# COMP 250

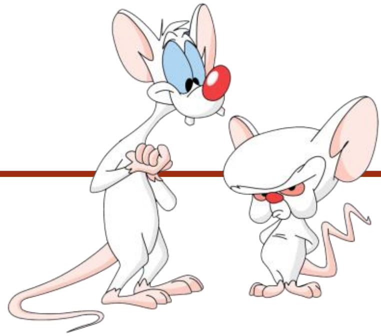
## INTRODUCTION TO COMPUTER SCIENCE

Week 13-1 : Maps

Giulia Alberini, Fall 2020

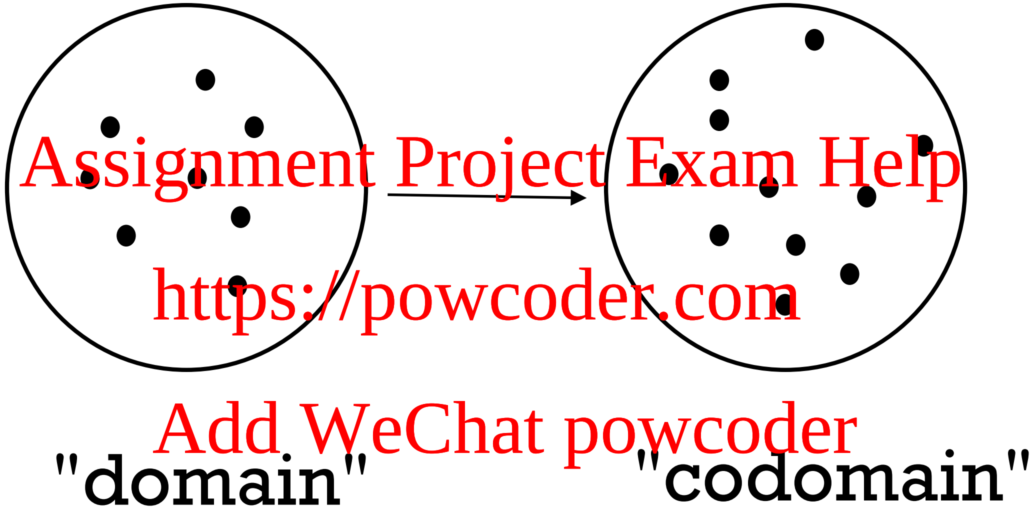
Slides adapted from Michael Langer's

# WHAT ARE WE GOING TO DO IN THIS VIDEO?



- Maps

# MAP (MATHEMATICS)



A map is a set of pairs $\{ (x, f(x)) \}$.

Each $x$ in domain maps to exactly one $f(x)$ in codomain, but it can happen that $f(x1) = f(x2)$ for different $x1, x2$, i.e. many-to-one.

## FAMILIAR EXAMPLES

Calculus 1 and 2 ("functions"):

$$f : \mathbb{R}^n \to \mathbb{R}^n$$

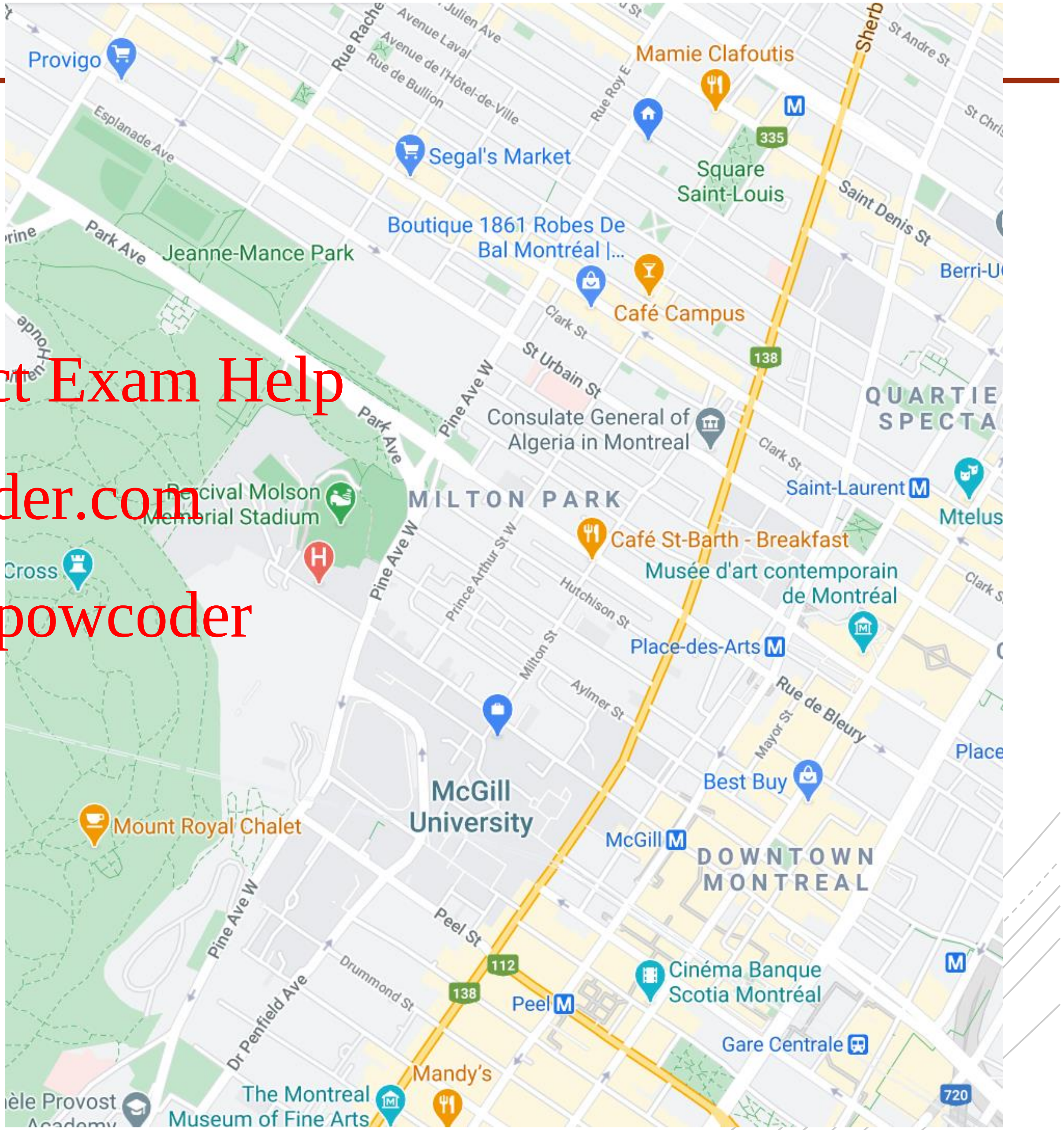
Asymptotic complexity in CS:

$t$ : input size → number of steps in a algorithm.

