

lecture 1

- two's complement
 - floating point numbers
 - hexadecimal
- <https://powcoder.com>
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Car odometer (fixed number of digits)



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$$\begin{array}{r} 0000999 \\ + 0000001 \\ \hline 0001000 \end{array}$$

9 9 9 9 9 9

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0 0 0 0 0 0

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9 9 9 9 9 9

≡

1 1

3 2 8 7 6 9

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$$\begin{array}{r}
 328769 \\
 + 671231 \\
 \hline
 000000
 \end{array}$$

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Add WeChat powcoder 328769

If you know what "modular arithmetic" is (MATH 240), then you recognize this: addition of integers mod 10^6 .

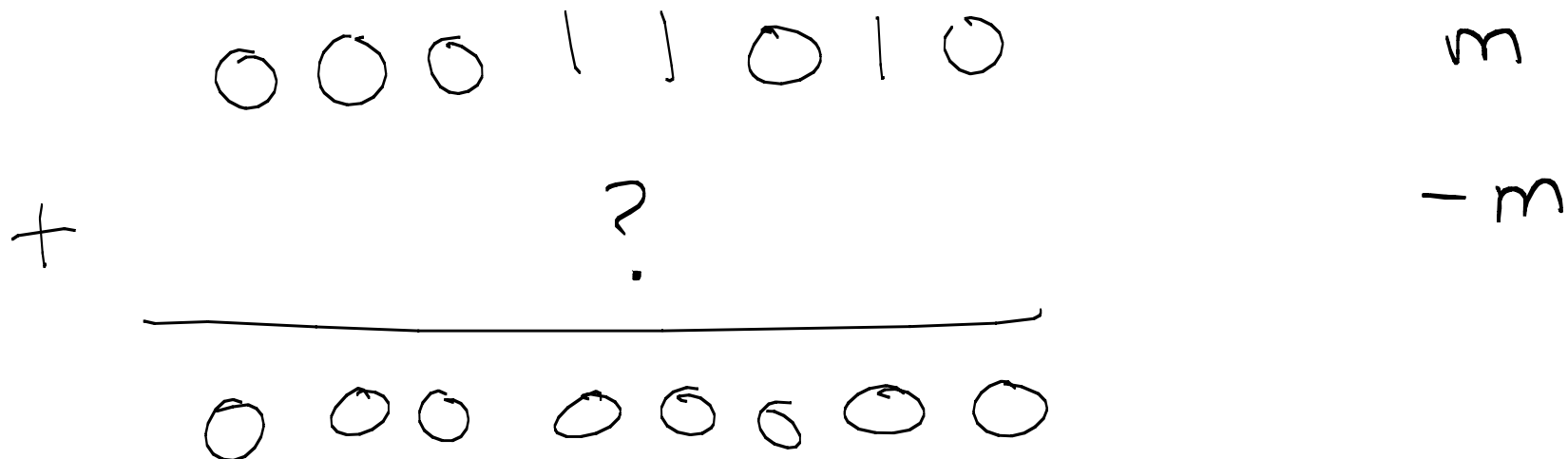
Q: How to represent negative numbers in binary ?

A: Given an 8 bit binary number m ,
define $-m$ so that $m + (-m) = 0$.

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Two's complement representation of integers

Example: How to represent -26 ?

Use a trick!

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$n = 26$

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← invert bits

$$\begin{array}{r} 00011010 \\ + 11100101 \\ \hline 11111111 \end{array}$$

$$\begin{array}{r}
 00011010 \quad m=2b \\
 + 11100101 \quad \leftarrow \text{invert bits} \\
 \hline
 11111111
 \end{array}$$

$$\begin{array}{r}
 + \quad \quad \quad 1 \quad \leftarrow \text{add } 1 \\
 \hline
 \end{array}$$

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$$\begin{array}{r}
 + 11100101 \quad \leftarrow \text{inverted bits} \\
 \hline
 11100110 \quad \leftarrow \text{add } 1 \\
 \quad \quad \quad \leftarrow -2b
 \end{array}$$

Another example: What is -0 ?

