

Data Structures and Algorithms (DSA)

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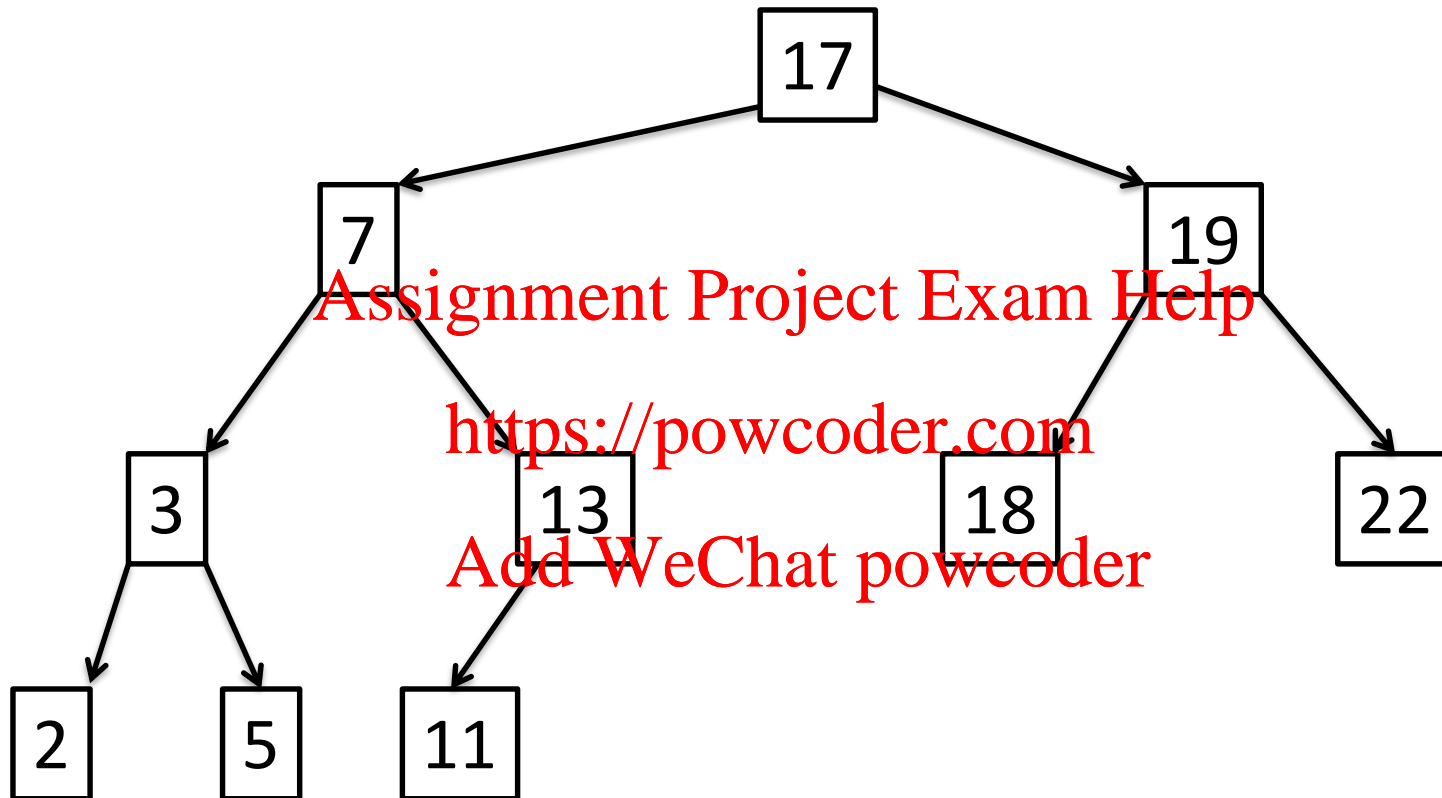
Lecture 10: Binary Search Trees

Overview

- Trees
 - Sorted Sequences
 - Binary Search Trees
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Tree



Representing Trees

- Heaps are special trees (can be stored efficiently in an array)
- Want to have a pointer based representation of trees
- Each node stores
 - an element
 - has a pointer to the left subtree (might be empty)
 - has a point to the right subtree (might be empty)

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Treeltem

Class Handle = Pointer to Treeltem

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Class Treeltem of Element

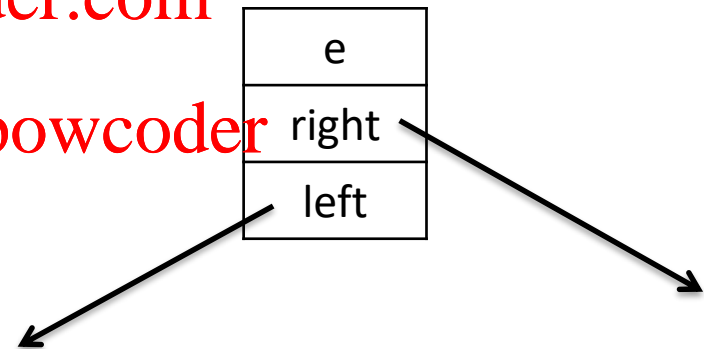
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e: Element

right: Handle

left: Handle

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Tree Traversal

- Want to visit every node in the tree (and print out the elements).

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- Recursive formulation for tree traversal

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Preorder Traversal

Preorder(Tree T)

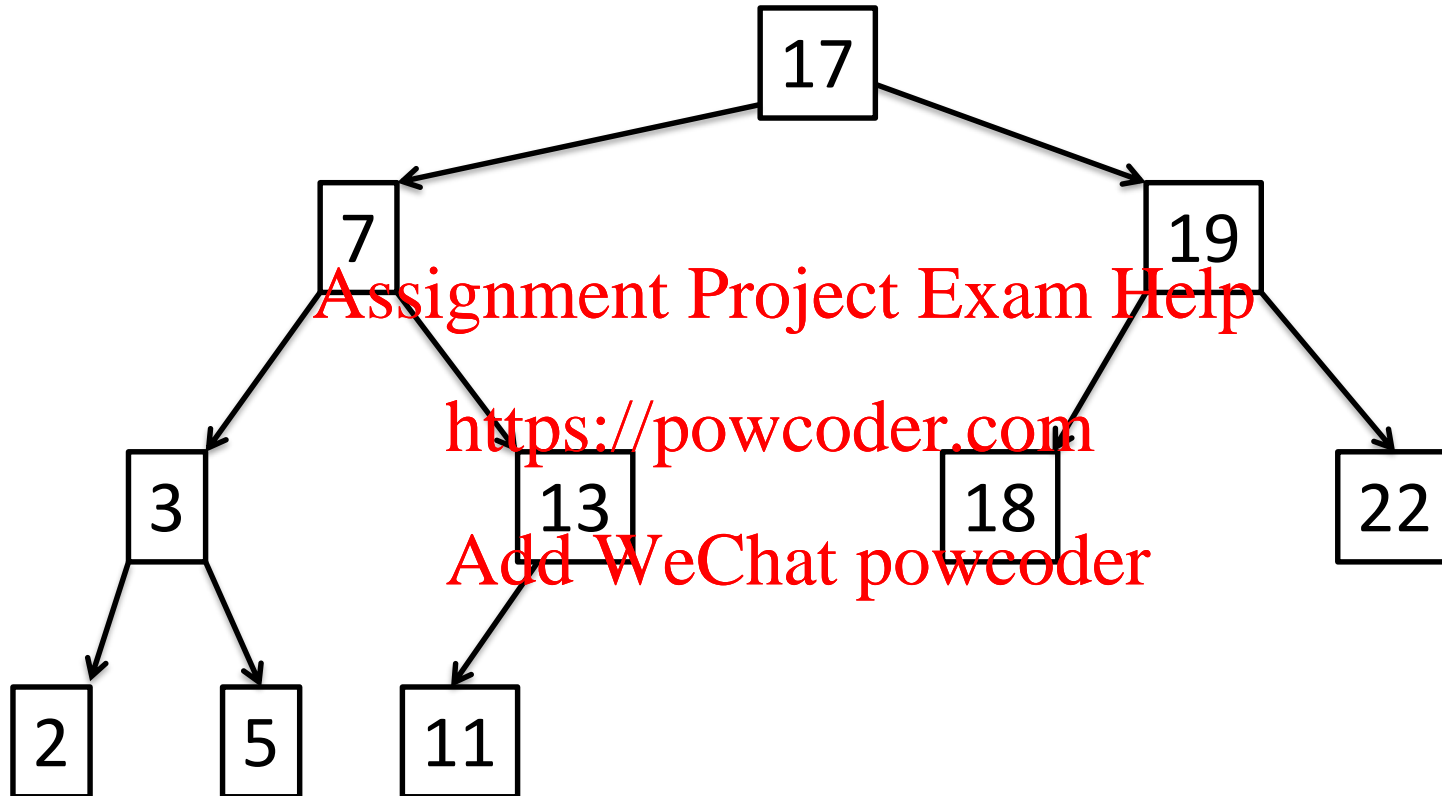
1. Visit the root (and print out the element)
2. If (T->left !=null) Preorder(T->left)
3. If (T->right !=null) Preorder(T->right)

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Preorder Traversal



Order nodes are visited: 17, 7, 3, 2, 5, 13, 11, 19, 18, 22

Postorder Traversal

Postorder(Tree T)