# MAPS IN EVERYDAY LIFE

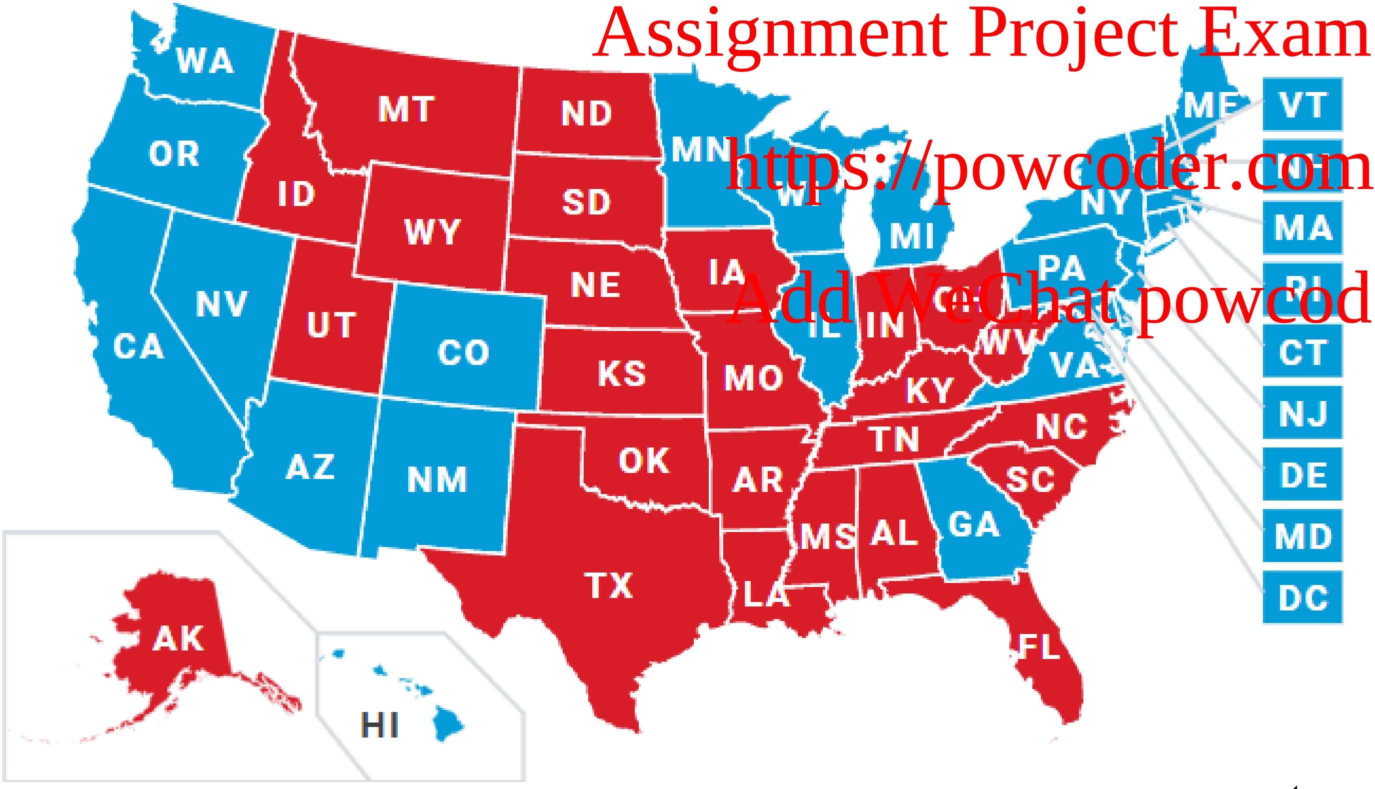
The term "map" commonly refers to a 2D spatial representation of a region of the earth's surface.

map(x, y) : position in image → position in 2D Montreal



# COLOR MAP

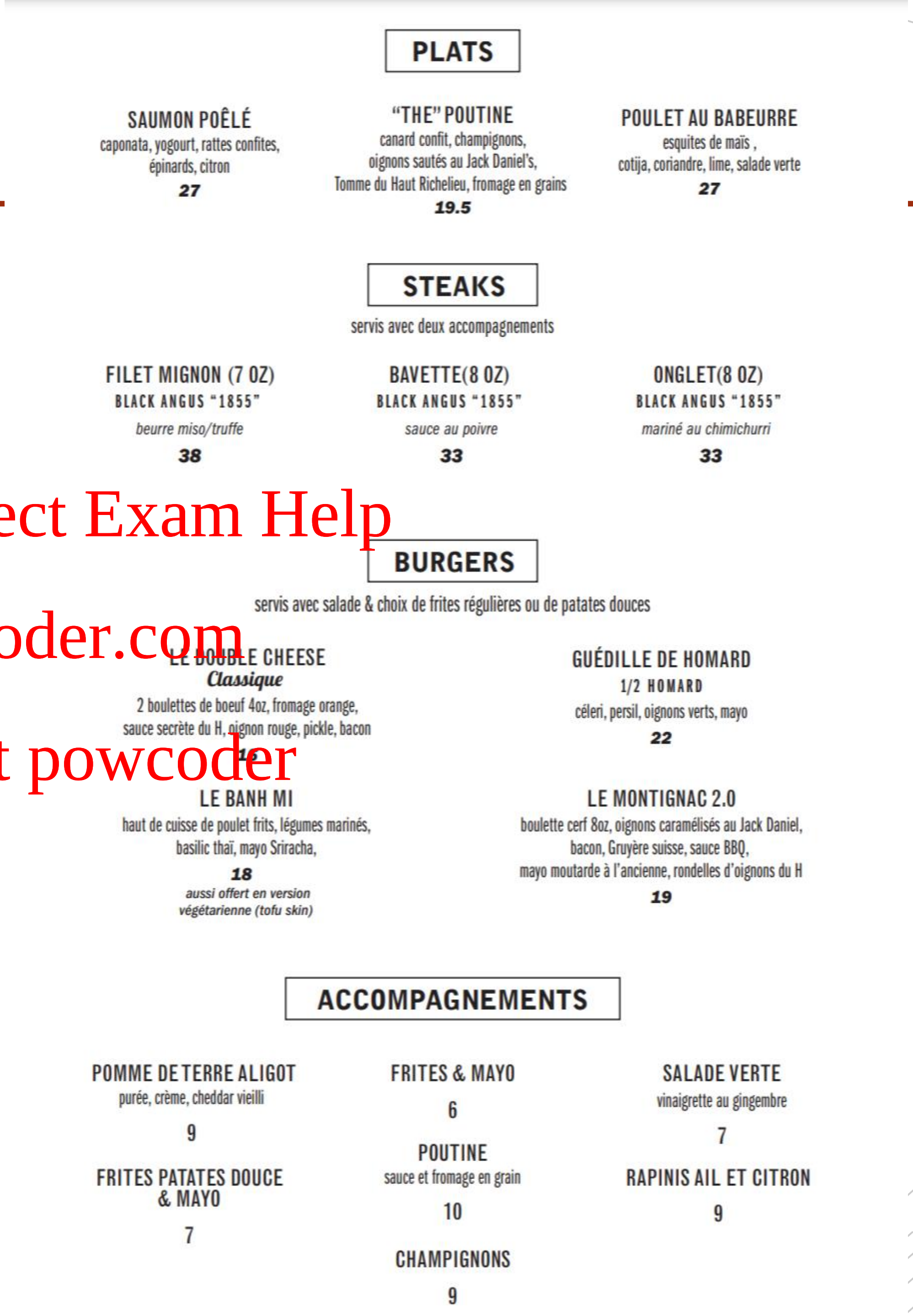
The color map representing the USA election results in 2020.



vote_result : US_state → {D, R}

# RESTAURANT MENU

menu : dish_name → price



## PLATS

**SAUMON POÊLÉ**
caponata, yogourt, rattes confites, épinards, citron
**27**

**"THE" POUTINE**
canard confit, champignons, oignons sautés au Jack Daniel's, Tomme du Haut Richelieu, fromage en grains
**19.5**

**POULET AU BABEURRE**
esquites de maïs , cotija, coriandre, lime, salade verte
**27**

## STEAKS

servis avec deux accompagnements

**FILET MIGNON (7 OZ)**
BLACK ANGUS "1855"
*beurre miso/truffe*
**38**

**BAVETTE(8 OZ)**
BLACK ANGUS "1855"
*sauce au poivre*
**33**

**ONGLET(8 OZ)**
BLACK ANGUS "1855"
*mariné au chimichurri*
**33**

## BURGERS

servis avec salade & choix de frites régulières ou de patates douces

**LE DOUBLE CHEESE**
***Classique***
2 boulettes de boeuf 4oz, fromage orange, sauce secrète du H, oignon rouge, pickle, bacon
**16**

**GUÉDILLE DE HOMARD**
1/2 HOMARD
céleri, persil, oignons verts, mayo
**22**

**LE BANH MI**
haut de cuisse de poulet frits, légumes marinés, basilic thaï, mayo Sriracha,
**18**
*aussi offert en version végétarienne (tofu skin)*

**LE MONTIGNAC 2.0**
boulette cerf 8oz, oignons caramélisés au Jack Daniel, bacon, Gruyère suisse, sauce BBQ, mayo moutarde à l'ancienne, rondelles d'oignons du H
**19**

