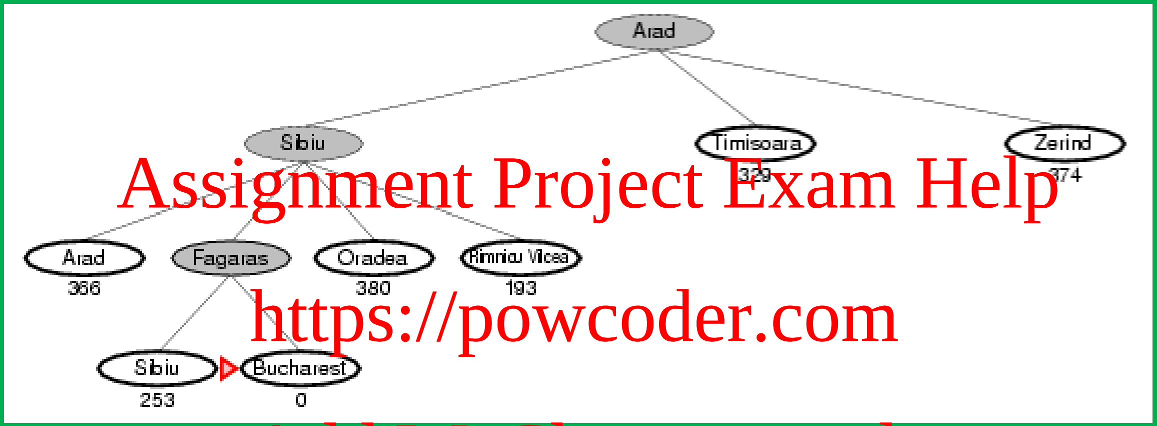

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Properties of greedy best-first search

- Complete? No – can get stuck in loops,

E.g., Consider Iasi to Fagaras:

Iasi → Neamt → Iasi → Neamt → ……..

- Time? $O(b^m)$, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ -- keeps all nodes in memory
- Optimal? No

A* search

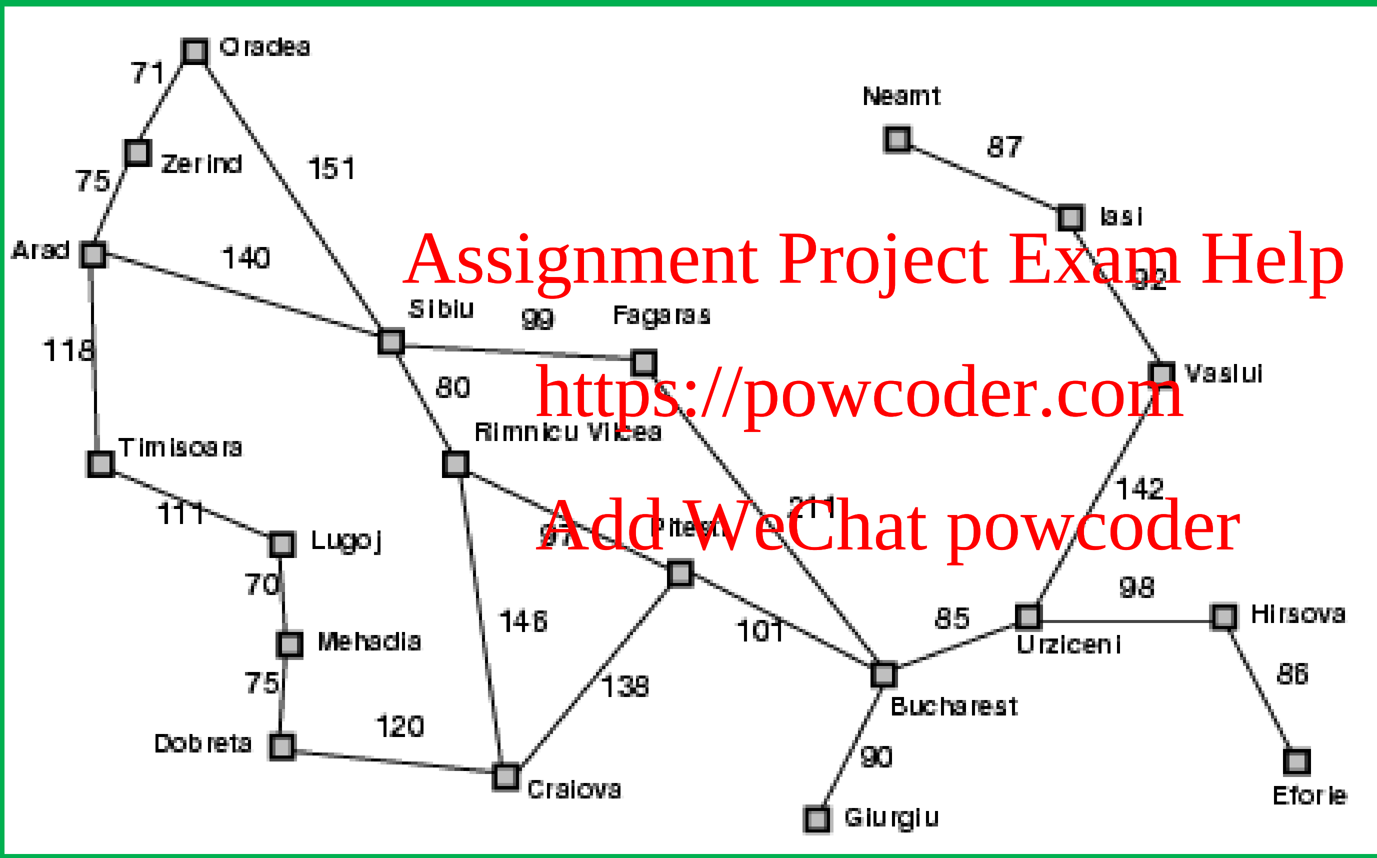
- Idea: avoid expanding paths that are <u>already</u> expensive

- Evaluation function $f(n) = g(n) + h(n)$

- $g(n)$ = cost so far to reach n

- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal

A*: Romania with step costs in km



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A* Evaluation: f(n) = g(n) + h(n)

A* search example

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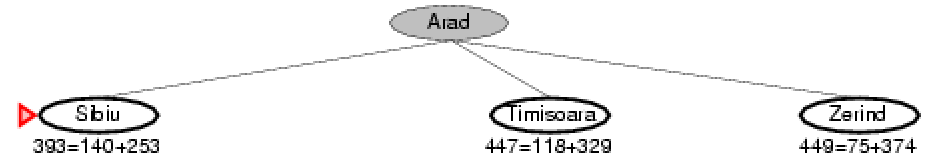
A* search example



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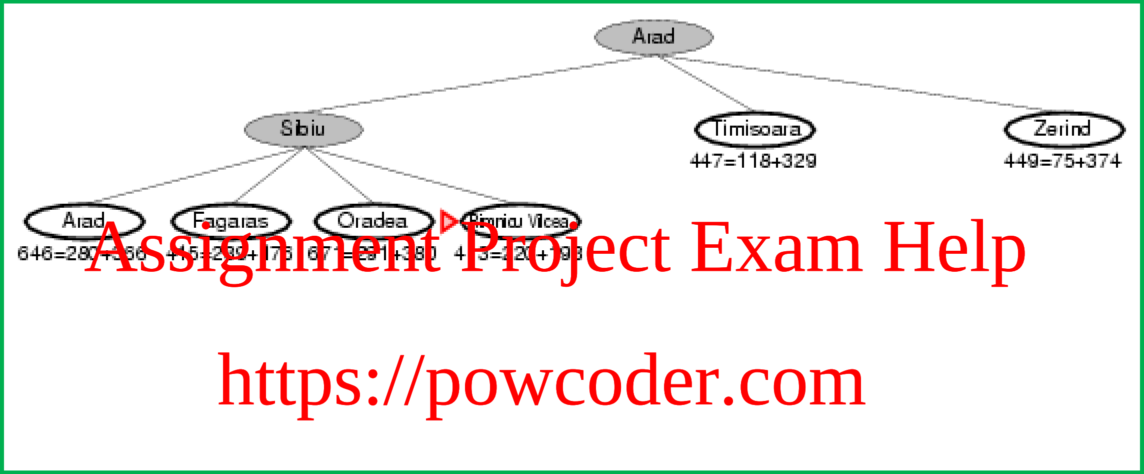
A* search example



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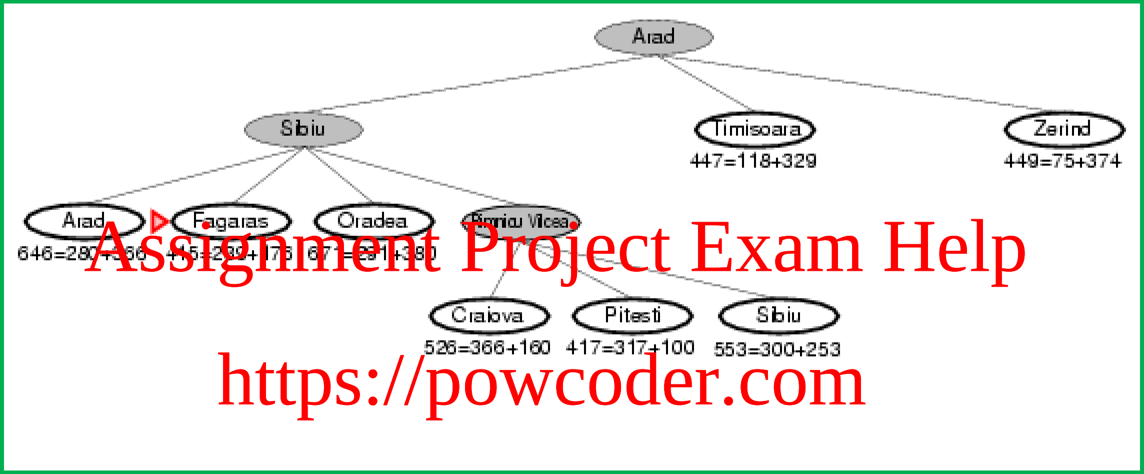

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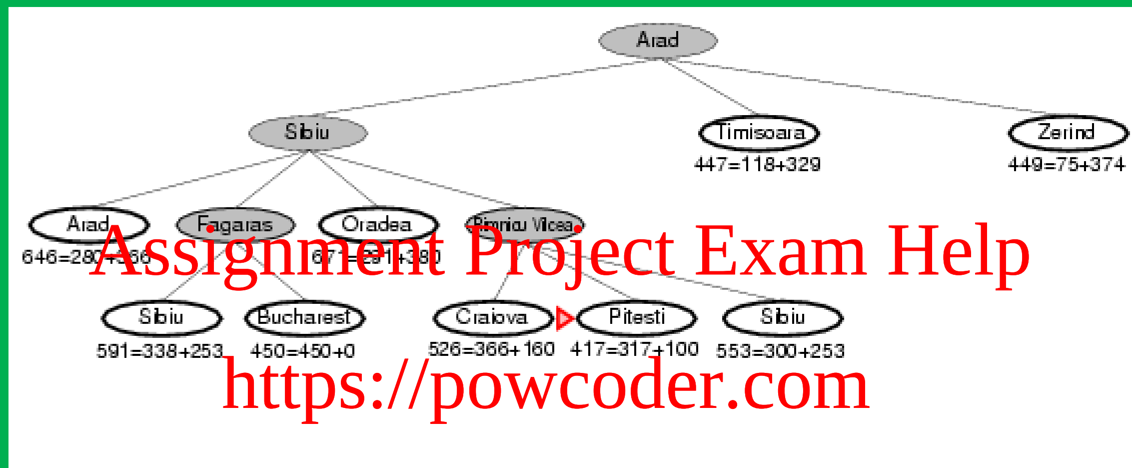

