CMPSC 461: Programming Language Concepts
Midterm 1 Solution

Lambda Calculus

Problem 1 [10pt] Consider a λ-term

$$\lambda x.\ \lambda y.\ x\ z\ \lambda x.\ x$$

1. (6pt) Compute the set of free variables in the term. **Show the detailed derivation in your answer**.

   only z is free

2. (4pt) Based on the results above, connect all bound variables to their definitions with lines. For example, the answer for $\lambda x.\ x\ x\ y$ should be written as $\lambda \underline{x.\ x}\ \ x\ \ y$.

   $\lambda \underline{x.\ \lambda y.\ x}\ \ z\ \lambda \underline{x.\ x}$

Problem 2 [10pt] Fully evaluate: $(\lambda x\ y\ z.\ x\ z)\ (\lambda x.\ x)\ v$. **Show the detailed derivation in your answer**.

$$\begin{aligned}
&(\lambda x\ y\ z.\ x\ z)\ (\lambda x.\ x)\ v\\
=&(\lambda x.\ \lambda y.\ \lambda z.\ x\ z)\ (\lambda x.\ x)\ v\\
=&(\lambda y.\ (\lambda z.\ ((\lambda x.\ x)\ z)))\ v\\
=&\lambda z.\ ((\lambda x.\ x)\ z)\\
=&\lambda z.\ z
\end{aligned}$$

Problem 3 [12pt] Recall the following encodings defined in lectures:
$\texttt{TRUE} \triangleq \lambda x\ y.\ x$
$\texttt{FALSE} \triangleq \lambda x\ y.\ y$
One way of encoding the logical "and" in λ-calculus is as follows:

$$\texttt{AND} \triangleq \lambda x\ y.\ x\ y\ \texttt{FALSE}$$

1. (6pt) Write down an encoding of the logical "or" in λ-calculus so that it computes on `TRUE` and `FALSE` (e.g., `OR FALSE TRUE` is `TRUE`; `OR FALSE FALSE` is `FALSE`).

$$\texttt{OR} \triangleq \lambda x\ y.\ x\ \texttt{TRUE}\ y$$

2. (6pt) Based on your encoding, show that `OR FALSE FALSE` evaluates to `FALSE`. **Show the detailed derivation in your answer**.

$$
\begin{aligned}
&\texttt{OR FALSE FALSE}\\
=&(\lambda x\ y.\ x\ \texttt{TRUE}\ y)\ \texttt{FALSE FALSE}\\
=&\texttt{FALSE TRUE FALSE}\\
=&(\lambda x\ y.\ y)\ \texttt{TRUE FALSE}\\
=&\texttt{FALSE}
\end{aligned}
$$

Scheme

Problem 4 [30pt]

1. (10pt) Consider the following Scheme program:

```
(define (foo l)  (map (lambda (x) (/ x 2)) l))
```


- What is the result of `(foo '(2 4 6 8))`?
- In general, given a parameter `l`, describe what does this function return in one sentence.

(1 2 3 4)
In general, this function returns a list where each element is divided by 2.

2. (10pt) Consider the following Scheme program:

```
(define (goo f l)
  (cond ((null? l) '())
        (else (let ((result (goo f (cdr l))))
                (append (f (car l)) result)))))
```

- What is the result of `(goo (lambda (x) (list x x)) '(1 4 9 16))`?
- In general, given parameters `f l`, describe what does this function return in one sentence.

(1 1 4 4 9 9 16 16)
In general, this function returns a list where elements are the concatenation of the results of applying `f` to elements in `l`, in the same order.

3. (10pt) Use the `foldl` function in Scheme to implement a function `newfilter`, which has the same functionality as the `filter` function in Scheme.

```
(define (newfilter f l)
  (foldl (lambda (x c) (if (f x) (append c (list x))
                           c)) '() l))
```

Syntax

Problem 5 [16pt]

1. (8pt) Write down a regular expression that captures all numbers from 1 to 33.

   [1-9] | [12] [0-9] | 3 [0-3]

2. (8pt) Write down a regular expression that captures nonempty binaries (strings with 0's and 1's) that start and end with **different digits** and with **at least three digits**. For example, 011 and 1100 are such binaries, while 1 and 010 are not.

   1(0|1)+0 | 0(0|1)+1

Problem 6 [6pt] For the following grammar, state whether it is ambiguous or unambiguous. If it is ambiguous, give an equivalent unambiguous grammar. If it is unambiguous, state all the precedences and associativities enforced by the grammar. Assume Id is a terminal that generates identifiers.

$E ::= E + E \mid F$
$F ::= G * F \mid G$
$G ::= Id \mid (E)$

Ambiguous. It is equivalent to the grammar below:

$E ::= E + F \mid F$
$F ::= G * F \mid G$
$G ::= Id \mid (E)$

Problem 7 [16pt] Consider the context free grammar for (simplified) λ-calculus:

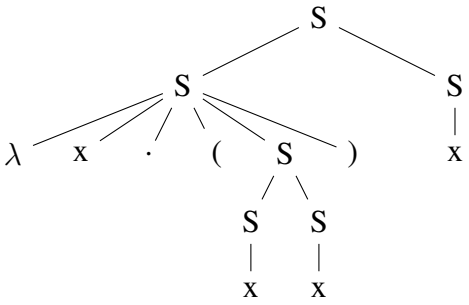
$$S ::= \mathtt{x} \mid \lambda \mathtt{x}.\ (S) \mid S\ S$$

where x is a terminal for letter x, symbols 'λ', '.', '(', ')' are terminals.

1. (8pt) Write down the **leftmost derivation** for the string "λx.(x x) x". **Show the detailed derivation in your answer**.

S ⇒ S S ⇒ λx.(S) S ⇒ λx. (S S) S⇒ λx. (x S) S ⇒ λx. (x x) S ⇒ λx. (x x) x

2. (8pt) Draw the *concrete* parse tree for the string "λx.(x x) x".



Bonus Questions

Bonus Problem 1 [5pt] Implement a function `findMax` **using foldl** in Scheme to get the largest element from a list of numbers. If the list is empty, your implementation should return $-$`inf.0`, which represents negative infinity in Scheme. Here are a couple of examples:

```
(findMax  '())              ; returns -inf.0
(findMax  '(1 -3 9 13 -2))  ; returns 13
(findMax  '(-4 -5 -1))      ; returns -1
```

```
(define (findMax lst)
   (foldl (lambda (x c) (if (> x c) x c)) -inf.0 lst))
```

Bonus Problem 2 [5pt] A list can be encoded in the λ-calculus by its `fold` function (the `foldr` function in Scheme). For example, a list with x,y,z can be encoded as $\lambda c.\ \lambda n.\ (c\ x\ (c\ y\ (c\ z\ n)))$. Moreover, an empty list `nil` is simply encoded as `nil` $\triangleq \lambda c.\ \lambda n.\ n$.

Under such encoding, define a function `cons` (the `cons` function in Scheme) that takes an element h, a list t and returns a list where h is the head, and t is the tail.

`cons` $\triangleq (\lambda h.\ \lambda t.\ (\lambda c.\ \lambda n.\ c\ h\ (t\ c\ n)))$