## ACCOMPAGNEMENTS

**POMME DE TERRE ALIGOT**
purée, crème, cheddar vieilli
9

**FRITES PATATES DOUCE & MAYO**
7

**FRITES & MAYO**
6

**POUTINE**
sauce et fromage en grain
10

**CHAMPIGNONS**
9

**SALADE VERTE**
vinaigrette au gingembre
7

**RAPINIS AIL ET CITRON**
9

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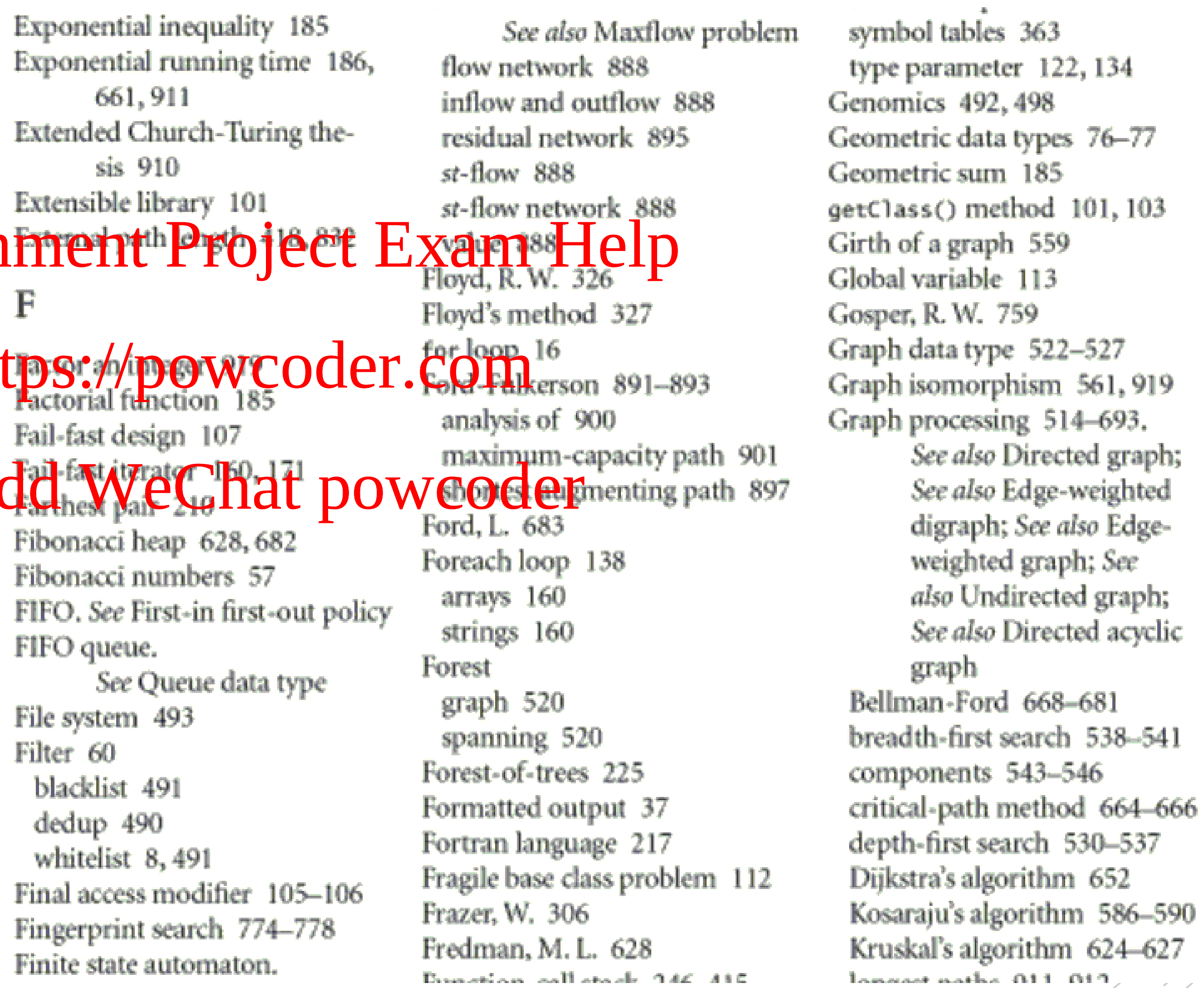
index : term → list of pages



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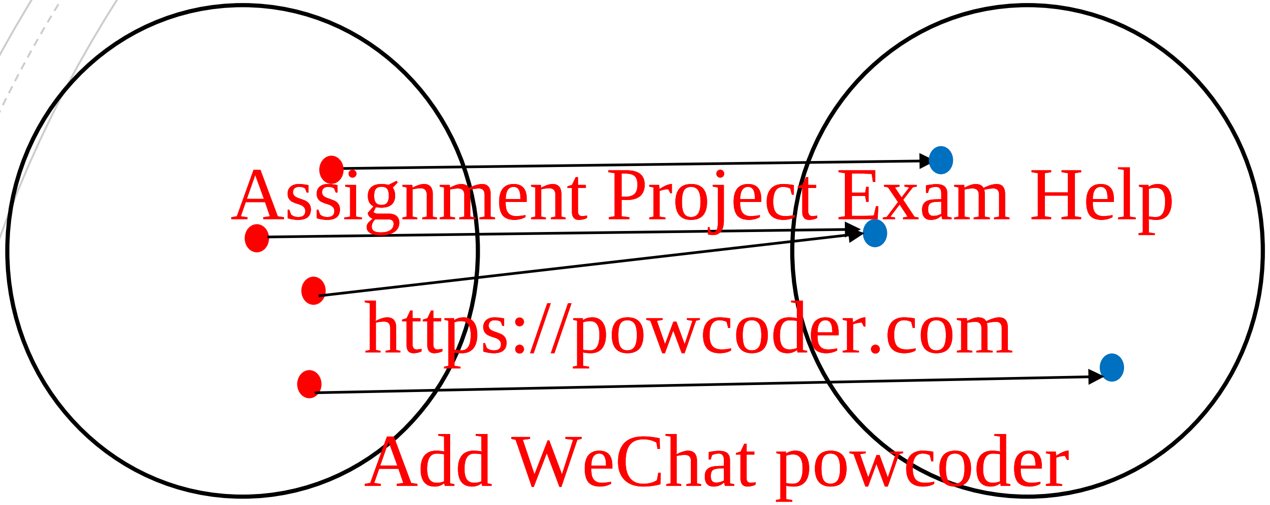
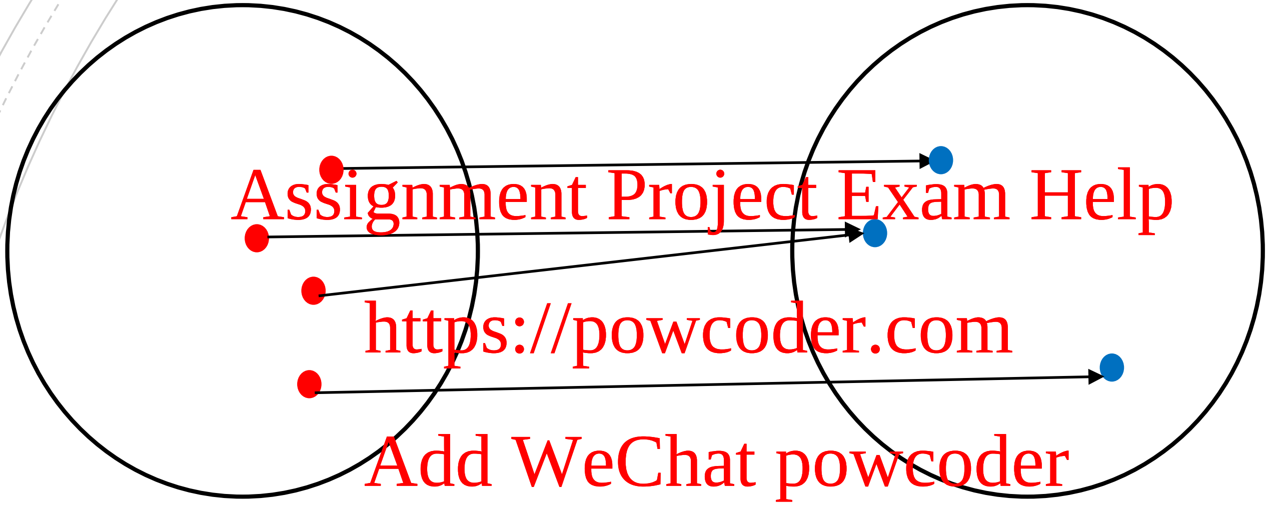
# MAP (ADT)



keys (type K) values (type V)

A map is a set of (key, value) pairs.
For each key, there is at most one value.