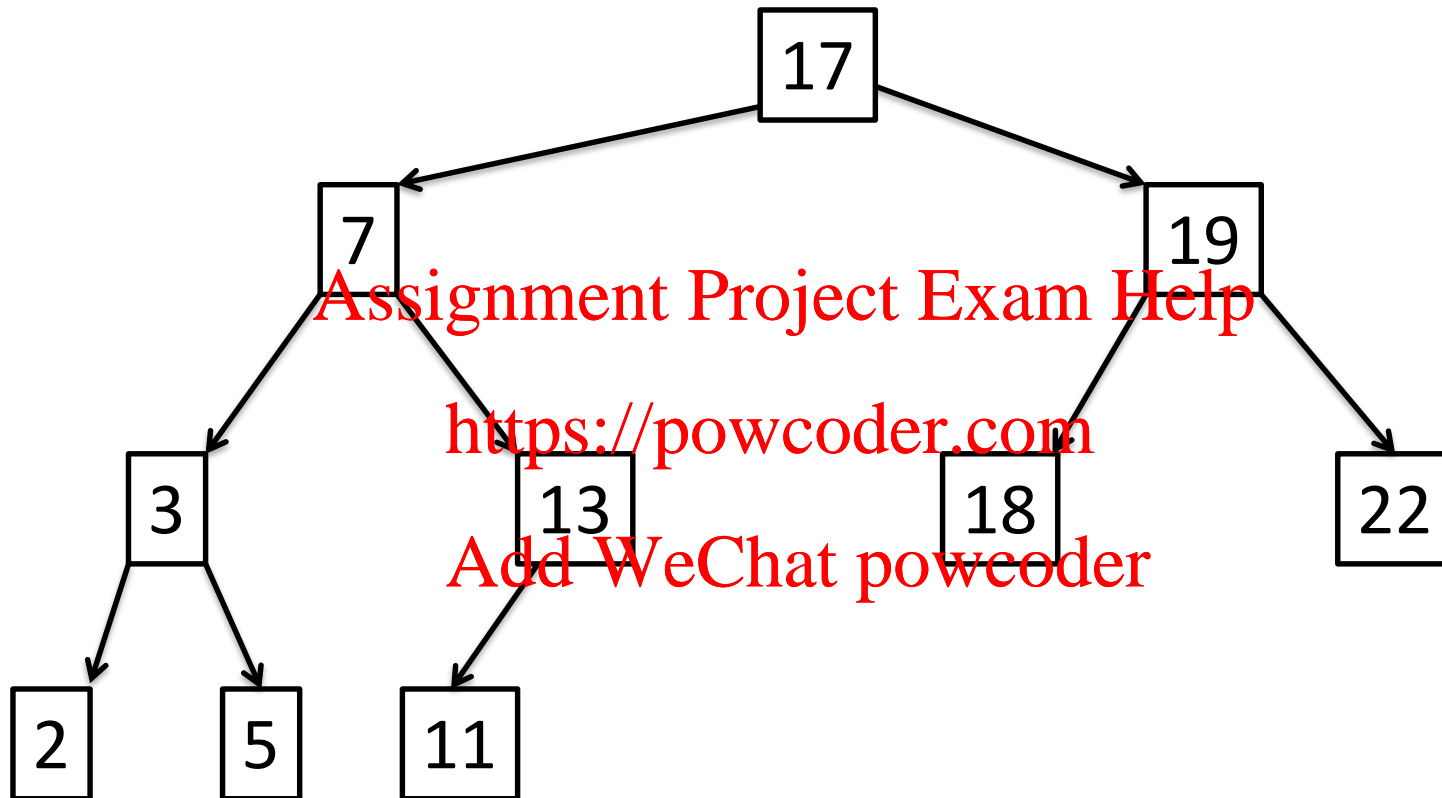
1. If (T->left !=null) Postorder(T->left)
2. If (T->right !=null) Postorder(T->right)
3. Visit the root (and print out the element)

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Postorder Traversal



Order nodes are visited: 2, 5, 3, 11, 13, 7, 18, 22, 19, 17

Inorder Traversal

Inorder(Tree T)

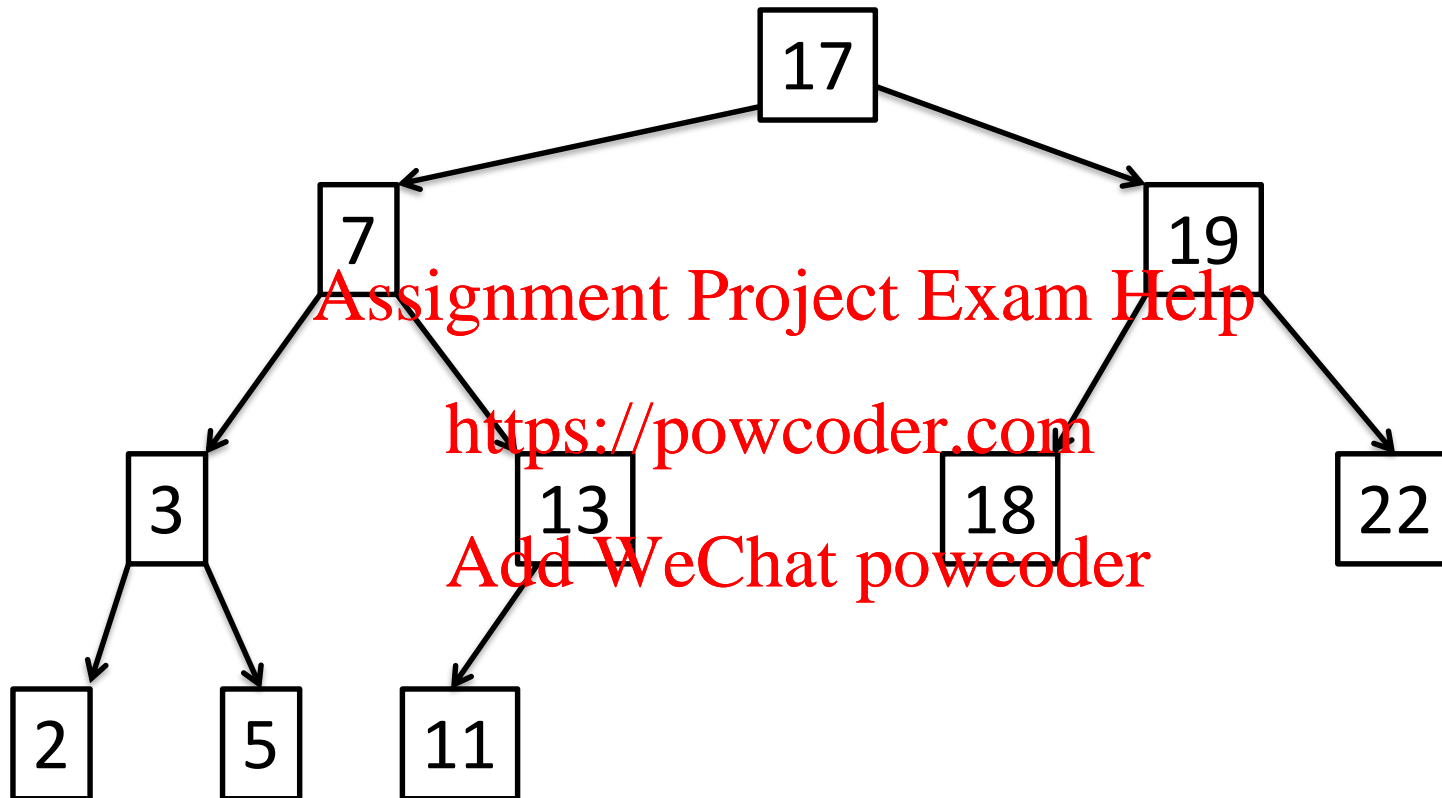
1. If (T->left != null) Inorder(T->left)
2. Visit the root (and print out the element)
3. If (T->right != null) Inorder(T->right)

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Inorder Traversal



Order nodes are visited: 2, 3, 5, 7, 11, 13, 17, 18, 19, 22

Observation: This sequence is sorted

Sorted Sequences

Operations for Sorted Sequences

- Find an element e in the sorted sequence
- Insert an element e into the sorted sequence
- Delete an element e from the sorted sequence.

