Let's get started !

Q: How do people and computers represent numbers ?

Base 10

$$238 = 2 \cdot 10^2 + 3 \cdot 10^1 + 8 \cdot 10^0$$

$$m = \sum_{i=0}^{k-1} a_i 10^i$$

$$= (a_{k-1} \, a_{k-2} \, \cdots \, a_2 \, a_1 \, a_0)_{ten}$$

Base 2

$$11010 = 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2$$

$$+ 1 \cdot 2^1 + 0 \cdot 2^0$$

$$= 16 + 8 + 2 = 26$$

$$m = \sum_{i=0}^{n-1} b_i 2^i$$

$$= \left(b_{n-1} b_{n-2} \cdots b_2 b_1 b_0 \right)_{two}$$

Counting in binary

decimal
0
1
2
3
4
5
6
7
8
.

binary
0
1
10
11
100
101
110
111
1000
.

binary
0000
0001
0010
0011
0100
0101
0110
0111
1000

To convert from binary to decimal, you need to know the powers of 2.

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
⋮	⋮

memorize

How can we convert **m** from decimal to binary?

Idea 1:

Find the biggest power of two less than or equal to m, and subtract it from m.

Repeat until done.

(Requires memorizing powers of 2.)

How can we convert **m** from decimal to binary?

Idea 2: Consider familiar idea from base 10:

$$238 = 230 + 8$$

$$m = (m / 10) \times 10 + m \% 10$$

↑
integer division

Same idea works in base 2.

$$m = (m/2) \times 2 + m \% 2$$

Example:

$$m = (100101)_{two}$$

$$m/2 = 10010$$

$$m \% 2 = 1$$

$$m = \sum_{i=0}^{n-1} b_i 2^i$$

$$= \left(b_{n-1} \; b_{n-2} \; \cdots \; b_2 \; b_1 \; b_0 \right)_{two}$$

$$m/2 = \sum_{i=1}^{n-1} b_i 2^{i-1}$$

$$= \left(b_{n-1} \; b_{n-2} \cdots b_2 \; b_1 \right)_{two}$$

$$m \% 2 = b_0$$

Algorithm:
given m in decimal, convert it to binary.

$$i \leftarrow 0$$

$$\text{while } m > 0 \; \{$$

$$b_i \leftarrow m \% 2$$

$$m \leftarrow m / 2$$

$$i \leftarrow i + 1$$

$$\}$$

Example

$$m = 241 = (1111\ 0001)_{two}$$

$$\underline{m} \qquad\qquad \underline{bi}$$

$$\boxed{241}$$

$$= 120 \times 2 +\ 1$$

$$60 \qquad\qquad 0$$

$$30 \qquad\qquad 0$$

$$15 \qquad\qquad 0$$

$$7 \qquad\qquad 1$$

$$3 \qquad\qquad 1$$

$$1 \qquad\qquad 1$$

$$0$$

$$\underline{m}$$

$$\underline{b_i}$$

$$(1111\ 0001)_{two}$$

1111000

1111 0

1111 0

1111

111

11

1

1

0

0

0

1

1

1

Q: How to add two numbers in binary ?

Addition (base 10)

$$
\begin{array}{r}
2\ 3\ 4\ 3 \\
+\ 5\ 8\ 1\ 9 \\
\hline
8\ 1\ 6\ 2
\end{array}
$$

You need to memorize single digit sums to do this.

Addition (base 2)

0 0 1 1 0 1 0

0 0 0 1 1 0 1 0 ← 26

0 0 0 1 1 0 1 1 ← 27

0 0 1 1 0 1 0 1 ← 53

Subtraction (base 10)

$$
\begin{array}{r}
2\ 3\ 4\ 3 \\
-\ 5\ 8\ 1\ 7 \\
\hline
\end{array}
$$

ASIDE: the grade school algorithm doesn't work when the bigger number is on the bottom.   To take the difference using the grade school algorithm, you put the bigger number on top and take the negative of the result.

$$a - b$$

$$= a + (-b)$$

Next class we will learn how to represent *negative numbers in binary* which allows us to perform this sum.