CMSC5741 Big Data Tech. & Apps.

Lecture 8: Massive Link Analysis

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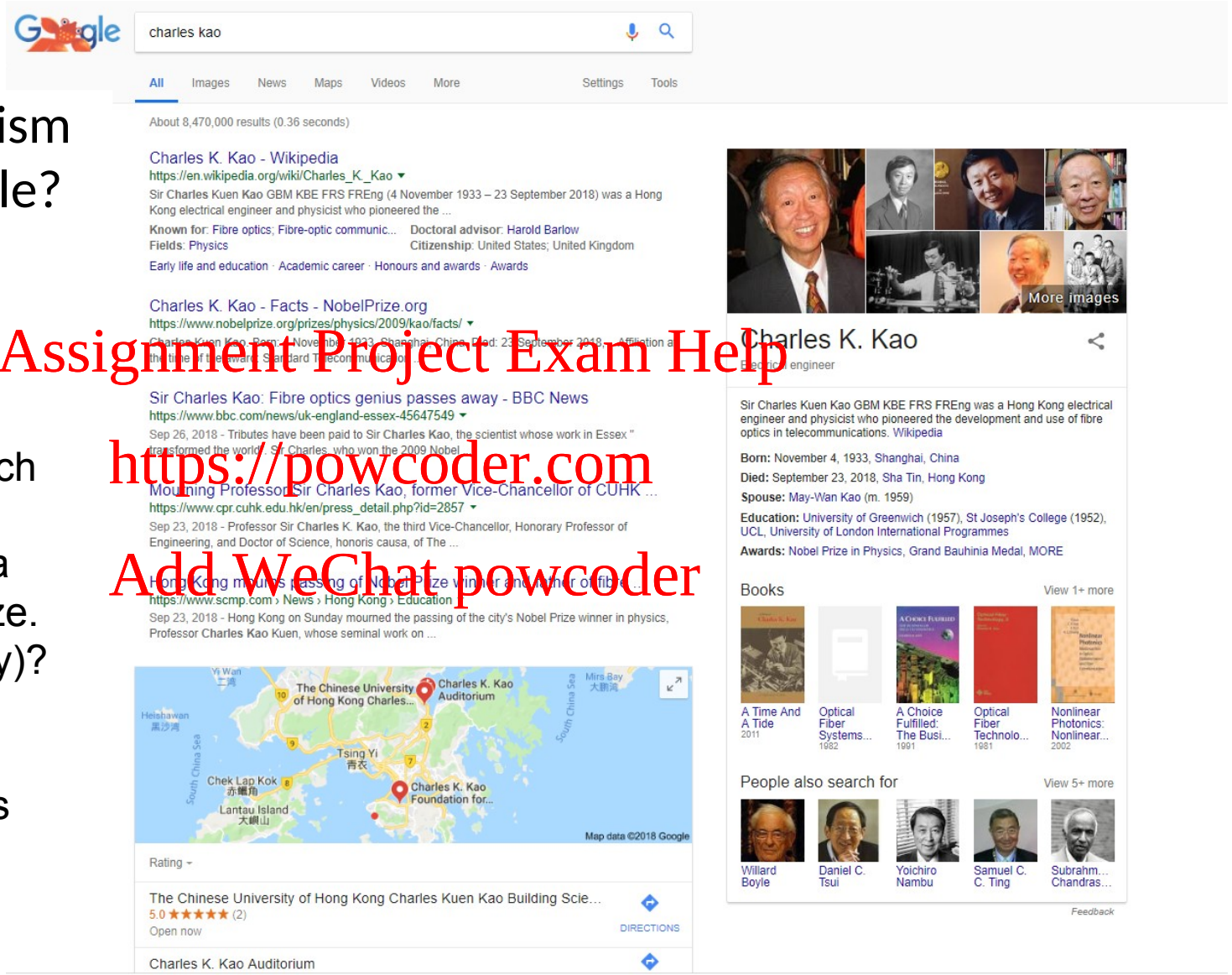
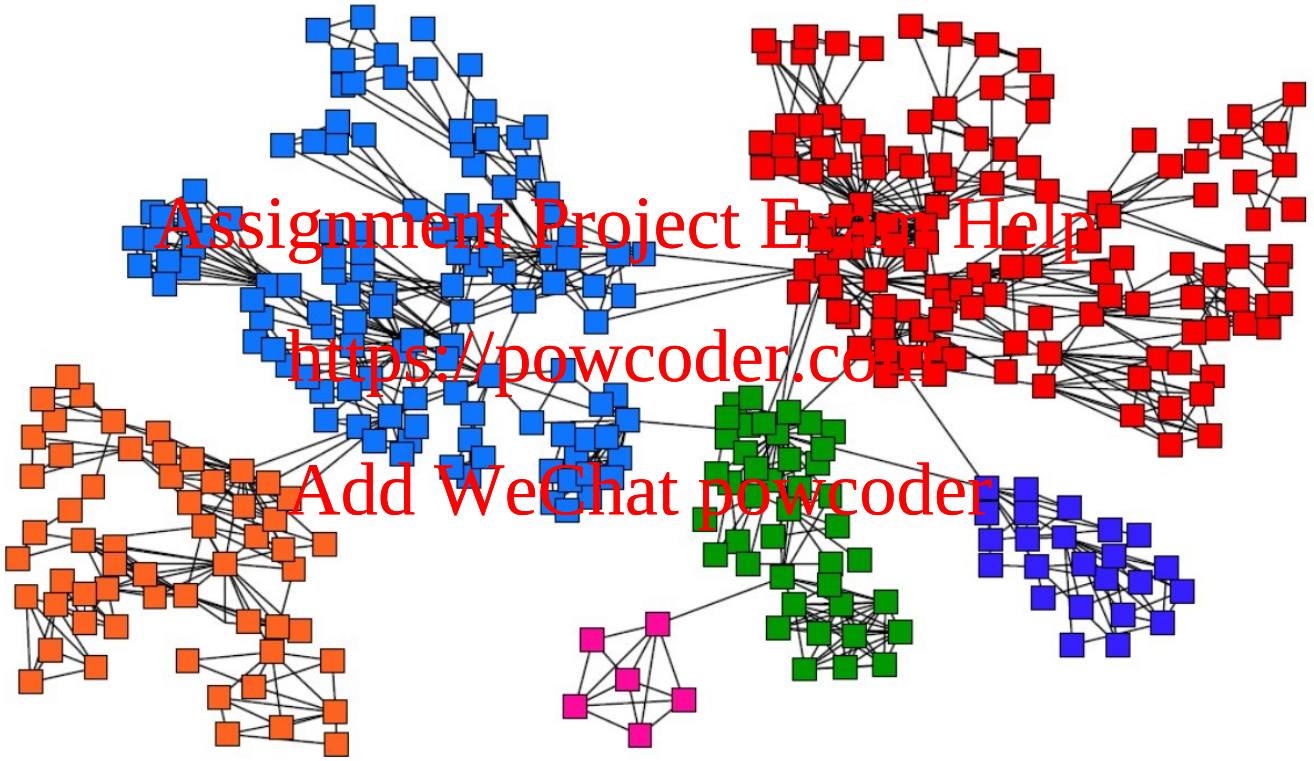
What's the Mechanism Distinguished Google?

How Google return such kind of rankings (e.g. Charles Kao Wikipedia first then his NobelPrize. Even his passing away)?

One important factor is PageRank score.



How to make PageRank computation scalable



- There are billions of web pages and hyperlinks between them, how to compute their ranking score (e.g., PageRank) efficiently?