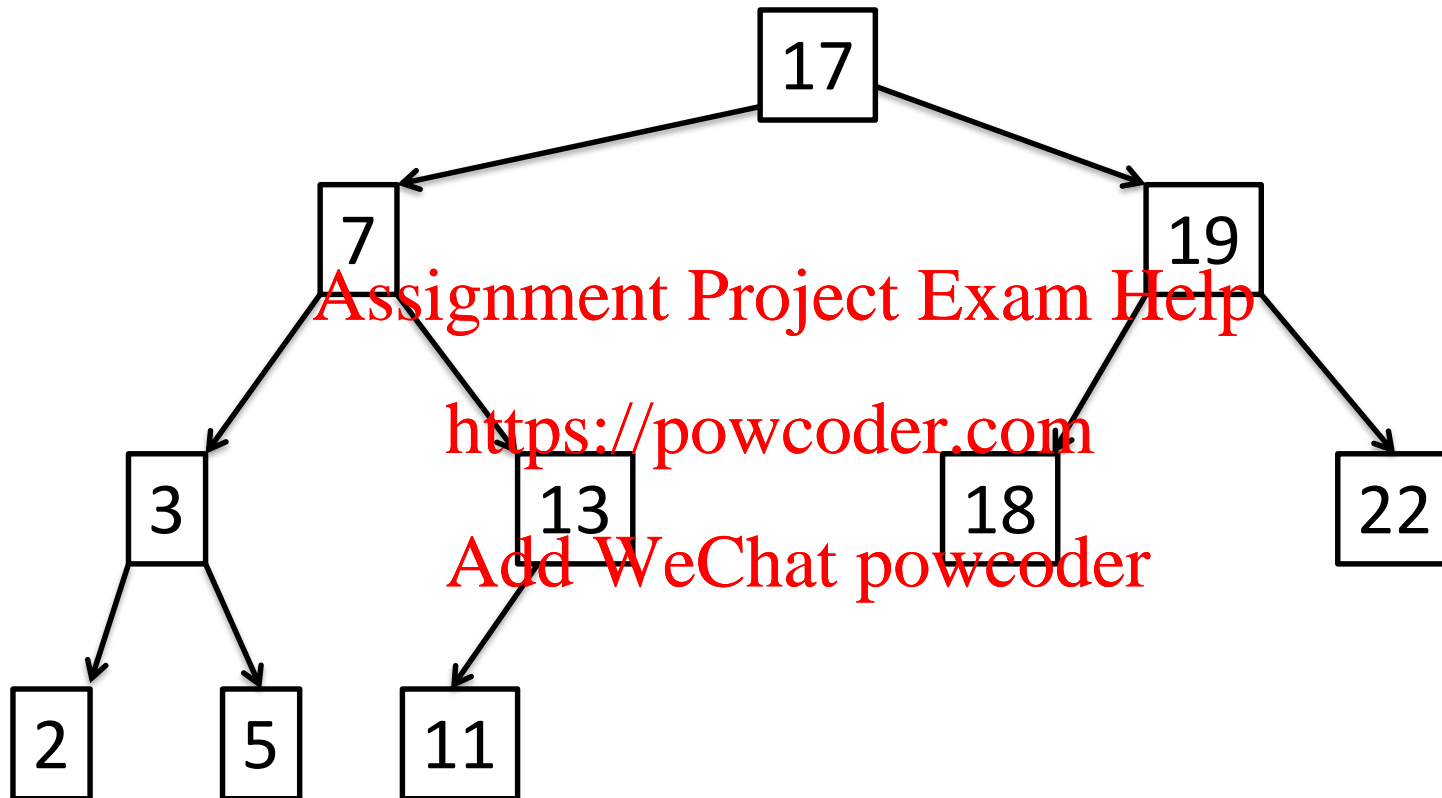
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Want to have all these operations implemented in time $O(\log n)$.

Binary Search Tree



Sorted sequence by Inorder Traversal: 2, 3, 5, 7, 11, 13, 17, 18, 19, 22

Properties of Binary Search Trees

- All elements in the left subtree of a node k have value smaller than k .
- All elements in the right subtree of a node k have value larger than k .

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Method find

find(k)

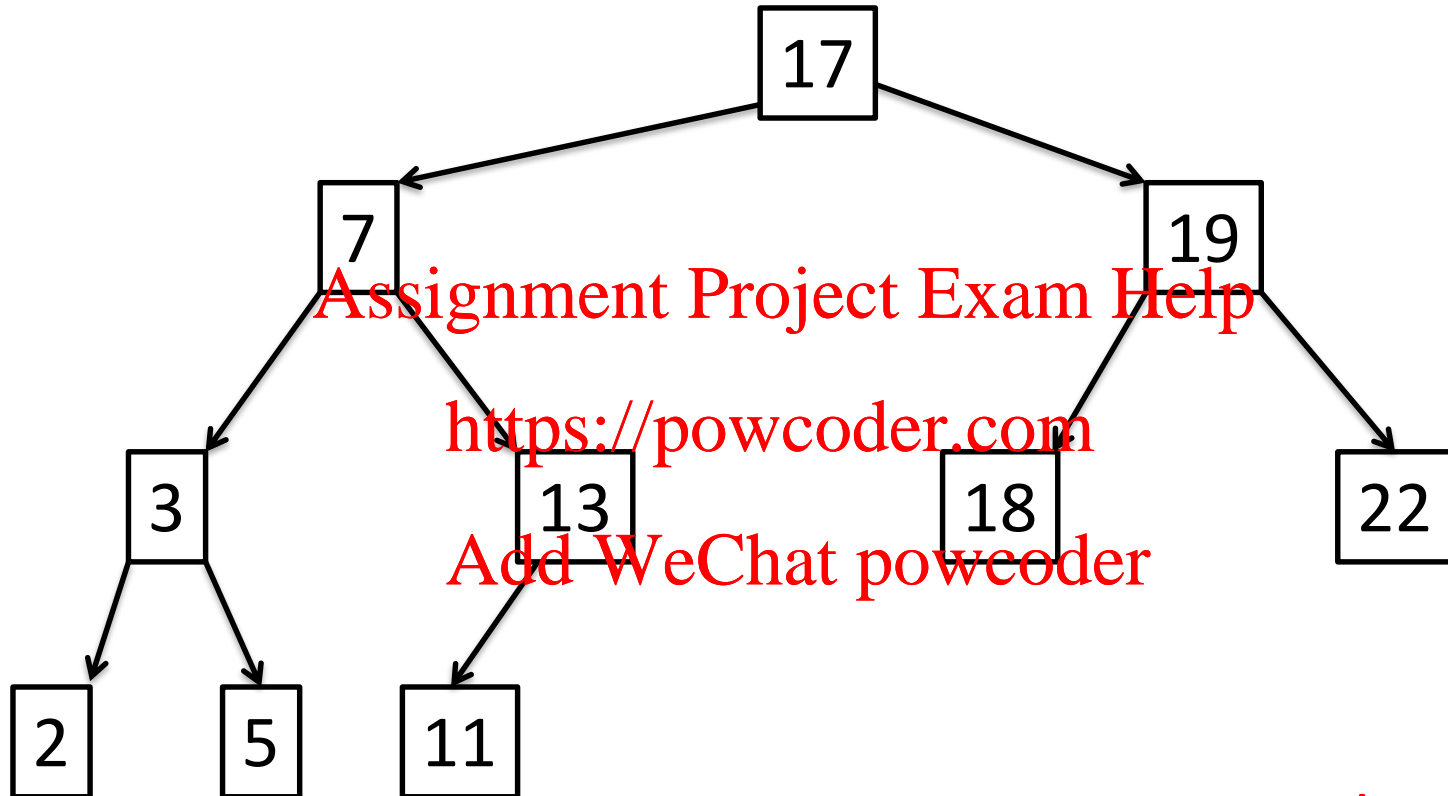
- Start at the root.
- At a node x, compare x and k.
 1. If $k=x$, then found
 2. If $k < x$, search in the left subtree of x. If subtree does not exist return not found.
 3. If $k > x$, search in the right subtree of x. If subtree does not exist, return not found

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Find



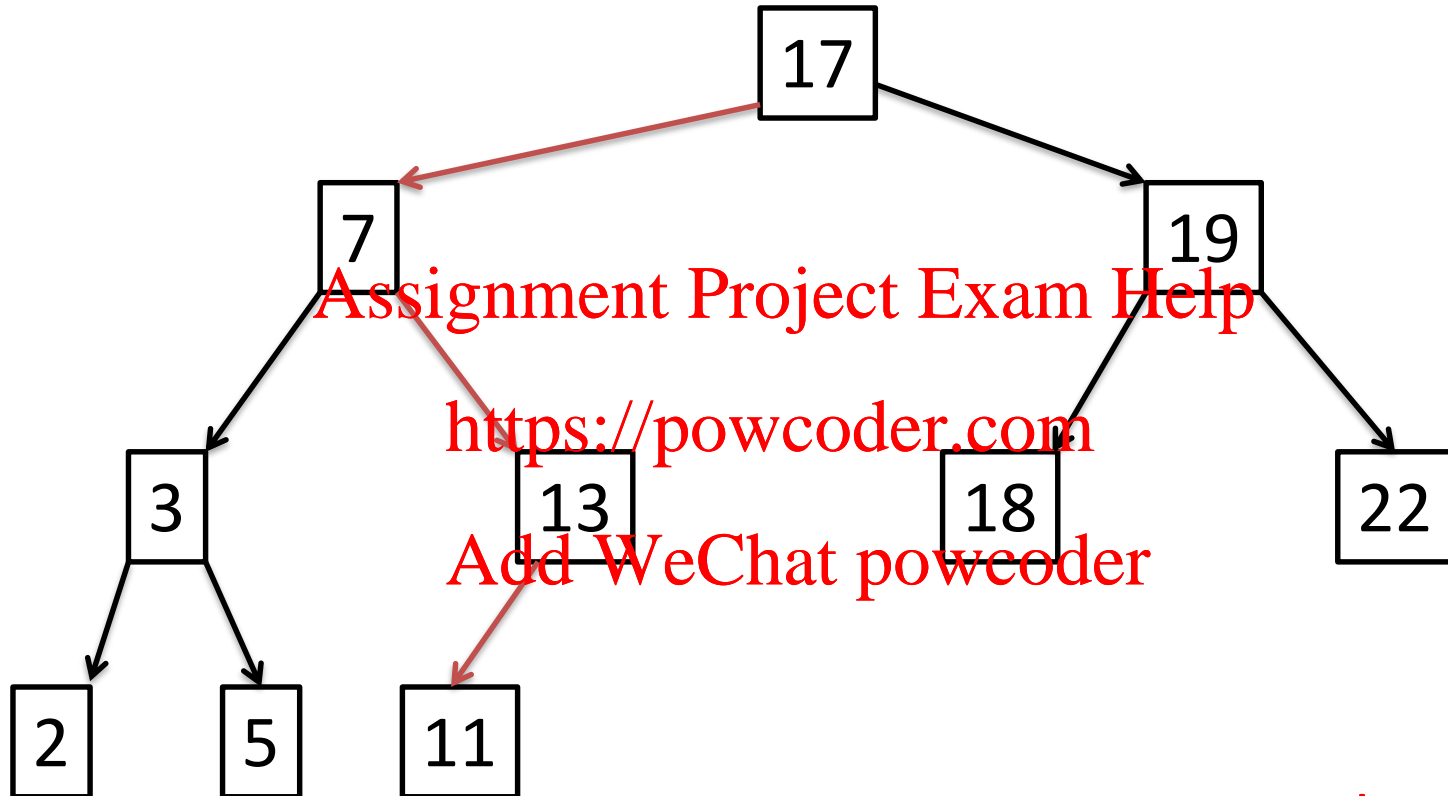
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Find 11