0 0 0 0 0 0 0 0

$m = 0$

1 1 1 1 1 1 1 1

invert bits

+

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0 0 0 0 0 0 0 0

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+

1 1 1 1 1 1 1 1

1

0 0 0 0 0 0 0 0

We have verified that $-0 = 0$.

What about $m = 128$? What is -128 ?

| 0 0 0 0 0 0 0 0

$m = 128$

0 | | | | | | |

invert bits

+

0 0 0 0 0 0 0 0

add 1

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0 | | | | | | |

+

| 0 0 0 0 0 0 0 0

$m = -128$

Thus, 128 is equivalent to -128.

binary

0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 1

0 0 0 0 0 0 1 0

0 0 0 0 0 0 1 1

0 0 0 0 0 1 0 0

⋮

0 1 1 1 1 1 1 1

1 0 0 0 0 0 0 0

⋮

1 1 1 1 1 1 1 0

1 1 1 1 1 1 1 1

"unsigned"

0

1

2

3

4

⋮

⋮

1 2 7

1 2 8

⋮

⋮

2 5 4

2 5 5

"signed"

0

1

2

3

4

⋮

⋮

⋮

1 2 7

- 1 2 8

⋮

⋮

- 2

- 1

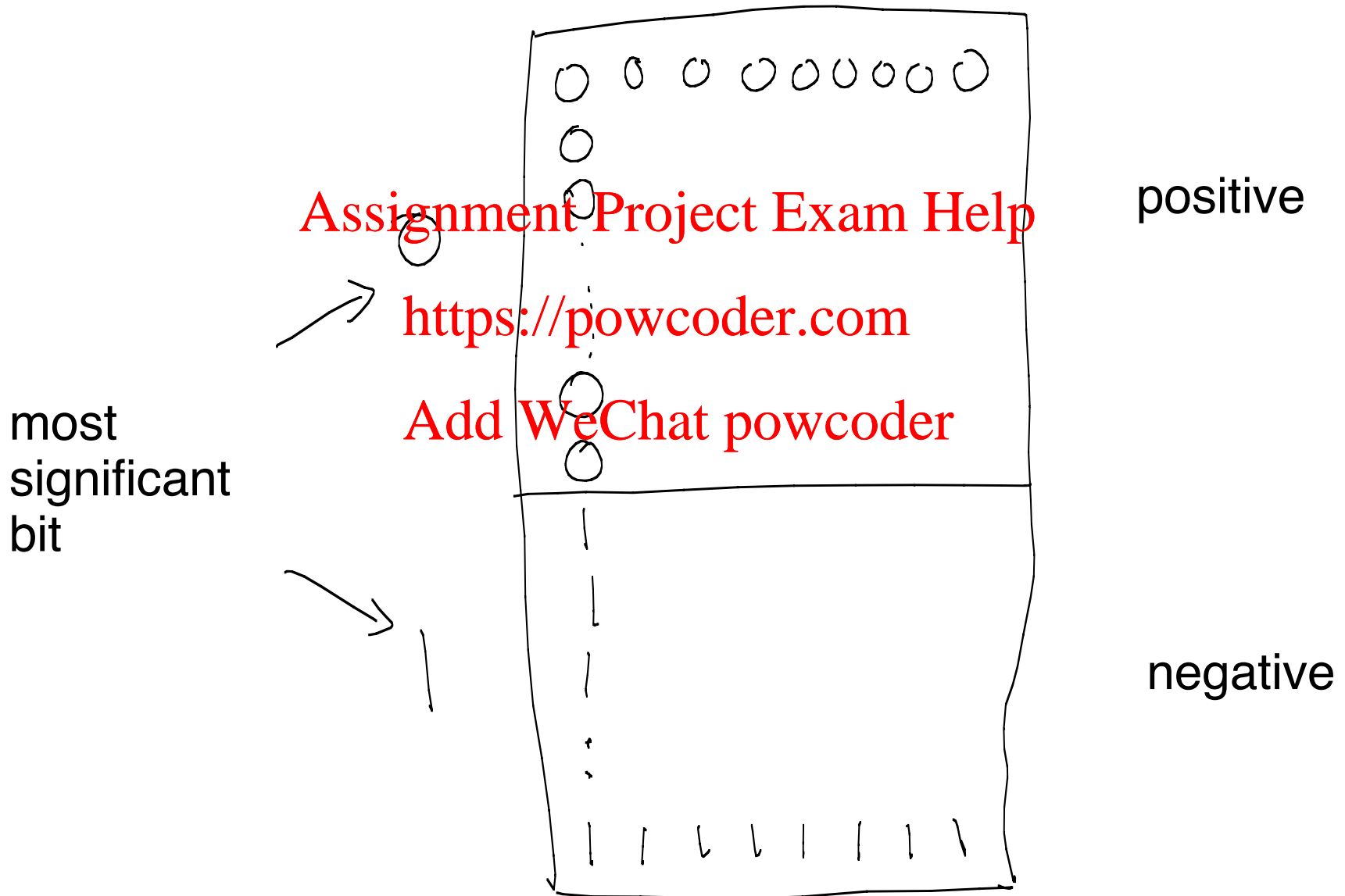
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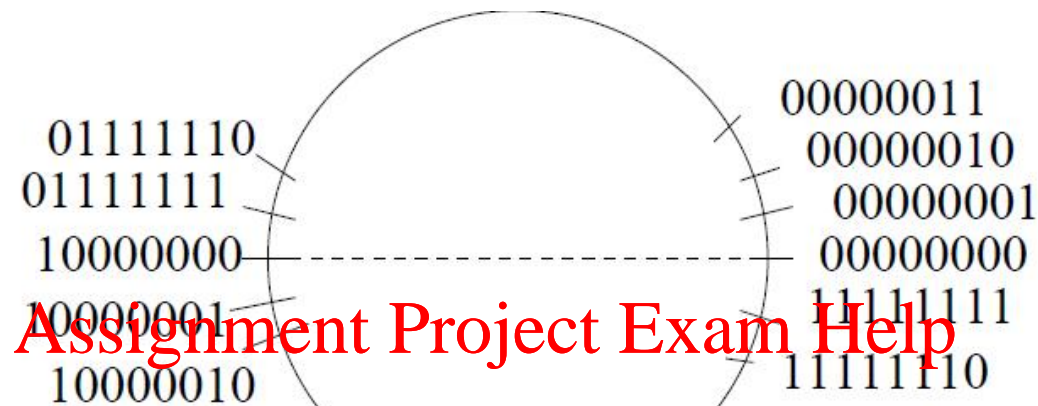
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} →

