Week 5 Workshop

Alice: Your model reduces the most interesting information to something flat and boring.

Vittorio: You're right, and this causes a lot of problems.

Sergio: Designing the schema for a complex application is tough, and it is easy to make mistakes when updating a database.

Riccardo: Also, the system knows so little about the data that it is hard to obtain good performance.

Alice: Are you telling me that the model is bad?

Vittorio: No, wait, we are going to fix it!

(Foundations of Databases, S. Abiteboul, R. Hull, V. Vianu, Addison-Wesley, 1995)

Housekeeping

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2. **More consultation hours in Week 6**
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Update Anomalies

- **What could happen to insert, delete and update operations?**

ENROLMENT					
Name	StudentID	DoB	CourseNo	Semester	Unit
Tom	123456	25/01/1989	COMP2400	2010 S2	6
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Australian National University

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- **Insertion anomalies**: If inserting a new course COMP3000, then ... (i.e., cannot insert NULL values into Course because of the entity integrity constraint).
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- **Insertion anomalies**: If inserting a new course COMP3000, then ... (i.e., cannot insert NULL values into Course because of the entity integrity constraint).
- **Deletion anomalies**: If deleting the enrolled course COMP2400 of Fran, then ... the personal information of Fran, such as DoB, will be lost as well.
- **Modification anomalies**: If changing the DoB of Michael, then ...

Update Anomalies

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- **Insertion anomalies**: If inserting a new course COMP3000, then ... (i.e., cannot insert NULL values into Course because of the entity integrity constraint).
- **Deletion anomalies**: If deleting the enrolled course COMP2400 of Fran, then ... the personal information of Fran, such as DoB, will be lost as well.
- **Modification anomalies**: If changing the DoB of Michael, then ... update every tuple that records the DoB of this student.

Update Anomalies?

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⇓

STUDENT

Name	StudentID	DoB
Tom	123456	25/01/1988
Michael	123458	21/04/1985
Fran	123457	11/09/1987

COURSE

CourseNo	Unit
COMP2400	6
COMP8740	12

ENROL

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123456	COMP8740	2011 S2
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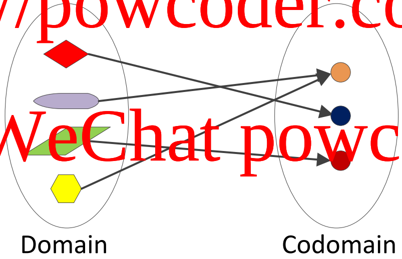
Why Functional Dependencies?

FDs tell us "relationship between and among attributes"!

- FDs are developed to define the **goodness** and **badness** of (relational) database design in a formal way.
  - **Top down**: start with a relation schema and FDs, and produce smaller relation schemas in certain normal form (called *normalisation*).
  - **Bottom up**: start with attributes and FDs, and produce relation schemas (*not popular in practice*).

What is "Functional" about Functional Dependencies?

- The notion of functional dependency is very close to the notion of function.
- A (total) **function** $f : X \rightarrow Y$ describes a relationship between two sets X and Y such that each element of X is mapped to a unique element of Y.



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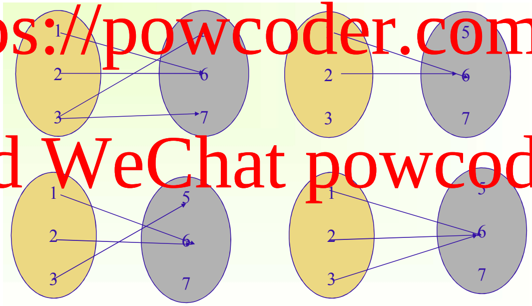
- **Exercise:** *which of them represent a function?*