Find



Find 11

Insertion

insert(k)

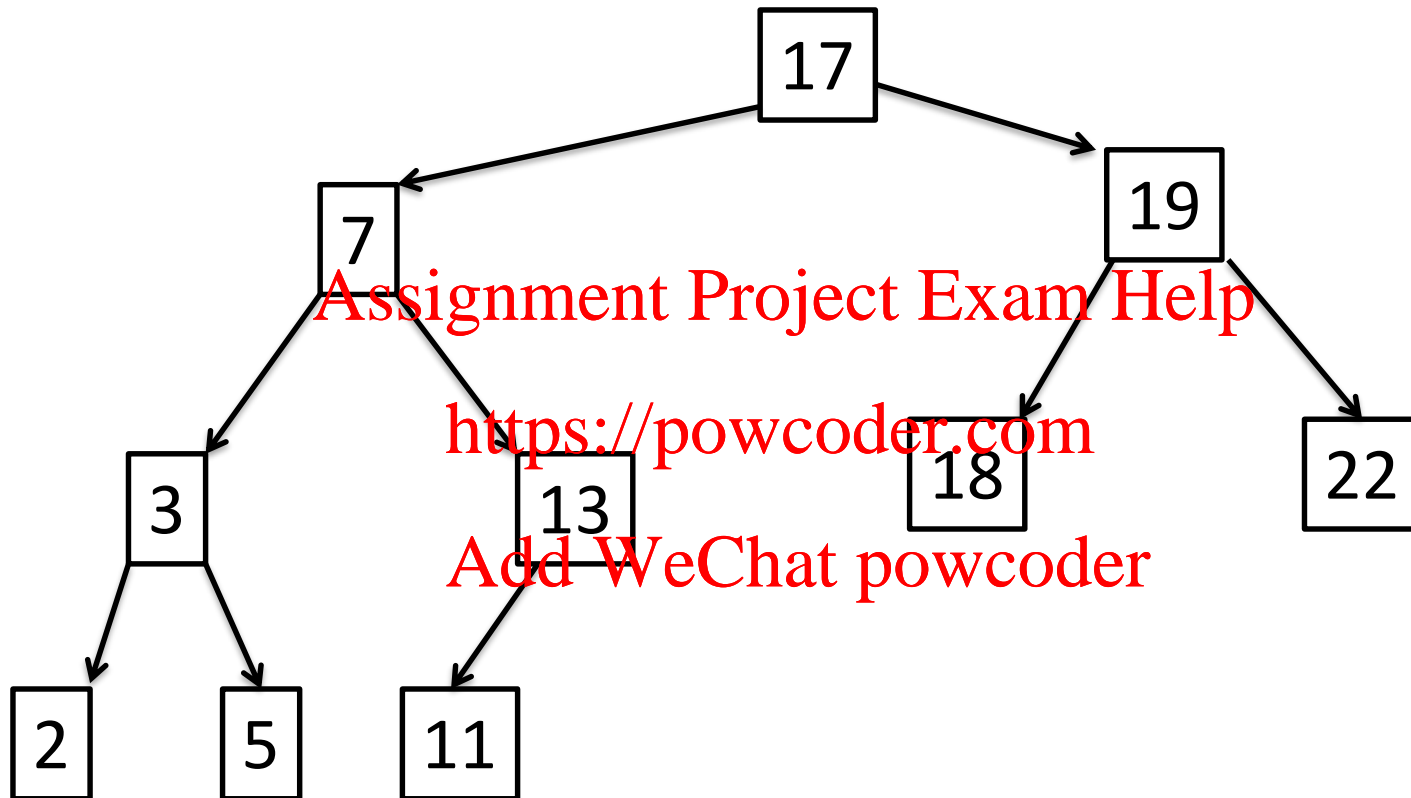
1. find(k)
2. If not found, element with key k becomes child of the last visited node of find(k).

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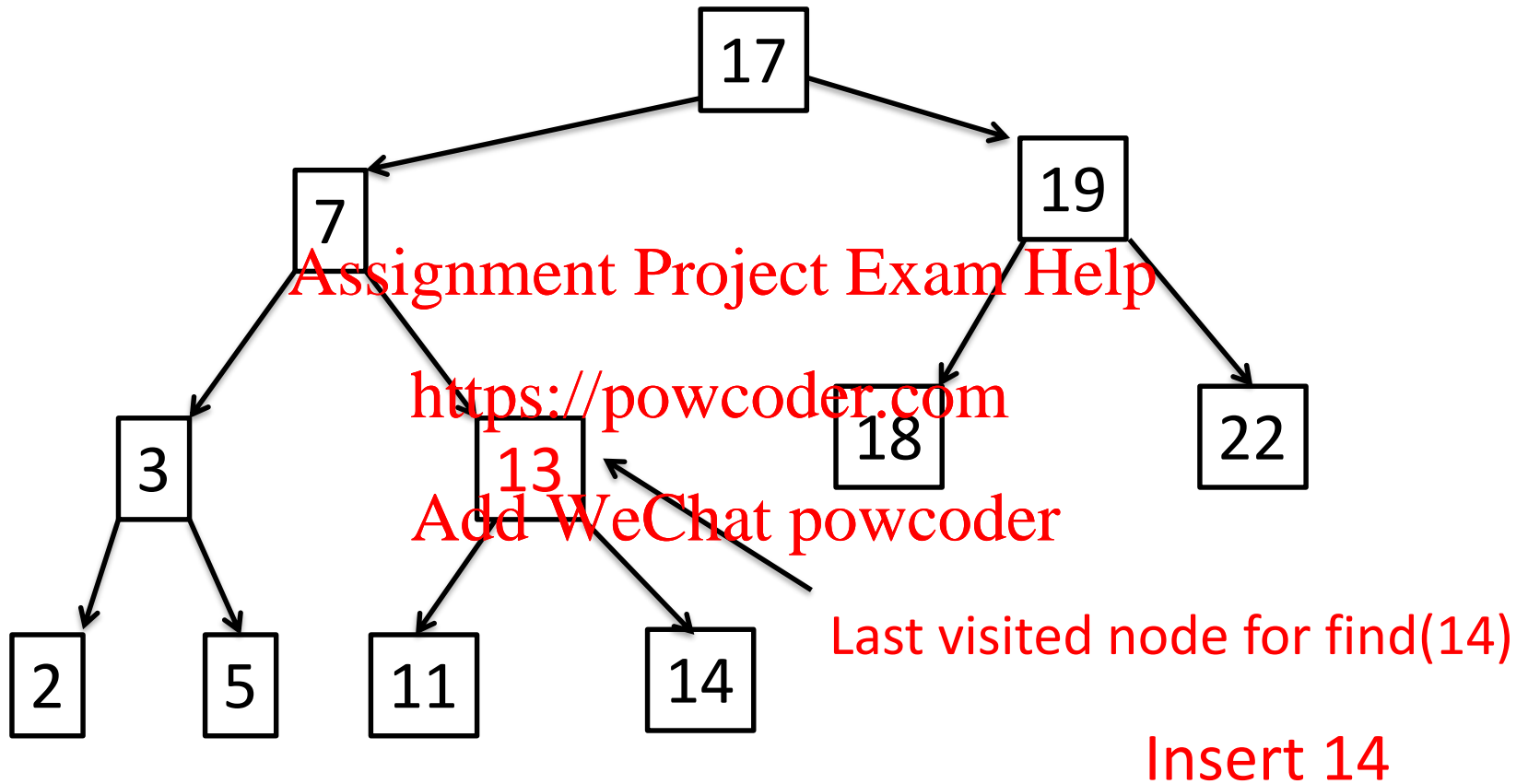
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Insert