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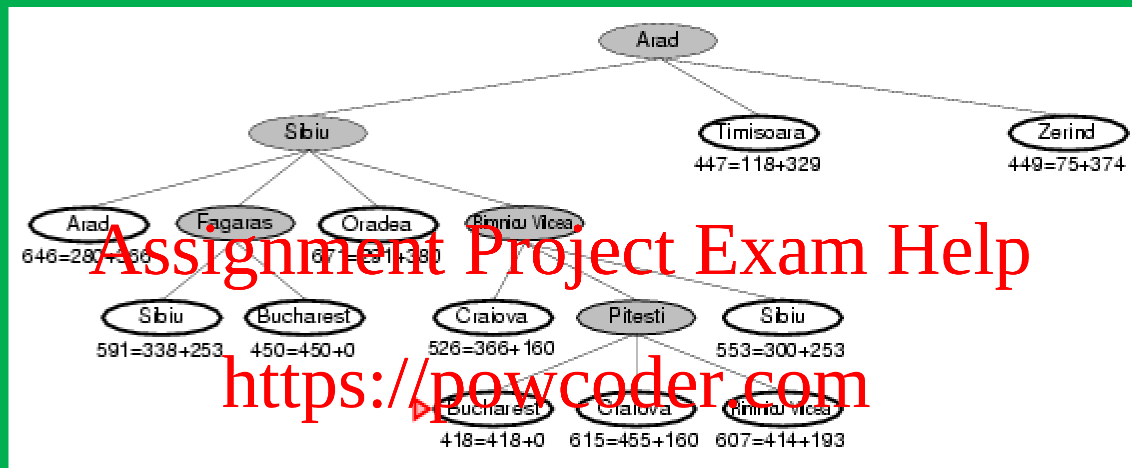

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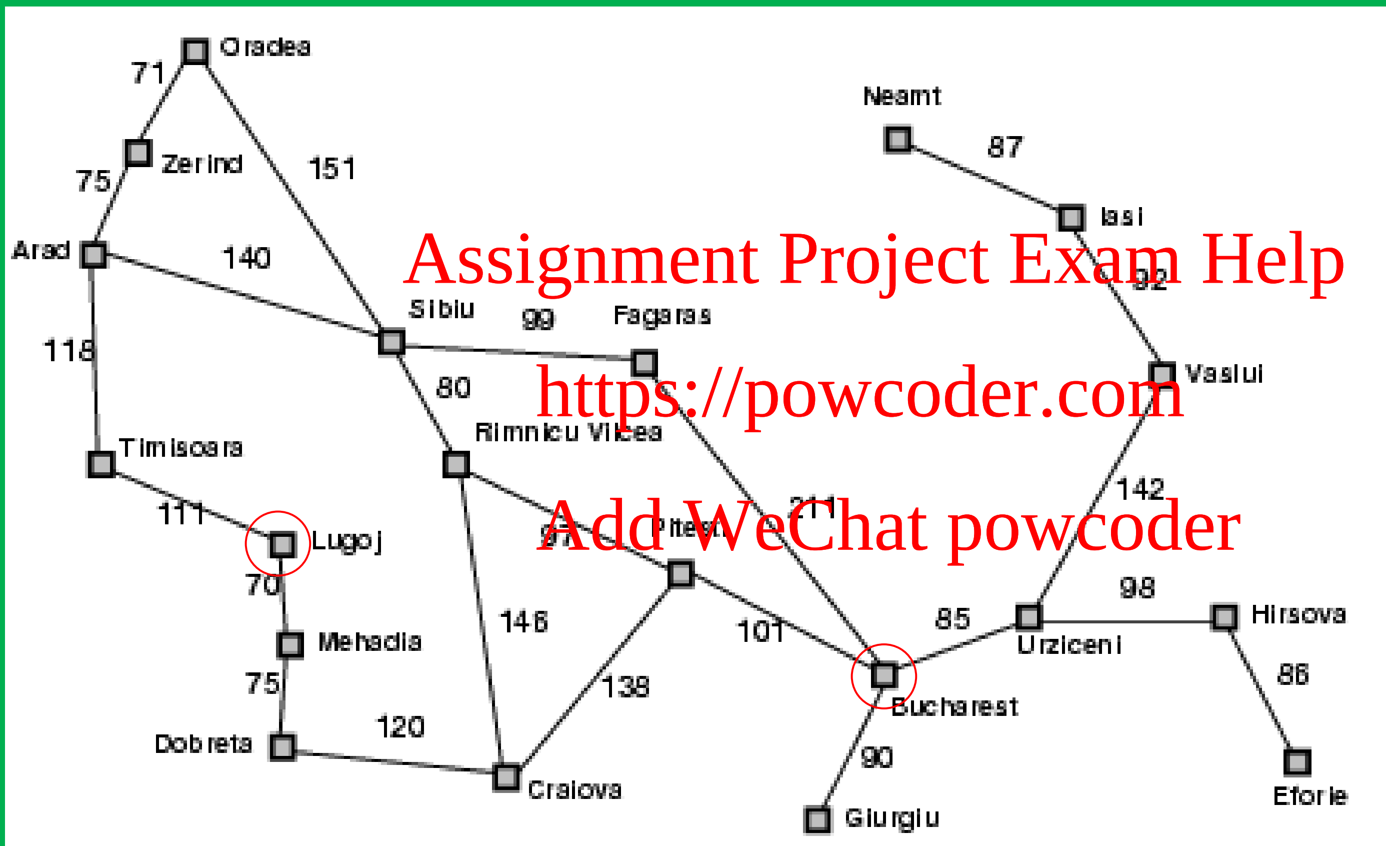
A*: In Class Exercise



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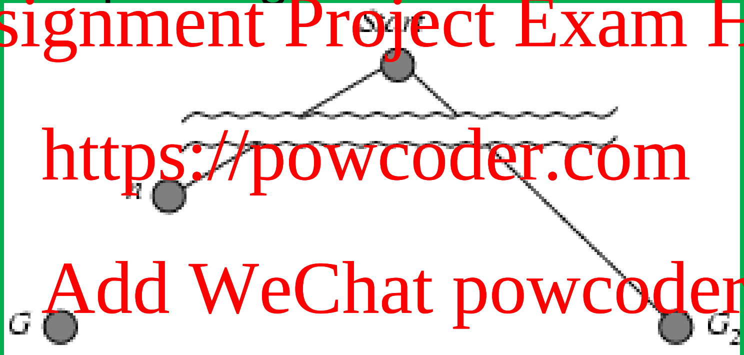
A* Evaluation: $f(n) = g(n) + h(n)$

Admissible heuristics

- A heuristic *h(n)* is admissible if for every node *n*, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from *n*.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If *h(n)* is admissible, A* using `TREE-SEARCH` is optimal

Optimality of A* (proof)

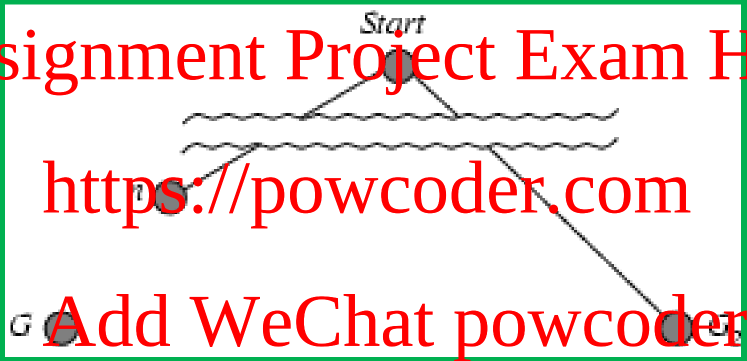
- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.



- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $g(G_2) > g(G)$ since G_2 is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above

Optimality of A* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.



• $f(G_2)$	$> f(G)$	from above
• $h(n)$	$\leq h^*(n)$	since h is admissible
• $g(n) + h(n)$	$\leq g(n) + h^*(n)$	
• $f(n)$	$\leq f(G)$	

Hence $f(G_2) > f(n)$, and A* will never select G_2 for expansion

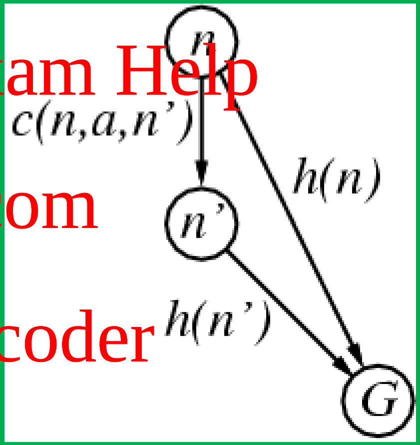
Consistent heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

  $h(n) \leq c(n,a,n') + h(n')$

- If *h* is consistent, we have

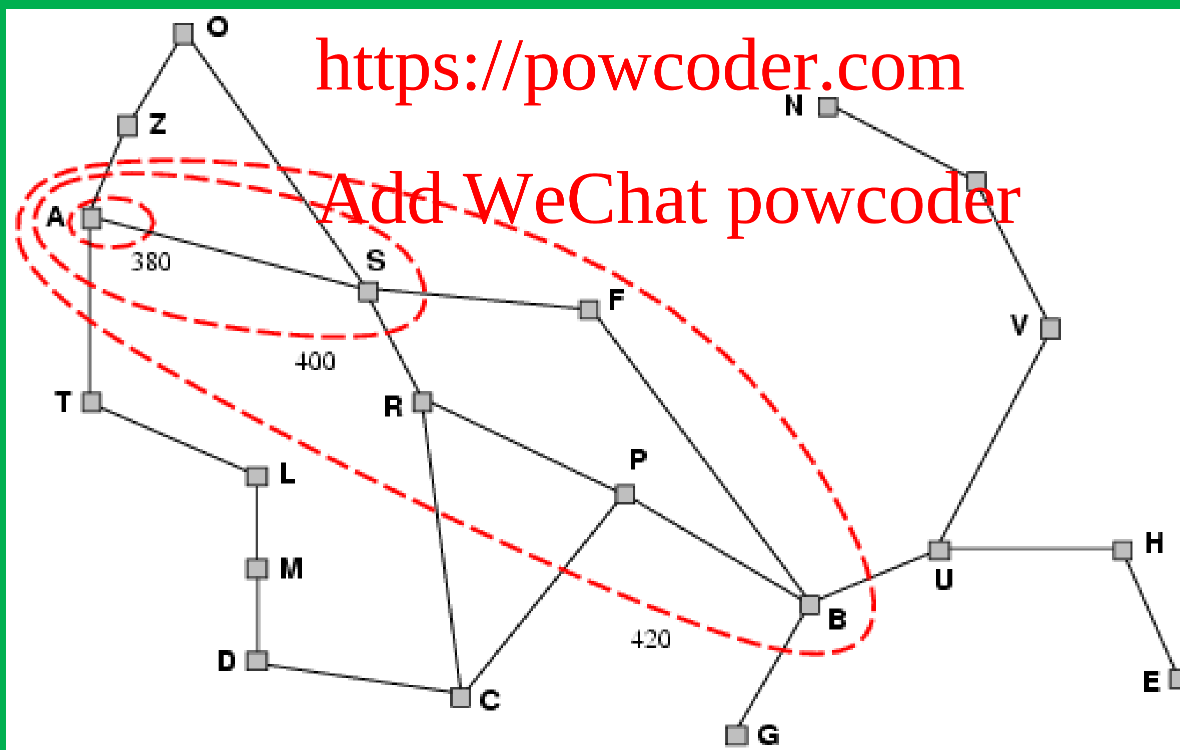
$$
\begin{aligned}
f(n') &= g(n') + h(n') \\
&= g(n) + c(n,a,n') + h(n') \\
&\geq g(n) + h(n) \\
&= f(n)
\end{aligned}
$$