signed integers



8 bit integers (unsigned vs. signed)



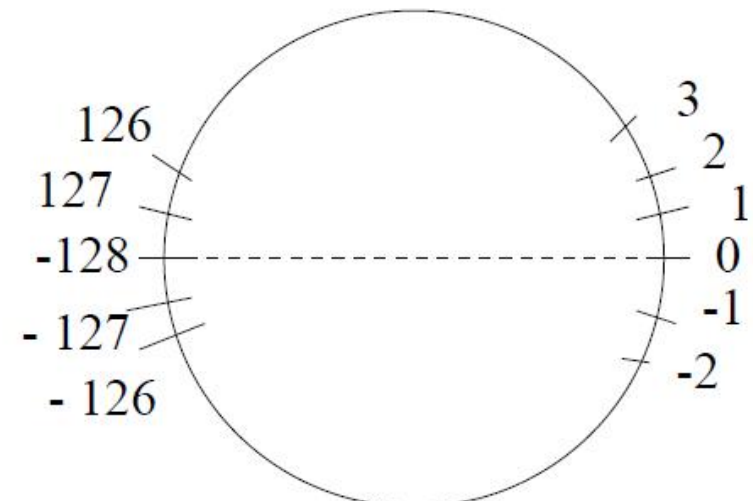
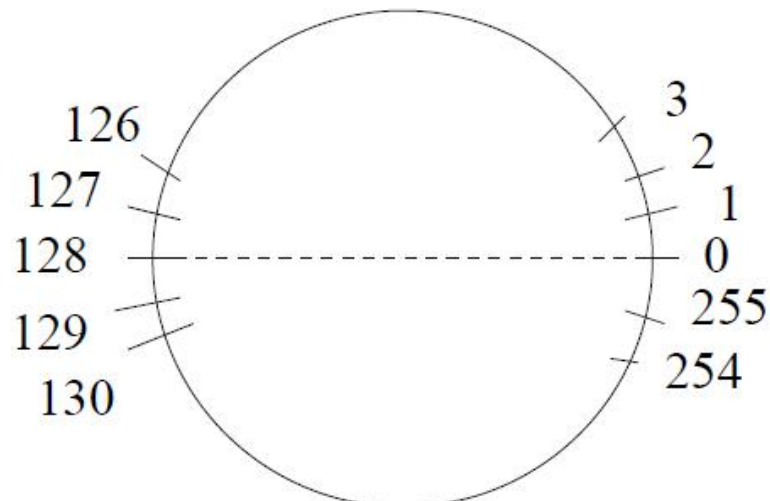
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unsigned