Outline

- Web as a Graph
- PageRank
- Topic-Specific PageRank
- Appendix: Trust-Rank

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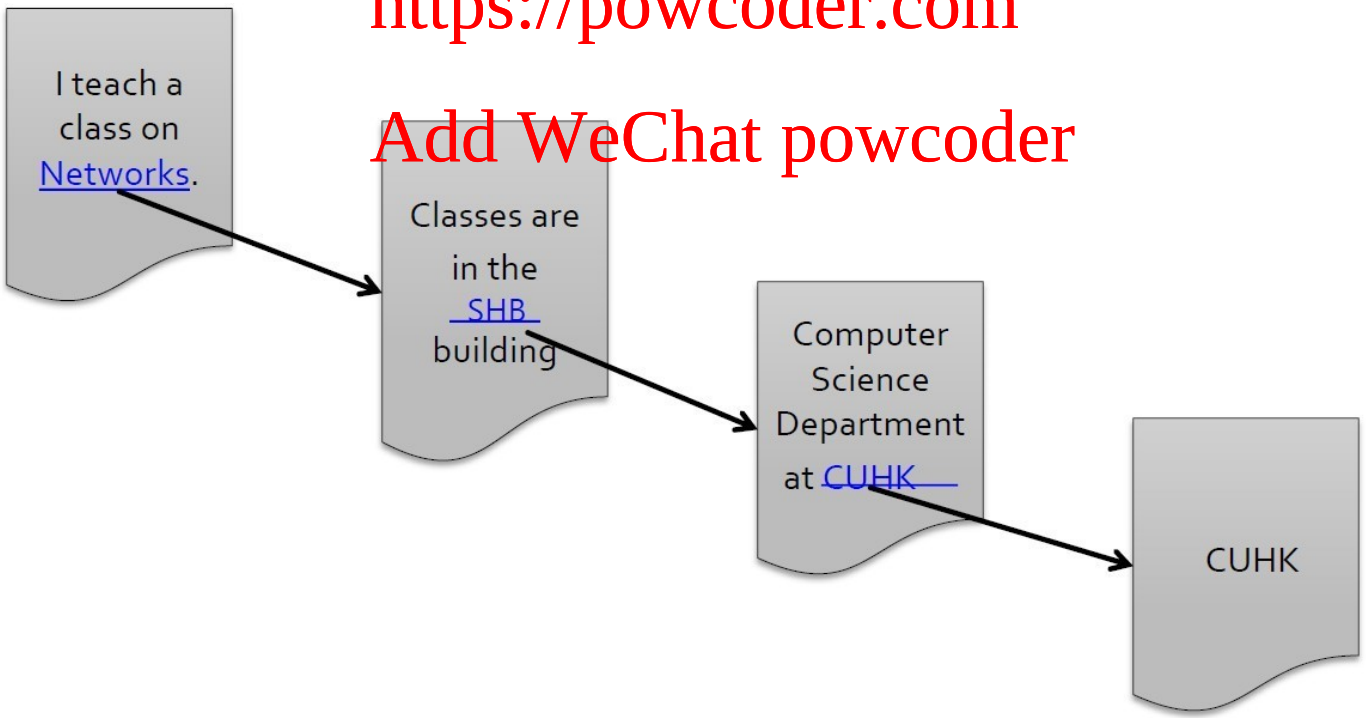
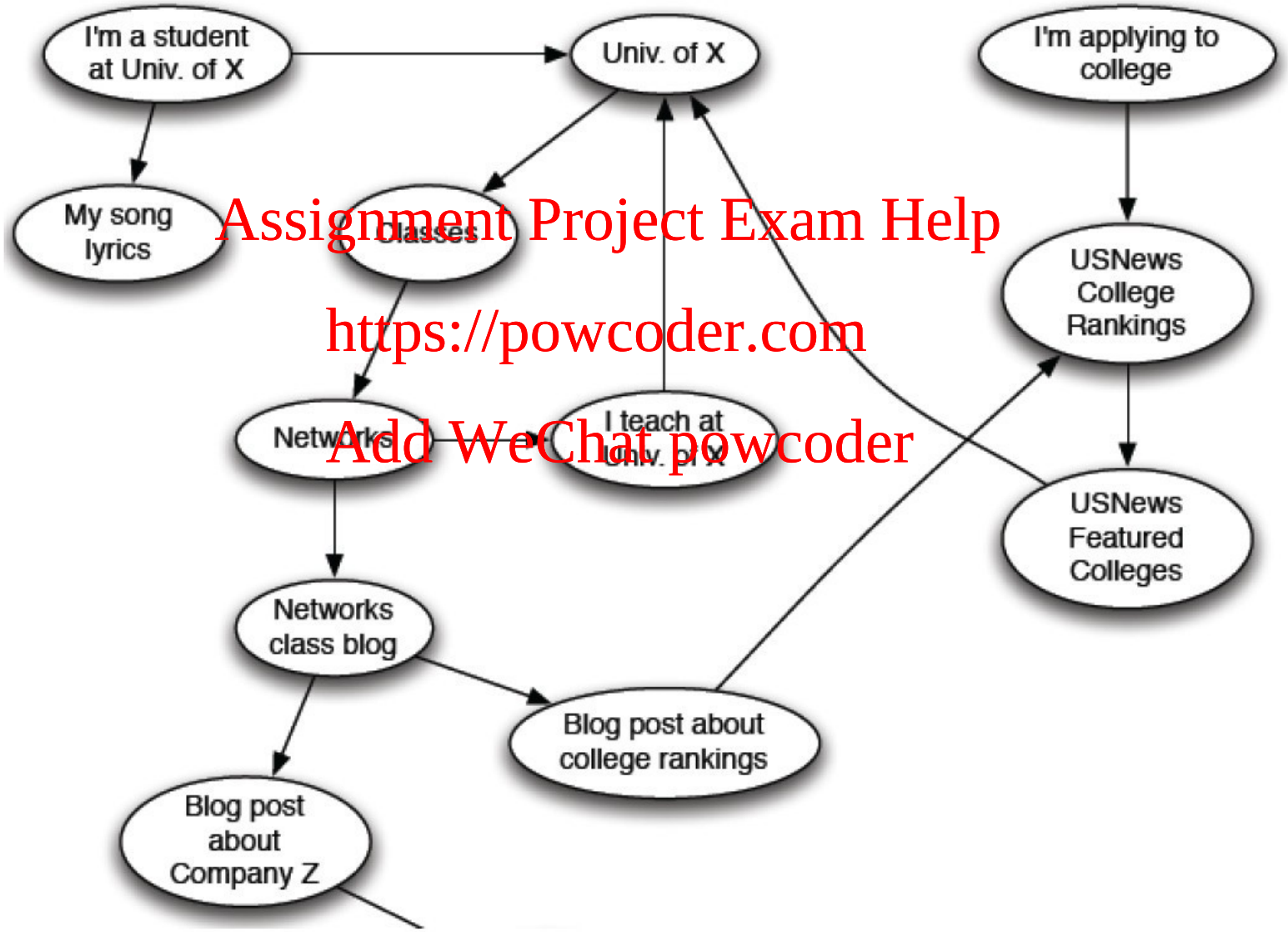
Web as a Graph

- Web as a directed graph:
  - Nodes: Web pages
  - Edges: Hyperlinks



Web as a Directed Graph



Broad Question

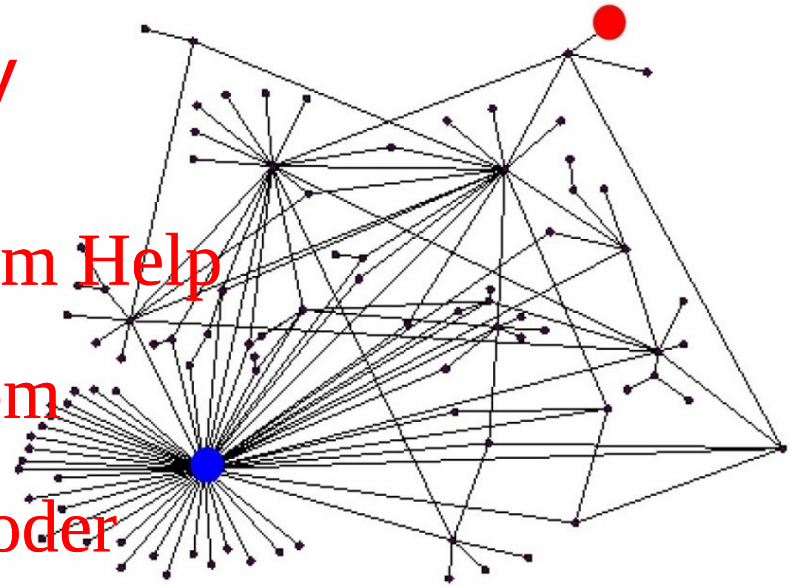
- How to organize the Web?
- First try: Human curated Web directories
  - Yahoo, DMOZ, LookSmart
- Second try: Web Search
  - Information Retrieval investigates: Find relevant docs in a small and trusted set
    - Newspaper articles, Patents, etc.
  - But: Web is huge, full of untrusted documents, random things, web spam, etc.

Web Search: 2 Challenges

- Web contains many sources of information: Who to "trust"?
  - Trick: Trustworthy pages may point to each other
- What is the "best" answer to query "newspaper"?
  - No single right answer
  - Trick: Pages that actually know about newspapers might all be pointing to many newspapers

Ranking Nodes on the Graph

- All web pages are not equally "important"
  - www.cuhk.edu.hk vs. www.joe-schmoe.com
- There is large diversity in the web-graph node connectivity.
  - Rank the pages by the link structure!



Link Analysis Algorithms

- We will cover the following Link Analysis approaches for computing importance of nodes in a graph
  - PageRank
  - Topic-Specific (Personalized) PageRank
  - Web Spam Detection Algorithms, e.g. TrustRank

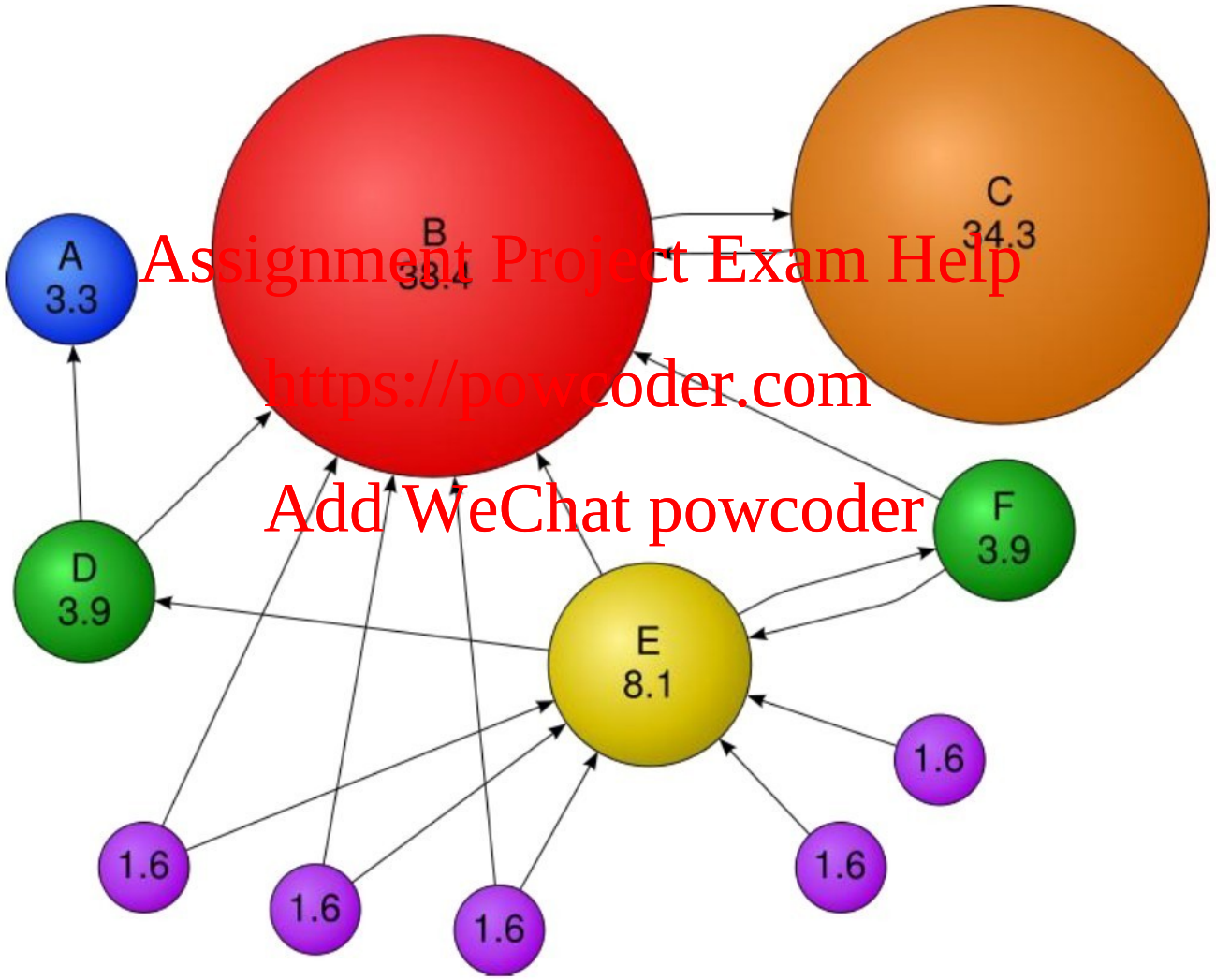
Outline

- Web as a Graph
- PageRank
- Topic-Specific PageRank
- Appendix: Trust-Rank

Links as Votes

- Idea: Links as votes
  - Page is more important if it has more links
    - In-coming links? Out-going links?
- Think of in-links as votes:
  - www.cuhk.edu.hk has 15,432 in-links
  - www.joe-schmoe.com has 1 in-link
- Are all in-links equal?
  - Link from important pages count more
  - Recursive question!