- i.e., *f(n)* is non-decreasing along any path.
- Theorem: If *h(n)* is consistent, A* using `GRAPH-SEARCH` is optimal

Optimality of A*

- A* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Contour *i* has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A*

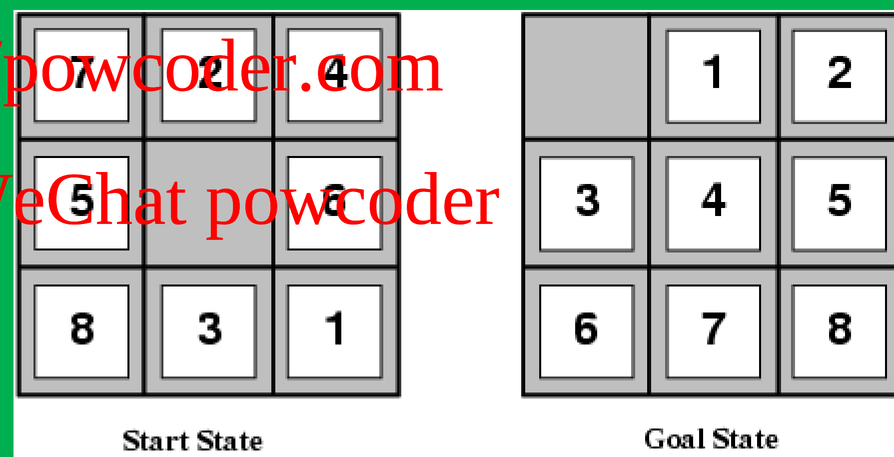
- Complete? Yes (unless there are infinitely many nodes with f ≤ *f(G)*)
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



- $h_1(S) = ?$
- $h_2(S) = ?$

Admissible heuristics

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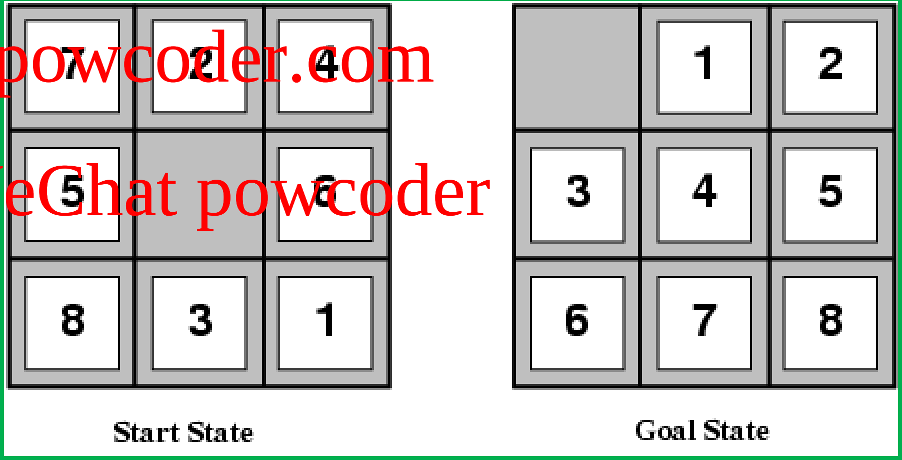


- $h_1(S) = ?$ 8
- $h_2(S) = ?$ 3+1+2+2+2+3+3+2 = 18

Effective Branching Factor, b*

- Performance measure for a heuristic

$$N+1 = 1 + b^* + b^{*2} \ldots + b^{*d}$$

$$(b^*)^{d+1} = N\text{, so } b^* = (N)^{1/(d+1)}$$

- If A* generates solution at depth d=5 and expands N=52 nodes, then

$$b^{*6} = 52,$$
$$b^* = (52)^{1/6} = 1.92$$

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
- then h_2 dominates h_1
- h_2 is better for search

- Typical search costs (average number of nodes expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Local search algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution

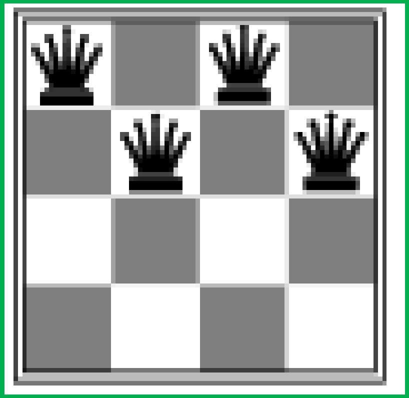
- State space = set of "complete" configurations

- Find configuration satisfying constraints, e.g., n-queens

- In such cases, we can use local search algorithms
- keep a single "current" state, try to improve it

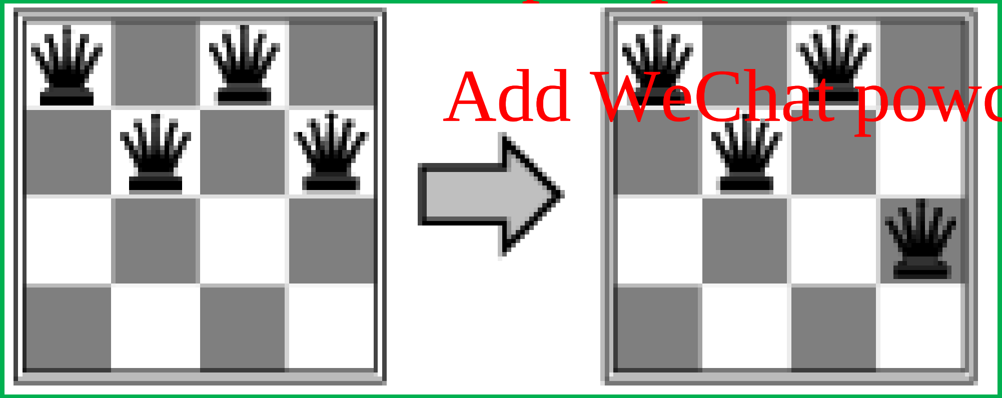
Example: *n*-queens

- Put *n* queens on an *n* × *n* board with no two queens on the same row, column, or diagonal



Example: *n*-queens

- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal



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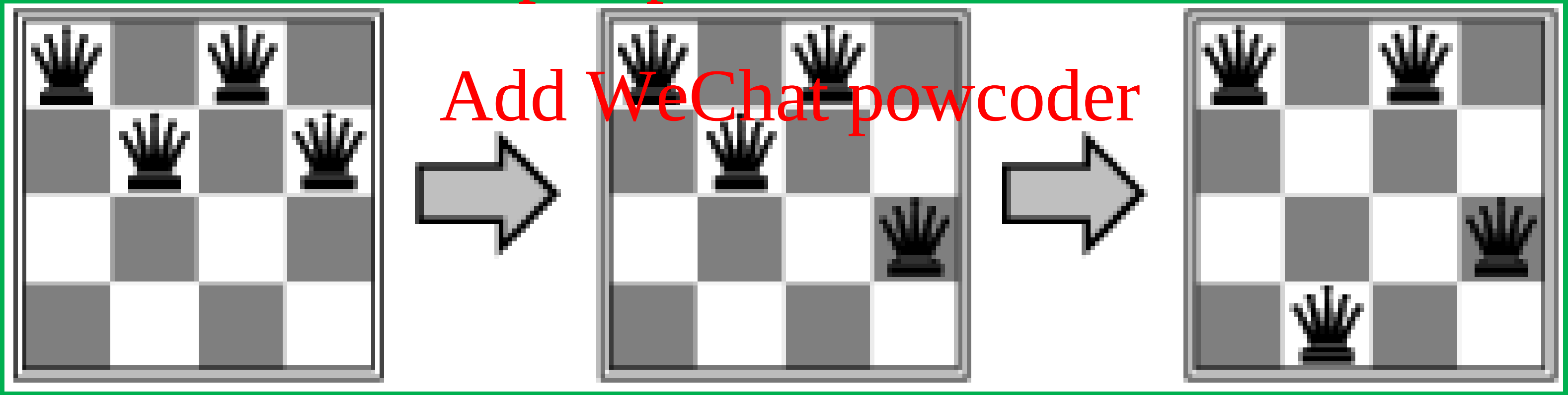

Hill-climbing search

- "Like climbing Everest ..


```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node

   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
      neighbor ← a highest-valued successor of current
      if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
      current ← neighbor
```

Hill-climbing search

- "Like climbing Everest in thick fog …


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Hill-climbing search

- "Like climbing Everest in thick fog with amnesia."