signed



n bits defines 2^n integers

unsigned

0, 1, ...

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$2^n - 1$

signed

-2^{n-1} , ...

0, ...

$2^{n-1} - 1$

Take $n = 32$.

The largest signed integer is $2^{31} - 1$.

$2^{10} = 1024 \sim 10^3 = \text{one thousand.}$

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$2^{20} \sim 10^6 = \text{one million}$

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$2^{30} \sim 10^9 = \text{one billion}$

$2^{31} \sim 2,000,000,000 = \text{two billion}$

Java Example

```
int j = 4000000000;    // 4 billion > 2^31
```

This gives a compiler error. "The literal of type int is out of range."

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```
int j = 2000000000;    // 2 billion < 2^31
```

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```
System.out.println( 2 * j );
```

// This prints out -294967296.

// To understand why these particular digits are printed, you
// would need to convert 4000000000 to binary, which I don't
// recommend.)

lecture 1

- two's complement

- floating point numbers

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- hexadecimal

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Floating Point

"decimal point"



$$26.375 = 2 \times 10^1 + 6 \times 10^0 + 3 \times 10^{-1} + 7 \times 10^{-2} + 5 \times 10^{-3}$$

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"binary point" Add WeChat powcoder

$$(11010.011)_2$$

$$\begin{aligned} &= 2^4 + 2^3 + 2^1 + 2^{-2} + 2^{-3} \\ &= 16 + 8 + 2 + 0.25 + 0.125 \\ &= 26.375 \end{aligned}$$

Convert from binary to decimal

We must use both positive and negative powers of 2.