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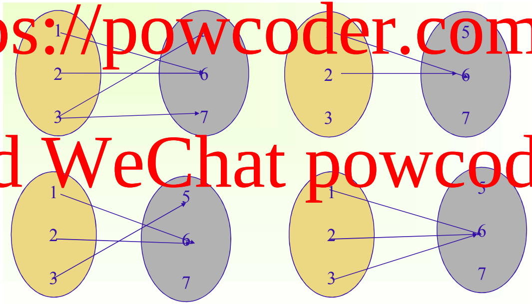
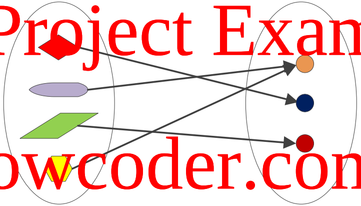


Answer: **The ones at the bottom.**

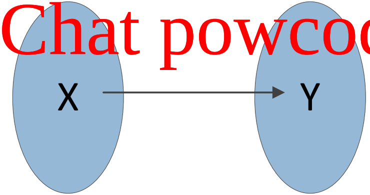
Functions vs Functional Dependencies

Function



Domain Codomain

Functional dependency



Determinant Dependant

Functions vs Functional Dependencies

$$f(x) = x^2$$

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x	$f(x)$
1	1
2	4
3	9
4	16
5	25
6	36
...	...

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$$X \rightarrow f(x)$$

Formal Definition

- Let R be a relation schema.
  - A **FD** on R is an expression $X \rightarrow Y$ with attribute sets $X, Y \subseteq R$.
  - A relation $r(R)$ **satisfies $X \rightarrow Y$ on R** if, for any two tuples $t_1, t_2 \in r(R)$, whenever the tuples t_1 and t_2 coincide on values of X, they also coincide on values of Y.

$$t_1[X] = t_2[X]$$
$$\Downarrow$$
$$t_1[Y] = t_2[Y]$$

- A FD is **trivial** if it can *always* be satisfied, e.g.,
  - $\{A, B\} \rightarrow \{A\}$
  - $\{A, B, C\} \rightarrow \{A, B, C\}$

- **Syntactical convention**: (1) Instead of $\{A, B, C\}$, we may use ABC. (2) $A, B, \ldots$ for individual attributes and $X, Y, \ldots$ for sets of attributes.

Exercise - Functional Dependencies

- A functional dependency specifies a constraint on the relation schema that must hold **at all times**.
- Consider the following relation with attributes {A,B,C,D,E}. Do they satisfy the given FDs?

r(*R*)

A	B	C	D	E
1	2	3	4	5
1	2	2	2	2
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3. $E \rightarrow ABCD$

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How to Identify FDs in General?

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  Can be provided in the form of discussion with application users and/or data requirement specifications

How to Identify FDs in General?

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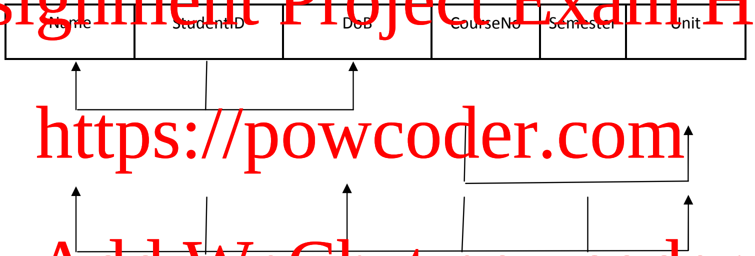
- In real-life applications, we often use the following approaches:

  (1) **Analyse data requirements**
  Can be provided in the form of discussion with application users and/or data requirement specifications.

  (2) **Analyse sample data**
  Useful when application users are unavailable for consultation and/or the document is incomplete.

(1) Analyse Data Requirements and FD Diagram



- StudentID $\rightarrow$ Name, DoB;
- CourseNo $\rightarrow$ Unit;
- StudentID, CourseNo, Semester $\rightarrow$ Name, DoB, Unit.

(2) Analyse Sample Data

- Can you find some FDs on ENROLMENT based on the sample data?