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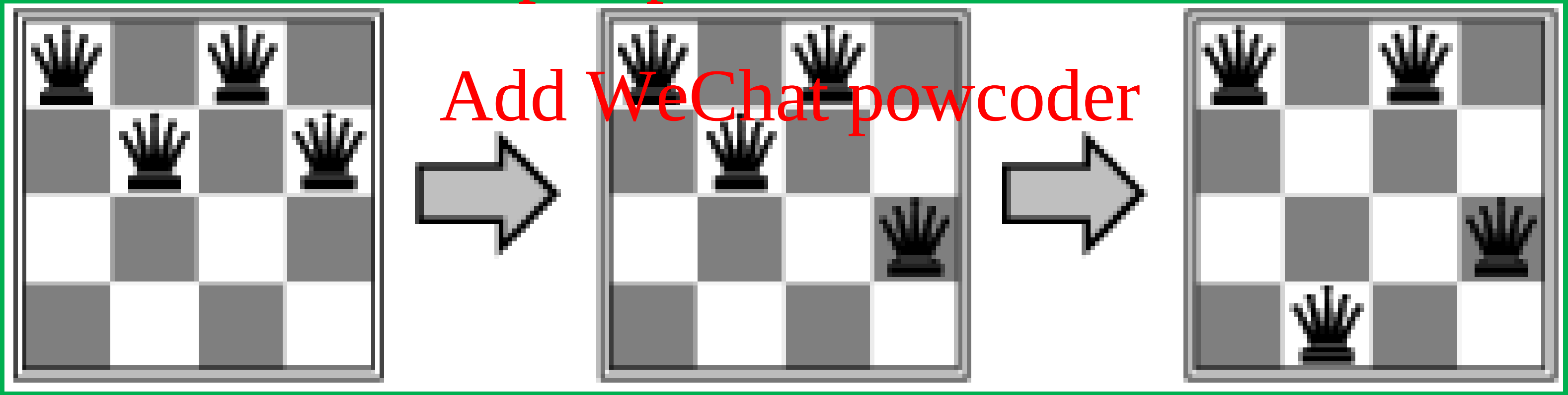
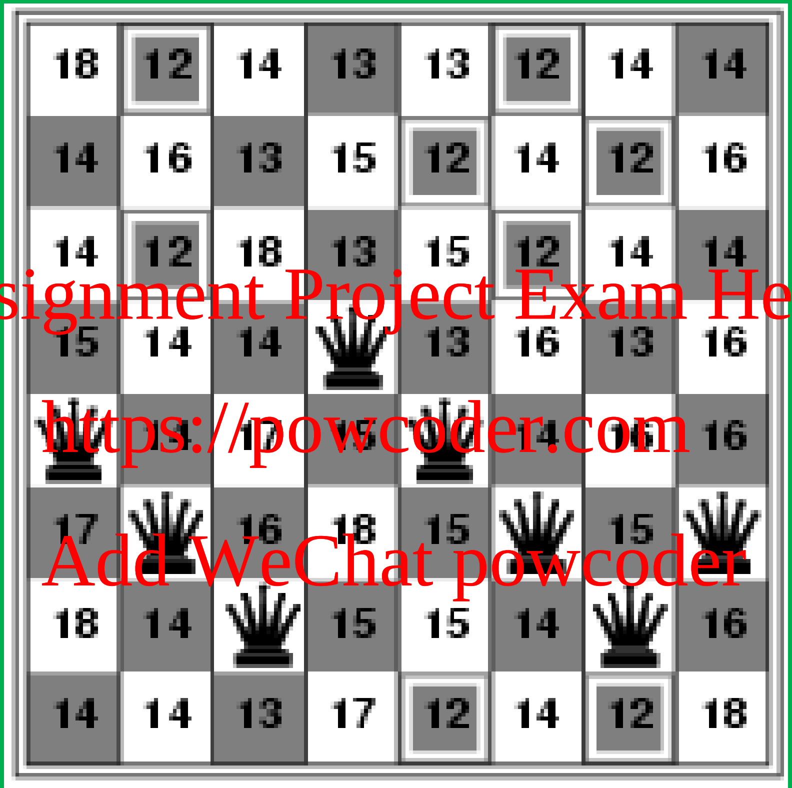
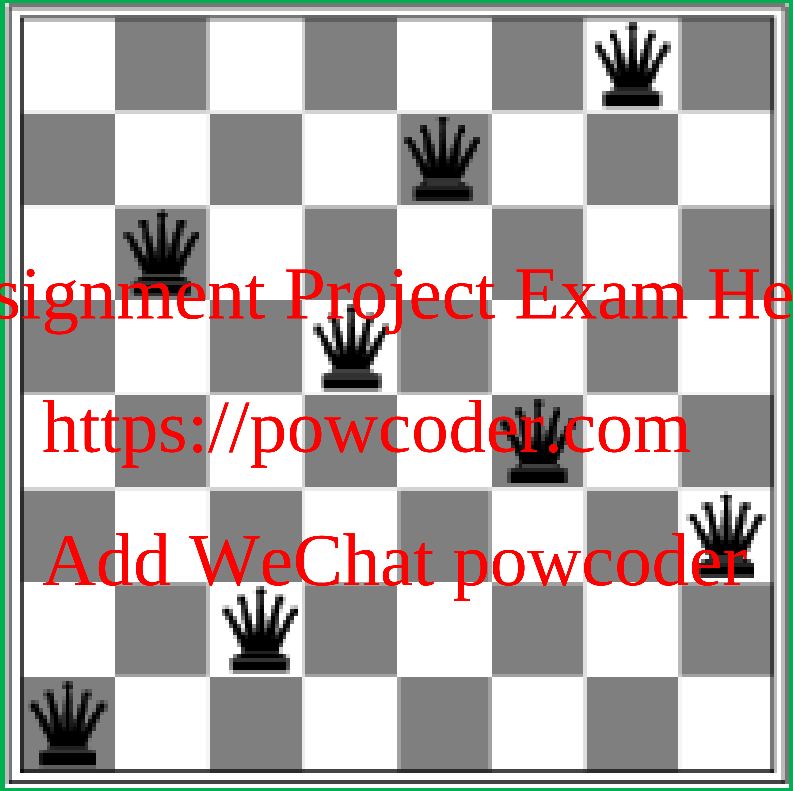


Hill-climbing search: 8-queens problem



- *h* = number of pairs of queens that are attacking each other, either directly or indirectly

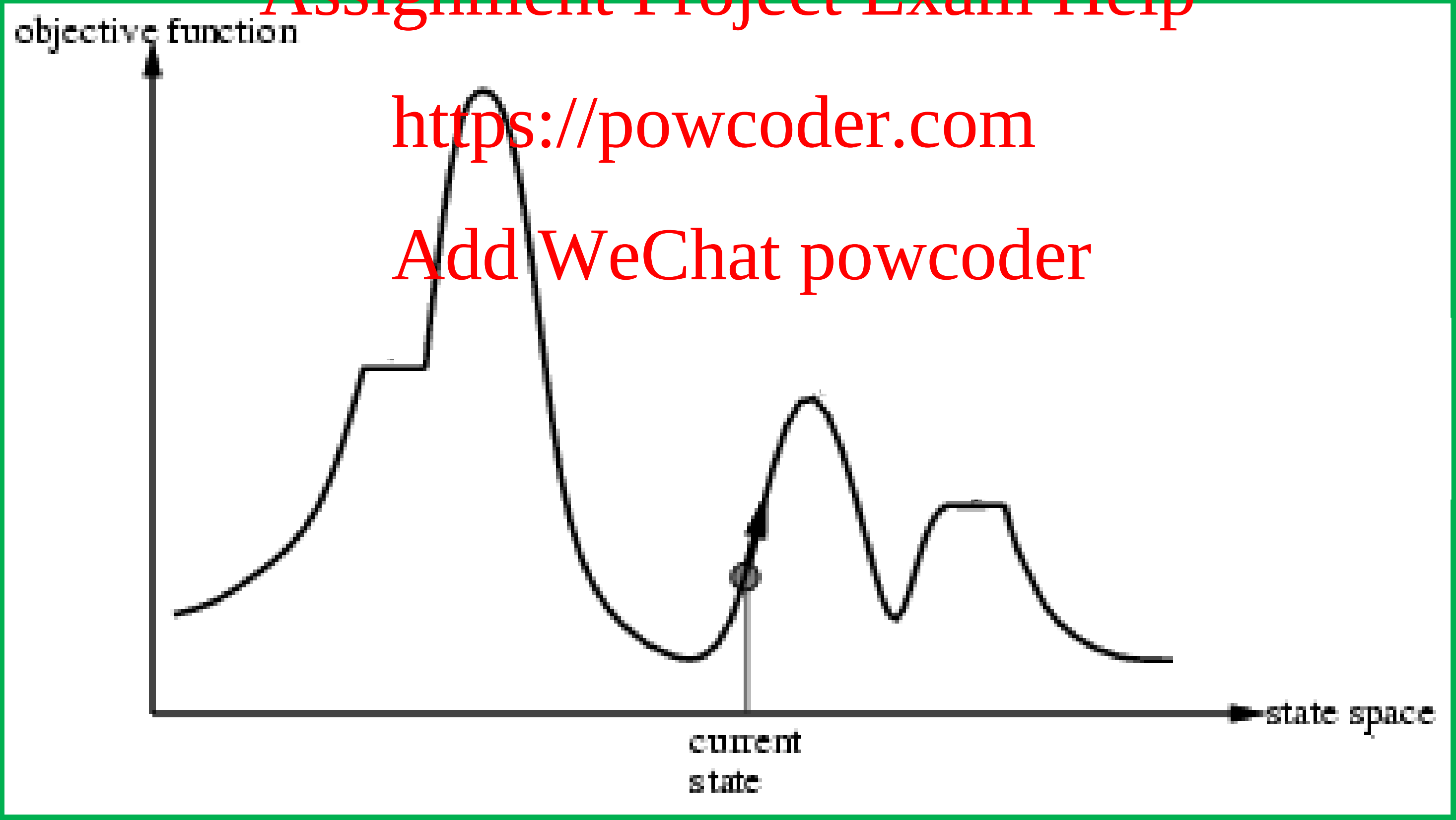
Hill-climbing search: 8-queens problem



- A local minimum with $h = 1$

Hill-climbing search

- Problem: depending on initial state, can get stuck in local maxima



Hill-climbing search

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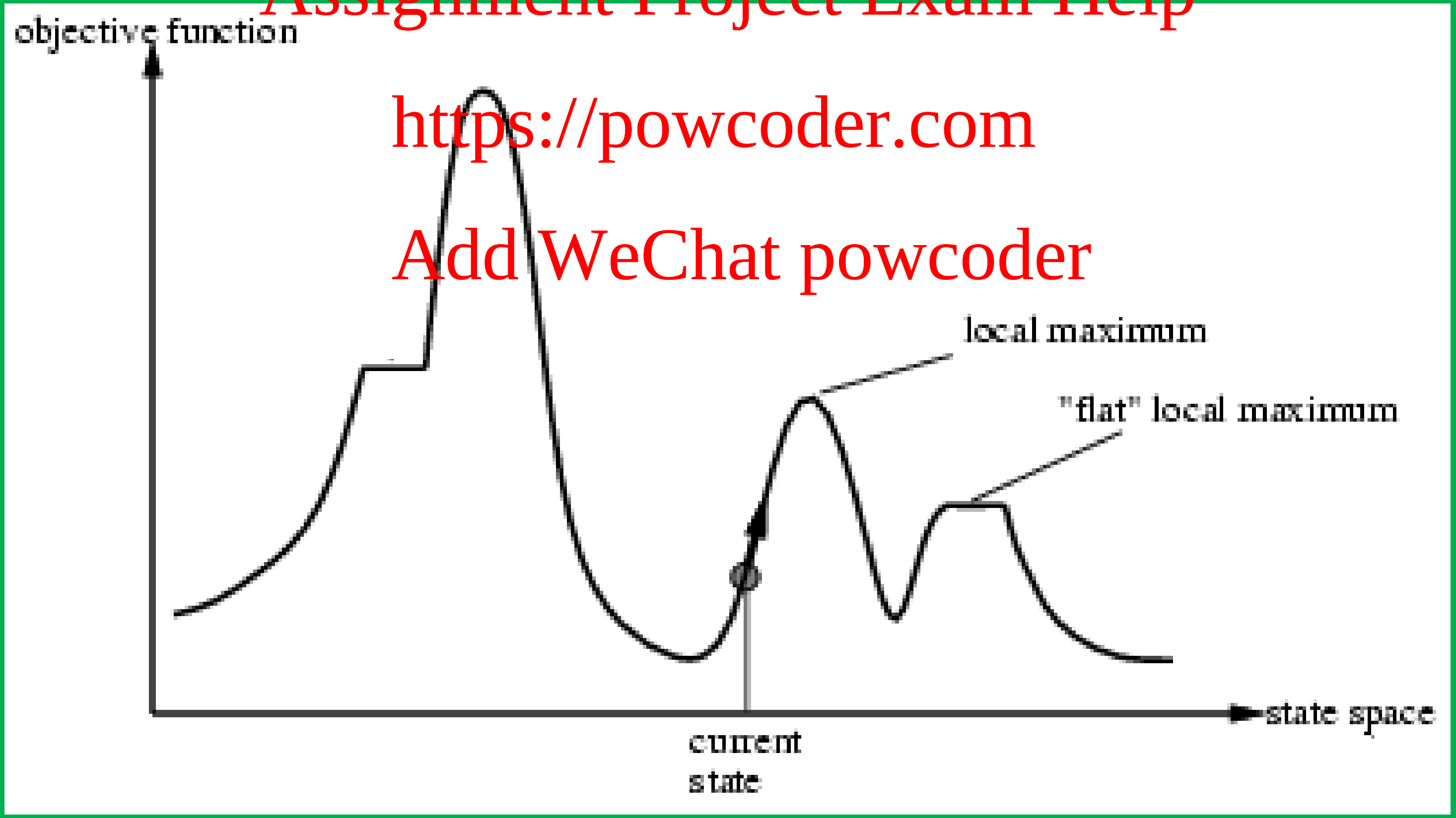