$\frac{1}{2}$	$\frac{2^0}{2^1}$
$\frac{1}{4}$	$\frac{2^0}{2^2}$
$\frac{1}{8}$	$\frac{2^0}{2^3}$
$\frac{1}{16}$	$\frac{2^0}{2^4}$
$\frac{1}{32}$	$\frac{2^0}{2^5}$
$\frac{1}{64}$	$\frac{2^0}{2^6}$
$\frac{1}{128}$	$\frac{2^0}{2^7}$
$\frac{1}{256}$	$\frac{2^0}{2^8}$
$\frac{1}{512}$	$\frac{2^0}{2^9}$
$\frac{1}{1024}$	$\frac{2^0}{2^{10}}$
$\frac{1}{2048}$	$\frac{2^0}{2^{11}}$
$\frac{1}{4096}$	$\frac{2^0}{2^{12}}$
$\frac{1}{8192}$	$\frac{2^0}{2^{13}}$
$\frac{1}{16384}$	$\frac{2^0}{2^{14}}$
$\frac{1}{32768}$	$\frac{2^0}{2^{15}}$
$\frac{1}{65536}$	$\frac{2^0}{2^{16}}$
$\frac{1}{131072}$	$\frac{2^0}{2^{17}}$
$\frac{1}{262144}$	$\frac{2^0}{2^{18}}$
$\frac{1}{524288}$	$\frac{2^0}{2^{19}}$
$\frac{1}{1048576}$	$\frac{2^0}{2^{20}}$
$\frac{1}{2097152}$	$\frac{2^0}{2^{21}}$
$\frac{1}{4194304}$	$\frac{2^0}{2^{22}}$
$\frac{1}{8388608}$	$\frac{2^0}{2^{23}}$
$\frac{1}{16777216}$	$\frac{2^0}{2^{24}}$
$\frac{1}{33554432}$	$\frac{2^0}{2^{25}}$
$\frac{1}{67108864}$	$\frac{2^0}{2^{26}}$
$\frac{1}{134217728}$	$\frac{2^0}{2^{27}}$
$\frac{1}{268435456}$	$\frac{2^0}{2^{28}}$
$\frac{1}{536870912}$	$\frac{2^0}{2^{29}}$
$\frac{1}{1073741824}$	$\frac{2^0}{2^{30}}$
$\frac{1}{2147483648}$	$\frac{2^0}{2^{31}}$
$\frac{1}{4294967296}$	$\frac{2^0}{2^{32}}$
$\frac{1}{8589934592}$	$\frac{2^0}{2^{33}}$
$\frac{1}{17179869184}$	$\frac{2^0}{2^{34}}$
$\frac{1}{34359738368}$	$\frac{2^0}{2^{35}}$
$\frac{1}{68719476736}$	$\frac{2^0}{2^{36}}$
$\frac{1}{137438953472}$	$\frac{2^0}{2^{37}}$
$\frac{1}{274877906944}$	$\frac{2^0}{2^{38}}$
$\frac{1}{549755813888}$	$\frac{2^0}{2^{39}}$
$\frac{1}{1099511627776}$	$\frac{2^0}{2^{40}}$
$\frac{1}{2199023255552}$	$\frac{2^0}{2^{41}}$
$\frac{1}{4398046511104}$	$\frac{2^0}{2^{42}}$
$\frac{1}{8796093022208}$	$\frac{2^0}{2^{43}}$
$\frac{1}{17592186044416}$	$\frac{2^0}{2^{44}}$
$\frac{1}{35184372088832}$	$\frac{2^0}{2^{45}}$
$\frac{1}{70368744177664}$	$\frac{2^0}{2^{46}}$
$\frac{1}{140737488355328}$	$\frac{2^0}{2^{47}}$
$\frac{1}{281474976710656}$	$\frac{2^0}{2^{48}}$
$\frac{1}{562949953421312}$	$\frac{2^0}{2^{49}}$
$\frac{1}{1125899906842624}$	$\frac{2^0}{2^{50}}$
$\frac{1}{2251799813685248}$	$\frac{2^0}{2^{51}}$
$\frac{1}{4503599627370496}$	$\frac{2^0}{2^{52}}$
$\frac{1}{9007199254740992}$	$\frac{2^0}{2^{53}}$
$\frac{1}{18014398509481984}$	$\frac{2^0}{2^{54}}$
$\frac{1}{36028797018963968}$	$\frac{2^0}{2^{55}}$
$\frac{1}{72057594037927936}$	$\frac{2^0}{2^{56}}$
$\frac{1}{144115188075855872}$	$\frac{2^0}{2^{57}}$
$\frac{1}{288230376151711744}$	$\frac{2^0}{2^{58}}$
$\frac{1}{576460752303423488}$	$\frac{2^0}{2^{59}}$
$\frac{1}{1152921504606846976}$	$\frac{2^0}{2^{60}}$
$\frac{1}{2305843009213693952}$	$\frac{2^0}{2^{61}}$
$\frac{1}{4611686018427387904}$	$\frac{2^0}{2^{62}}$
$\frac{1}{9223372036854775808}$	$\frac{2^0}{2^{63}}$
$\frac{1}{18446744073709551616}$	$\frac{2^0}{2^{64}}$
$\frac{1}{36893488147419103232}$	$\frac{2^0}{2^{65}}$
$\frac{1}{73786976294838206464}$	$\frac{2^0}{2^{66}}$
$\frac{1}{147573952589676412928}$	$\frac{2^0}{2^{67}}$
$\frac{1}{295147905179352825856}$	$\frac{2^0}{2^{68}}$