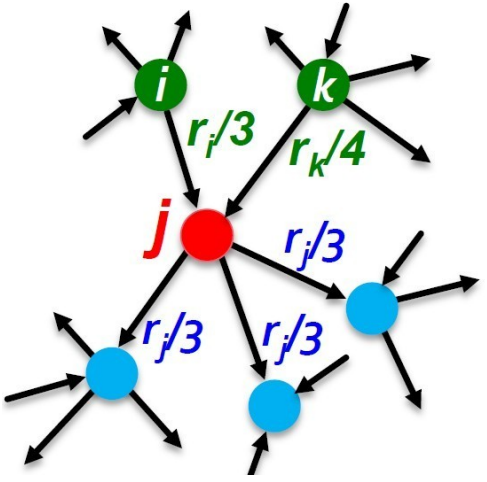
Example: PageRank Scores



Simple Recursive Formulation

- Each link's vote is proportional to the importance of its source page
- If page *j* with importance r_j has *n* out-links, each link gets r_j/n votes.
- Page *j*'s own importance is the sum of the votes on its in-links

$$r_j = r_i/3 + r_k/4$$

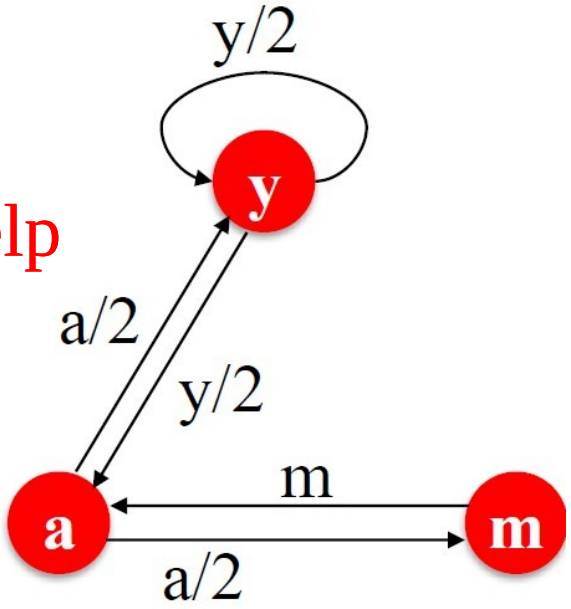


PageRank: The "Flow" Model

- A "vote" from an important page is worth more
- A page is important if it is pointed to by other important pages r_j
- Define a "rank" for page *j*

$$r_j = \sum_{i \to j} \frac{r_i}{d_i}$$

$d_i \ldots$ out-degress of node i



"Flow" equations:

$r_y = r_y/2 + r_a/2$

$r_a = r_y/2 + r_m$

$r_m = r_a/2$

Solving the Flow Equations

- 3 equations, 3 unknowns, no constants
  - No unique solution
  - All solutions equivalent modulo the scale factor
- Additional constraint forces uniqueness:
  - $r_y + r_a + r_m = 1$
  - Solution: $r_y = \frac{2}{5}, r_a = \frac{2}{5}, r_m = \frac{1}{5}$
- Gaussian Elimination method works for small examples, but we need a better method for large web-size graphs

Flow equations:

$$r_y = r_y/2 + r_a/2$$
$$r_a = r_y/2 + r_m$$
$$r_m = r_a/2$$

PageRank: Matrix Formulation

- Stochastic adjacency matrix M
  - Let page i have d_i out-links
  - If $i \to j$, then $M_{ji} = \frac{1}{d_i}$, else $M_{ji} = 0$
    - M is a column stochastic matrix, i.e., columns sum to 1
- Rank vector r: vector with an entry per page
  - r_i is the importance score of page i
  - $\sum_i r_i = 1$
- The flow equations can be written $r_j = \sum_{i \to j} \frac{r_i}{d_i}$

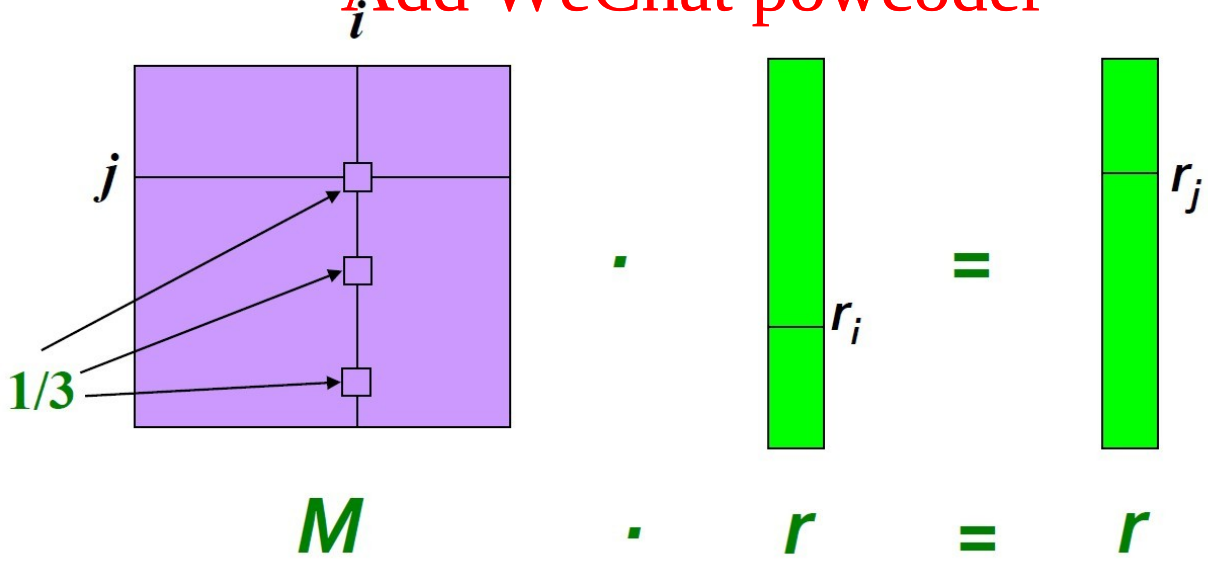
$$r = M \cdot r$$

Example

- Remember the flow equation: $r_j = \sum_{i \to j} \frac{r_i}{d_i}$
- Flow equation in the matrix form:

$$r = M \cdot r$$

– Suppose page i links to 3 pages, including j



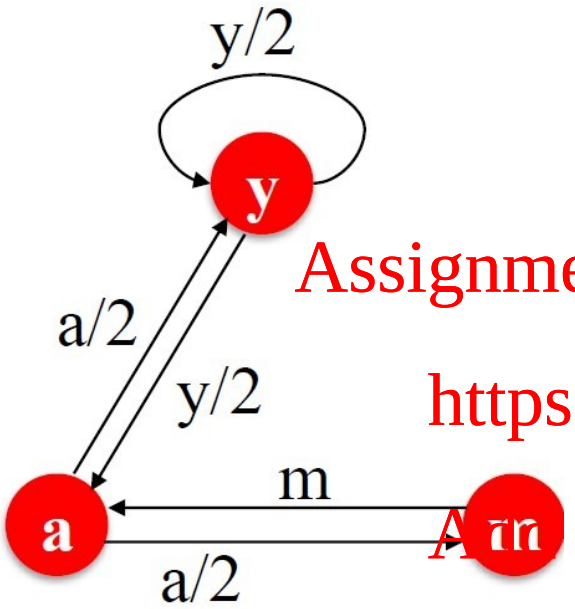
Eigenvector Formulation

- The flow equations can be written $r = M \cdot r$
- So the rank vector r is an eigenvector of the stochastic web matrix M
  - In fact, its first or principal eigenvector, with corresponding eigenvalue 1
    - Largest eigenvalue of M is 1 since M is column stochastic
- We can now efficiently solve for r!
  - The method is called Power iteration.

NOTE: x is an eigenvector with the corresponding eigenvalue λ if:

$$Ax = \lambda x$$

Example: Flow Equation & *M*



	y	a	m
y	½	½	0
a	½	0	1
m	0	½	0

$r = M \cdot r$

"Flow" equations:

$$r_y = r_y/2 + r_a/2$$
$$r_a = r_y/2 + r_m$$
$$r_m = r_a/2$$

$$\begin{bmatrix} y \\ a \\ m \end{bmatrix} = \begin{bmatrix} ½ & ½ & 0 \\ ½ & 0 & 1 \\ 0 & ½ & 0 \end{bmatrix} \begin{bmatrix} y \\ a \\ m \end{bmatrix}$$

Power Iteration Method

- Given a web graph with *n* nodes, where the nodes are pages and edges are hyperlinks
- Power Iteration: a simple iterative scheme
  - Suppose there are *N* web pages
  - Initialize: $\mathbf{r}^{(0)} = [1/N, \ldots, 1/N]^T$
  - Iterate: $\mathbf{r}^{(t+1)} = \mathbf{M} \cdot \mathbf{r}^{(t)}$
  - Stop when $|\mathbf{r}^{(t+1)} - \mathbf{r}^{(t)}|_1 < \epsilon$
    - $|\mathbf{x}|_1 = \sum_{1 \le i \le N} |x_i|$ is the L1 norm

$$r_j = \sum_{i \to j} \frac{r_i}{d_i}$$

$d_i \ldots$ out-degress of node i

Why Power Iteration Works?

- Power iteration:
  - A method for finding dominant eigenvector (the vector corresponding to the largest eigenvalue)
    - $\mathbf{r}^{(1)} = \mathbf{M} \cdot \mathbf{r}^{(0)}$
    - $\mathbf{r}^{(2)} = \mathbf{M} \cdot \mathbf{r}^{(1)} = \mathbf{M}(\mathbf{M}\mathbf{r}^{(1)}) = \mathbf{M}^{\mathbf{2}} \cdot \mathbf{r}^{(0)}$

      $\mathbf{r}^{(3)} = \mathbf{M} \cdot \mathbf{r}^{(2)} = \mathbf{M}(\mathbf{M}^{\mathbf{2}}\mathbf{r}^{(0)}) = \mathbf{M}^{\mathbf{3}} \cdot \mathbf{r}^{(0)}$
- Claim:
  - Sequence $\mathbf{M} \cdot \mathbf{r}^{(0)}, \mathbf{M}^{\mathbf{2}} \cdot \mathbf{r}^{(0)}, \ldots, \mathbf{M}^{\mathbf{k}} \cdot \mathbf{r}^{(0)}, \ldots$ approaches the dominant eigenvector of ***M***

Why Power Iteration Works?