# MAP (ADT)

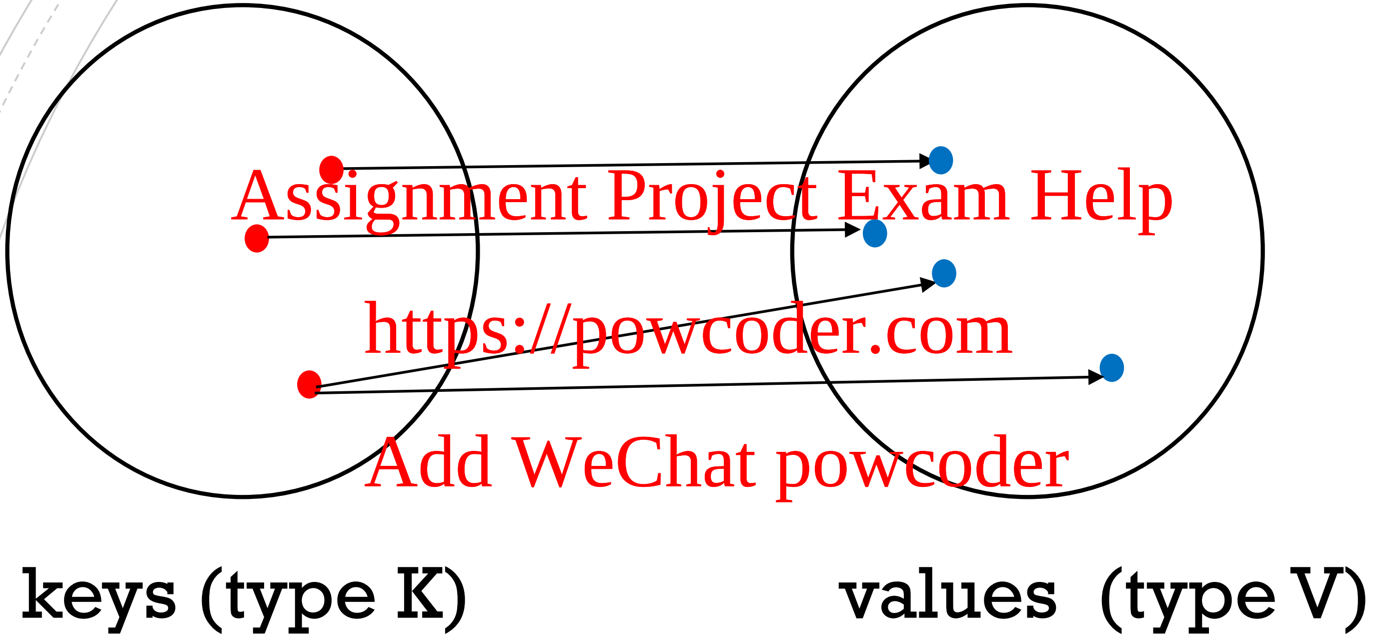


keys (type K)

values (type V)

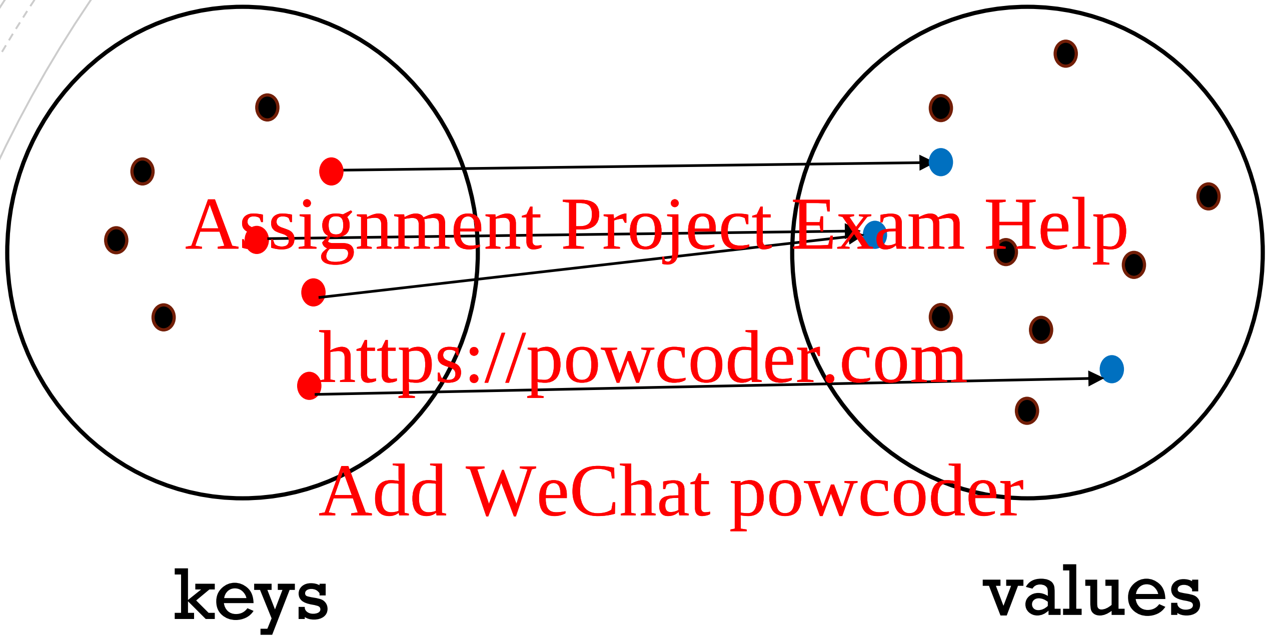
Note that it is possible for two keys to map to the same value.

# MAP (ADT)



It is NOT allowed for one key to map to two different values! The example above is NOT a map.

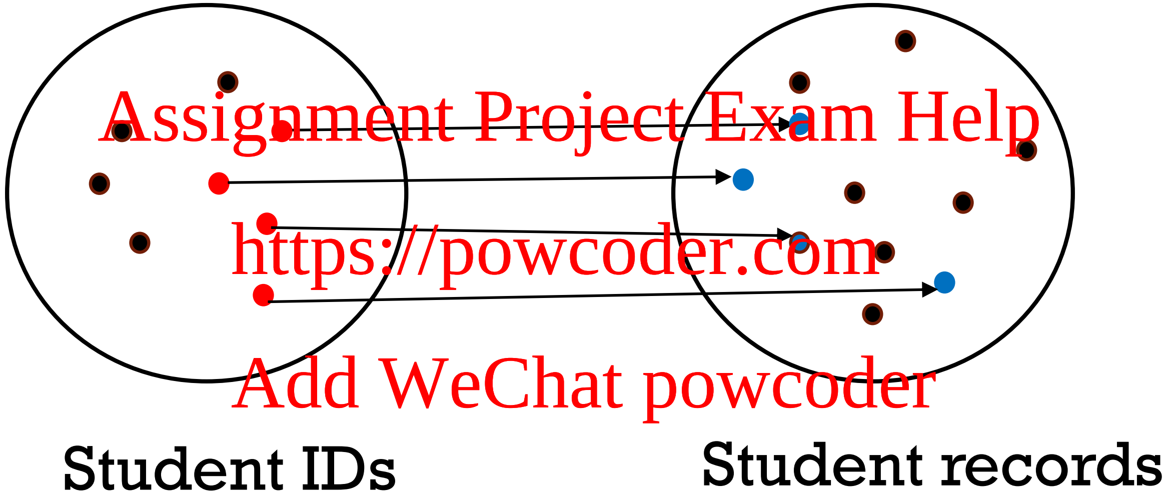
# MAP ENTRIES



The black dots here indicate objects (or potential objects) of type K or V that are *not* in the map.

