

Algorithm and Data Structure

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Analysis

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(ADSA)

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AVL-Trees

Overview

AVL-Trees:

- Find, insert, remove

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Runtimes for Binary Search Tree

Find, insert, remove:

Worst case: $\Theta(n)$

Best case: $\Theta(\log n)$

Average case: $\Theta(\log n)$

Aim: Time $O(\log n)$ in the worst case

AVL-Tree

Observation:

- Binary search trees can get imbalanced when applying insert and/or remove operations.

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Idea:

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- Whenever a subtree rooted at a node v gets imbalanced, apply operations that balance it out in time $O(\log n)$.

AVL Tree

Let $h(T)$ be the height of a tree T .

Let v be a node in T and T_l and T_r be the left and right subtree of v .

We denote by $b(v) = h(T_l) - h(T_r)$ the balance degree of v .

Definition: A binary search tree T is called an AVL-tree if for each $v \in T$, $b(v) \in \{-1, 0, 1\}$ holds.

Height of an AVL-tree

Theorem(without proof) Let T be an AVL-tree consisting of n nodes. Then $h(T) \leq 1.44 \log n$

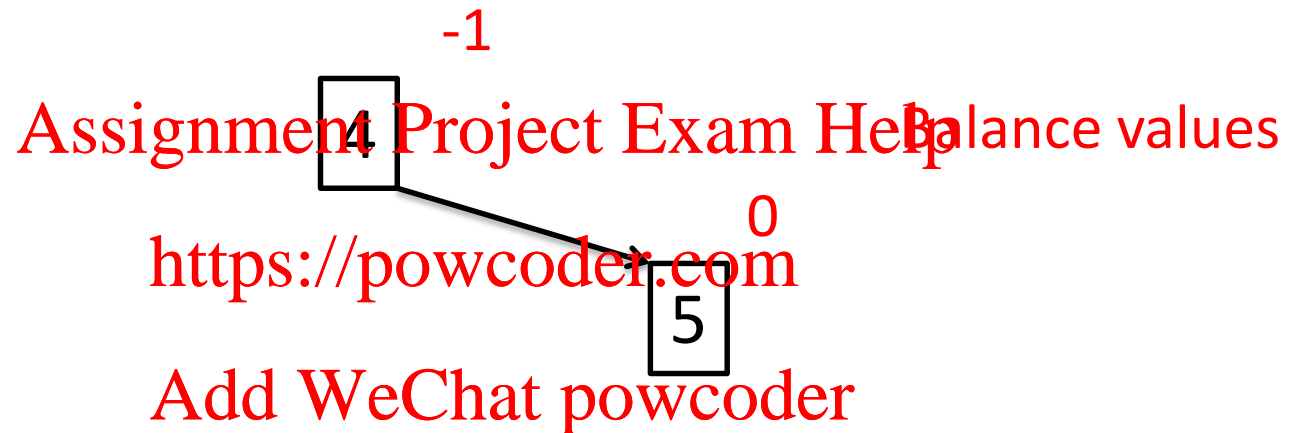
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We have to consider the operations find, insert, and delete for AVL-trees.

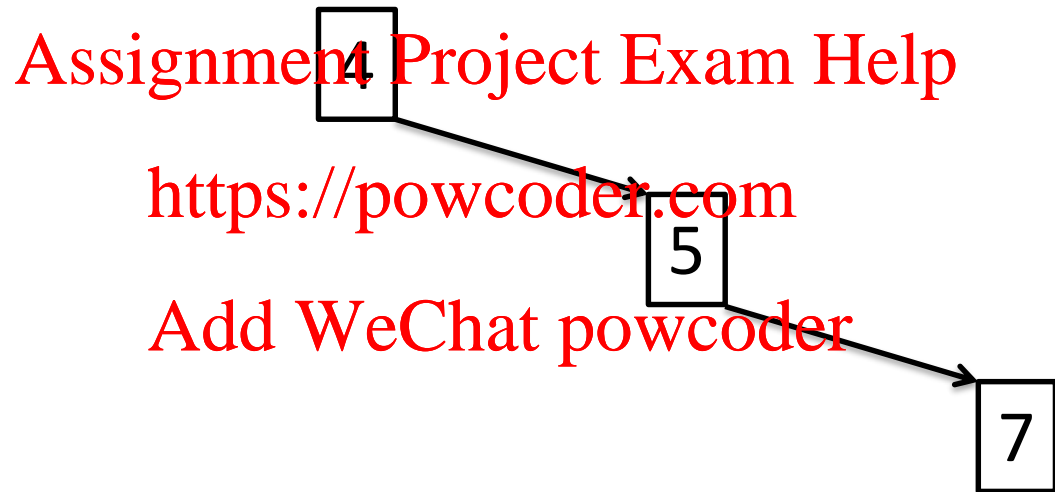
- Find is as for Binary Search Trees.
- For insert and remove we might have to rebalance the tree.