Insert 14

Insert



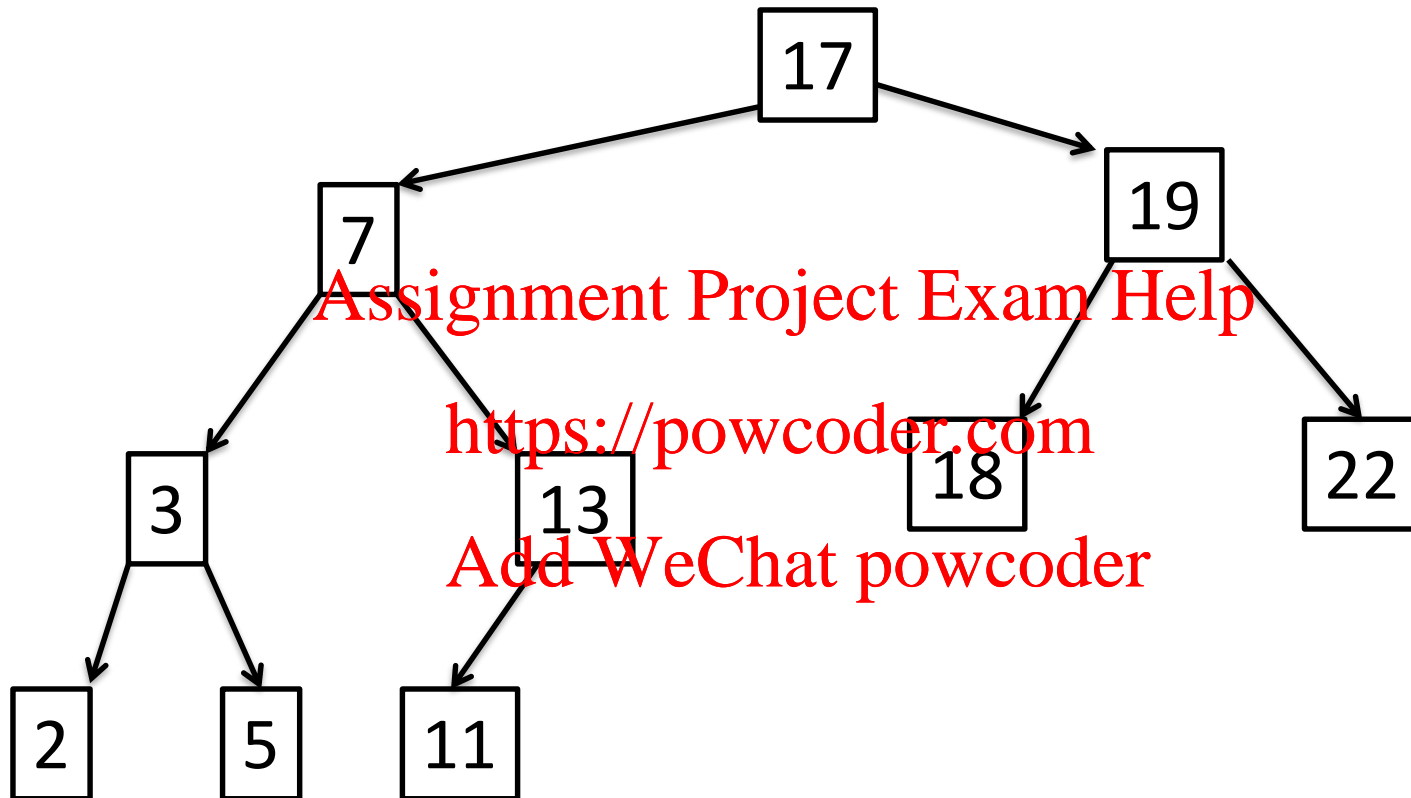
Remove

remove(k)

1. find(k)

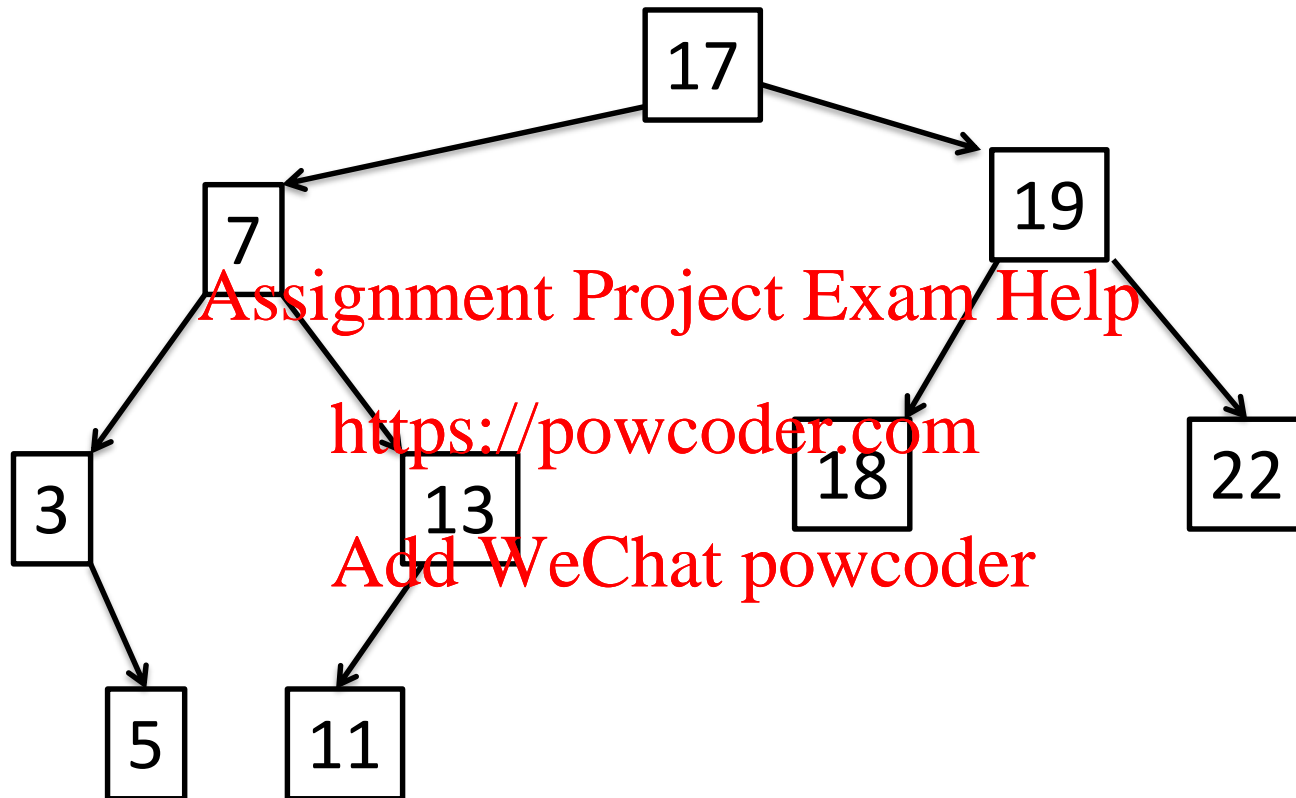
- If k is stored at a leaf, delete this leaf and the incoming edge.
- If k has one child x, redirect pointer pointing to k to x and delete k.
- If k has two children
 - search in the tree for the largest element x smaller than k.
 - swap x and k and delete(k)