Hill-climbing search

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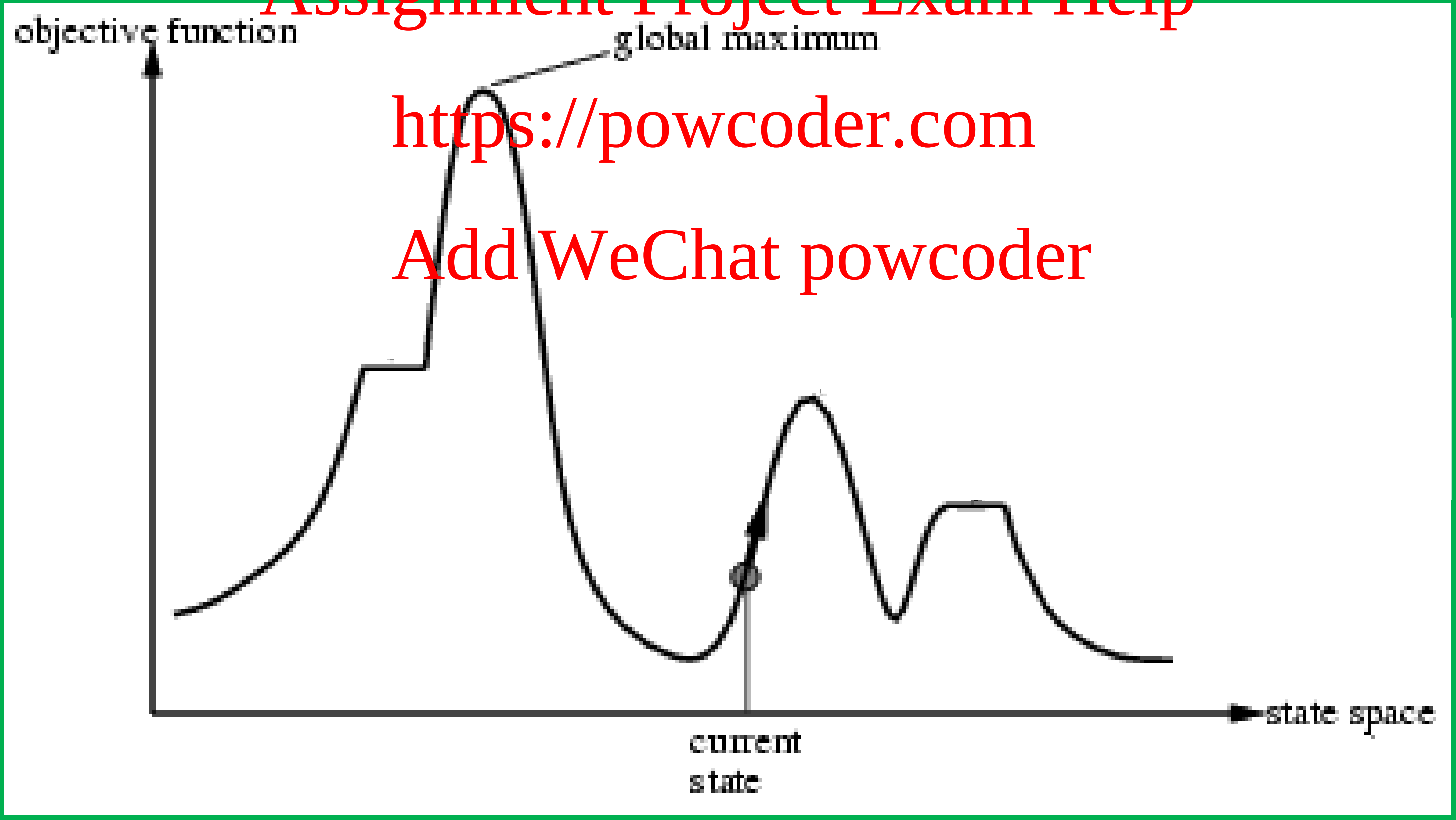

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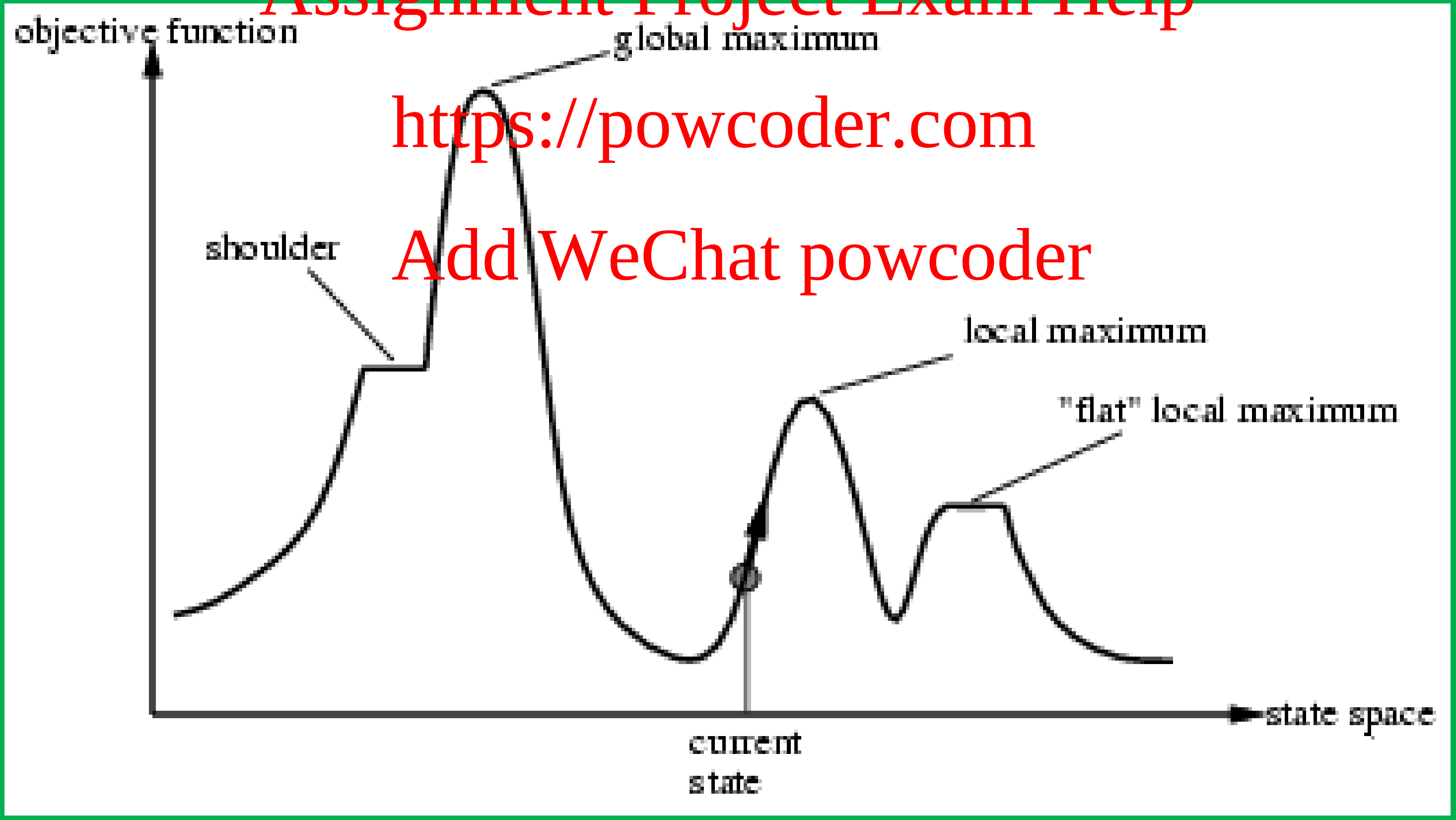


Local beam search

- Keep track of *k* states rather than just one
- Start with *k* randomly generated states

- At each iteration, all the successors of all *k* states are generated
- If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat.

Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency


```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← VALUE[next] – VALUE[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(ΔE/T)
```

Properties of simulated annealing search

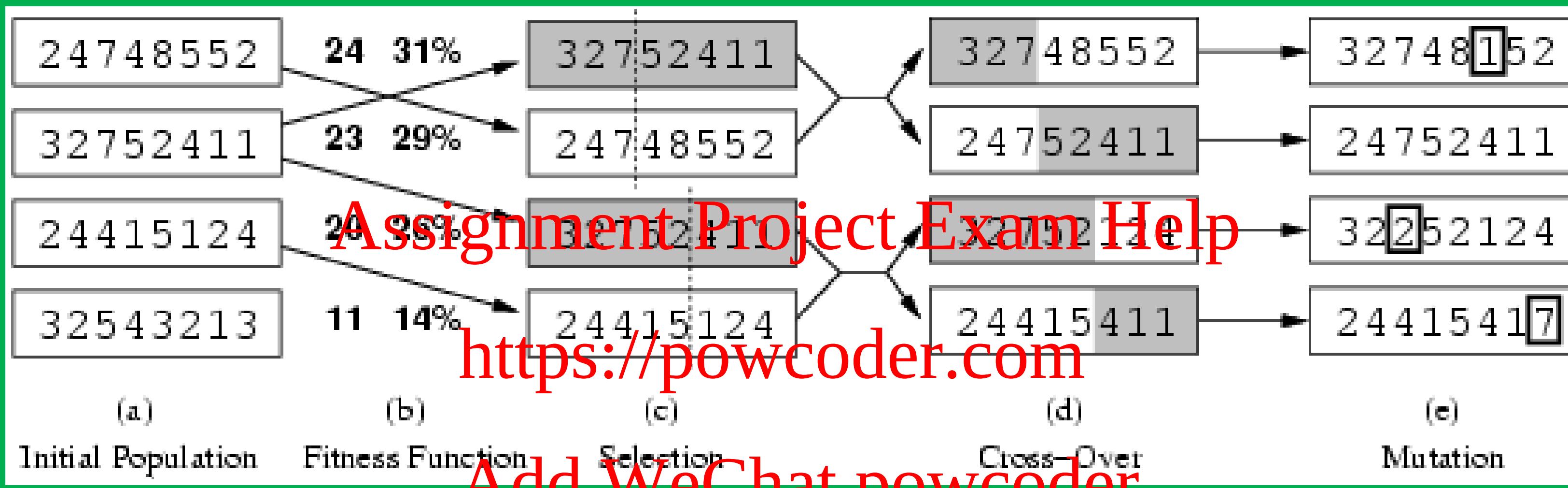
- One can prove: If *T* decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1

- Widely used in VLSI layout, airline scheduling, etc

Genetic algorithms

- A successor state is generated by combining two parent states
- Start with *k* randomly generated states (population)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (fitness function). Higher values for better states.
- Produce the next generation of states by selection, crossover, and mutation

Genetic algorithms



- Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)
- 24/(24+23+20+11) = 31%
- 23/(24+23+20+11) = 29% etc

Genetic algorithms

Adversarial Search

- Games!