Example Insert



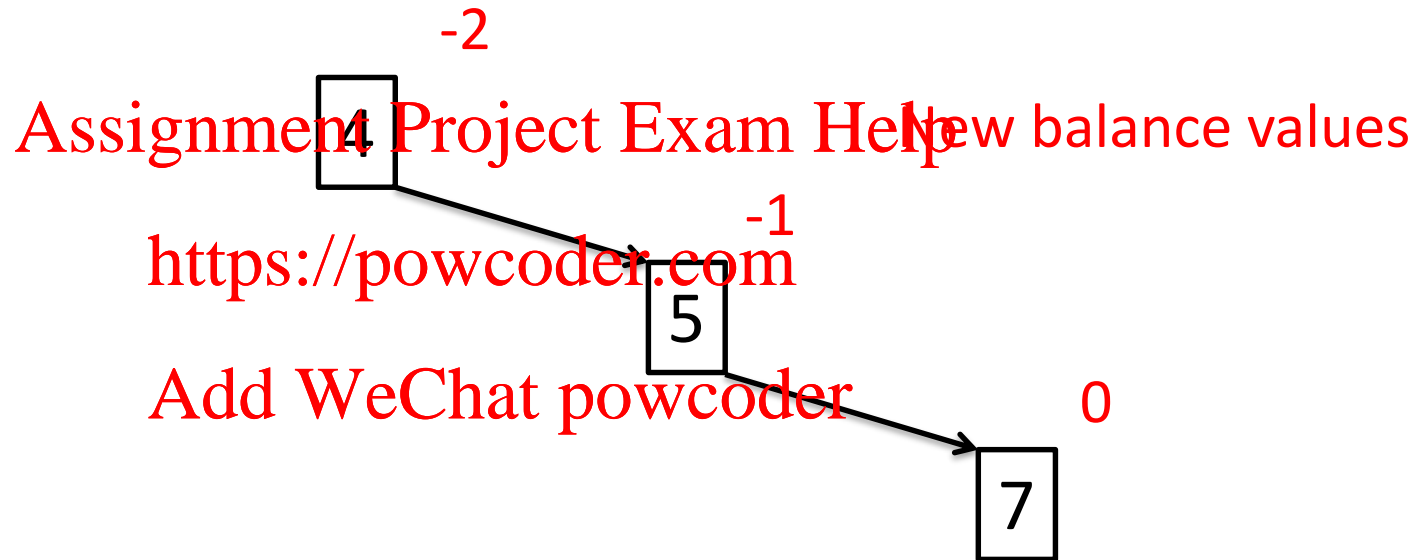
Insert 7

Example Insert

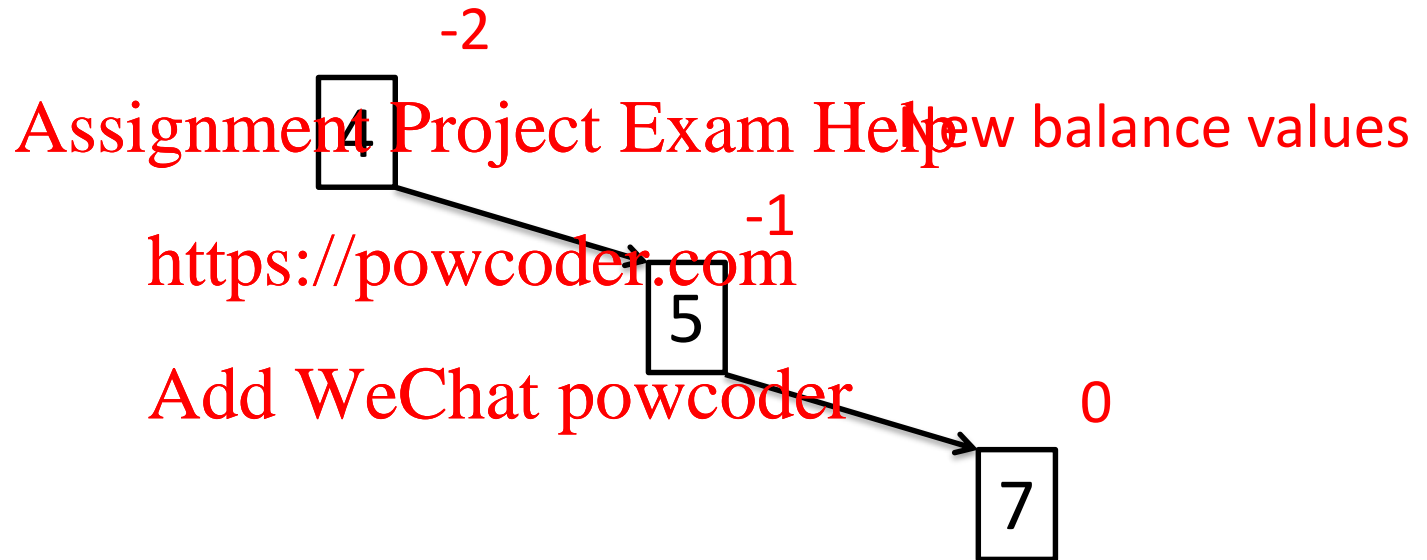


Consider path from new leaf
to the root and check balance values

Example Insert

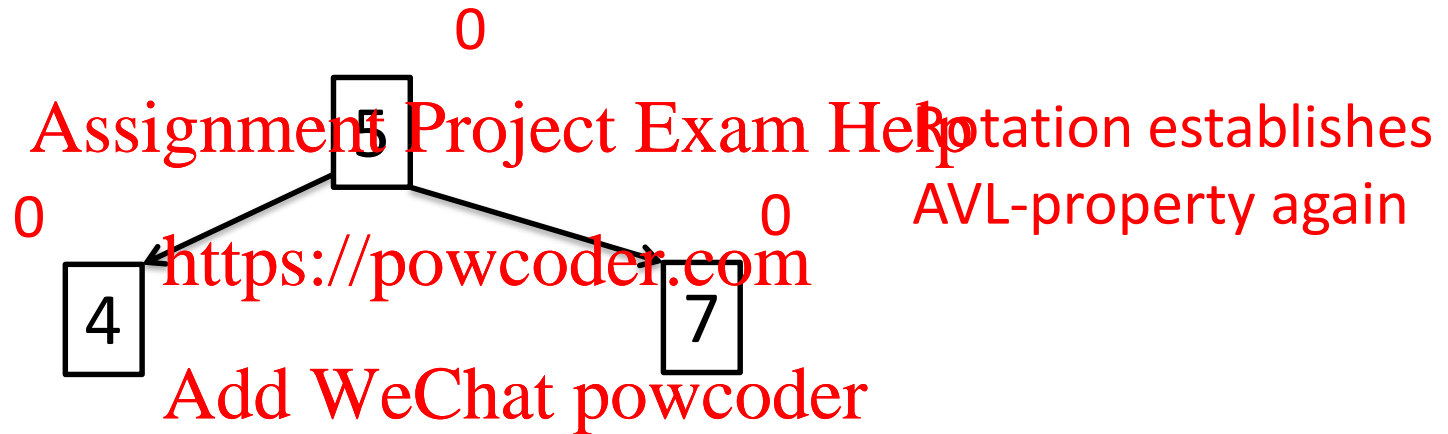


Example Insert



AVL-property at node 4 violated

Example Insert



Insertion

Inserting a new element z can violate the AVL-property.

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Consider path from the newly inserted leaf z to the root and repair AVL-property.

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Rebalancing

Let z be the newly inserted leaf.

Consider the path from z to the root (reverse the insertion path).

Update the balance values.

Repair AVL-property (if necessary).

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Insert

- we insert new node z as for Binary Search Trees.

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- $\text{bal}(z)=0$ holds after insertion.

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- $\text{bal}(v)$ might change by 1 for a node v on the path from z to the root.

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- If $b(v) \notin \{-1, 0, 1\}$ rebalance

Rebalancing

Start examining for v , where v is the parent of z , and continue with the parent of v (if necessary).

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Assume that the right child x of node v is on the path from z to the root.

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Before insertion -> After insertion:

- $\text{bal}(v) = 1 \rightarrow \text{bal}(v) = 0$ (height of tree rooted at v has not changed, stop rebalancing)
- $\text{bal}(v) = 0 \rightarrow \text{bal}(v) = -1$ (height of tree rooted at v has increased by 1, stop rebalancing only if v is root, otherwise examine parent of v)
- $\text{bal}(v) = -1 \rightarrow \text{bal}(v) = -2$ (AVL-property violated, carry out rotation)

Left Rotation

Assume node v and right child x on the path.