$\frac{1}{590295810358705651712}$	$\frac{2^0}{2^{69}}$
$\frac{1}{1180591620717411303424}$	$\frac{2^0}{2^{70}}$
$\frac{1}{2361183241434822606848}$	$\frac{2^0}{2^{71}}$
$\frac{1}{4722366482869645213696}$	$\frac{2^0}{2^{72}}$
$\frac{1}{9444732965739290427392}$	$\frac{2^0}{2^{73}}$
$\frac{1}{18889465931478580854784}$	$\frac{2^0}{2^{74}}$
$\frac{1}{37778931862957161709568}$	$\frac{2^0}{2^{75}}$
$\frac{1}{75557863725914323419136}$	$\frac{2^0}{2^{76}}$
$\frac{1}{151115727451828646838272}$	$\frac{2^0}{2^{77}}$
$\frac{1}{302231454903657293676544}$	$\frac{2^0}{2^{78}}$
$\frac{1}{604462909807314587353088}$	$\frac{2^0}{2^{79}}$
$\frac{1}{1208925819614629174706176}$	$\frac{2^0}{2^{80}}$
$\frac{1}{2417851639229258349412352}$	$\frac{2^0}{2^{81}}$
$\frac{1}{4835703278458516698824704}$	$\frac{2^0}{2^{82}}$
$\frac{1}{9671406556917033397649408}$	$\frac{2^0}{2^{83}}$
$\frac{1}{19342813113834066795298816}$	$\frac{2^0}{2^{84}}$
$\frac{1}{38685626227668133590597632}$	$\frac{2^0}{2^{85}}$
$\frac{1}{77371252455336267181195264}$	$\frac{2^0}{2^{86}}$
$\frac{1}{154742504910672534362390528}$	$\frac{2^0}{2^{87}}$
$\frac{1}{309485009821345068724781056}$	$\frac{2^0}{2^{88}}$
$\frac{1}{618970019642690137449562112}$	$\frac{2^0}{2^{89}}$
$\frac{1}{1237940039285380274899124224}$	$\frac{2^0}{2^{90}}$
$\frac{1}{2475880078570760549798248448}$	$\frac{2^0}{2^{91}}$
$\frac{1}{4951760157141521099596496896}$	$\frac{2^0}{2^{92}}$
$\frac{1}{9903520314283042199192993792}$	$\frac{2^0}{2^{93}}$
$\frac{1}{19807040628566084398385987584}$	$\frac{2^0}{2^{94}}$
$\frac{1}{39614081257132168796771975168}$	$\frac{2^0}{2^{95}}$
$\frac{1}{79228162514264337593543950336}$	$\frac{2^0}{2^{96}}$
$\frac{1}{158456325028528675187087900672}$	$\frac{2^0}{2^{97}}$
$\frac{1}{316912650057057350374175801344}$	$\frac{2^0}{2^{98}}$
$\frac{1}{633825300114114700748351602688}$	$\frac{2^0}{2^{99}}$
$\frac{1}{1267650600228229401496703205376}$	$\frac{2^0}{2^{100}}$
$\frac{1}{2535301200456458802993406410752}$	$\frac{2^0}{2^{101}}$
$\frac{1}{5070602400912917605986812821504}$	$\frac{2^0}{2^{102}}$
$\frac{1}{10141204801825835211973625643008}$	$\frac{2^0}{2^{103}}$
$\frac{1}{20282409603651670423947251286016}$	$\frac{2^0}{2^{104}}$
$\frac{1}{40564819207303340847894502572032}$	$\frac{2^0}{2^{105}}$
$\frac{1}{81129638414606681695789005144064}$	$\frac{2^0}{2^{106}}$
$\frac{1}{162259276829213363391578010288128}$	$\frac{2^0}{2^{107}}$
$\frac{1}{324518553658426726783156020576256}$	$\frac{2^0}{2^{108}}$
$\frac{1}{649037107316853453566312041152512}$	$\frac{2^0}{2^{109}}$
$\frac{1}{1298074214633706907132624082305024}$	$\frac{2^0}{2^{110}}$
$\frac{1}{2596148429267413814265248164610048}$	$\frac{2^0}{2^{111}}$
$\frac{1}{5192296858534827628530496329220096}$	$\frac{2^0}{2^{112}}$
$\frac{1}{10384593717069655257060992658440192}$	$\frac{2^0}{2^{113}}$
$\frac{1}{20769187434139310514121985316880384}$	$\frac{2^0}{2^{114}}$
$\frac{1}{41538374868278621028243970633760768}$	$\frac{2^0}{2^{115}}$
$\frac{1}{83076749736557242056487941267521536}$	$\frac{2^0}{2^{116}}$