- "Unpredictable" opponent → specify a move for every possible opponent reply

- Time limits → unlikely to find goal, must approximate

Deterministic games in practice

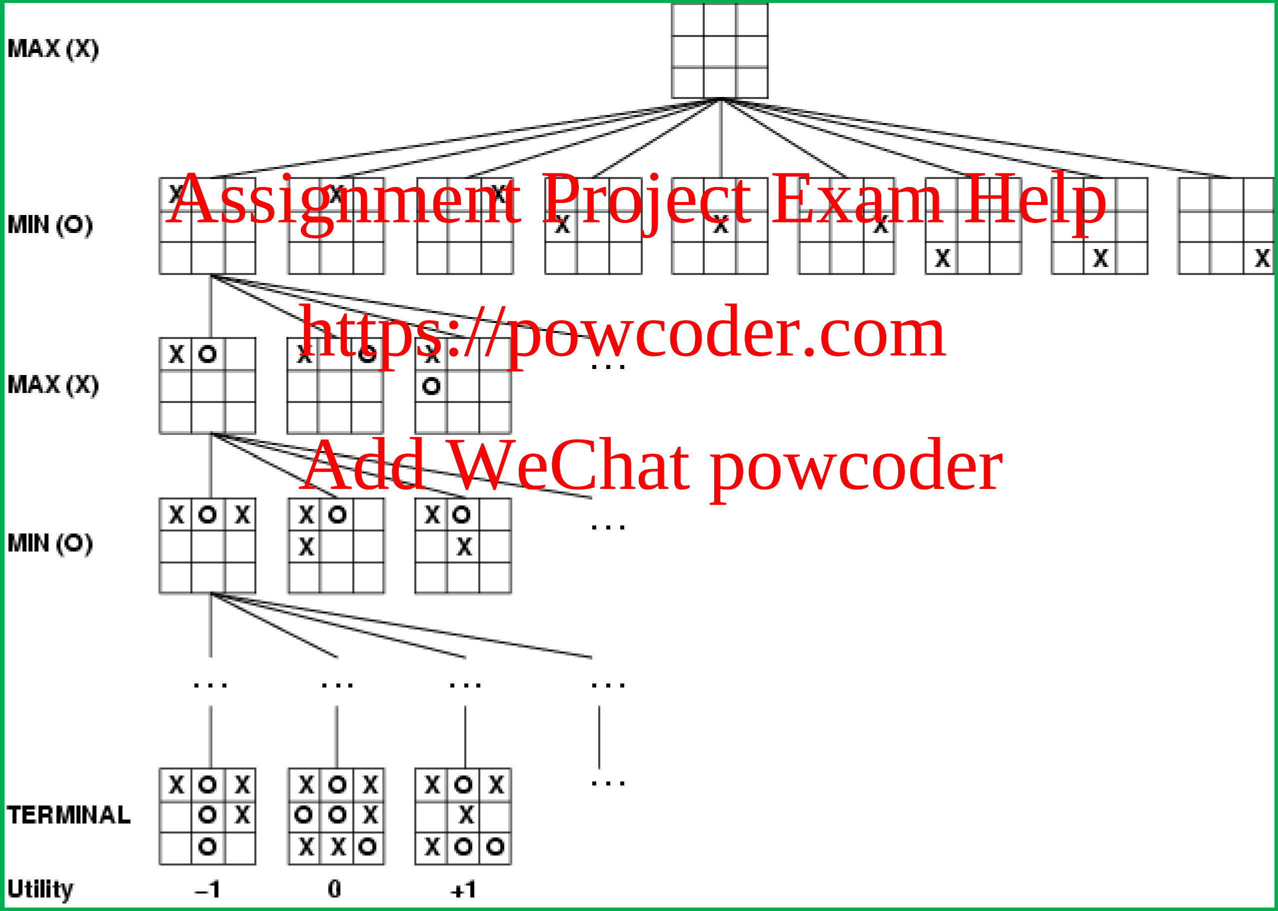
- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used a precomputed endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 444 billion positions.

- Chess: Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply. Current programs even better.

- Othello: human champions refuse to compete against computers, who are too good.

- Go: In the 2017 Future of Go Summit, AlphaGo beat Ke Jie, the world No.1 ranked player at the time, in a three-game match. alphaGo uses Monte Carlo and neural network techniques to learn how to refine its search to winning games.

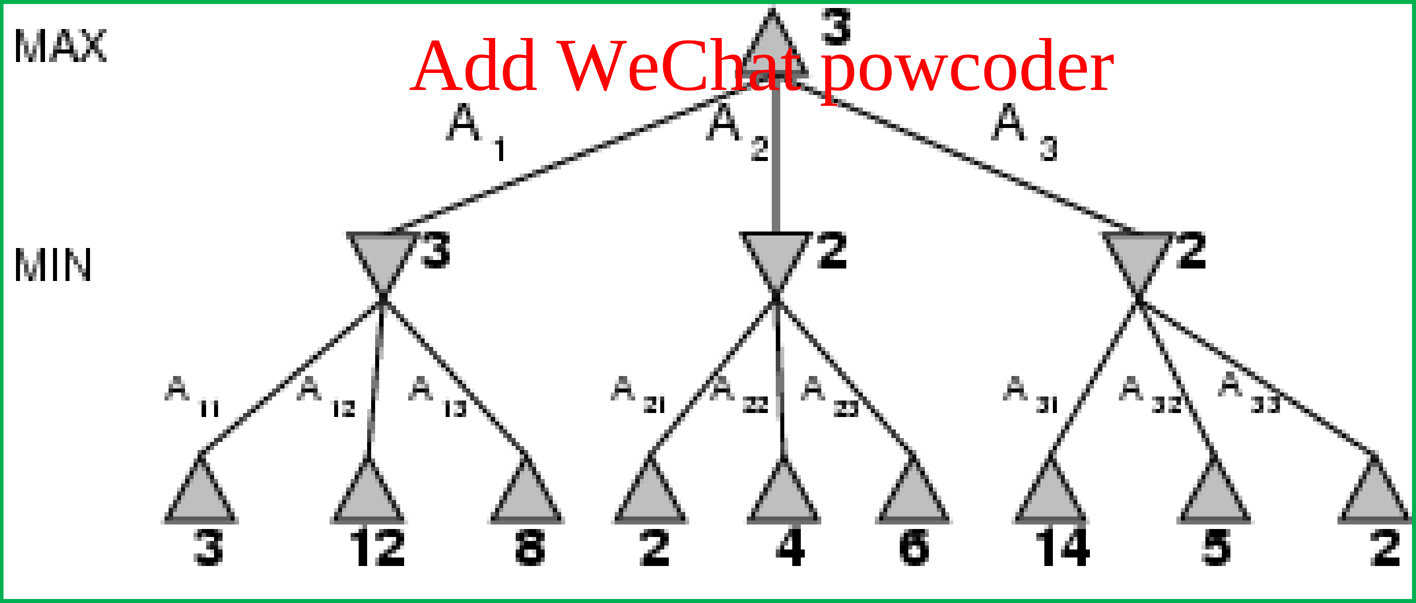
Game tree (2-player, deterministic, turns)



Minimax

- Perfect play for deterministic games
- Idea: choose move to position with highest minimax value
  = best achievable payoff against best play

- E.g., 2-**ply** game:



Minimax algorithm

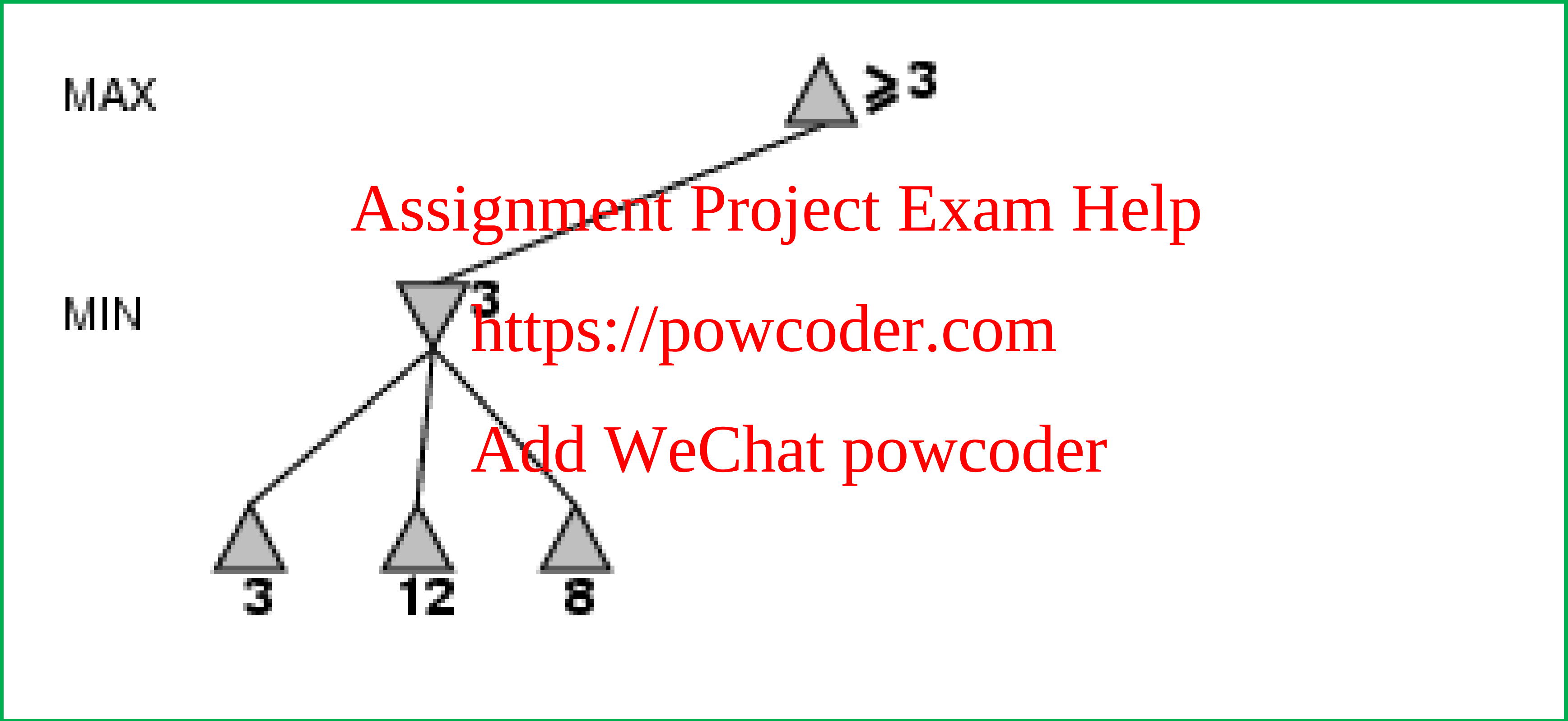
```
function MINIMAX-DECISION(state) returns an action
    v ← MAX-VALUE(state)
    return the action in SUCCESSORS(state) with value v

function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s))
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s))
    return v
```