- w is right child of x on the path

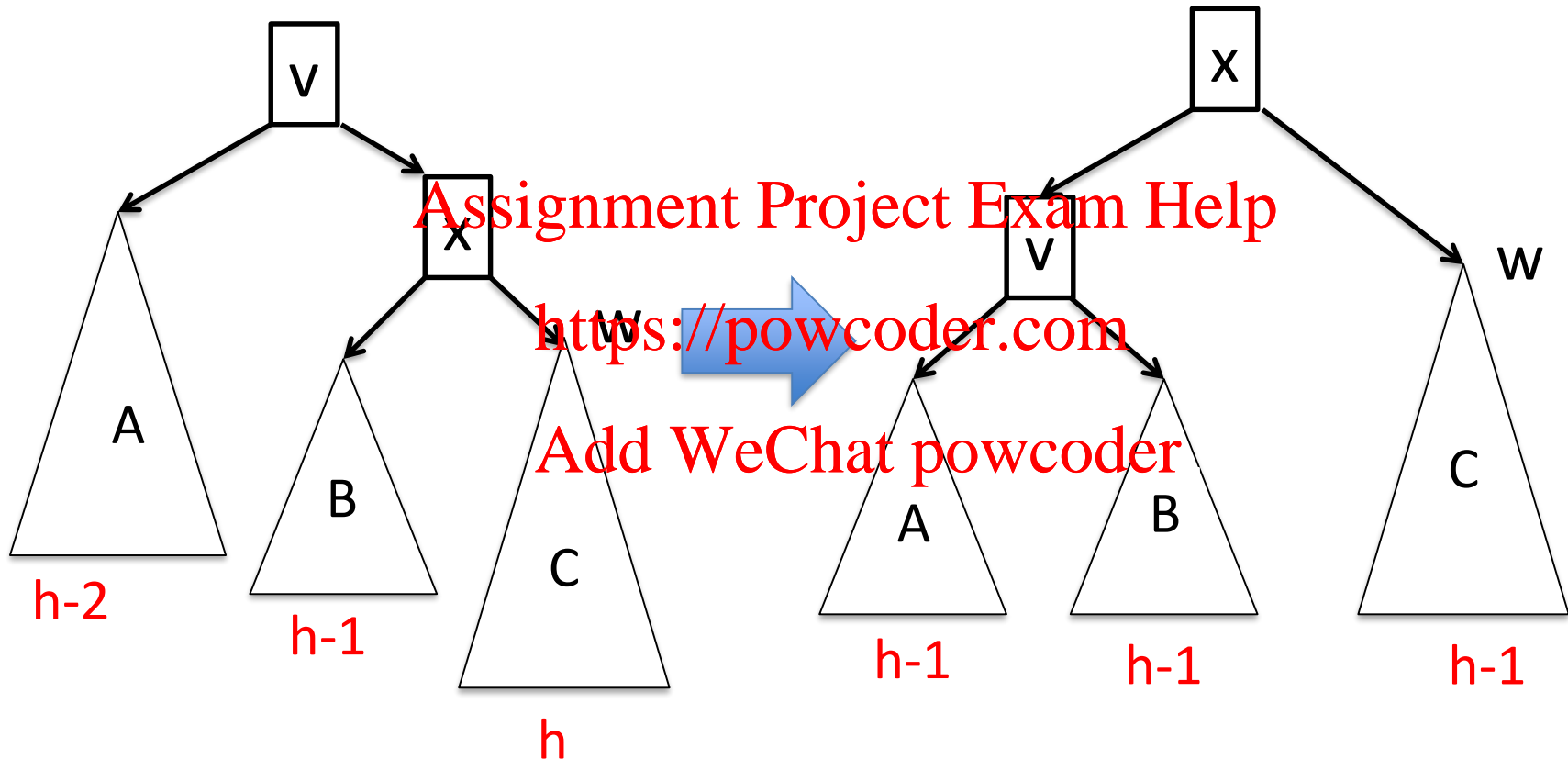
-> Left rotation

New balance values: $\text{bal}(x)=0$ and $\text{bal}(v)=0$

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Analogous: Right Rotation

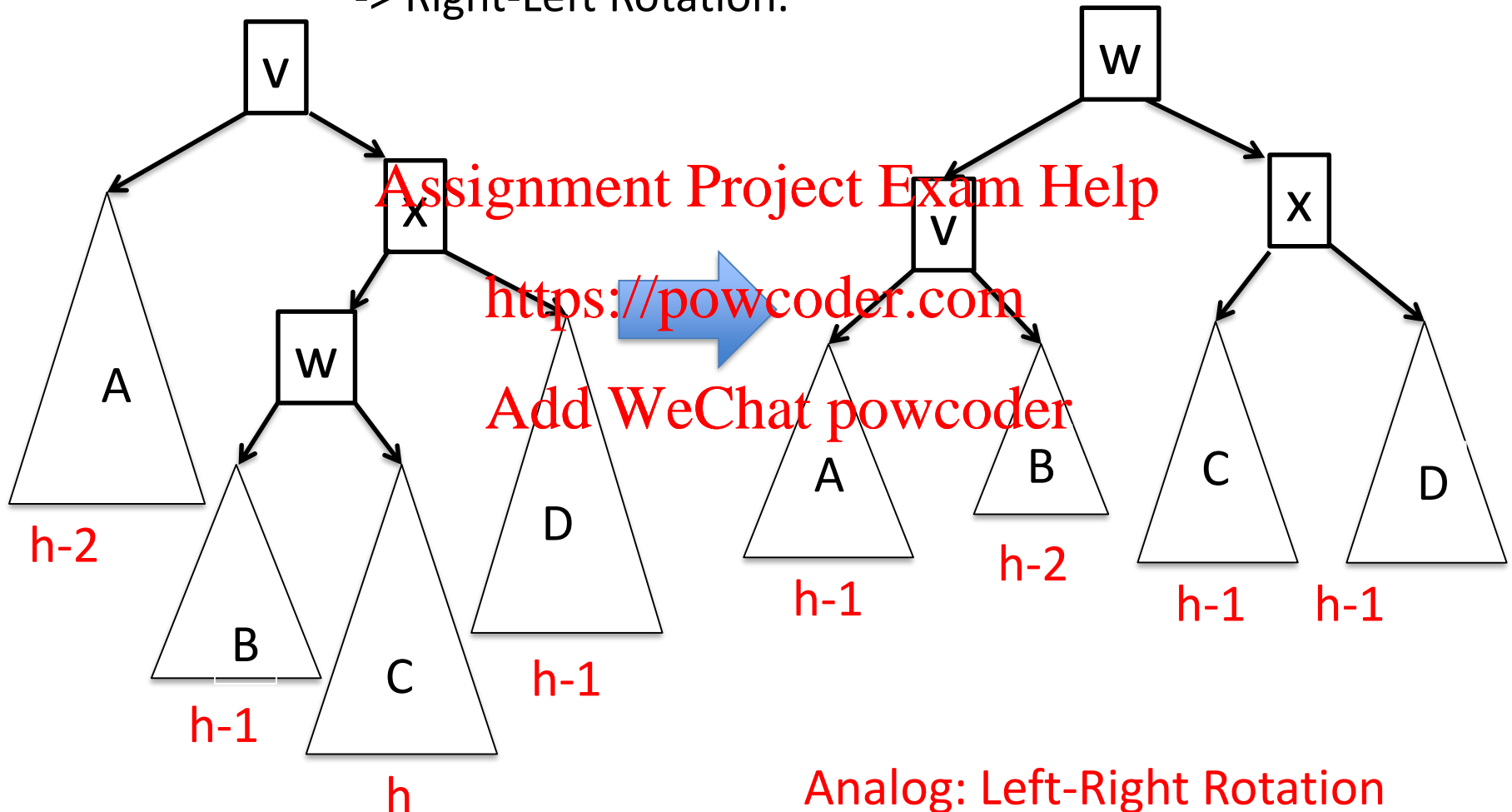
Left Rotation



Analog: Right Rotation

Right-Left Rotation

w is left child of x on the path
-> Right-Left Rotation.



Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6

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Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6

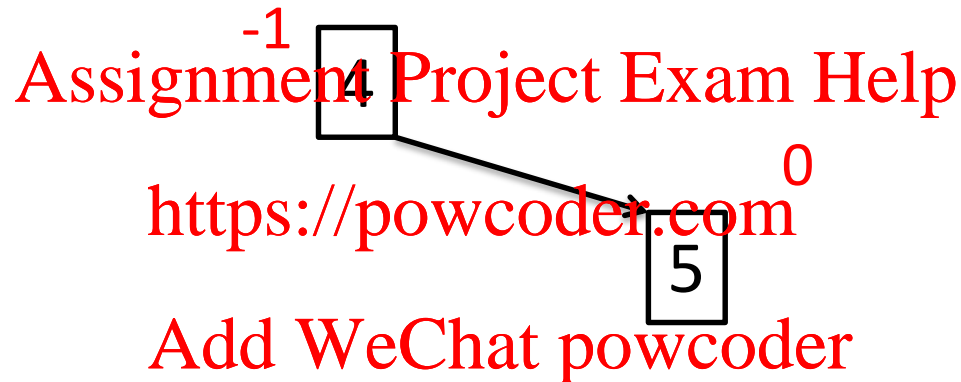
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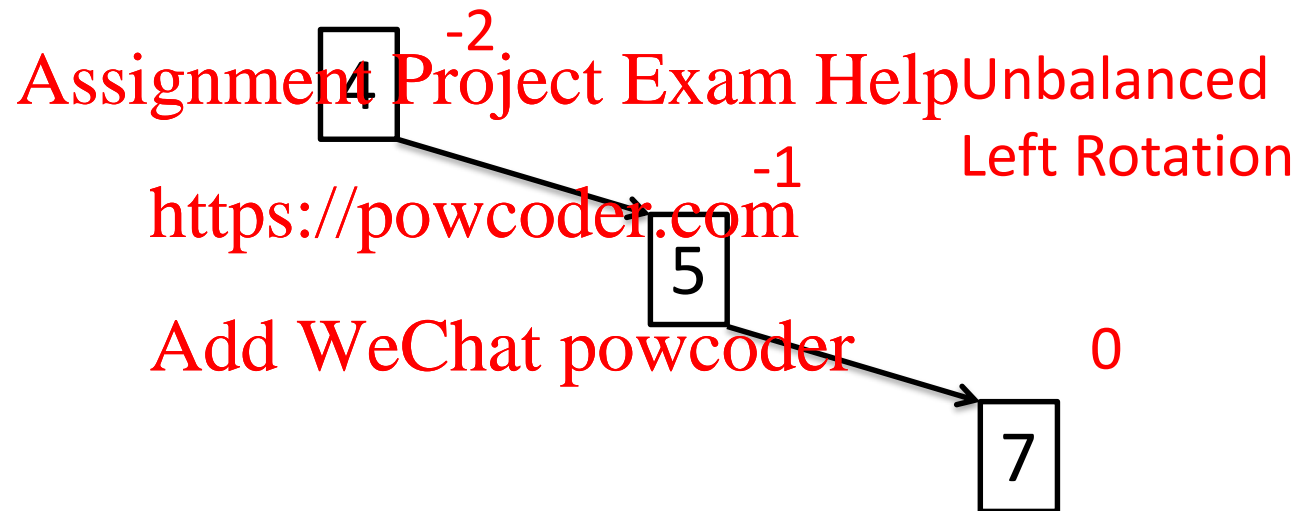
Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6



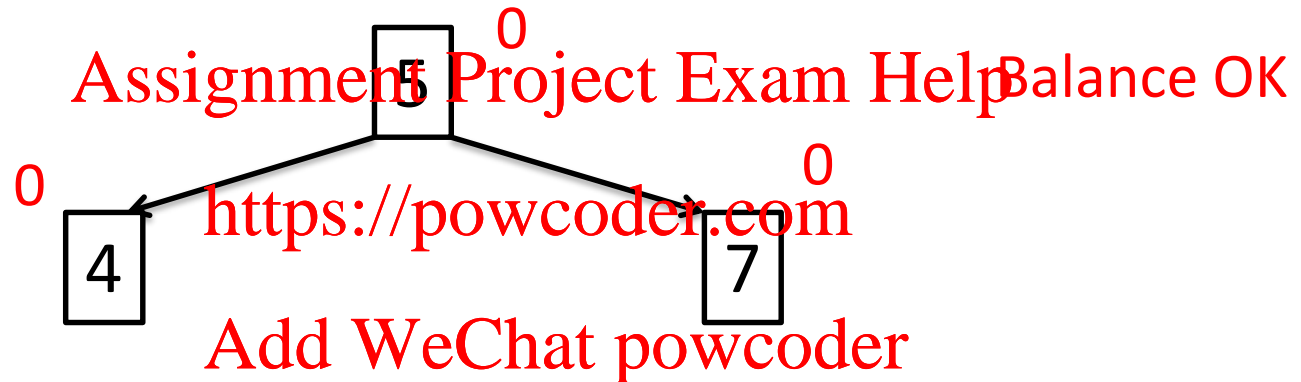
Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6



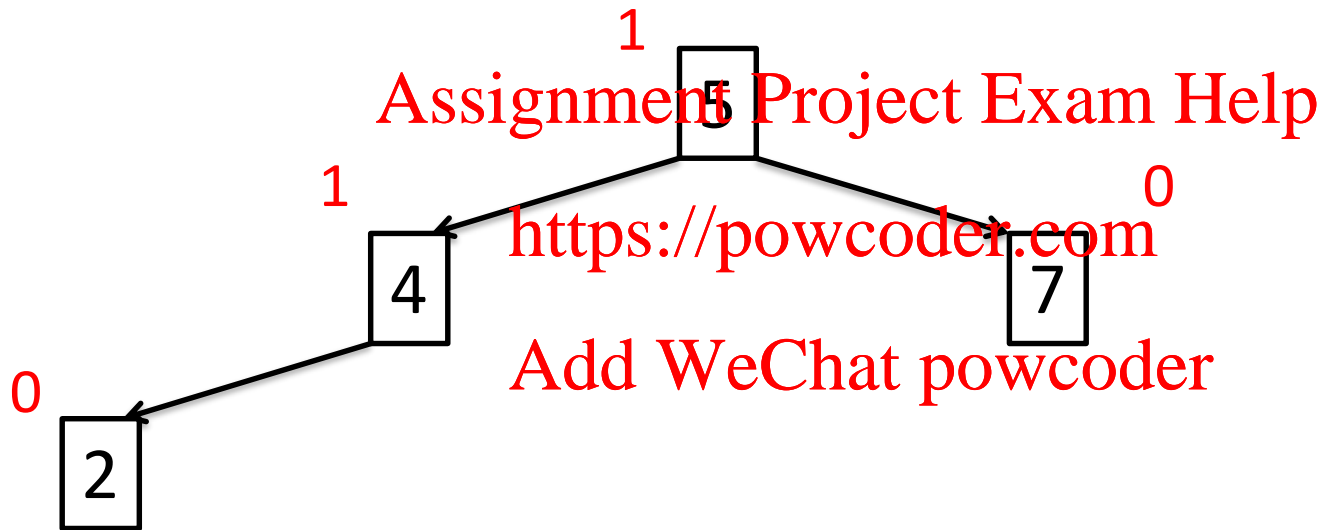
Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6