Remove leaf



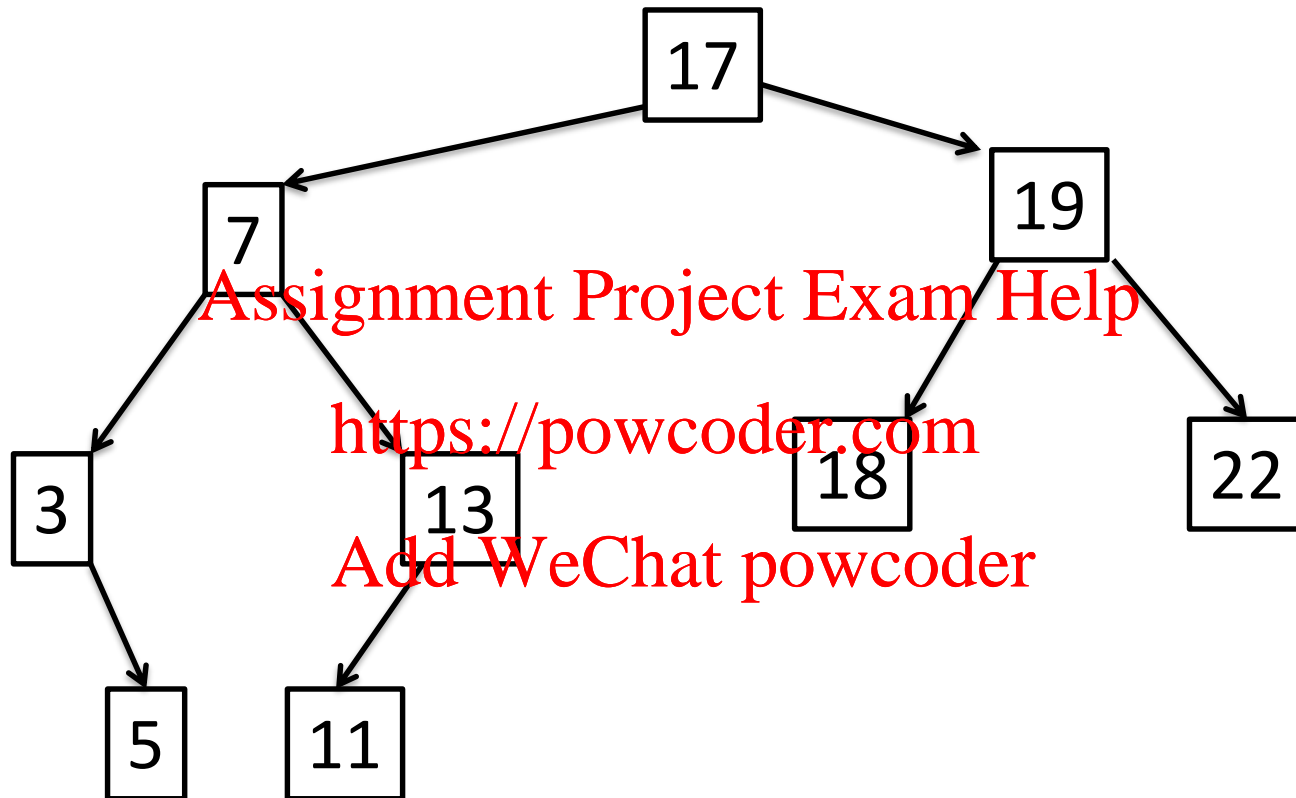
Remove 2

Remove leaf



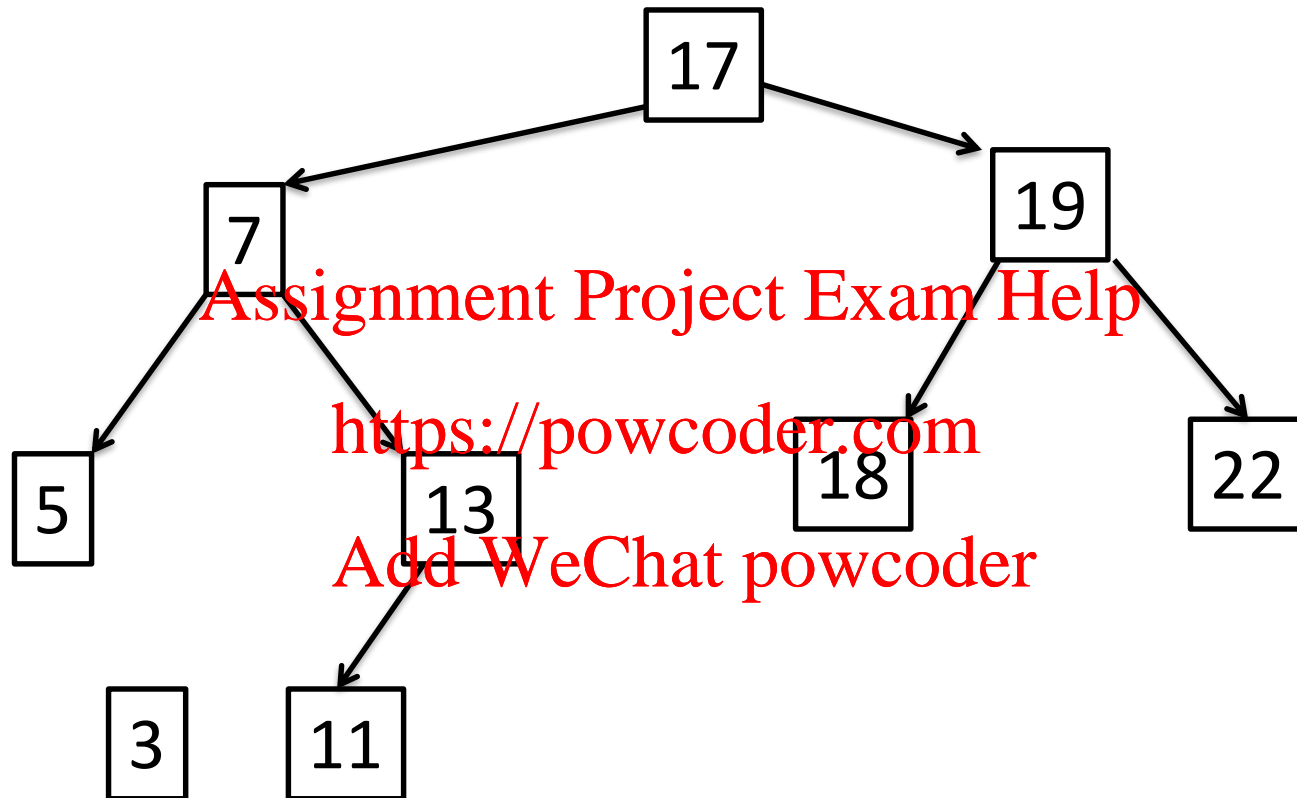
Remove 2

Remove node with 1 child



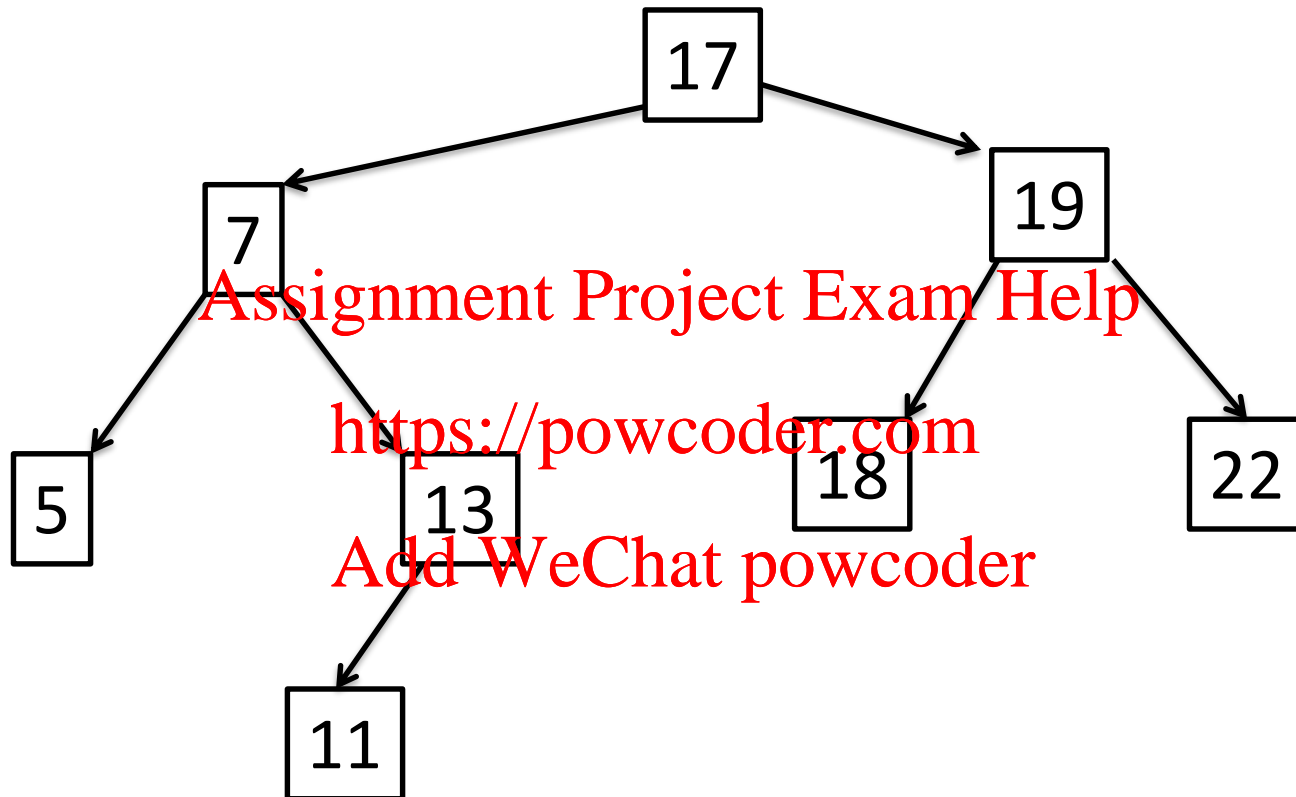
Remove 3

Remove node with 1 child



Remove 3

Remove node with 1 child



Remove 3

Remove node with 2 children

How to find the largest element smaller than k?