- Proof:
  - Assume ***M*** has ***n*** linearly independent eigenvectors $x_1, x_2, \ldots, x_n$ with corresponding eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, where $\lambda_1 > \lambda_2 > \ldots > \lambda_n$
  - Vectors $x_1, x_2, \ldots, x_n$ form a basis and thus we can write: $r^{(0)} = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$
  - $$\begin{aligned} Mr^{(0)} &= M(c_1 x_1 + c_2 x_2 + \ldots + c_n x_n) \\ &= c_1(Mx_1) + c_2(Mx_2) + \ldots + c_n(Mx_n) \\ &= c_1(\lambda_1 x_1) + c_2(\lambda_2 x_2) + \ldots + c_n(\lambda_n x_n) \end{aligned}$$
  - Repeated multiplication on both sides:

$$M^k r^{(0)} = c_1(\lambda_1^k x_1) + c_2(\lambda_2^k x_2) + \ldots + c_n(\lambda_n^k x_n)$$

Why Power Iteration Works?

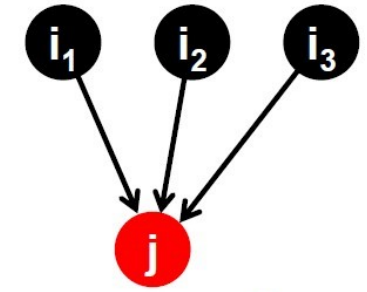
- Proof: (cont.)
  - Repeated multiplication on both sides produces

$$M^k r^{(0)} = \lambda_1^k \left[c_1 x_1 + c_2 \left(\frac{\lambda_2}{\lambda_1}\right)^k x_2 + \cdots + c_n \left(\frac{\lambda_2}{\lambda_1}\right)^k x_n \right]$$

  - Since $\lambda_1 > \lambda_2$ then $\frac{\lambda_2}{\lambda_1}, \frac{\lambda_3}{\lambda_1} \cdots < 1$, so $\frac{\lambda_i}{\lambda_1} = 0 \text{ as } k \to \infty$
  - Thus $M^k r^{(0)} \approx c_1(\lambda_1^k x_1)$
    - Note if $c_1 = 0$, then the method won't converge

Random Walk Interpretation

- Imagine a random web surfer:
  - At any time t, surfer is on some page *i*
  - At time t+1, the surfer follows an out-link from *i* uniformly at random
  - Ends up on some page *j* linked from *i*
  - Process repeats indefinitely
- Let:
  - p(t) … vector whose i^{th} coordinate is the prob. that the surfer is at page *i* at time *t*
  - So p(t) is a probability distribution over pages



$$r_j = \sum_{i \to j} \frac{r_i}{\mathrm{d}_{\mathrm{out}}(\mathrm{i})}$$

The Stationary Distribution

- Where is the surfer at time t+1
  - Follows a link uniformly at random

$$p(t+1) = M \cdot p(t)$$

- Suppose the random walk reaches a state

$$p(t+1) = M \cdot p(t) = p(t)$$

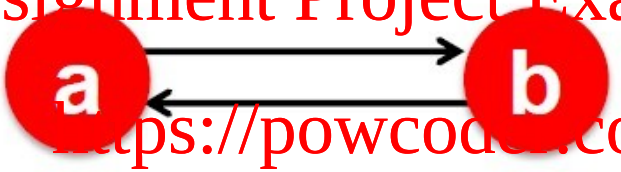
Then p(t) is stationary distribution of a random walk

- Our original rank vector r satisfies $r = M \cdot r$
  - So r is a stationary distribution for the random walk

PageRank

- Three questions:
  - Does this converge?
  - Does it converge to what we want?
  - Are results reasonable?

Does This Converge?



$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{d_i}$$

- **Example:**

$$\begin{matrix} r_a \\ r_b \end{matrix} = \begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{matrix}$$

Iteration 0, 1, 2, …

Does it Converge to What We Want?

$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{d_i}$$

- **Example:**

$$\begin{matrix} r_a \\ r_b \end{matrix} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$$

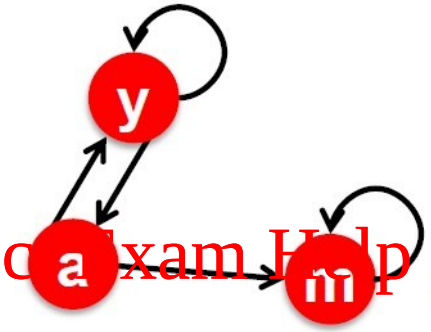
Iteration 0, 1, 2, …

PageRank: Problems

- Two problems:
  - Spider traps: all out-links are within the group
    - Eventually spider traps absorb all importance
  - Some pages are dead ends (have no out-links)
    - Such pages cause importance to "leak out"

Problem: Spider Traps

- Power Iteration:
  - Set $r_j = 1$
  - $r_j = \sum_{i \to j} \frac{r_i}{d_i}$
    - And iterate
- Example



	y	a	m
y	½	½	0
a	½	0	0
m	0	½	1

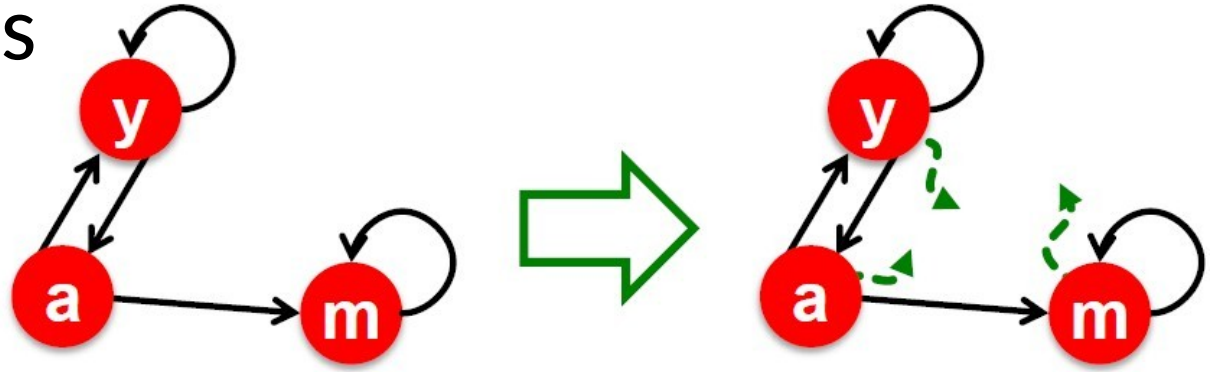
$r_y = r_y/2 + r_a/2$
$r_a = r_y/2$
$r_m = r_a/2 + r_m$

$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = \begin{matrix} 1/3 & 2/6 & 3/12 & 5/24 & & 0 \\ 1/3 & 1/6 & 2/12 & 3/24 & \ldots & 0 \\ 1/3 & 3/6 & 7/12 & 16/24 & & 1 \end{matrix}$$

Iteration 0, 1, 2, …

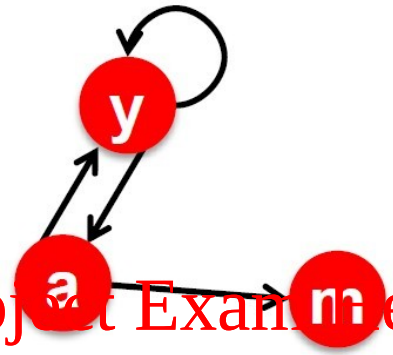
Solution: Random Teleports

- The Google solution for spider traps: At each time step, the random surfer has two options
  - With prob. β, follow a link at random
  - With prob. $1-\beta$, jump to some random page
  - Common values for β are in the range 0.8 to 0.9
- Surfer will teleport out of spider trap within a few time steps



Problem: Dead Ends

- Power Iteration:
  - Set $r_j = 1$
  - $r_j = \sum_{i \to j} \frac{r_i}{d_i}$
    - And iterate
- Example



	y	a	m
y	½	½	0
a	½	0	0
m	0	½	0

$r_y = r_y/2 + r_a/2$
$r_a = r_y/2$
$r_m = r_a/2$

$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = \begin{matrix} 1/3 & 2/6 & 3/12 & 5/24 & & 0 \\ 1/3 & 1/6 & 2/12 & 3/24 & \ldots & 0 \\ 1/3 & 1/6 & 1/12 & 2/24 & & 0 \end{matrix}$$

Iteration 0, 1, 2, …

Solution: Always Teleport

- Teleports: Follow random teleport links with probability 1.0 from dead-ends
  - Adjust matrix accordingly

	y	a	m
y	½	½	0
a	½	0	0
m	0	½	0

	y	a	m
y	½	½	⅓
a	½	0	⅓
m	0	½	⅓

Why Teleports Solve the Problem?

$$\mathbf{r}^{(\mathbf{t+1})} = Mr^{(t)}$$

- Markov chains

  - Set of states **X**

  - Transition matrix **P** where $P_{ij} = P(X_t = i | X_{t-1} = j)$
  - π specifying the stationary probability of being at each state $x \in X$
  - Goal is to find π such that $\pi = P\pi$

Why Is This Analogy Useful?

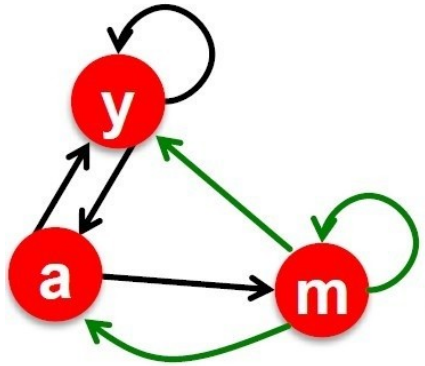
- Theory of Markov chains
- Fact: For any start vector, the power method applied to a Markov transition matrix **P** will converge to a unique positive stationary vector as long as **P** is stochastic, irreducible and aperiodic.