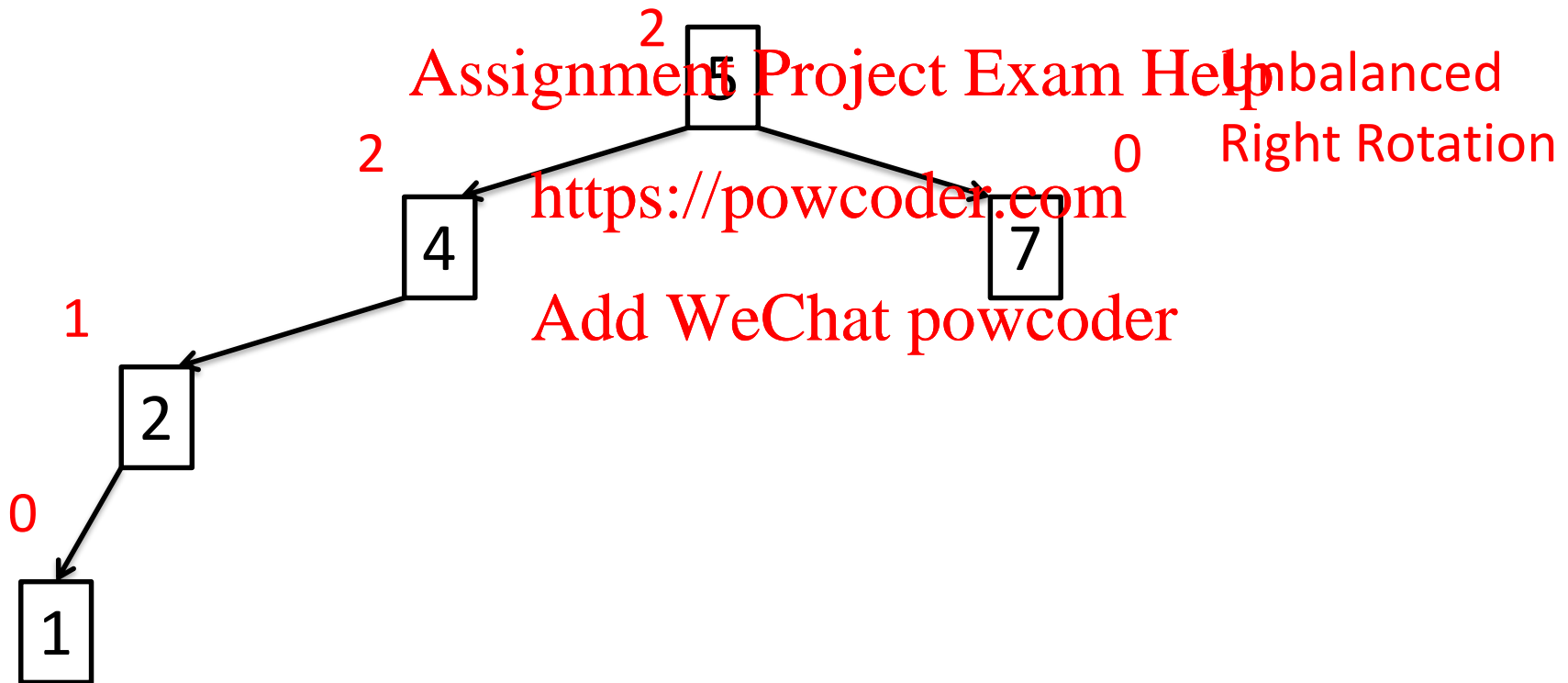
Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6



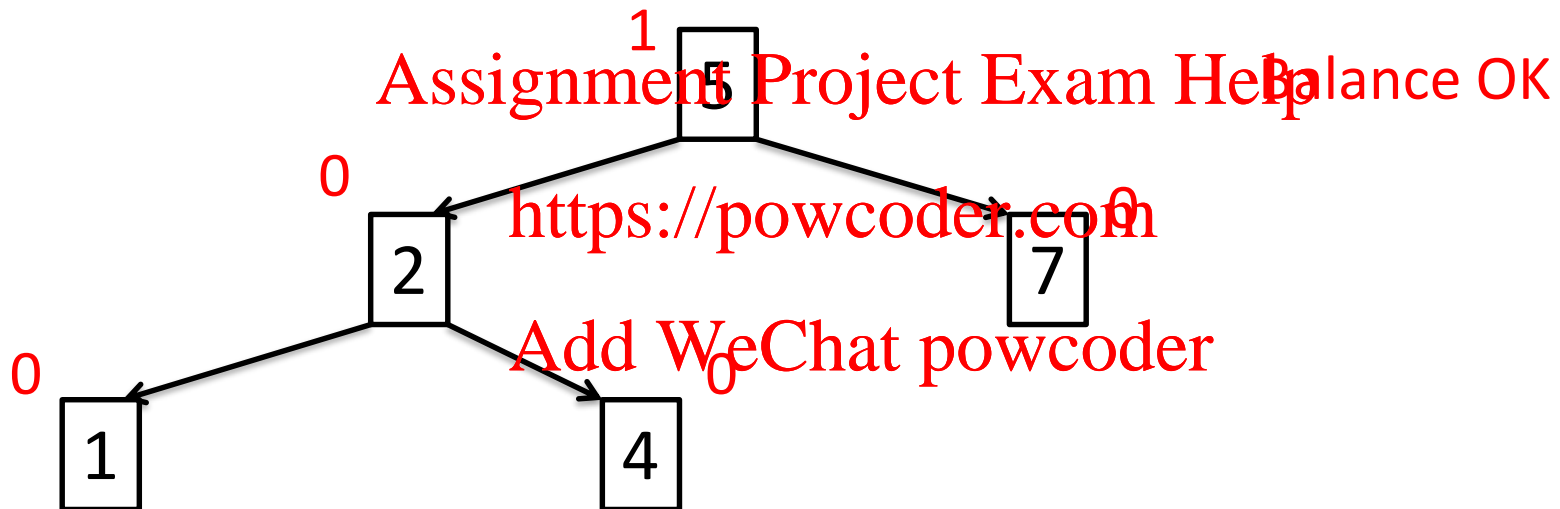
Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6