Each (key, value) pair is called an *entry*. In this example, there are four entries.

# EXAMPLE



In COMP 250 this semester, the above mapping has ~650 entries.
Most McGill students are not taking COMP 250 this semester.

Student ID also happens to be part of the student record.

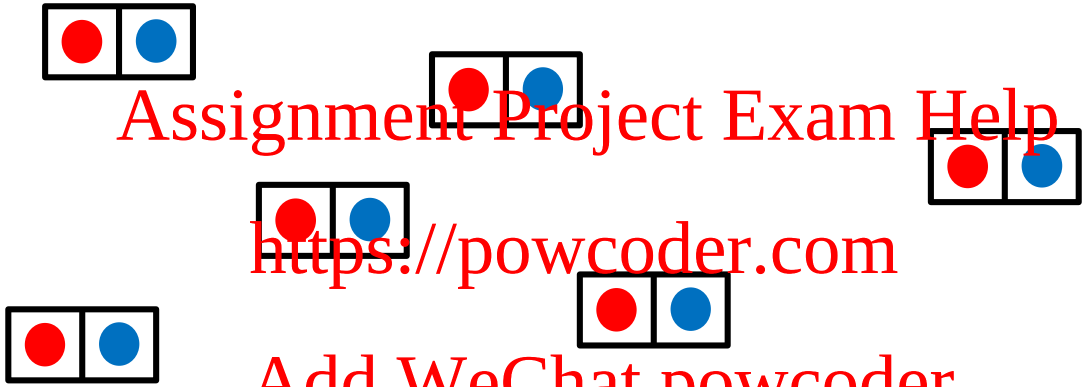
# MAP ADT

put( key, value ) // Add the entry (key, value) to the map. If the map previously contained an entry with key, the old value is replaced by the specified value.

get(key) // Returns the value to which the specified key is mapped. Why not get(key, value) ?

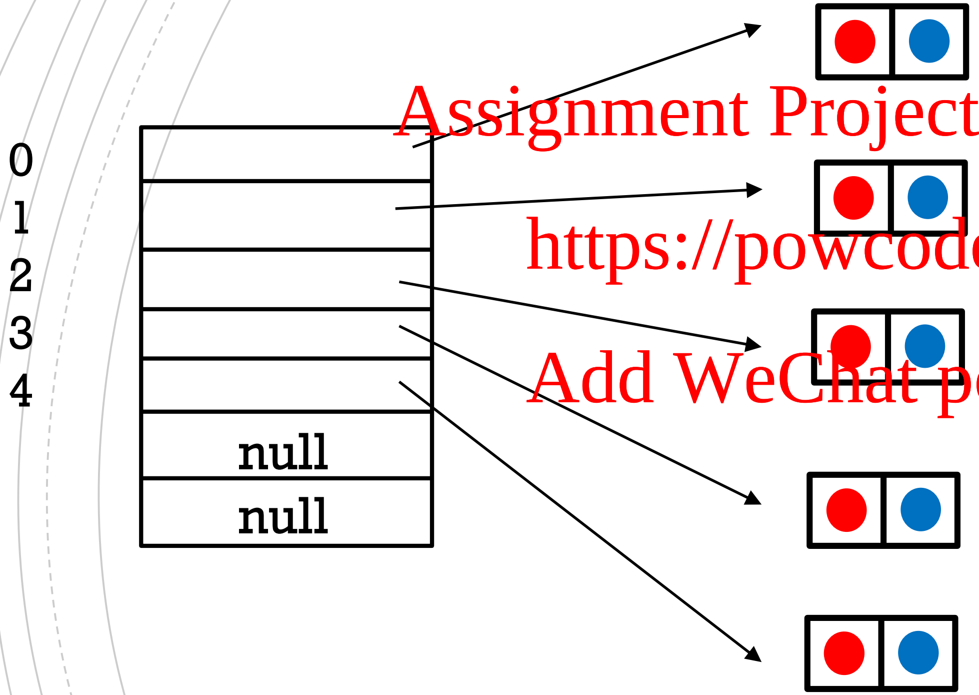
remove(key) // Removes the entry with the specified key. Returns true if the entry was removed, false otherwise.

# DATA STRUCTURES FOR MAPS



How to organize a set of (key, value) pairs, i.e. entries ?

## ARRAY LIST

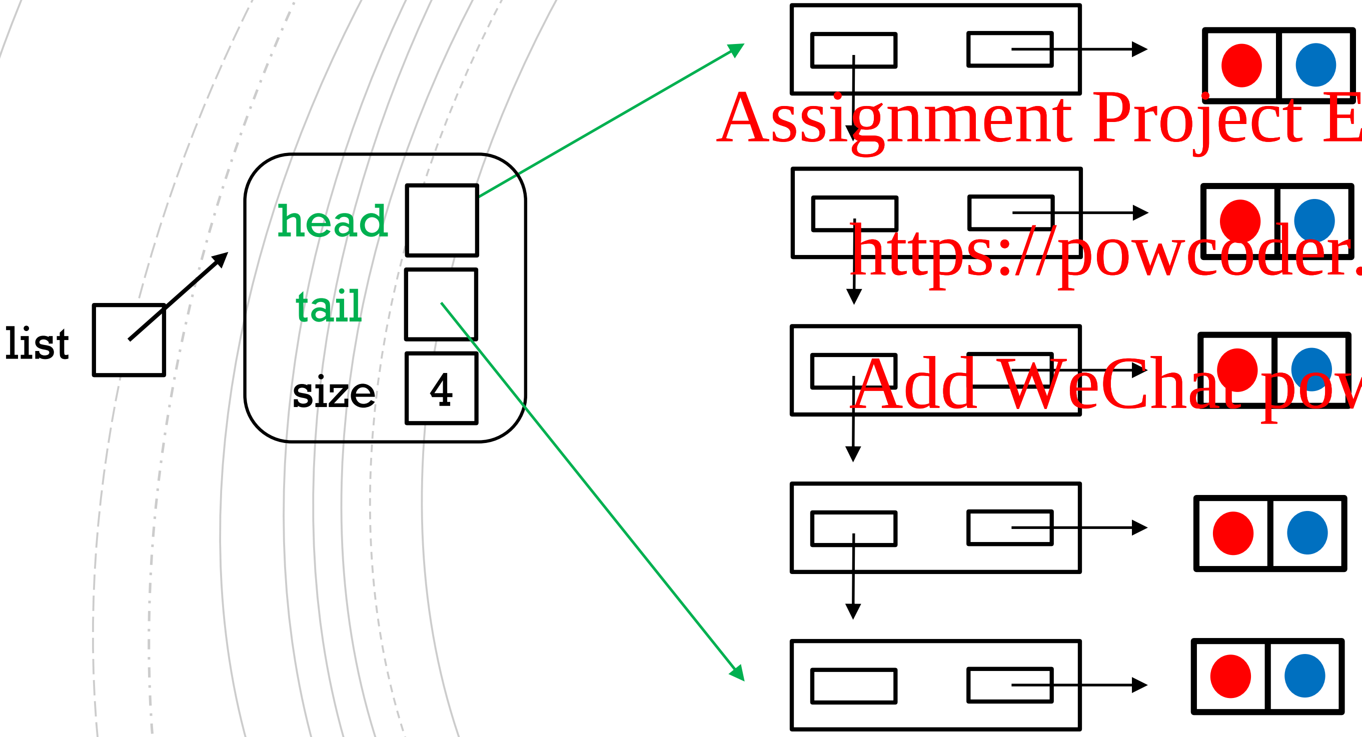


How can we implement the following methods?

put( key, value )
get(key)
remove(key)

put() would be O(1), while get() and remove() would be O(n)

# SINGLY (OR DOUBLY) LINKED LIST



How can we implement the following methods?