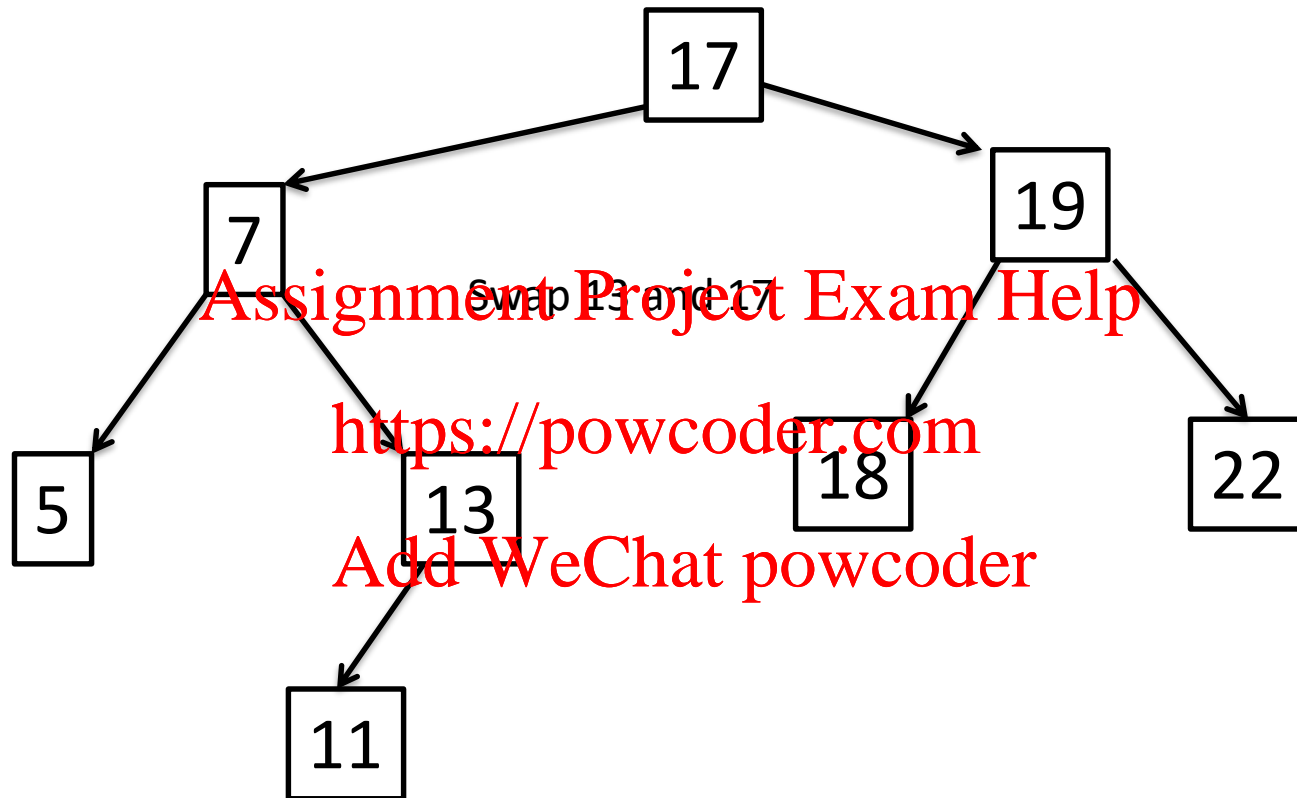
- Go to the left child of k (has to exist as k has two children.
- Follow the pointer to the right as long as possible.

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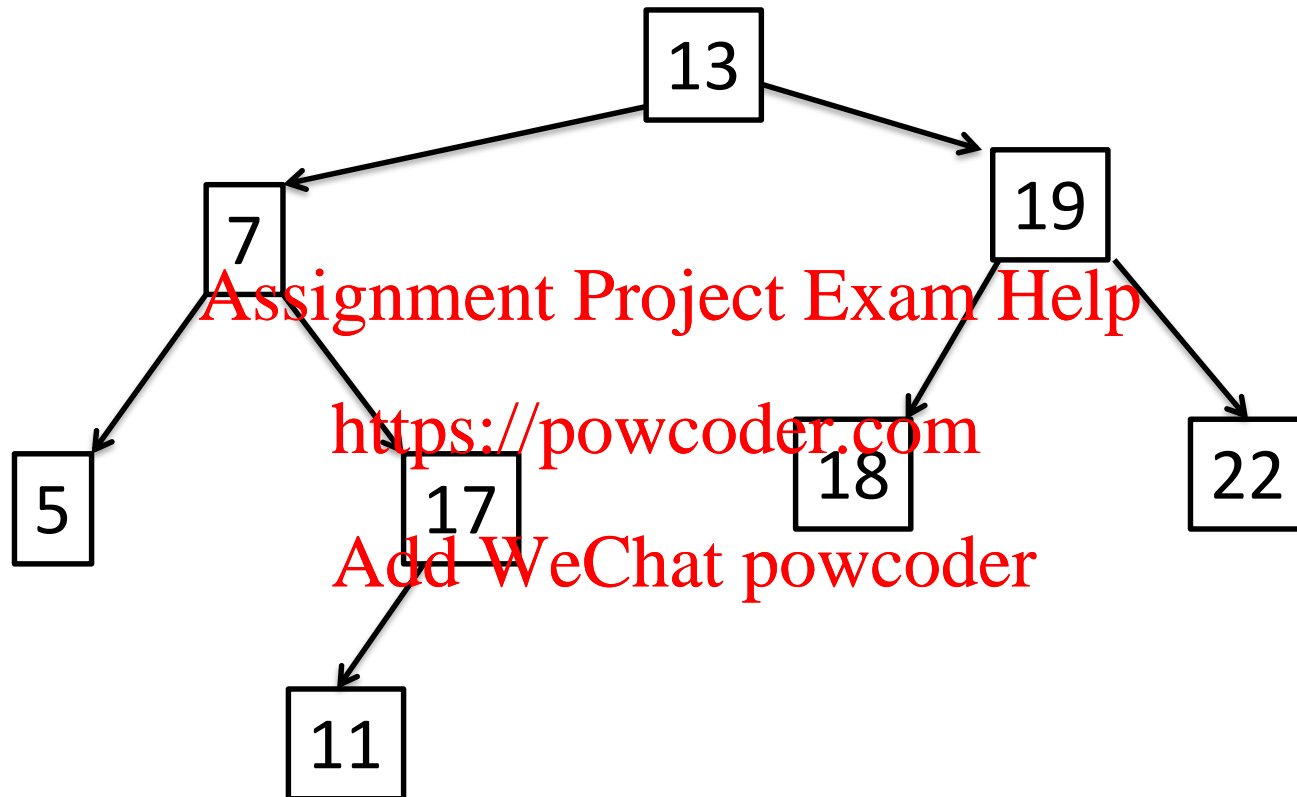
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Remove node with 2 children