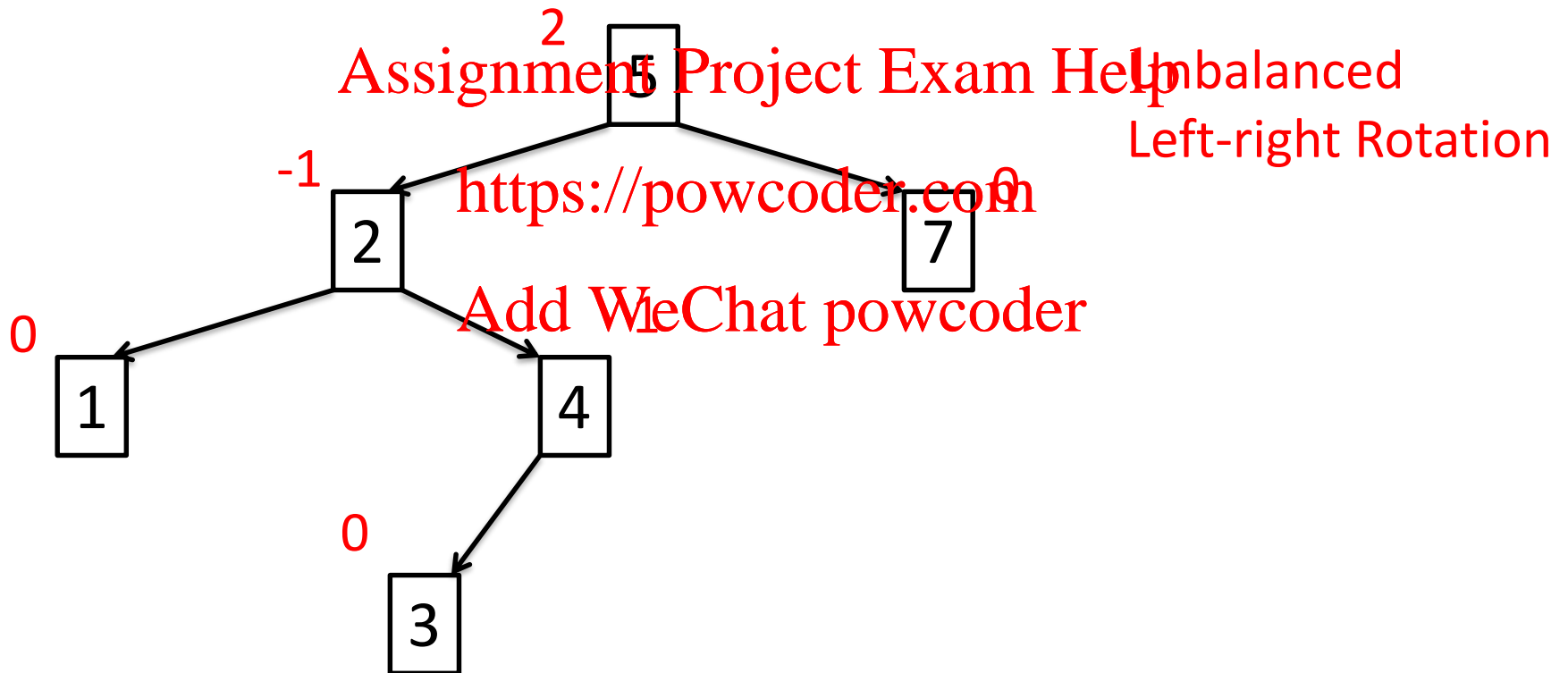
Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6



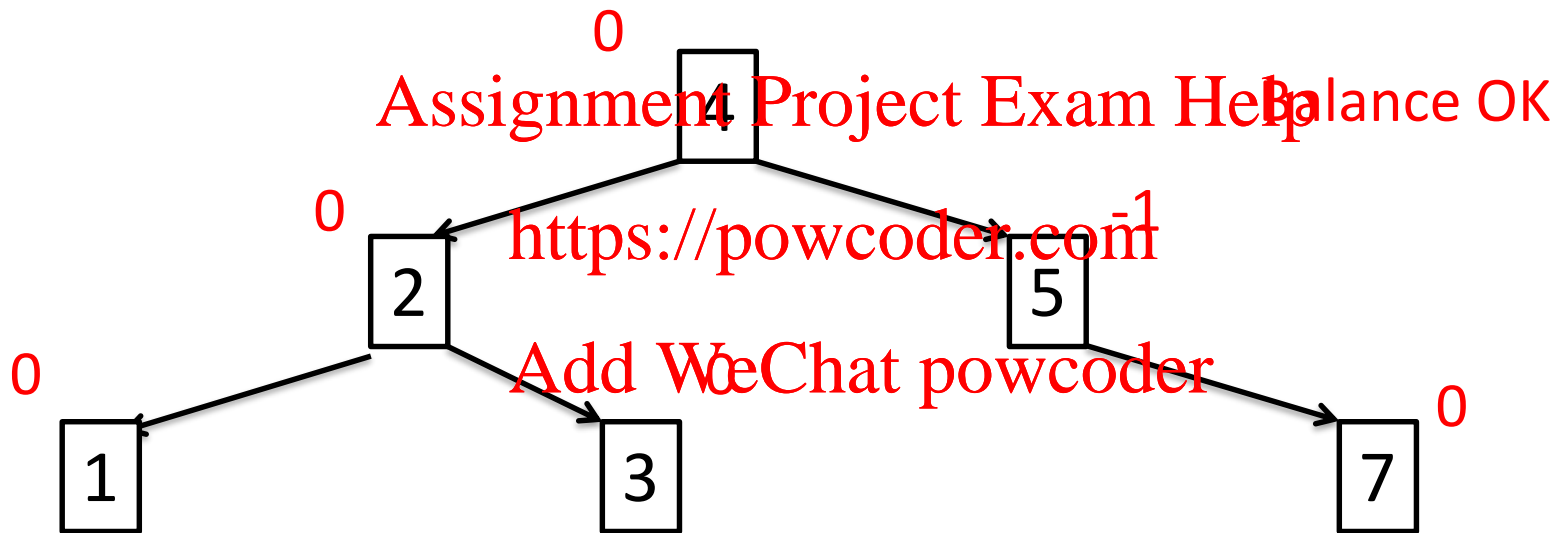
Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6