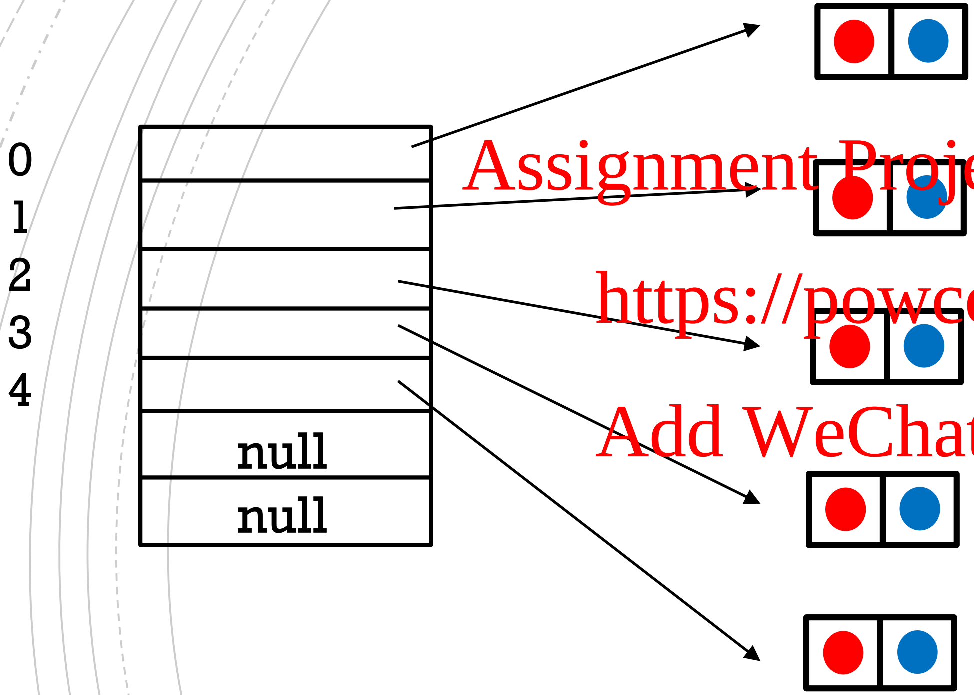
put( key, value )
get(key)
remove(key)

put() would be O(1), while get() and remove() would be O(n)

# LET'S ADD ASSUMPTIONS

- Special case #1: what if keys are *comparable* ?

# ARRAY LIST (SORTED BY KEY)



How can we implement the following methods?

put( key, value )
get(key)
remove(key)

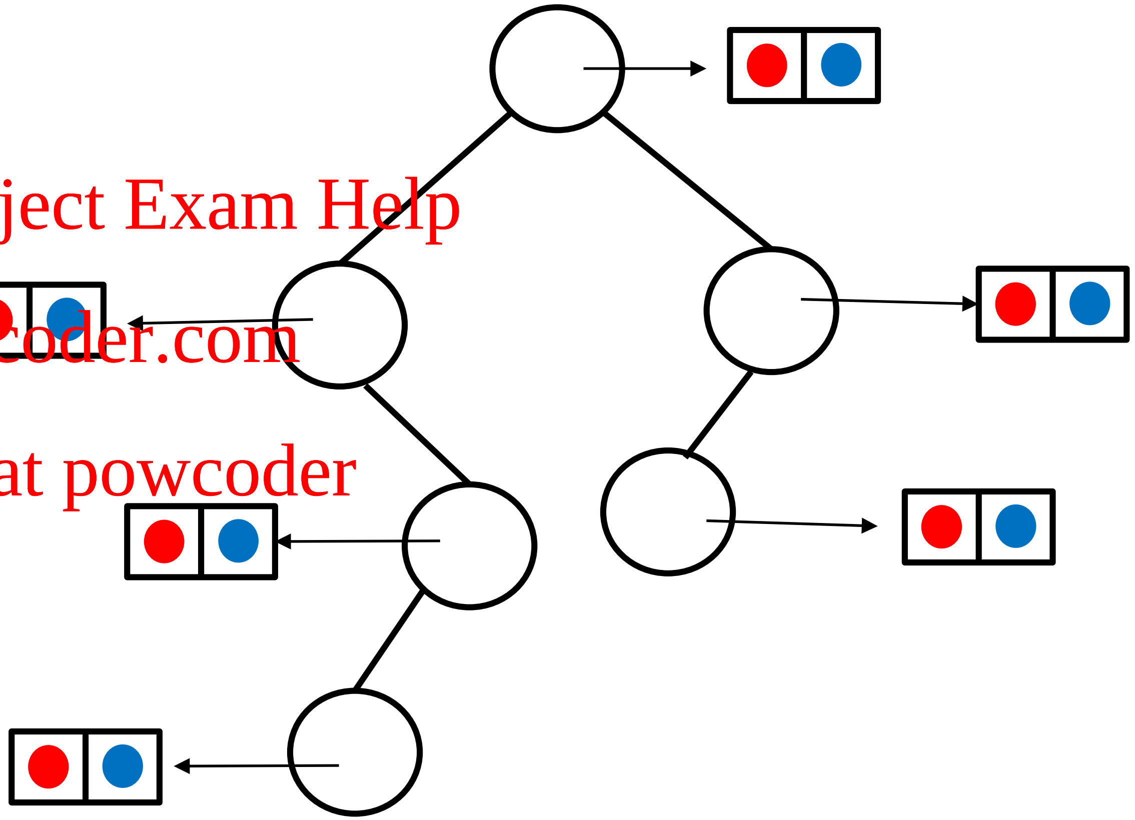
put() and remove() would be O(n), while get() could be performs in time O(log n) using binary search

# BINARY SEARCH TREE (SORTED BY KEY)

How can we implement the following methods?

put( key,  value )
get(key)
remove(key)

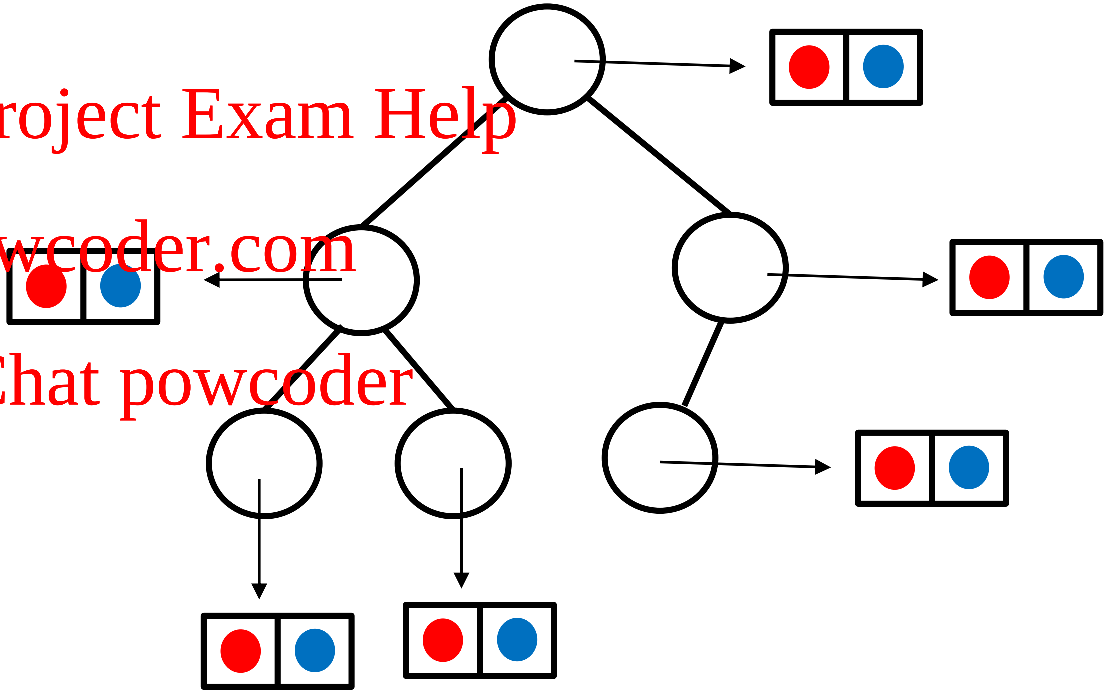
The performance of put(), get() and remove() depends on the tree. If we have a balances tree, then these operations would all take time O(log n) in worst case. You will learn more about balanced tree in COMP 251.