$\frac{1}{166153499473114484112975882535043072}$	$\frac{2^0}{2^{117}}$
$\frac{1}{332306998946228968225951765070086144}$	$\frac{2^0}{2^{118}}$
$\frac{1}{664613997892457936451903530140172288}$	$\frac{2^0}{2^{119}}$
$\frac{1}{1329227995784915872903807060280344576}$	$\frac{2^0}{2^{120}}$
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$\frac{1}{10633823966279326983230456482242756608}$	$\frac{2^0}{2^{123}}$
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$\frac{1}{89202980794122492566142873090593446023921664}$	$\frac{2^0}{2^{146}}$
$\frac{1}{178405961588244985132285746181186892047843328}$	$\frac{2^0}{2^{147}}$
$\frac{1}{356811923176489970264571492362373784095686656}$	$\frac{2^0}{2^{148}}$
$\frac{1}{713623846352979940529142984724747568191373312}$	$\frac{2^0}{2^{149}}$
$\frac{1}{1427247692705959881058285969449495136382746624}$	$\frac{2^0}{2^{150}}$
$\frac{1}{2854495385411919762116571938898990272765493248}$	$\frac{2^0}{2^{151}}$
$\frac{1}{5708990770823839524233143877797980545530986496}$	$\frac{2^0}{2^{152}}$
$\frac{1}{11417981541647679048466287755595961091061972992}$	$\frac{2^0}{2^{153}}$
$\frac{1}{22835963083295358096932575511191922182123945984}$	$\frac{2^0}{2^{154}}$
$\frac{1}{45671926166590716193865151022383844364247891968}$	$\frac{2^0}{2^{155}}$
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$\frac{1}{730750818665451459101842416358141509827966271488}$	$\frac{2^0}{2^{159}}$
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$\frac{1}{2923003274661805836407369665432566039311865085952}$	$\frac{2^0}{2^{161}}$
$\frac{1}{5846006549323611672814739330865132078623730171904}$	$\frac{2^0}{2^{162}}$
$\frac{1}{11692013098647223345629478661730264157247460343808}$	$\frac{2^0}{2^{163}}$
$\frac{1}{23384026197294446691258957323460528314494920687616}$	$\frac{2^0}{2^{164}}$
$\frac{1}{46768052394588893382517914646921056628989841375232}$	$\frac{2^0}{2^{165}}$
$\frac{1}{93536104789177786765035829293842113257979682750464}$	$\frac{2^0}{2^{166}}$
$\frac{1}{187072209578355573530071658587684226515959365500928}$	$\frac{2^0}{2^{167}}$
$\frac{1}{374144419156711147060143317175368453031918731001856}$	$\frac{2^0}{2^{168}}$
$\frac{1}{748288838313422294120286634350736906063837462003712}$	$\frac{2^0}{2^{169}}$
$\frac{1}{1496577676626844588240573268701473812127674924007424}$	$\frac{2^0}{2^{170}}$
$\frac{1}{2993155353253689176481146537402947624255349848014848}$	$\frac{2^0}{2^{171}}$
$\frac{1}{5986310706507378352962293074805895248510699696029696}$	$\frac{2^0}{2^{172}}$
$\frac{1}{11972621413014756705924586149611790497021399392059392}$	$\frac{2^0}{2^{173}}$
$\frac{1}{23945242826029513411849172299223580994042798784118784}$	$\frac{2^0}{2^{174}}$
$\frac{1}{47890485652059026823698344598447161988085597568237568}$	$\frac{2^0}{2^{175}}$
$\frac{1}{95780971304118053647396689196894323976171195136475136}$	$\frac{2^0}{2^{176}}$
$\frac{1}{191561942608236107294793378393788647952342390272950272}$	$\frac{2^0}{2^{177}}$
$\frac{1}{383123885216472214589586756787577295904684780545900544}$	$\frac{2^0}{2^{178}}$
<	