Make M Stochastic

- Stochastic: Every column sums to 1
- A possible solution: add green links

$$A = M + a^T \left(\frac{1}{n} e\right)$$

- $a_i = 1$ if node i has out degree 0, otherwise 0.
- e . . . vector of all 1s

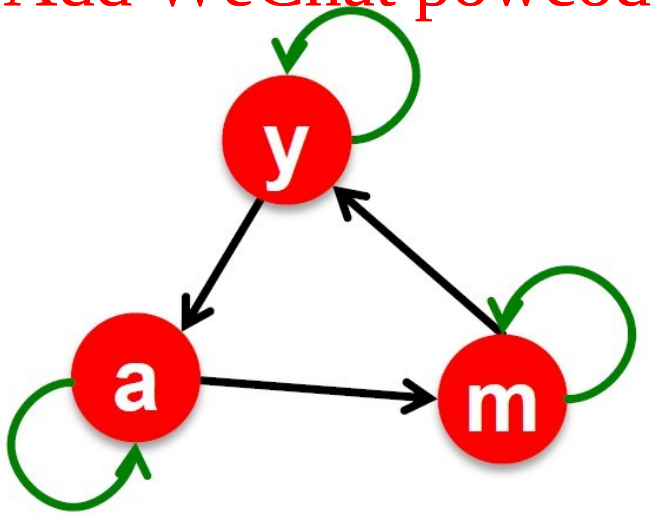


	y	a	m
y	½	½	1/3
a	½	0	1/3
m	0	½	1/3

$$r_y = r_y/2 + r_a/2 + r_m/3$$
$$r_a = r_y/2 + r_m/3$$
$$r_m = r_a/2 + r_m/3$$

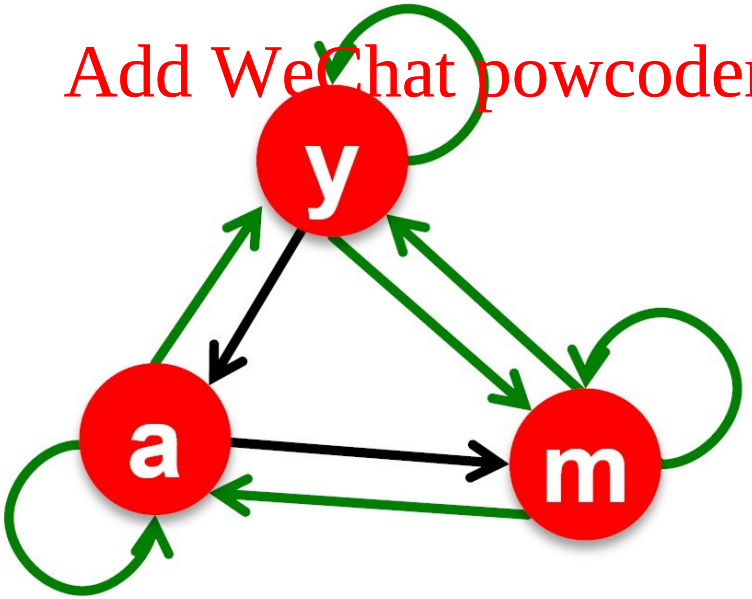
Make M Aperiodic

- A chain is periodic if there exists **k > 1** such that the interval between two visits to some state **s** is always a multiple of **k**.
- A possible solution: add green links



Make M Irreducible

- From any state, there is a non-zero probability of going from any one state to any another
- A possible solution: add green links



Solution: Random Jumps

- Google's solution that does it all:
  - Makes M stochastic, aperiodic, irreducible
- At each step, random surfer has two options:
  - With probability β, follow a link at random
  - With probability $1-\beta$, jump to some random page
- PageRank equation [Brin-Page,98]

$$r_j = \sum_{i \to j} \beta \frac{r_i}{d_i} + (1-\beta)\frac{1}{n}$$

This formulation assumes that **M** has no dead ends. We can either preprocess matrix **M** to remove all dead ends or explicitly follow random teleport links with probability 1.0 from dead-ends.

The Google Matrix

- PageRank equation [Brin-Page,98]

$$r_j = \sum_{i \to j} \beta \frac{r_i}{d_i} + (1 - \beta) \frac{1}{n}$$

- The Google Matrix A:

$$A = \beta M + (1 - \beta) \frac{1}{n} \mathbf{e} \cdot \mathbf{e}^{\mathbf{T}}$$

- A is stochastic, aperiodic and irreducible, so

$$\mathbf{r}^{(\mathbf{t+1})} = \mathbf{A} \cdot \mathbf{r}^{(\mathbf{t})}$$

- In practice $\beta = 0.8, 0.9$ (make 5 steps and jump)

In-class Practice

- Go to Practice

Computing PageRank

- Key step is matrix-vector multiplication
  - $r^{new} = A \cdot r^{old}$
- Easy if we have enough main memory to hold A, r^{old}, r^{new}
- Say N=1 billion pages
  - We need 4 bytes for each entry (say)
  - 2 billion entries for vectors, approx. 8GB
  - Matrix A has N^2 entries: 10^{18} is a large number!

Matrix Formulation

- Suppose there are ***N*** pages
  - Consider page j, with d_j out-links
  - We have $M_{ij} = 1/|d_j|$ when $j \to i$ and $M_{ij} = 0$ otherwise
- The random teleport is equivalent to:
  - Adding a teleport link from j to every other page and setting transition prob. to $(1-\beta)/N$
  - Reducing the prob. of following each out-link from $1/|d_j|$ to $\beta/|d_j|$

Rearranging the Equation

- $r = A \cdot r$, where $A_{ij} = \beta M_{ij} + \frac{1-\beta}{N}$
- $r_i = \sum_{j=1}^{N} A_{ij} \cdot r_j$
- $$r_i = \sum_{j=1}^{N} \left[\beta M_{ij} + \frac{1-\beta}{N}\right] \cdot r_j$$
  $$= \sum_{j=1}^{N} \beta M_{ij} \cdot r_j + \frac{1-\beta}{N} \sum_{j=1}^{N} r_j$$
  $$= \sum_{j=1}^{N} \beta M_{ij} \cdot r_j + \frac{1-\beta}{N}, \text{ since } \sum r_j = 1$$
- So we get: $r = \beta M \cdot r + \left[\frac{1-\beta}{N}\right]_N$

Note: Here we assumed **M** has no **dead-ends**

$[x]_N$ … a vector of length *N* with all entries x

Spare Matrix Formulation

- We just rearranged the PageRank equation

$$r = \beta M \cdot r + \left[\frac{1-\beta}{N}\right]_N$$

- M is a sparse matrix! (with no dead-ends)

  - 10 links per node, approx. 10*N* entries

- So in each iteration, we need to
  - Compute $r^{new} = A \cdot r^{old}$
  - Add a constant $(1-\beta)/N$ to each entry in r^{new}
    - Note: if *M* contains dead-ends then $\sum_i r_i^{new} < 1$ and we also have to renormalize r^{new} so that it sums to 1

PageRank: The Complete Algorithm

- **Input**: Graph $\mathbf{G}$ and parameter β
  - Directed graph $\mathbf{G}$ with **spider traps** and **dead ends**
  - Parameter β

- **Output**: PageRank vector r

  - Set: $r_j^{(0)} = \frac{1}{N}, t = 1$
  - do:

    * $\forall j$: $r'^{(t)}_j = \sum_{i \to j} \beta \frac{r_i^{(t-1)}}{d_i}$
      $r'^{(t)}_j = 0$ if in-deg. of j is 0
    * Now re-insert the leaked PageRank:
      $\forall j : r_j^{(t)} = r'^{(t)}_j + \frac{1-S}{N}$ where $S = \sum_j r'^{(t)}_j$
    * $t = t + 1$
  - while $\sum_j |r_j^{(t)} - r_j^{(t-1)}| > \epsilon$

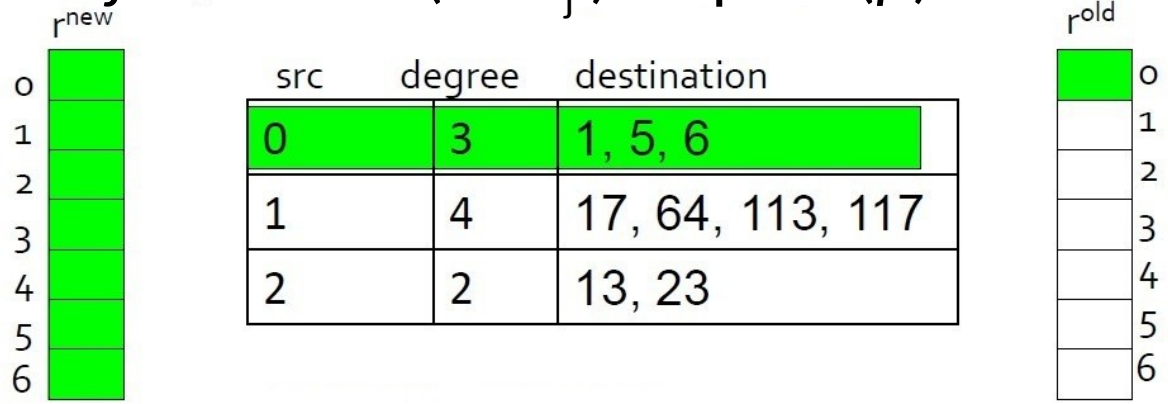
Sparse Matrix Encoding

- Encode sparse matrix using only nonzero entries
  - Space proportional roughly to number of links
  - Say 10N, or 4*10*1 billion = 40GB
  - Still won't fit in memory, but will fit on disk

source node	degree	destination nodes
0	3	1, 5, 7
1	5	17, 64, 113, 117, 245
2	2	13, 23