# MINHEAP (PRIORITY DEFINED BY KEY)

How can we implement the following methods?

put( key, value )
get(key)
remove(key)



The performance of put() would be O(log n). Implementing get() would require traversing the tree, so it would be O(n). Implementing remove() would be a little weird for heaps…

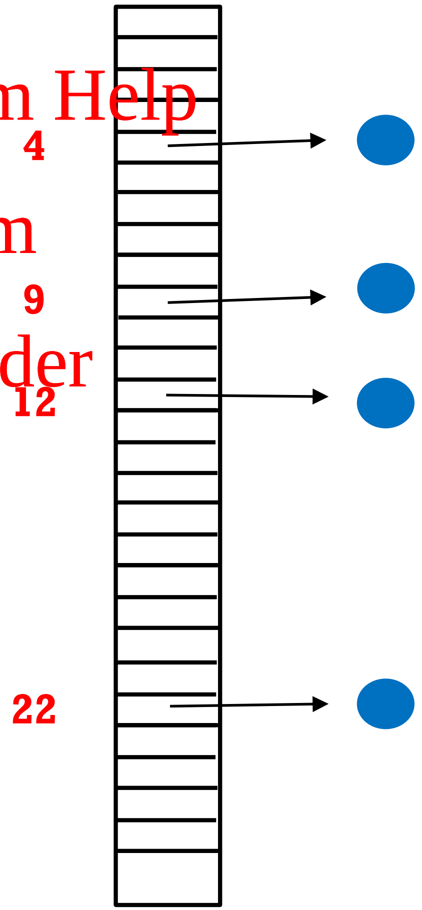
# LET'S ADD ASSUMPTIONS

- ~~Special case #1: what if keys are *comparable* ?~~
- Special case #2: what if keys are unique positive integers in small range ?

# ARRAYS OF VALUES

Then, we could use an array of type V (value) and have O(1) access.

This would not work well if keys are 9 digit student IDs.



# IN GENERAL

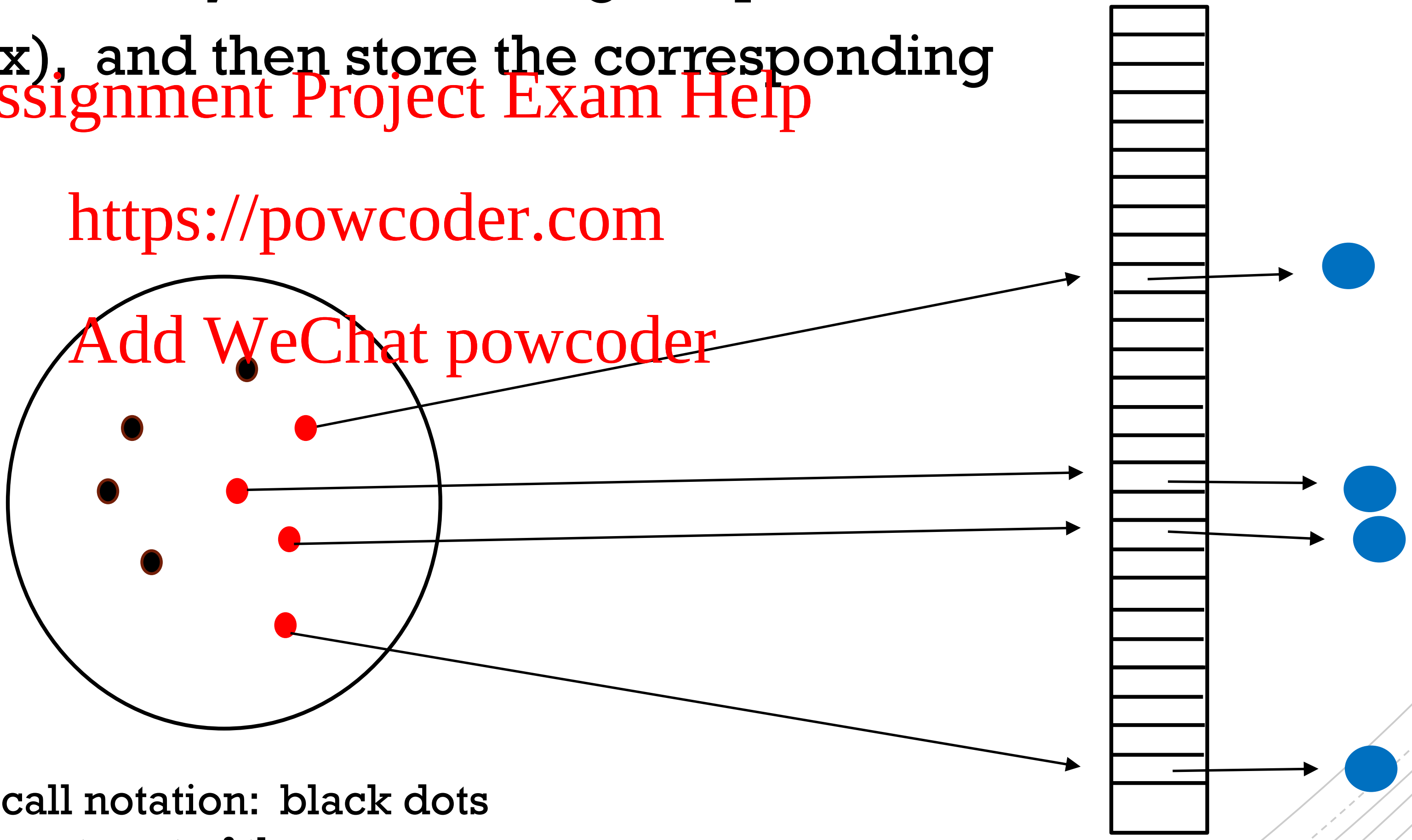
- Keys might not be comparable.

- Keys might be not be positive integers.

  e.g. Keys might be strings or some other type.

# STRATEGY IN THE GENERAL CASE

Try to define a map from keys to *small* range of positive integers (array index), and then store the corresponding values in the array.



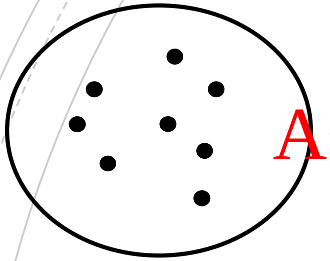
Recall notation: black dots are not part of the map.

# IN THIS VIDEO

Define a map from keys to *large* range of positive integers

Such map is called *hash code*.