How to convert from decimal to binary ?

$$26.375 = (\underline{\quad?} \cdot \underline{\quad?})_2$$

To find the bits for the positive powers of 2, use the algorithm from ~~Assignment Project Exam Help~~ ("repeated division").

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~~b.c~~
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$\begin{array}{r} m \\ \hline 26 \\ 13 \\ 6 \\ 3 \\ 1 \\ 0 \end{array}$

$\begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{array}$

\Rightarrow

$$26 = (11010)_2$$

What about negative powers of 2 ?

In general, note that multiplying by 2 shifts bits to the left (or shifts binary point to the right)

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Example: <https://powcoder.com>

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$$\begin{aligned} & (11010.011)_2 \times 2 \\ &= (110100.11)_2 \end{aligned}$$

Similarly....dividing by 2 and not ignoring remainder shifts bits to the right (or shifts binary point to the left)

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$$(11010.011)_2 / 2$$

$$= (1101.0011)_2$$

For the negative powers of 2, use "repeated multiplication"

$$.375$$

$$= .375 \times 2^1 \times 2^{-1}$$

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$$= .75 \times 2^{-2}$$

$$= 1.5 \times 2^{-3}$$

$$= 3.0 \times 2^{-3}$$

$$= (11)_2 \times 2^{-3}$$

$$= (.011)_2$$

convert
decimal
to binary



A more subtle example:

$$19.243 = (\quad?)_2$$

First, find the bits for the positive powers of 2 using "repeated division" (last lecture).

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m

19

9

4

2

1

0

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1

1

0

0

1

$$\therefore 19 = (10011)_2$$

Then find the bits for the negative powers of 2 using repeated multiplication.

243

$$= 243 \times 2^1 \times 2^{-1}$$

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$$= (5) 48.6 \times 2^{-1}$$

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?

Then find the bits for the negative powers of 2 using repeated multiplication.

$$.243$$

$$= (0)_2 .486 \times 2^{-1}$$

$$= (00)_2 .972 \times 2^{-2}$$

$$= (001)_2 .944 \times 2^{-3}$$

$$= (0011)_2 .888 \times 2^{-4}$$

$$\text{Thus } (.243)_{10} = (.0011)_2 + \sum_{i=-5}^{-\infty} b_i 2^i$$

Note the summation is over bits b_i from -5, -6, ..., -infinity.

$$19.243 = (10011.0011\text{---})_2$$

We cannot get an exact representation using a finite number of bits for this example.

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Can we say anything more general about what happens ?

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$$(05)_{10}$$

$$= (0)_2 \cdot 1 \times 2^{-1}$$

$$= (00)_2 \cdot 2 \times 2^{-2}$$

$$= (000)_2 \cdot 4 \times 2^{-3}$$

$$= (0000)_2 \cdot 8 \times 2^{-4}$$

$$= (00001)_2 \cdot 16 \times 2^{-5}$$

$$= (000011)_2 \cdot 2 \times 2^{-6}$$

$$= \begin{array}{ccccccc} & 00 & & 0011 & & 0011 & & 0011 & \text{etc.} \\ & \underline{\hspace{1cm}} & & & & \underline{\hspace{1cm}} & & & \end{array}$$

This will repeat over and over again.

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When we convert a floating point decimal number with a finite number of digits into binary, we get:

- a finite number of non-zero bits to left of binary point
- an infinitely repeating sequence of bits to the right of the binary point

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Why ?

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[Note: sometimes the infinite number of repeating bits are all 0's, as in the case of 0.375 a few slides back.]

Recall previous example...

$$\begin{aligned} & .243 \\ = & (0)_2 .486 \times 2^{-1} \\ = & (00)_2 .972 \times 2^{-2} \\ = & (001)_2 .944 \times 2^{-3} \\ = & (0011)_2 .888 \times 2^{-4} \\ = & \text{etc} \end{aligned}$$

Eventually, the three digits to the right of the decimal point will enter a cycle that repeats forever. This will produce a bit string that repeats forever.

Hexadecimal

Hexadecimal (base 16)

Writing down long strings of bits is awkward and error prone.

Hexadecimal simplifies the representation.

0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
a	1010
b	1011
c	1100
d	1101
e	1110
f	1111

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Examples of hexadecimal

1) 0010 1111 1010 0011

2 f a 3

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We write 0x2fa3 or 0X2FA3.

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2) 101100

We write 0x2c (10 1100), not 0xb0 (1011 00)