Basic Algorithm: Update Step

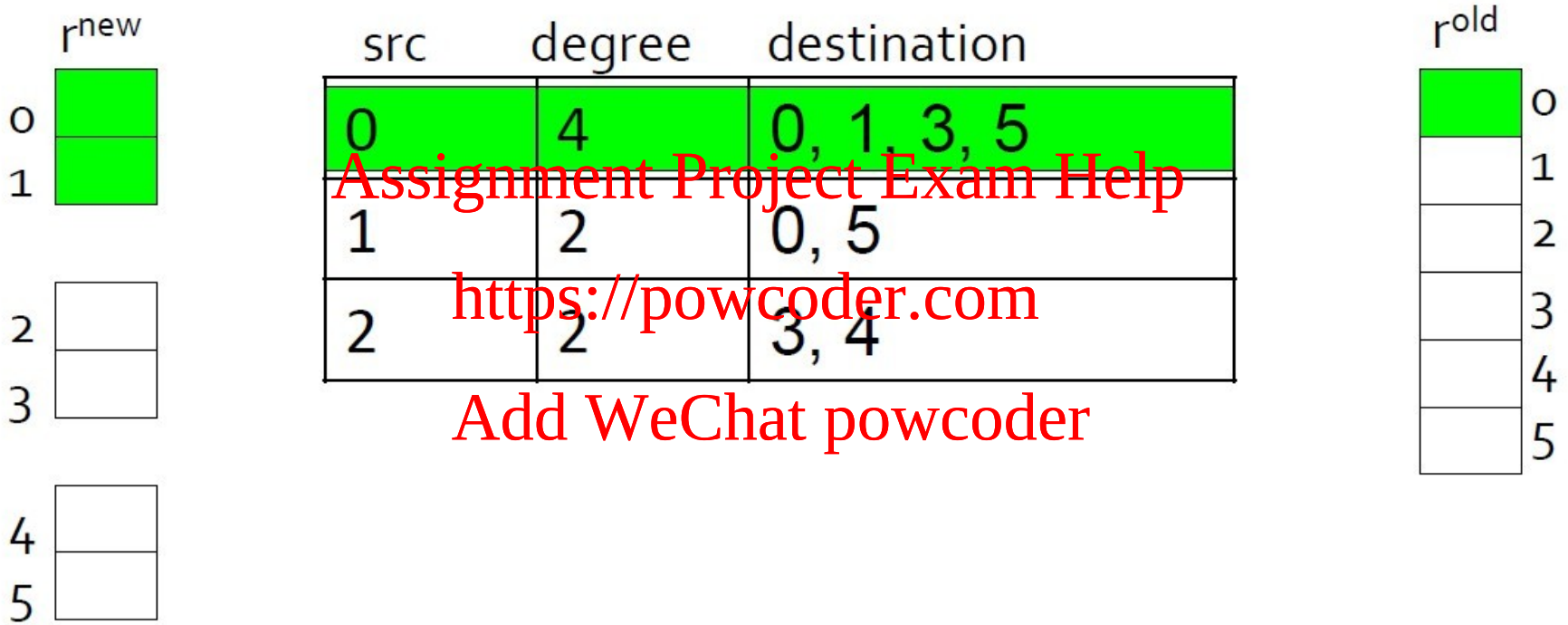
- Assume enough RAM to fit r^{new} into memory
  - Store r^{old} and matrix *M* on disk
- Then 1 step of power iteration is:
  - Initialize all entries of r^{new} to $(1-\beta)/N$
  - For each page p (of out-degree n):
    - Read into memory: *p*, *n*, $dest_1$, …, $dest_n$, $r^{old}(p)$
    - For *j*=1…*n*: $r^{new}(dest_j) \mathrel{+}= \beta\, r^{old}(p)/n$



Analysis

- Assume enough RAM to fit r^{new} into memory
  - Store r^{old} and matrix M on disk
- In each iteration, we have to:
  - Read r^{old} and M
  - Write r^{new} back to disk
  - IO cost = $2|r| + |M|$
- Question:
  - What if we could not even fit r^{new} in memory

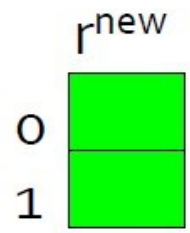
Block-based Update Algorithm



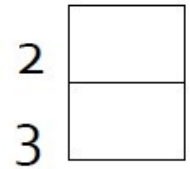
Analysis of Block Update

- Similar to nested-loop join in databases
  - Break r^{new} into *k* blocks that fit in memory
  - Scan *M* and r^{old} once for each block
- *k* scans of M and r^{old}
  - $k(|M|+|r|) + |r| = k|M| + (k+1)|r|$
- Can we do better?
  - Hint: *M* is much bigger than *r* (approx. 10-20x), so we must avoid reading it *k* times per iteration

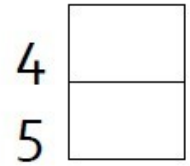
Block-Strip Update Algorithm



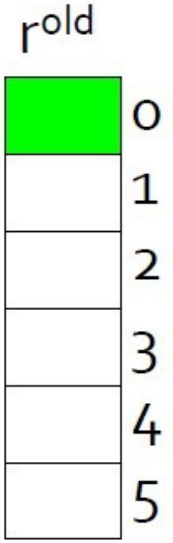
src	degree	destination
0	4	0, 1
1	3	0
2	2	1



0	4	3
2	2	3



0	4	5
1	3	5
2	2	4



Block-Strip Analysis

- Break *M* into stripes
  - Each strip contains only destination nodes in the corresponding block of r^{new}
- Some additional overhead per stripe
  - But it is usually worth it
- Cost per iteration

$$|M|(1+\epsilon) + (k+1)|r|$$

Some Problems with PageRank

- Measures generic popularity of a page
  - Biased against topic-specific authorities
  - Solution: Topic-Specific PageRank (next)
- Susceptible to Link spam
  - Artificial link topographies created in order to boost page rank
  - Solution: TrustRank (next)
- Uses a single measure of importance
  - Solution: Hubs-and-Authorities (in further reading)

Outline

- Web as a Graph
- PageRank
- Topic-Specific PageRank
- Appendix: Trust-Rank

Topic-Specific PageRank

- Instead of generic popularity, can we measure popularity within a topic?
- Goal: Evaluate Web pages not just according to their popularity, but by how close they are to a particular topic, e.g. "sports" or "history"
- Allows search queries to be answered based on interests of the user
  - Example: Query "Trojan" wants different pages depending on whether you are interested in sports, history and computer security

Topic-Specific PageRank

- Random walker has a small probability of teleporting at any step
- Teleport can go to:
  - Standard PageRank: Any page with equal probability
    - To avoid dead-end and spider-trap problems
  - Topic Specific PageRank: A topic-specific set of "relevant" pages (teleport set)

Topic-Specific PageRank

- Idea: Bias the random walk
  - When walker teleports, she picks a page from a set S
  - S contains only pages that are relevant to the topic
    - e.g., Open Directory (DMOZ) pages for a given topic/query
  - For each teleport set S, we get a different vector r_s

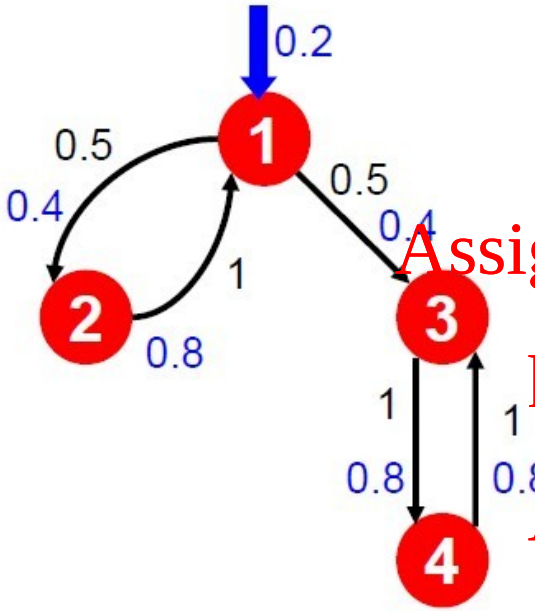
Matrix Formulation

- To make this work all we need is to update the teleportation part of the PageRank formulation:

$$A_{ij} = \begin{cases} \beta M_{ij} + (1-\beta)/|S| & \text{if } i \in S \\ \beta M_{ij} & \text{otherwise} \end{cases}$$

  - A is stochastic!
- We have weighted all pages in the teleport set S equally
  - Could also assign different weights to pages!
- Compute as for standard PageRank

Example



Suppose $S = \{1\}$, $\beta = 0.8$

Node	Iteration				
	0	1	2	…	stable
1	0.25	0.4	0.28		0.294
2	0.25	0.1	0.16		0.118
3	0.25	0.3	0.32		0.327
4	0.25	0.2	0.24		0.261

S={1}, β=0.90:
r=[0.17, 0.07, 0.40, 0.36]
S={1} , β=0.8:
r=[0.29, 0.11, 0.32, 0.26]
S={1}, β=0.70:
r=[0.39, 0.14, 0.27, 0.19]

S={1,2,3,4}, β=0.8:
r=[0.13, 0.10, 0.39, 0.36]
S={1,2,3} , β=0.8:
r=[0.17, 0.13, 0.38, 0.30]
S={1,2} , β=0.8:
r=[0.26, 0.20, 0.29, 0.23]
S={1} , β=0.8:
r=[0.29, 0.11, 0.32, 0.26]

Discovering the Topic

- Create different PageRanks for different topics
  - The 16 DMOZ top-level categories: arts, business, sports, …
- Which topic ranking to use?
  - User can pick from a menu
  - Classify query into a topic
  - Can use the context of the query
    - E.g., query is launched from a web page talking about a known topic
  - User context, e.g., user's bookmarks,…

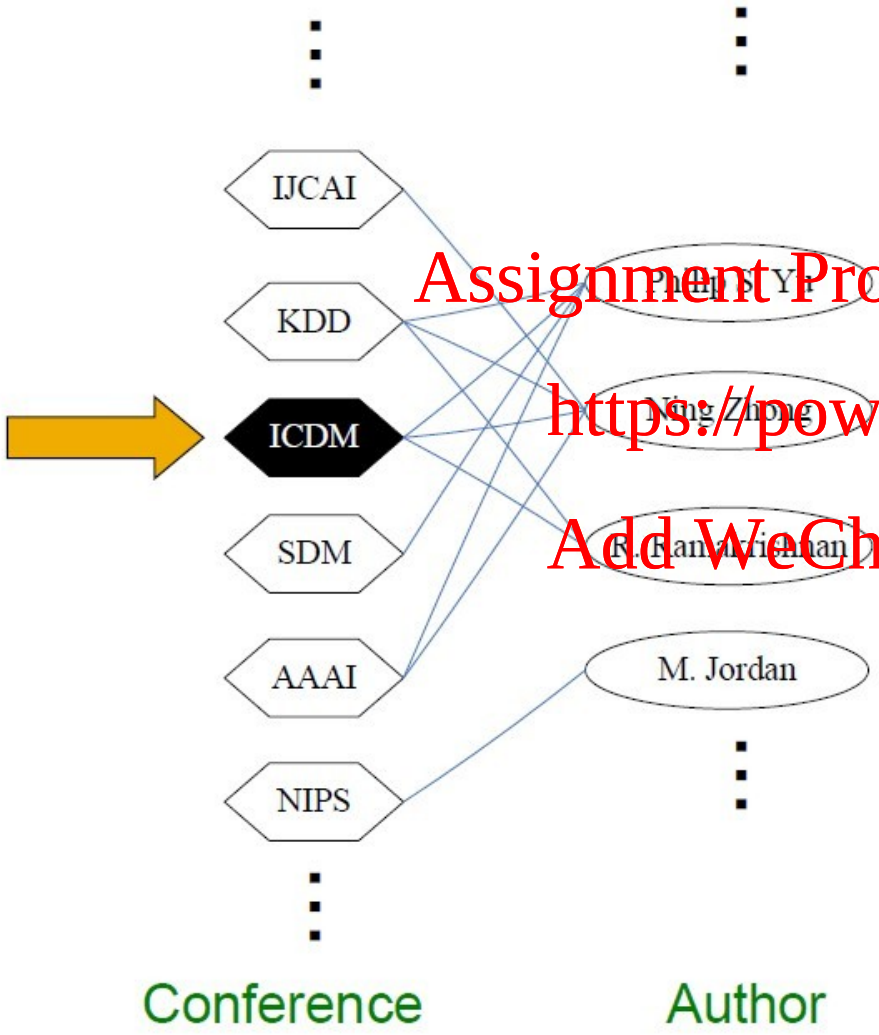
SimiRank: An Application of Personalized PageRank

- SimRank: Random walks from a fixed node on k-partite graphs
- Setting: k-partite graph with k types of nodes
  - Example: picture nodes and tag nodes
- How to find nodes similar to node *u*?
- Do a Random-Walk with Restarts from node *u*
  - i.e. teleport set $S = \{u\}$

SimiRank(cont.)