# JAVA'S `Object.hashcode()`



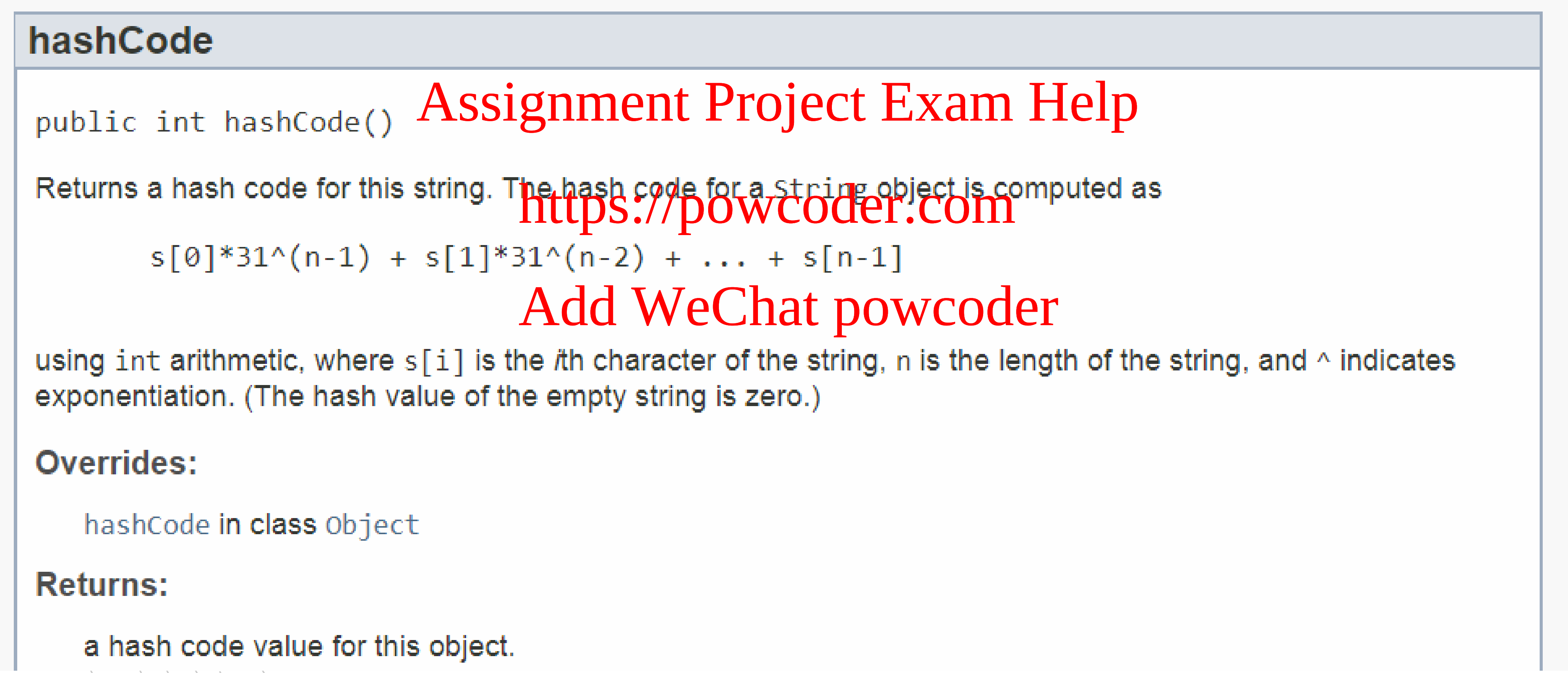
$\xrightarrow[\text{(not many-to-1)}]{\text{1-to-1}} \{0, 1, 2, ..., 2^{24} - 1\}$

objects in a Java program (runtime)

object's *address* in JVM memory (24 bits)

*By default,* "obj1 == obj2" means "obj1.hashcode() == obj2.hashcode()"

# JAVA'S `String.hashcode()`



## hashCode

```
public int hashCode()
```

Returns a hash code for this string. The hash code for a `String` object is computed as

```
s[0]*31^(n-1) + s[1]*31^(n-2) + ... + s[n-1]
```

using `int` arithmetic, where `s[i]` is the *i*th character of the string, `n` is the length of the string, and `^` indicates exponentiation. (The hash value of the empty string is zero.)

**Overrides:**

`hashCode` in class `Object`

**Returns:**

a hash code value for this object.

https://docs.oracle.com/javase/7/docs/api/java/lang/String.html

# EXAMPLE HASH CODE FOR STRINGS

**(not used in Java)**

$$h(s) = \sum_{i=0}^{s.length-1} s[i]$$

e.g.

$$h("eat") = h("ate") = h("tea")$$

ASCII values of 'a', 'e', 't' are 97, 101, 116.

## JAVA'S `String.hashcode()`

$$\texttt{s.hashCode()} \equiv \sum_{i=0}^{s.length-1} s[i] * x^{s.length-1-i}$$

where $x = 31$.

# JAVA'S `String.hashcode()`

$$\text{s.hashCode()} \equiv \sum_{i=0}^{s.length-1} s[i]\, x^{s.length-1-i}$$

where $x = 31$.

e.g. `s = "eat"` **then** `s.hashcode()` $= 101 * 31^2 + 97 * 31 + 116$

$$\underset{\substack{\texttt{'e'} \\ \texttt{s[0]}}}{101} * 31^2 + \underset{\substack{\texttt{'a'} \\ \texttt{s[1]}}}{97} * 31 + \underset{\substack{\texttt{'t'} \\ \texttt{s[2]}}}{116}$$

# JAVA'S `String.hashcode()`

$$\text{s.hashCode()} \equiv \sum_{i=0}^{s.length-1} s[i]\, x^{s.length-1-i}$$

where $x = 31$.

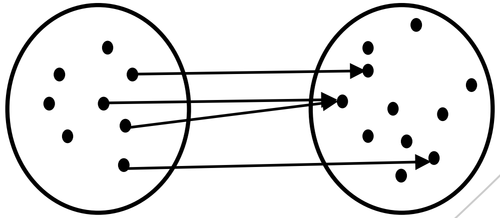
e.g. `s = "ate"` **then** `s.hashcode()` $= \underset{\underset{\texttt{s[0]}}{\texttt{'a'}}}{97} * 31^2 + \underset{\underset{\texttt{s[1]}}{\texttt{'t'}}}{116} * 31 + \underset{\underset{\texttt{s[2]}}{\texttt{'e'}}}{101}$

# JAVA'S `String.hashcode()`

$$\texttt{s.hashCode()} \equiv \sum_{i=0}^{s.length-1} s[i] * (31)^{s.length-1-i}$$

**If** `s1.hashCode() == s2.hashCode()` **then what can we conclude about** `s1.equals(s2)` **?**

`s1` *may or may not* be the same string as `s2`.

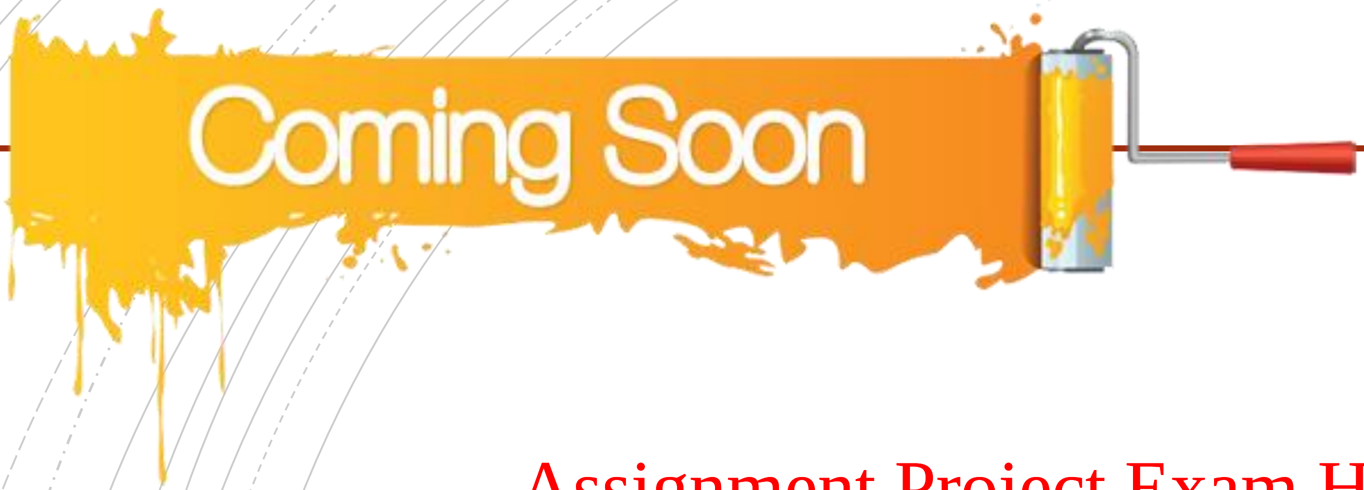


## JAVA'S `String.hashcode()`

$$\texttt{s.hashCode()} \equiv \sum_{i=0}^{s.length-1} s[i] * (31)^{s.length - 1 - i}$$

**If** `s1.hashCode() != s2.hashCode()` **then what can we conclude about** `s1.equals(s2)` **?**

`s1` **is a different string then** `s2`**.**



In the next video:

- Hash Maps