Properties of minimax

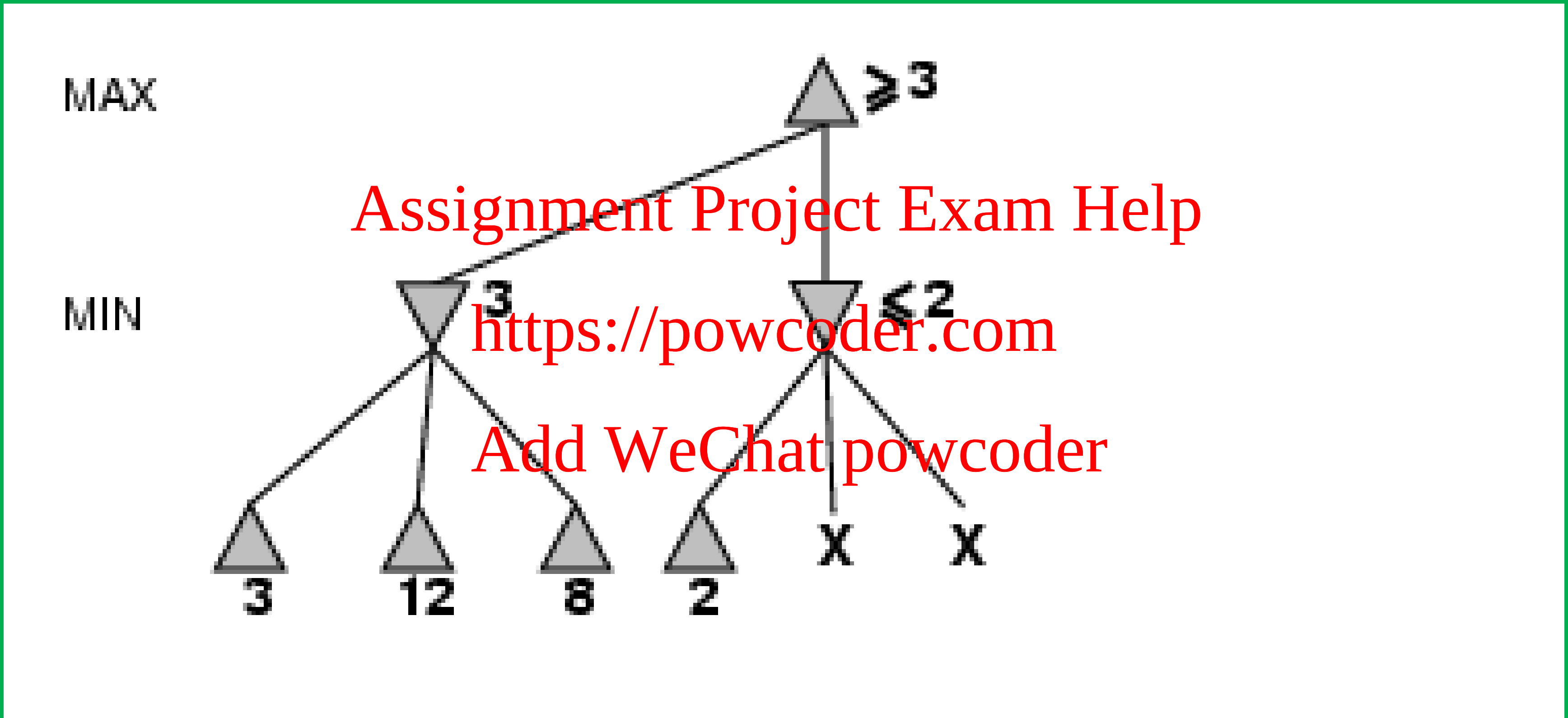
- Complete? Yes (if tree is finite)
- Optimal? Yes (against an optimal opponent)
- Time complexity? $O(b^m)$
- Space complexity? $O(bm)$ (depth-first exploration)

- For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
  → exact solution completely infeasible