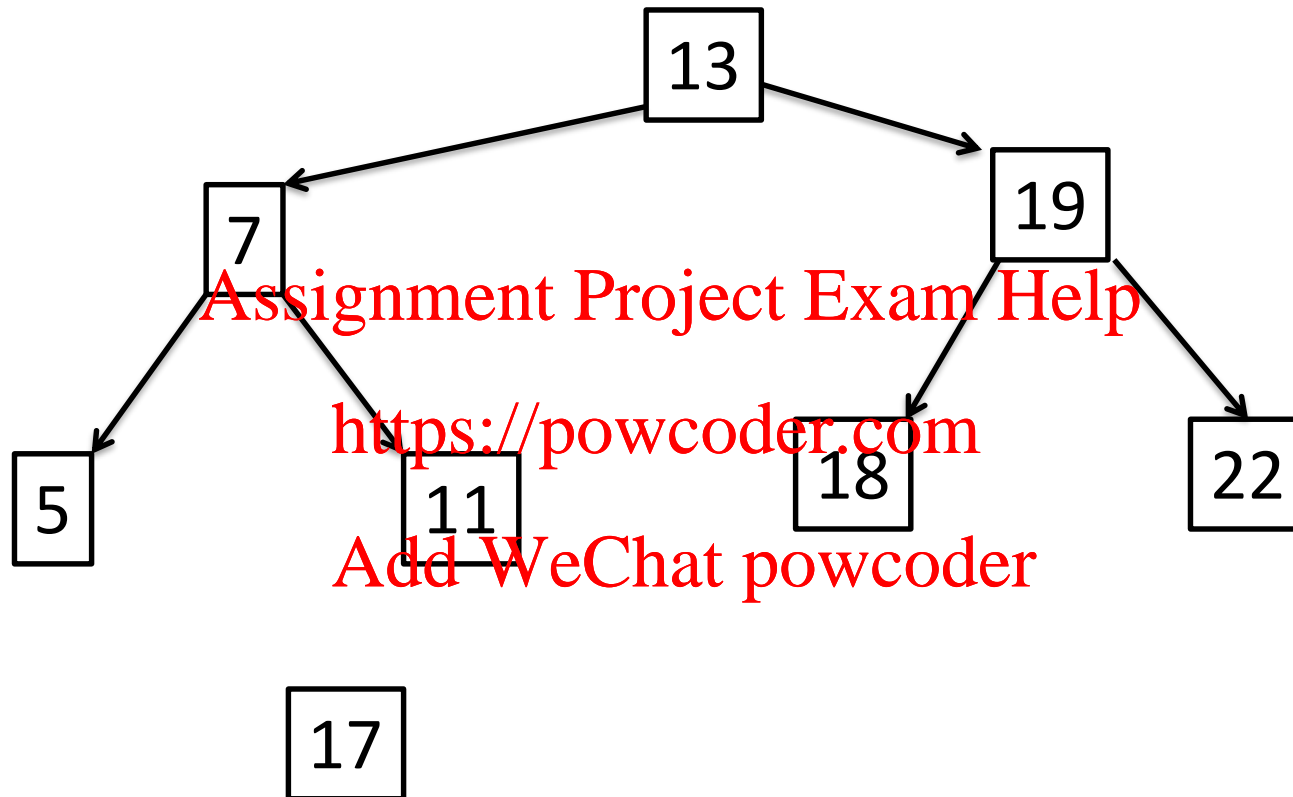
Remove 17

Remove node with 2 children

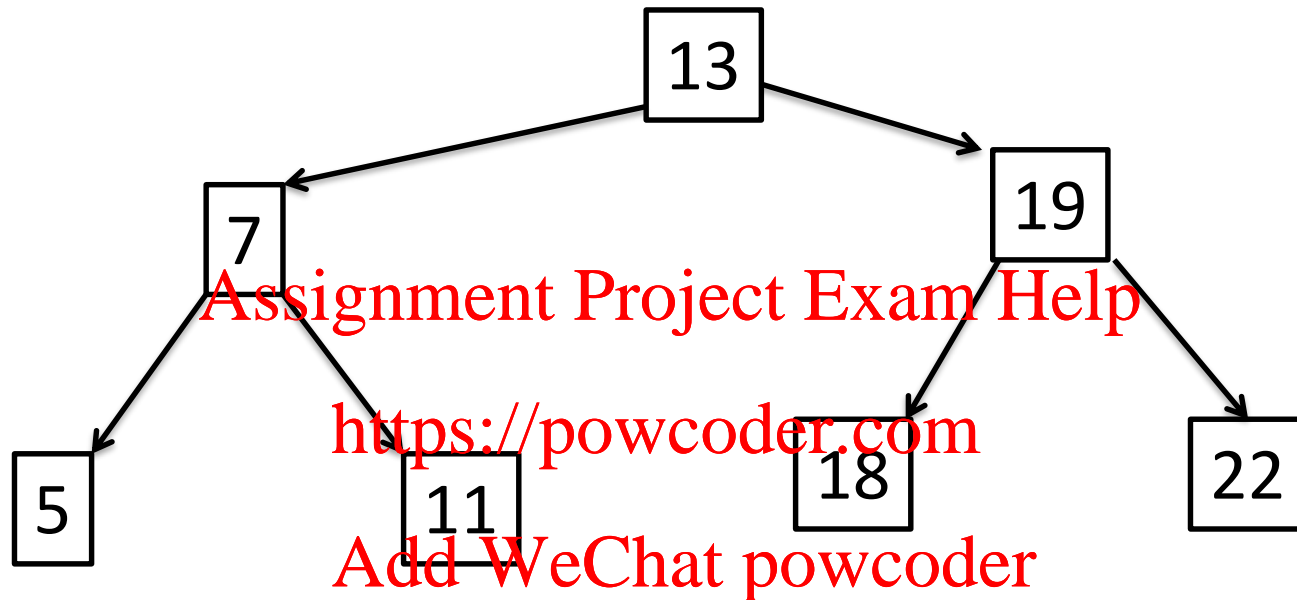


Remove 17

Remove node with 2 children



Remove node with 2 children



Remove 17

Perfectly Balanced Binary Search Trees

- A binary search tree is perfectly balanced if it has height $\lfloor \log n \rfloor$ (height is the length of the longest path from the root to a leaf)

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Insertion

19

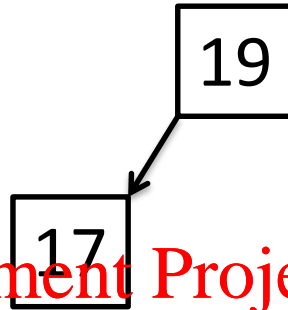
Insert 17

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Insertion



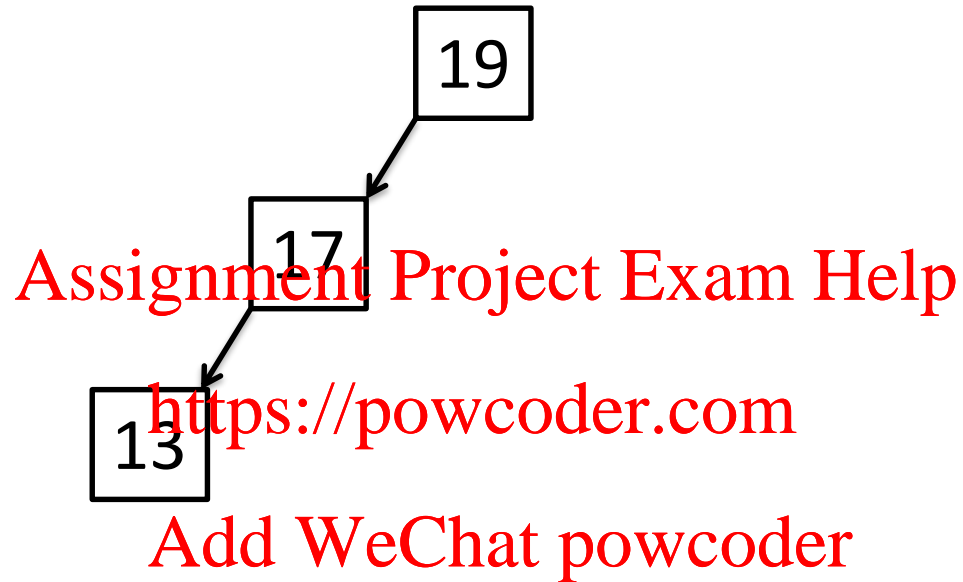
Insert 13

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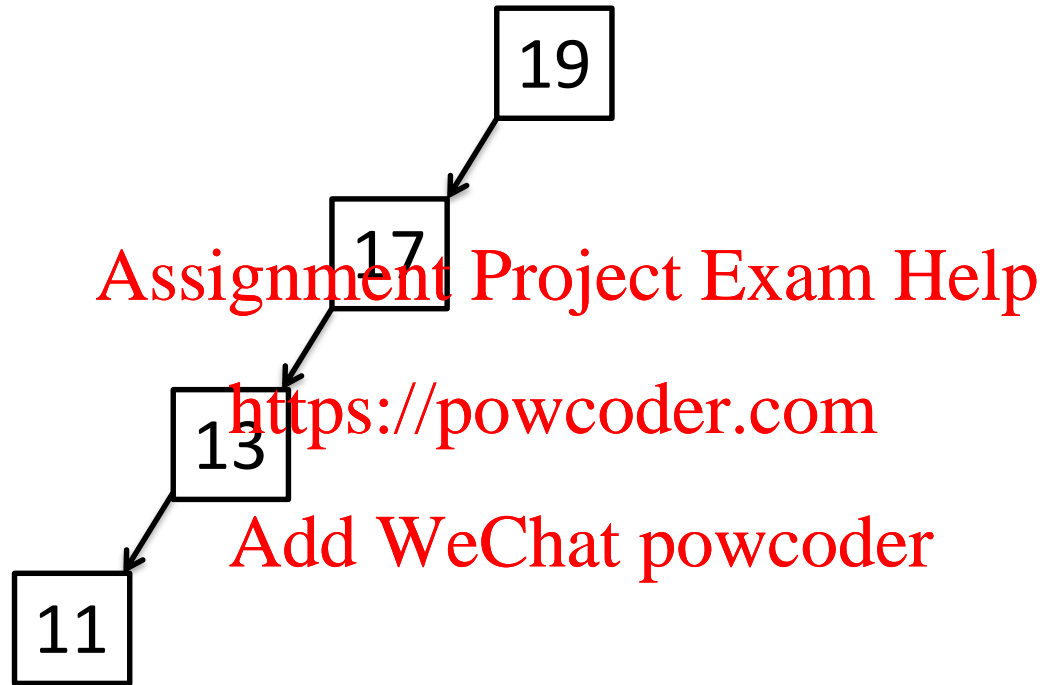
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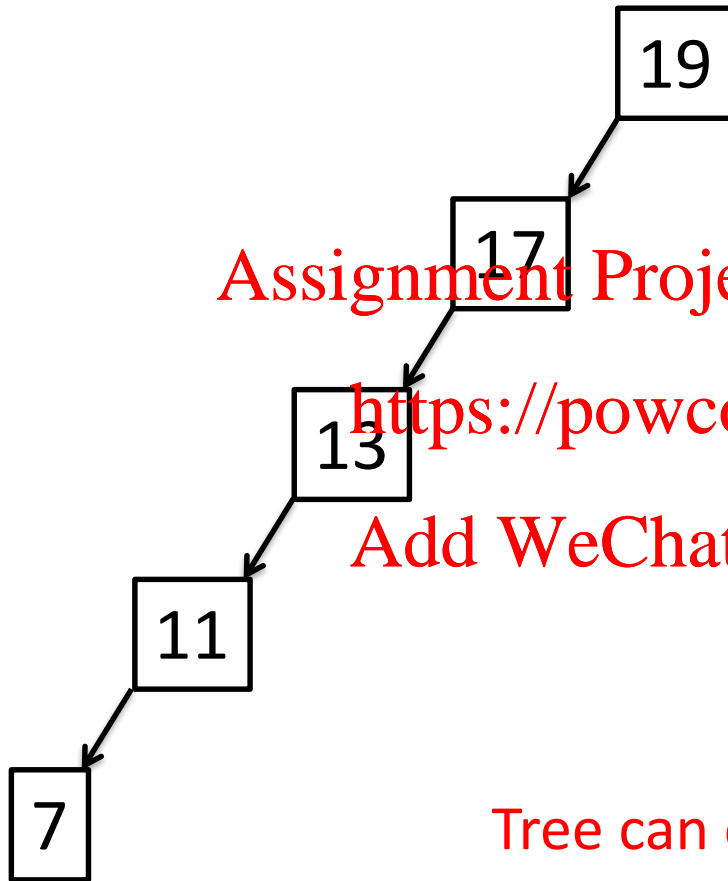
Insertion



Insertion



Insertion



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Problem?

Tree can degenerate to a list.

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