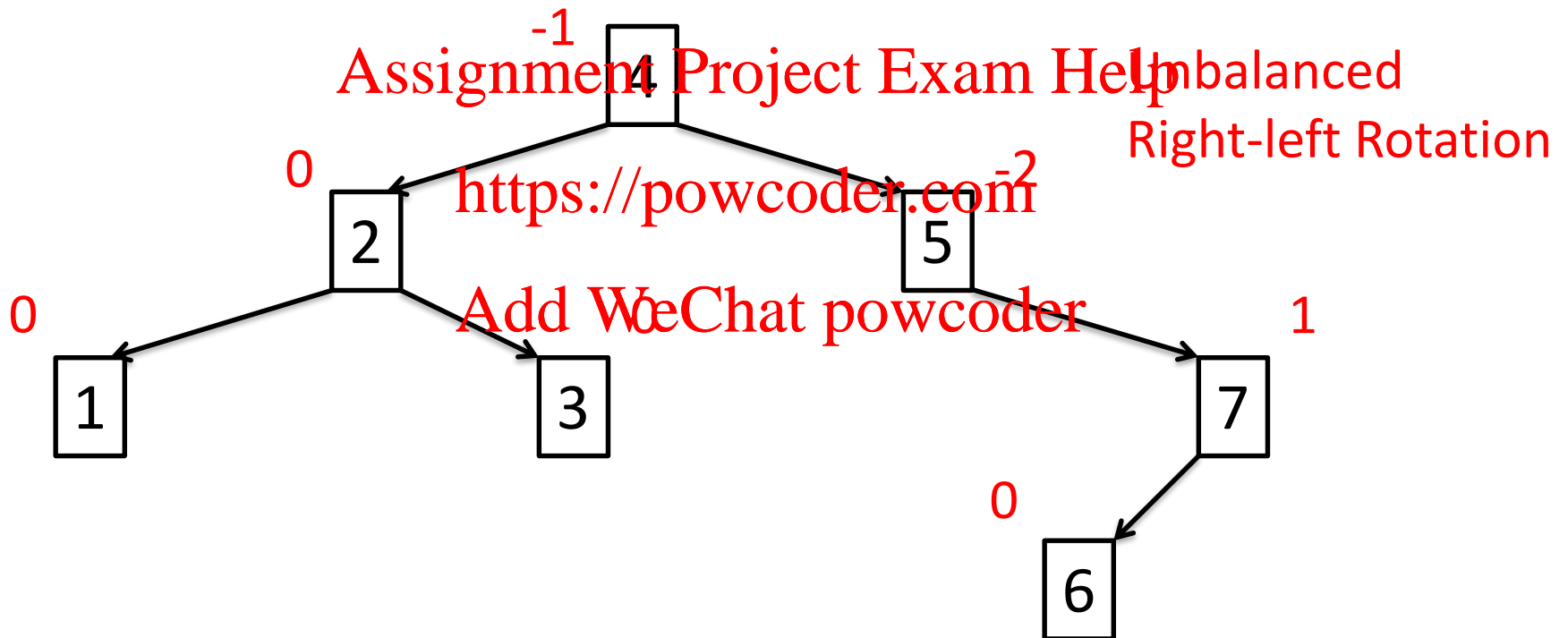
Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6



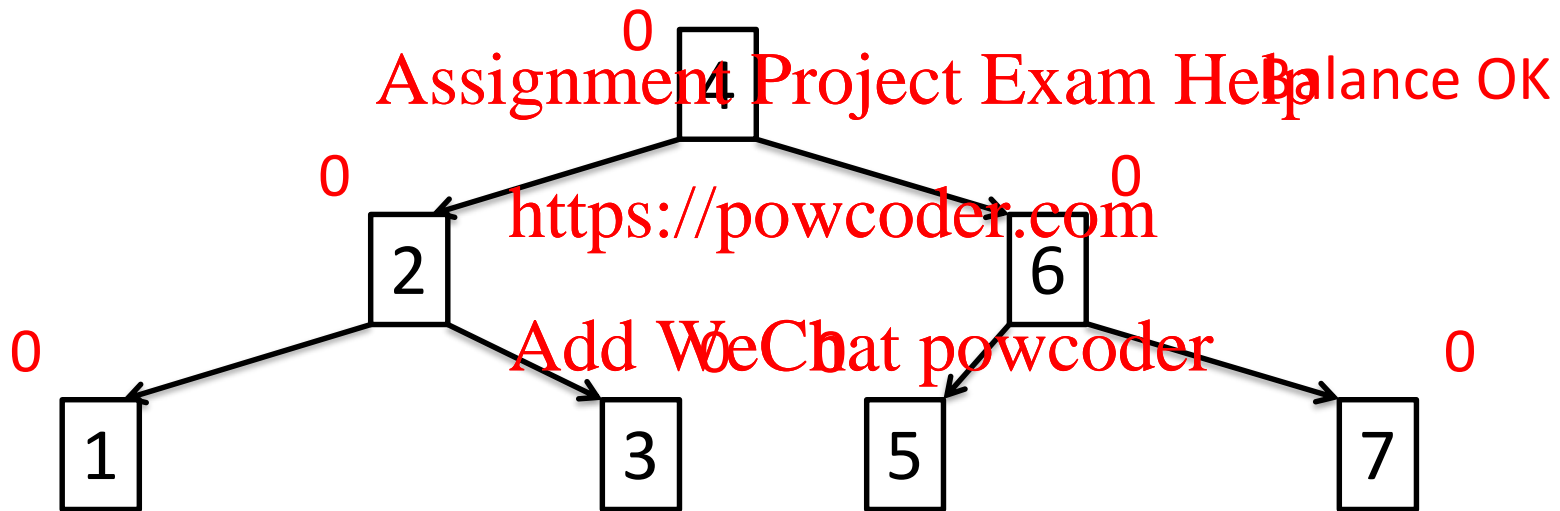
Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6