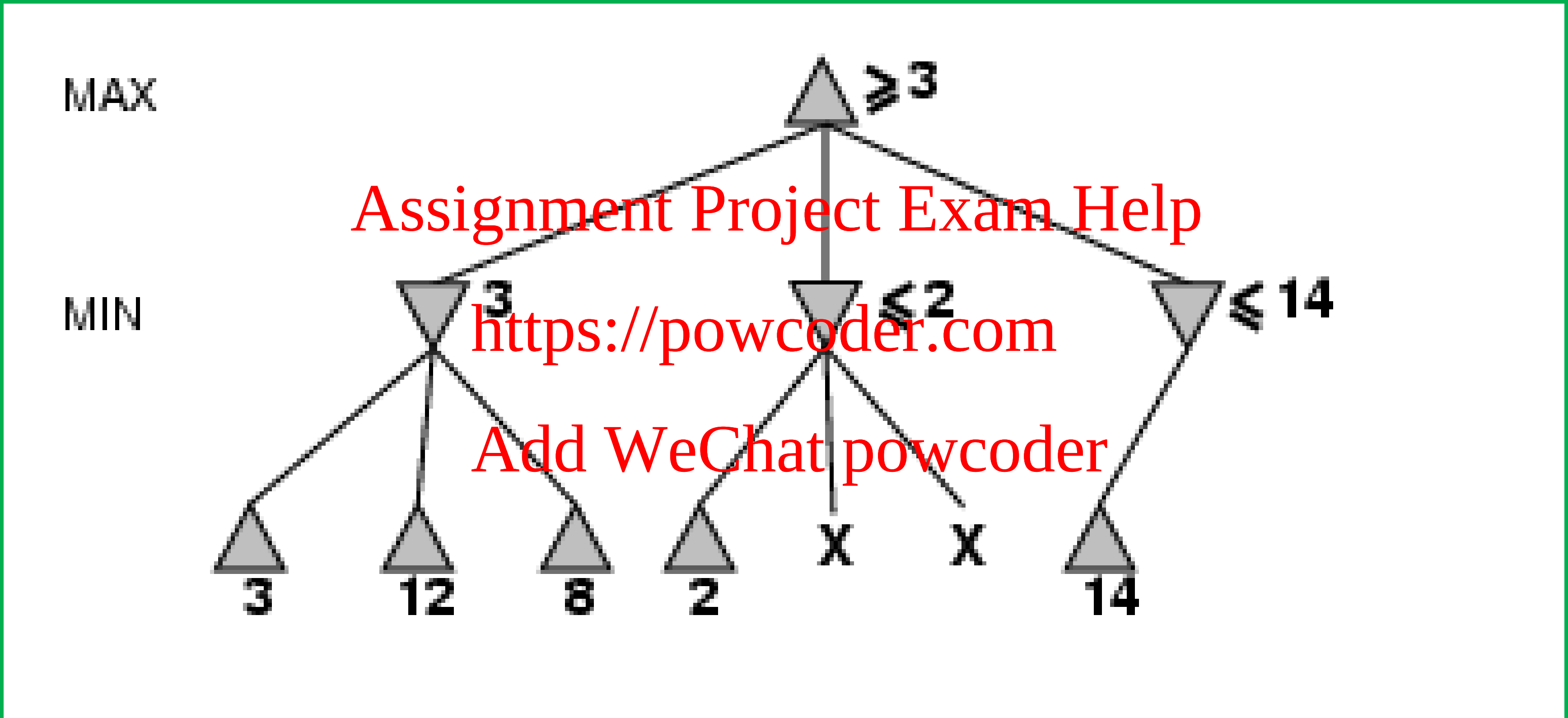
α-β pruning example



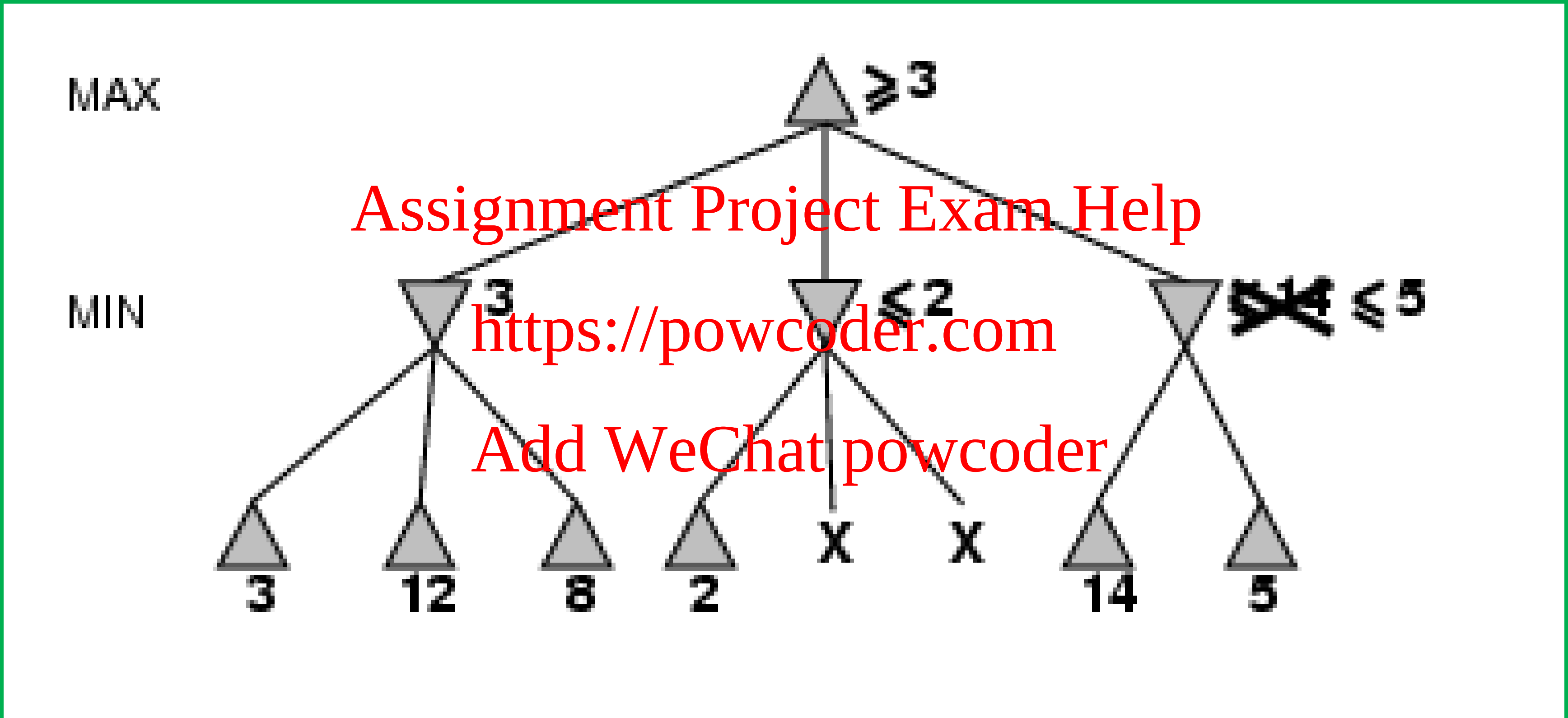
α-β pruning example



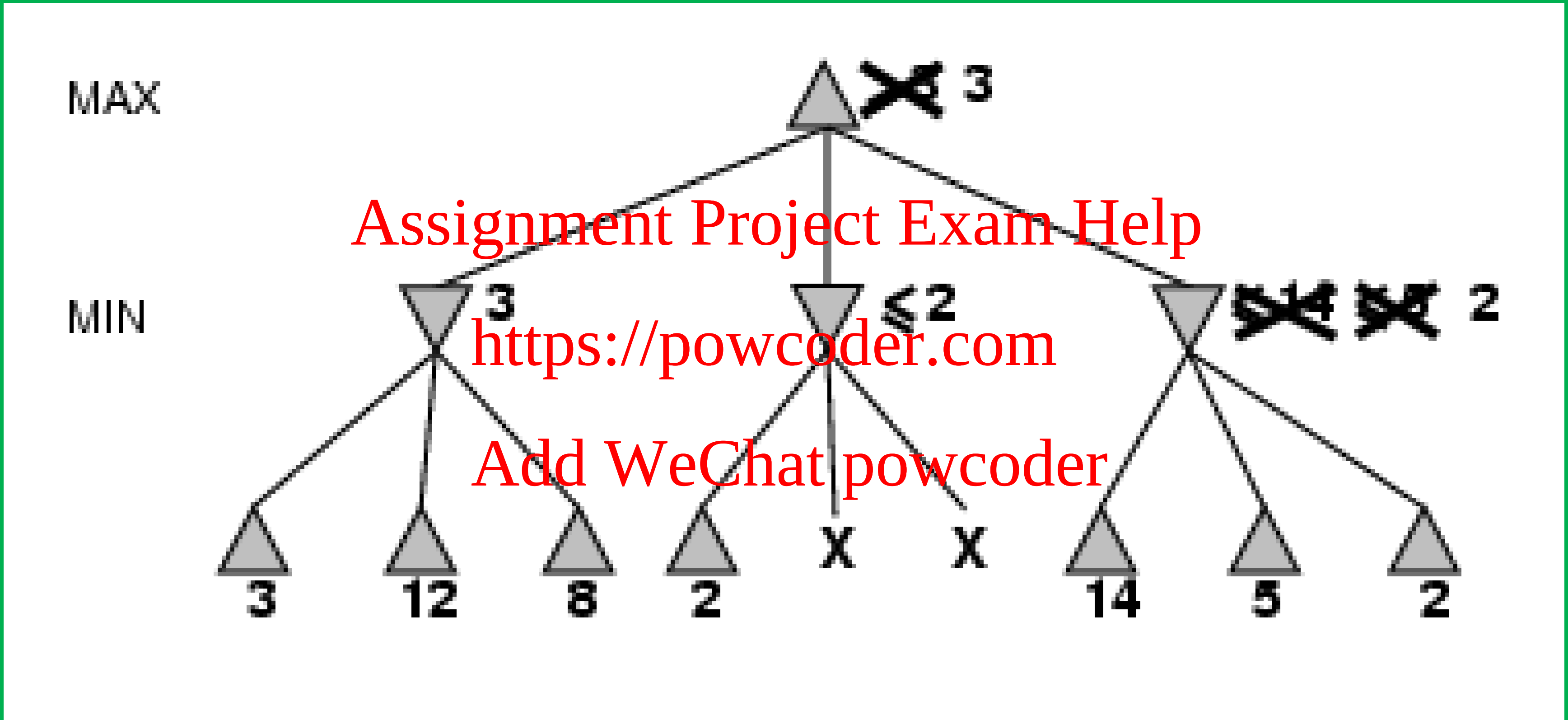
α-β pruning example



α-β pruning example



α-β pruning example



Properties of α-β

- Pruning does not affect final result

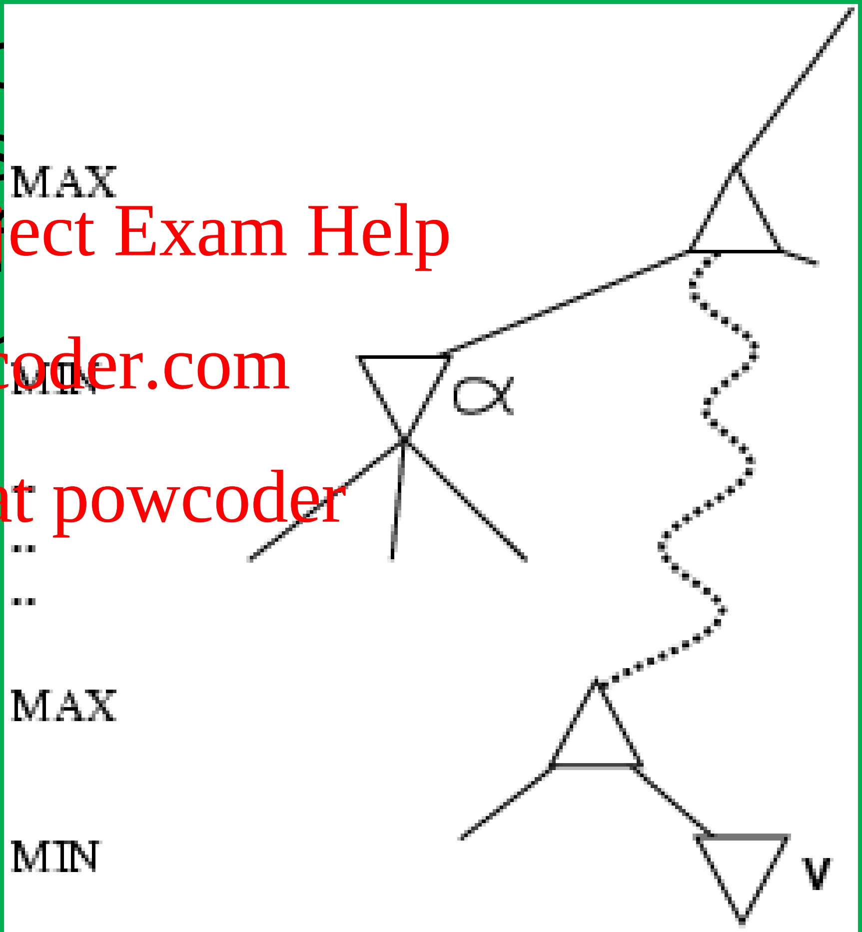
- Good move ordering improves effectiveness of pruning

- With "perfect ordering," time complexity = $O(b^{m/2})$
  → doubles depth of search

- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

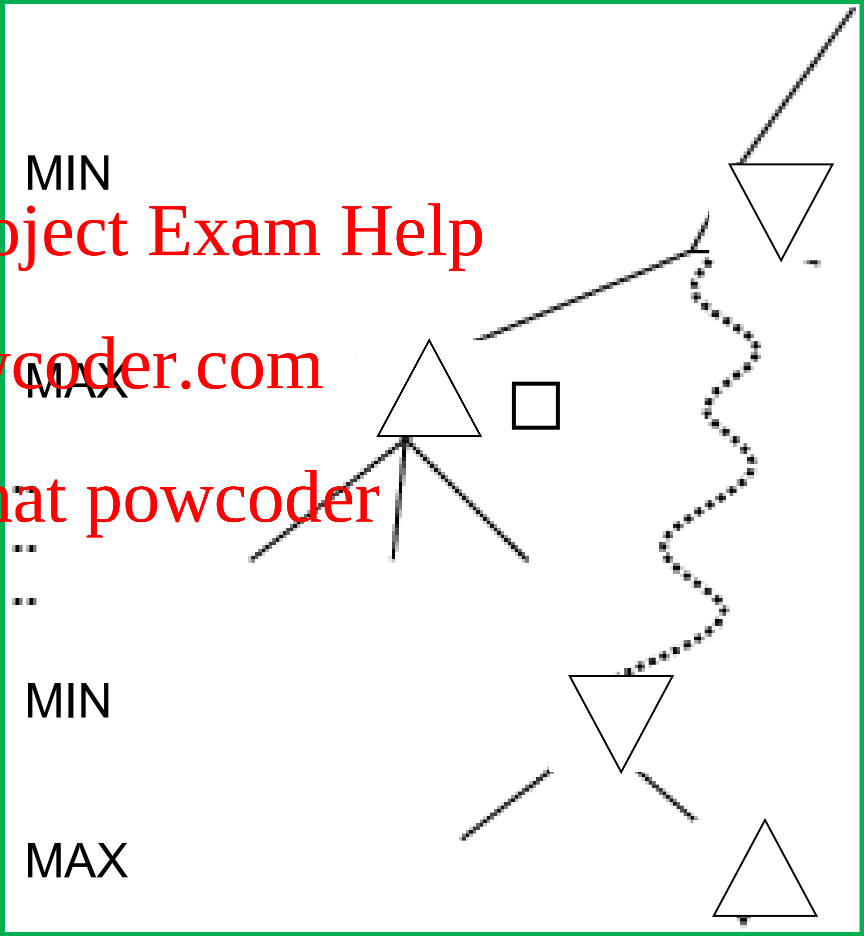
Why is it called α-β?

- α is the value of the b
  value) choice found s
  point along the path f
- If *v* is worse than α, r
  → prune that branch
- Define β similarly for



Why is it called α-β?

- β is the value of the value) choice found point along the path
- If v is worse than α
  → prune that branch



The α-β algorithm

function ALPHA-BETA-SEARCH(*state*) **returns** *an action*
  inputs: *state*, current state in game

  $v \leftarrow$ MAX-VALUE(*state*, $-\infty, +\infty$)
  return the *action* in SUCCESSORS(*state*) with value v

---

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s, α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

The α-β algorithm

```
function MIN-VALUE(state, α, β) returns a utility value
   inputs: state, current state in game
           α, the value of the best alternative for MAX along the path to state
           β, the value of the best alternative for MIN along the path to state
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← +∞
   for a, s in SUCCESSORS(state) do
      v ← MIN(v, MAX-VALUE(s, α, β))
      if v ≤ α then return v
      β ← MIN(β, v)
   return v
```

Resource limits

Suppose we have 100 secs, explore 10^4 nodes/sec
→ 10^6 nodes per move

Standard approach:

- cutoff test:
- e.g., depth limit (perhaps add quiescence search)

- evaluation function
- = estimated desirability of position

Evaluation functions

- For chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g., $w_1 = 9$ with

$f_1(s)$ = (number of white queens) – (number of black queens), etc.

Cutting off search

MinimaxCutoff is identical to *MinimaxValue* except

1. *Terminal?* is replaced by *Cutoff?*
2. *Utility* is replaced by *Eval*

Does it work in practice?

$b^m = 10^6$, $b=35 \rightarrow m=4$

4-ply lookahead is a hopeless chess player!

- 4-ply ≈ human novice
- 8-ply ≈ typical PC, human master
- 12-ply ≈ Deep Blue, Kasparov