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- We may have:
  - $\{StudentID\} \rightarrow \{Name, DoB\}$;

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- We may have:
  - $\{$StudentID$\} \rightarrow \{$Name, DoB$\}$;
  - $\{$StudentID, Name$\} \rightarrow \{$DoB$\}$;

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- We may have:
  - {StudentID} → {Name, DoB};
  - {StudentID, Name} → {DoB};
  - {Name} → {StudentID} ×;
  - ......

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- We may have:
  - {StudentID} $\rightarrow$ {Name, DoB};
  - {StudentID, Name} $\rightarrow$ {DoB};
  - {Name} $\rightarrow$ {StudentID} $\times$;
  - ......

Limitations:
(1) Sample data needs to be a true representation of **all possible values** in the database.
(2) Do we need all FDs?

Inference?

- To design a good database, we need to consider **all possible FDs**.

Example:

If $\{StudentID\} \rightarrow \{ProjectNo\}$ and $\{ProjectNo\} \rightarrow \{Supervisor\}$, we can infer $\{StudentID\} \rightarrow \{Supervisor\}$.

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If $\{StudentID\} \rightarrow \{ProjectNo\}$ and $\{ProjectNo\} \rightarrow \{Supervisor\}$, we can infer $\{StudentID\} \rightarrow \{Supervisor\}$.

If each student works on one project and each project has one supervisor, then each student must have one project supervisor.

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  If each student works on one project and each project has one supervisor, then each student must have one project supervisor.

- **Can we systematically infer all possible FDs?**

Armstrong's Inference Rules (Slides 16-25 will not to be assessed)

- The **Armstrong's inference rules** consist of the following three rules:

  - **Reflexive rule**: $XY \rightarrow Y$
  - **Augmentation rule**: $\{X \rightarrow Y\} \models XZ \rightarrow YZ$

  - **Transitive rule**: $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$

- We use the notation $\Sigma \models X \rightarrow Y$ to denote that $X \rightarrow Y$ is **inferred** from the set Σ of functional dependencies.

Rule 1 – Reflexive Rule

- $XY \rightarrow Y$.

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- **Example**:
  {StudentID, CourseNo, Semester} → {CourseNo, Semester}, where
    - X={StudentID};
    - Y={CourseNo, Semester}.

Rule 2 – Augmentation Rule

- $\{X \rightarrow Y\} \models XZ \rightarrow YZ$

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- **Example**:
$\{\{\text{CourseNo}\} \rightarrow \{\text{Unit}\}\} \models \{\text{CourseNo, Semester}\} \rightarrow \{\text{Unit, Semester}\}$,
where
  - X={CourseNo};
  - Y={Unit};
  - Z={Semester}.

Rule 3 – Transitive Rule

- $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$.

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- **Example**: {StudentID, CourseNo} $\rightarrow$ {CourseNo}, {CourseNo} $\rightarrow$ {Unit} $\models$ {StudentID, CourseNo} $\rightarrow$ {Unit}, where
  - X={StudentID, CourseNo};
  - Y={CourseNo};
  - Z={Unit}.

Other Derived Rules

- From Armstrong's axioms (i.e. reflexive, augmentation, transitive rules), we can derive the following rules:

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  - **Union rule**: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
  - **Example:** If StudentID $\rightarrow$ Name and StudentID $\rightarrow$ DoB hold, then we have StudentID $\rightarrow$ Name, DoB, where

    - X=StudentID;
    - Y=Name;
    - Z=DoB.

Other Derived Rules

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  - **Decomposition rule**: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$
  - **Example:** If StudentID $\rightarrow$ Name, DoB holds, then we have StudentID $\rightarrow$ Name and StudentID $\rightarrow$ DoB, where
    - X=StudentID;
    - Y=Name;
    - Z=DoB.