Example Insert

Create AVL-Tree for sequence 4, 5, 7, 2, 1, 3, 6



Time complexity of for insertion

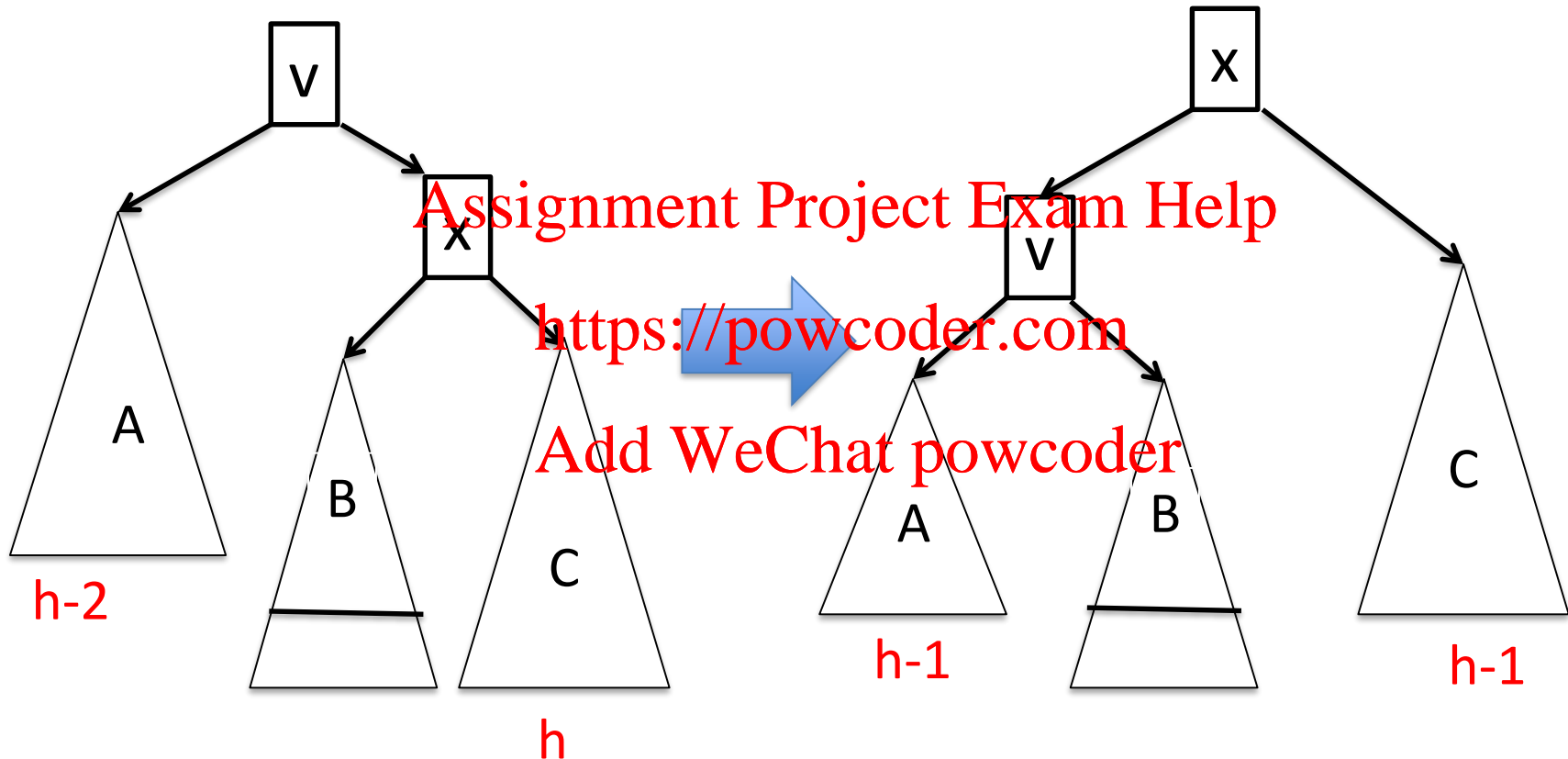
Inserting a node involves:

- Finding the location and adding the node to the tree
- Moving up the AVL tree see whether AVL property is violated and perform rotation as needed

$$O(\log n) + O(\log n) + O(1)$$

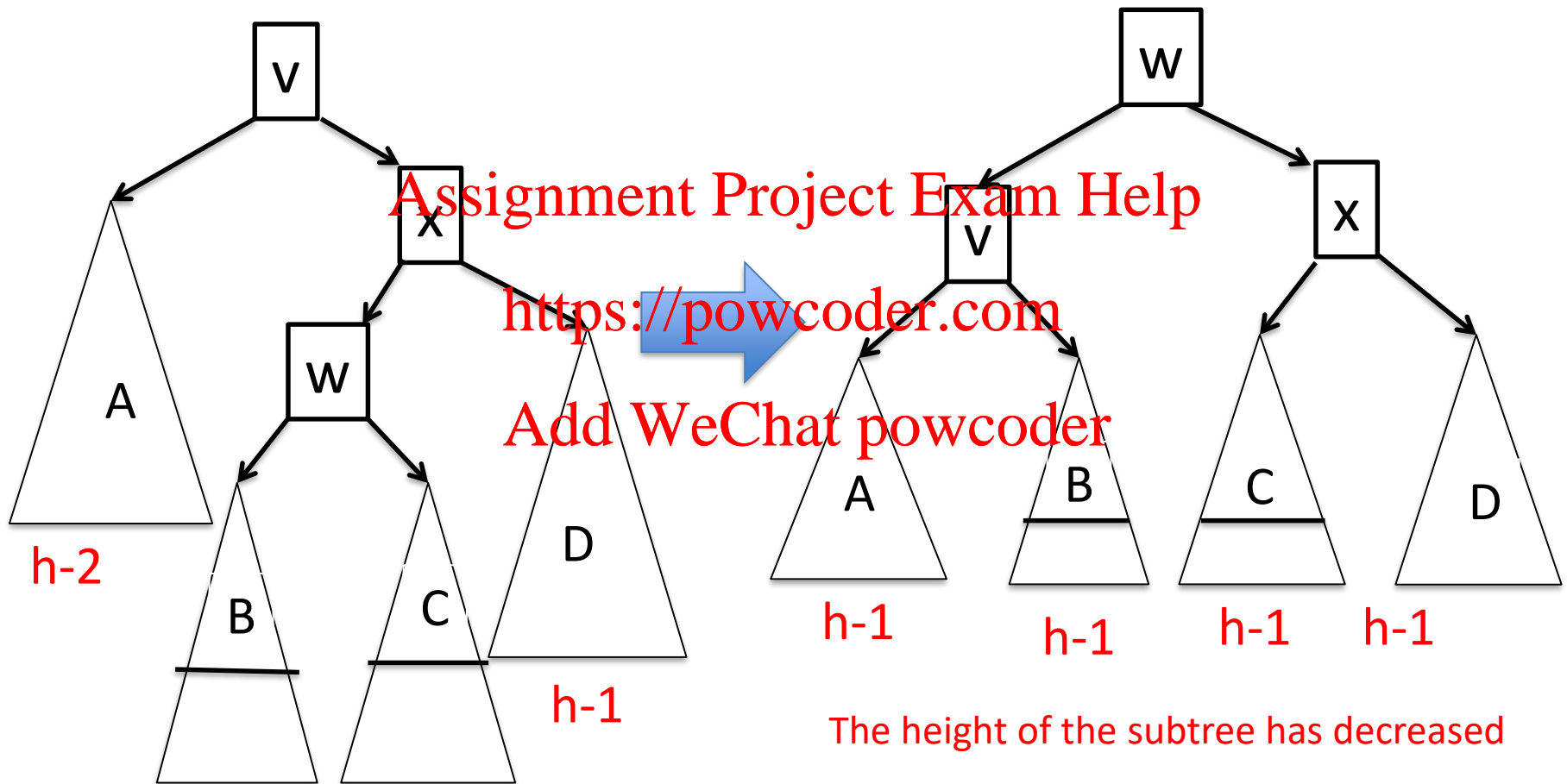
Remove – Left Rotation

W.l.o.g. assume that the deleted node was in the left subtree of v and height of this tree has decrease by 1.



If B had height $h-1$ before deletion,
the height of the subtree has decreased

Right-Left Rotation



h Either B or C might have height $h-1$

Analog: Left-Right Rotation

Rebalancing after Deletion

- After having rebalanced for node v the height of the tree previously rooted at v might have decreased after deleting and rebalancing.
- If this is the case old parent of v might be imbalanced.
- We might have to continue rebalancing until the root has been reached.

Runtime AVL-trees

Theorem: The operations find, insert, and delete can be implemented for AVL-trees in worst-case time $O(\log n)$.

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