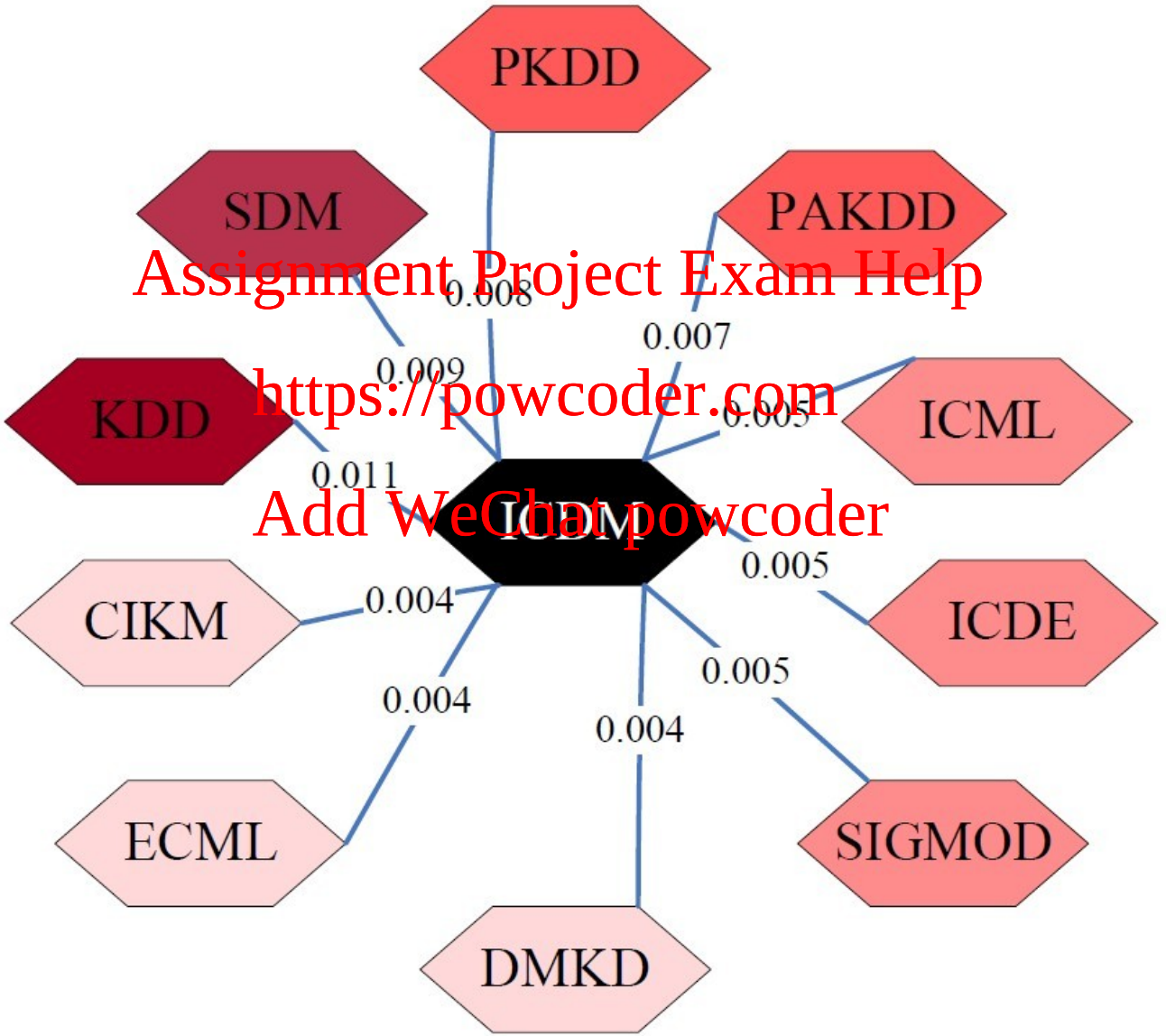
- Resulting scores measures similarity/proximity to node *u*
- Generally applicable to social networks (typically undirected graphs)
- Problems:
  - Must be done once for each node *u*
  - Suitable for sub-Web-scale applications

SimRank: Example



Q: What is most related conference to **ICDM**?

SimRank: Example



Outline

- Web as a Graph
- PageRank
- Topic-Specific PageRank
- Appendix: Trust-Rank
- Skip to conclusion

What is Web Spam?

- Spamming:
  - Any deliberate action to boost a web page's position in search engine results, incommensurate with page's real value
- Spam:
  - Web pages that are the result of spamming
- Approximately 10-15% of web pages are spam

Web Search

- Early Search Engines
  - Crawl the Web
  - Index pages by the words they contained
  - Respond to search queries with pages containing those words

Web Search

- Early page ranking
  - Attempt to order pages matching a search query by "importance"
  - First search engines considered:
    - Number of times query words appeared
    - Prominence of word position, e.g. title, header

First Spammers

- Those with commercial interests tried to exploit search engines to bring people to their own site – whether they wanted to be there or not
- Example
  - Shirt-seller might pretend to be about “movies”
- Techniques for achieving high relevance/importance for a web page

First Spammers: Term Spam

- How do you make your page appear to be about movies?
  - Add the word movie 1,000 times to your page, set text color to the background color, so only search engines would see it
  - Or, run the query "movie" on your target search engine, see what page came first in the listings, copy it into your page, make it "invisible"
- These and similar techniques are term spam

Google's Solution to Term Spam

- Believe what people say about you, rather than what you say about yourself
  - Use words in the anchor text (words that appear underlined to represent the link) and its surrounding text
  - PageRank as a tool to measure the "importance" of Web pages

Why It Works?

- Our hypothetical shirt-seller looses
  - Saying he is about movies doesn't help, because others don't say he is about movies, his page isn't very important, so it won't be ranked high for shirts or movies
- Example:
  - Shirt-seller creates 1,000 pages, each links to his with "movie" in the anchor text, these pages have no links in, so they get little PageRank
  - So the shirt-seller can't beat truly important movie, pages, like IMDB

Spam Farms

- Once Google became the dominant search engine, spammers began to work out ways to fool Google
- Spam farms were developed to concentrate PageRank on a single page
- Link spam:
  - Creating link structures that boost PageRank of a particular page

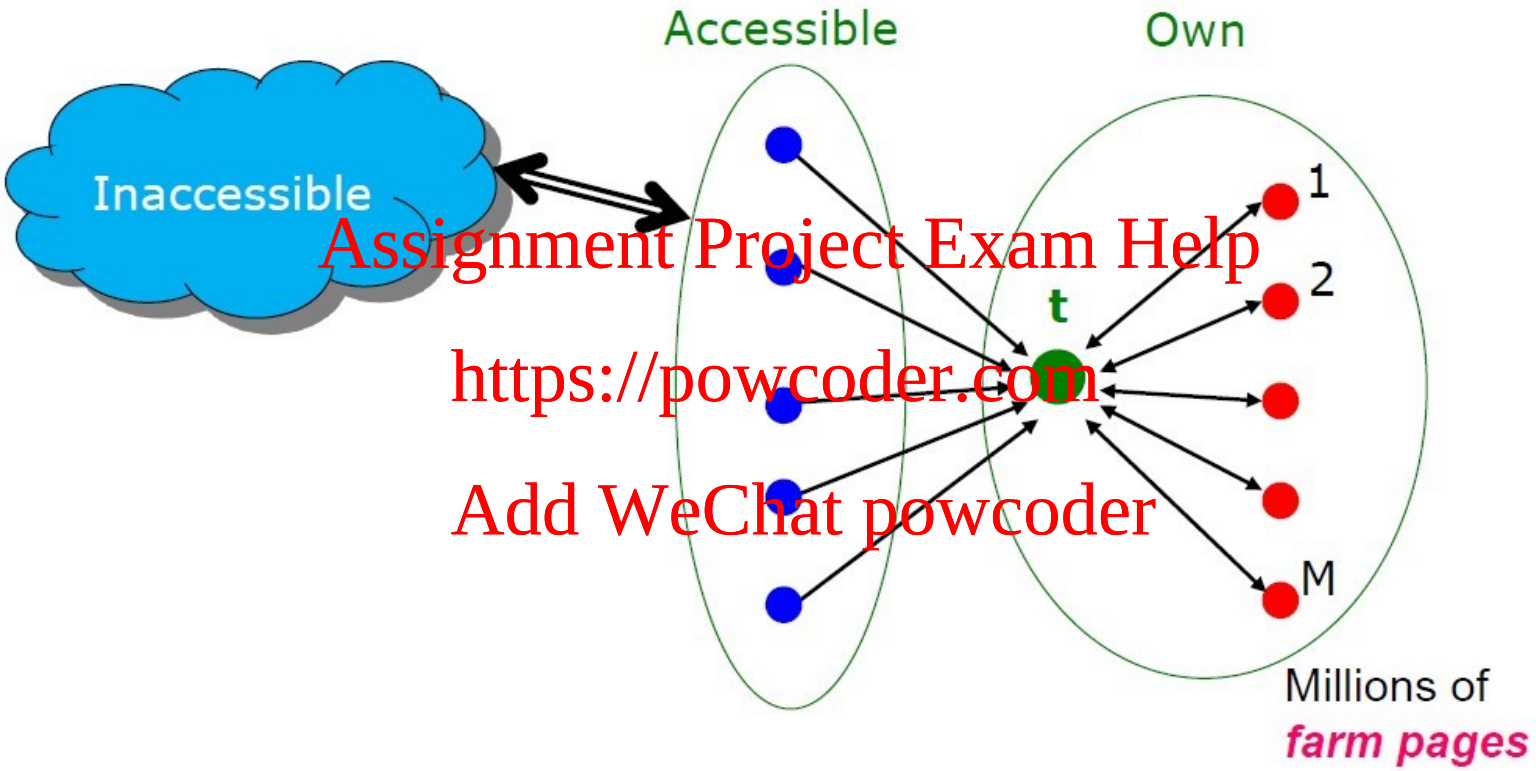
Link Spamming

- Three kinds of web pages from a spammer's point of view
  - Inaccessible pages
  - Accessible pages
    - E.g. blog comments pages
    - Spammer can post links to his pages
  - Own pages
    - Completely controlled by spammer
    - May span multiple domain names

Link Farms

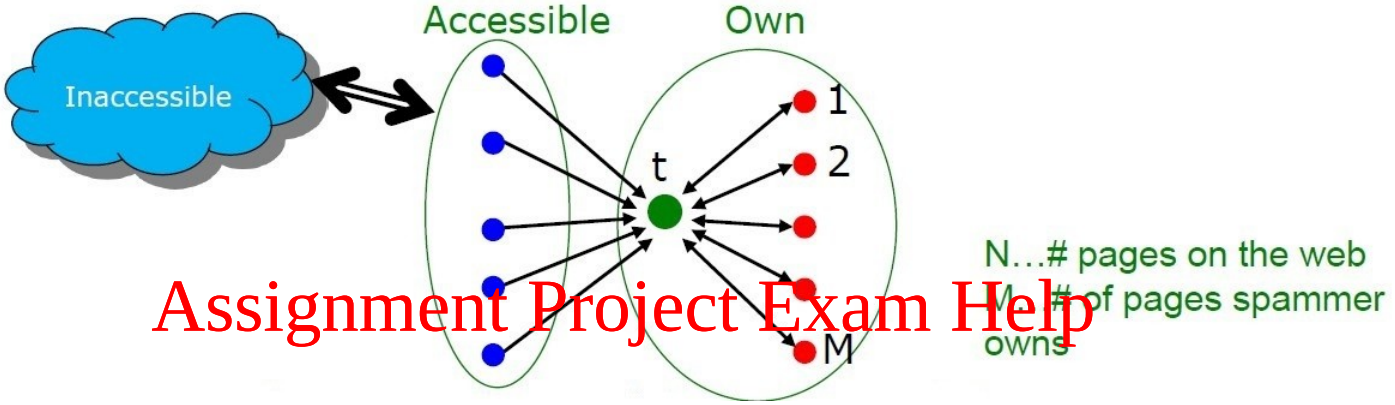
- Spammer's goal:
  - Maximize the PageRank of target page *t*
- Technique:
  - Get as many links from accessible pages as possible to target page *t*
  - Construct "link farm" to get PageRank multiplier effect

Link Farms



One of the most common and effective organizations for a link farm

Analysis

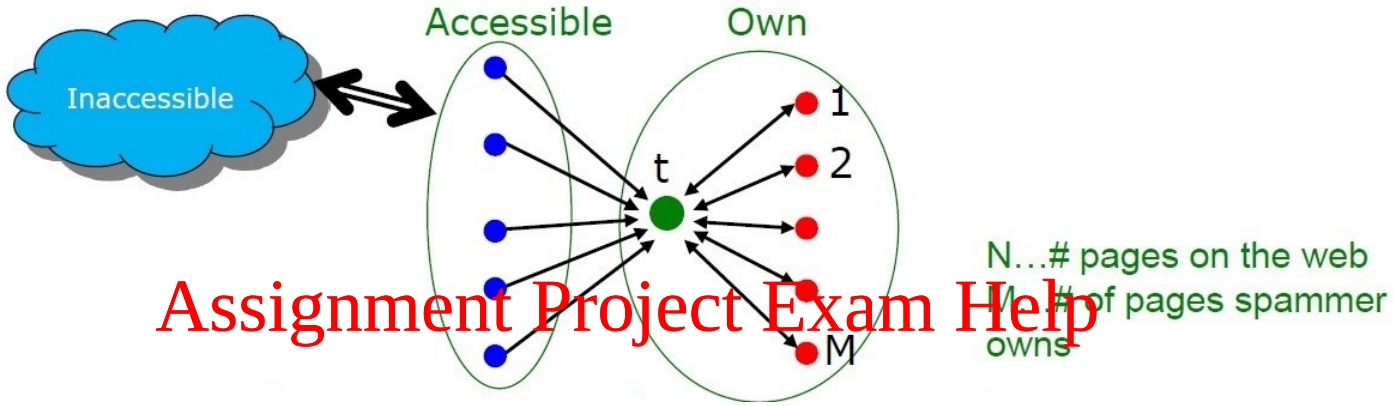


- *x*: PageRank contributed by accessible pages
- *y*: PageRank of target page t
- Rank of each "farm" page = $\frac{\beta y}{M} + \frac{1-\beta}{N}$
-

$$
\begin{aligned}
y &= x + \beta M \left[\frac{\beta y}{M} + \frac{1-\beta}{N}\right] + \frac{1-\beta}{N} \\
&= x + \beta^2 y + \frac{\beta(1-\beta)M}{N} + \frac{1-\beta}{N} \\
&= \frac{x}{1-\beta^2} + c\frac{M}{N}, where\ c = \frac{\beta}{1+\beta}
\end{aligned}
$$

Very small: ignore now we solve for y

Analysis



- $y = \frac{x}{1-\beta^2} + c\frac{M}{N}$, where $c = \frac{\beta}{1+\beta}$
- Multiplier effect for "acquired" PageRank
- By making *M* large, we can make y as large as we want

TrustRank: Combating Spam

- Combating term spam
  - Analyze text using statistical methods
  - Similar to email spam filtering
  - Also useful: Detecting approximate duplicate pages
- Combating link spam
  - Detection and blacklisting of structures that look like spam farms
    - Leads to another war – hiding and detecting spam farms
  - TrustRank = topic-specific PageRank with a teleport set of "trusted" pages
    - Example: .edu domains, similar domains for non-US schools

TrustRank: Idea

- Basic principle: Approximate isolation
  - It is rare for a "good" page to point to a "bad" (spam) page
- Sample a set of seed pages from the web
- Have an oracle (human) to identify the good pages and the spam pages in the seed set
  - Expensive task, so we must make seed set as small as possible

Trust Propagation

- Call the subset of seed pages that are identified as good the trusted pages
- Perform a topic-sensitive PageRank with teleport set = trusted pages
  - Propagate trust through links:
    - Each page gets a trust value between 0 and 1
- Use a threshold value and mark all pages below the trust threshold as spam

Why is It a Good Idea?

- Trust attenuation:
  - The degree of trust conferred by a trusted page decreases with the distance in the graph
- Trust splitting:
  - The larger the number of out-links from a page, the less scrutiny the page author gives each out-link
  - Trust is split across out-links

Picking the Seed Set

- Two conflicting considerations:
  - Human has to inspect each seed page, so seed set must be as small as possible
  - Must ensure every good page gets adequate trust rank, so need make all good pages reachable from seed set by short paths

Approaches to Picking Seed Set

- Suppose we want to pick a good set of *k* pages
- How to do that?
- PageRank
  - Pick the top k pages by PageRank
  - Theory is that you can't get a bad page's rank really high
- Use trusted domains whose membership is controlled, like .edu, .mil, .gov

Spam Mass

- In the TrustRank model, we start with good pages and propagate trust.
- Complementary view:
  - What fraction of a page's PageRank comes from spam pages?
- In practice, we don't know all the spam pages, so we need to estimate.

Spam Mass Estimation

- r_p = PageRank of page p
- r_p^+ = PageRank of p with teleport into trusted pages only

- Then: What fraction of a page's PageRank comes from spam pages?

$$r_p^- = r_p - r_p^+$$

- Spam mass of

$$p = \frac{r_p^-}{r_p}$$

One-slide Takeaway

- Web as a Graph
  - Denote the web structure as a graph
- PageRank
  - PageRank score reflect the importance of web pages
- Topic-Specific PageRank
  - Evaluate web pages by their popularity as well as particular topic
- Trust-Rank
  - Deal with link spams

Further Reading

- Original PageRank paper: http://ilpubs.stanford.edu:8090/422/1/1999-66.pdf
- An Analytical Comparison of Approaches to Personalizing PageRank: http://www-cs-students.stanford.edu/~taherh/papers/comparison.pdf

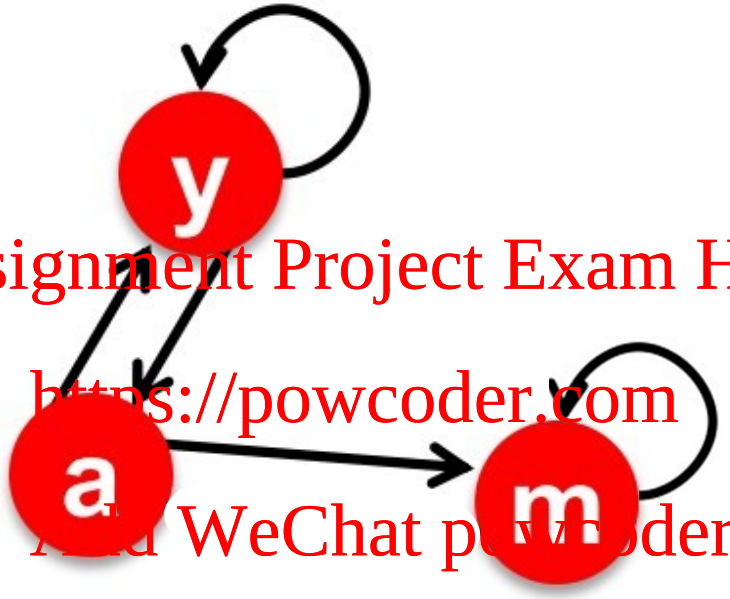
Further Reading

- HITS algorithm: http://www.cs.cornell.edu/home/kleinber/auth.pdf
- Parallel PageRank: http://link.springer.com/content/pdf/10.1007%2F11735106_22.pdf

Reference

- http://www.stanford.edu/class/cs246/slides/09-pagerank.pdf
- http://www.stanford.edu/class/cs246/slides/10-spam.pdf
- http://i.stanford.edu/~ullman/mmds/ch5.pdf
- http://en.wikipedia.org/wiki/PageRank
- http://en.wikipedia.org/wiki/HITS_algorithm

In-class Practice



- Compute the final PageRank Score of the given graph, with Google matrix A and assume $\beta = 0.8$
- Show the matrix A, and solve by both Gaussian Elimination and Power Iteration methods