Example on Armstrong's Inference Rules

- If each student works on one project and each project has one supervisor, does each student have one project supervisor?

$$\{\{\text{StudentID}\}\rightarrow\{\text{ProjectNo}\},\ \{\text{ProjectNo}\}\rightarrow\{\text{Supervisor}\}\} \models \{\text{StudentID}\}\rightarrow\{\text{Supervisor}\}$$

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- This can be proven by using the Transitive rule:

$$\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$$

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No. See the counter-example below:

X	Y
0	2
1	2

Armstrong's Inference Rules

- **Two questions:**

[1] William Ward Armstrong: Dependency Structures of Data Base Relationships, page 580-583. IFIP Congress, 1974.

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    ⇝ **completeness** (you can prove anything that is right)

- **Theorem** (W. W. Armstrong, 1974[1])
  - The Armstrong's inference rules are both **sound** and **complete**.

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Implied Functional Dependencies

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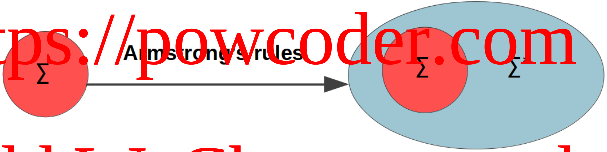

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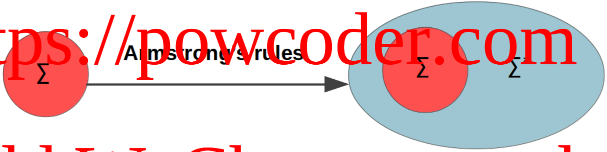

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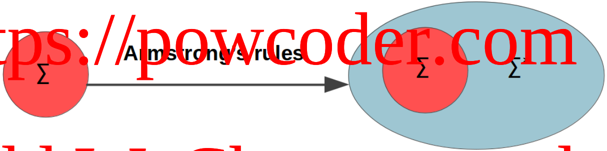

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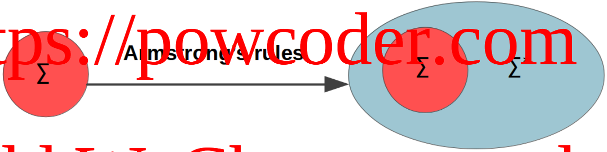

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- **How can we derive the proof more efficiently?**

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- Let Σ be a set of FDs. Check whether or not $\Sigma \models X \rightarrow W$ holds?

[2] See Algorithm 15.1 on Page 538 in [Elmasri & Navathe, 7th edition] or Algorithm 1 on Page 555 in [Elmasri & Navathe, 6th edition]

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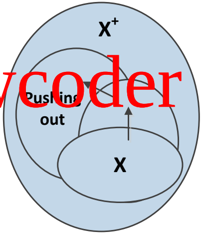
- **Algorithm**[2]
  - $X^+ := X$;
  - repeat until no more change on X^+
    - for each $Y \to Z \in \Sigma$ with $Y \subseteq X^+$, add all the attributes in Z to X^+, i.e., replace X^+ by $X^+ \cup Z$.



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- Consider a relation schema $R = \{A, B, C, D, E, F\}$, a set of FDs $\Sigma = \{AC \rightarrow B, B \rightarrow CD, C \rightarrow E, AF \rightarrow B\}$ on R.
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  2. Then we check that $DE \subseteq (AC)^+$. Hence $\Sigma \models AC \rightarrow DE$.

- **Can you quickly tell whether or not $\Sigma \models AC \rightarrow EF$ holds?**

  $\Sigma \models AC \rightarrow EF$ does not hold because $EF \not\subseteq (AC)^+$

Exercise – Implied Functional Dependencies

- Consider a relation schema $R = \{A, B, C, D, E\}$ and a set of functional dependencies $\Sigma = \{A \rightarrow C, B \rightarrow C, CD \rightarrow E\}$ on R.
- Decide whether or not

  1. $\Sigma \models AD \rightarrow CE$ holds
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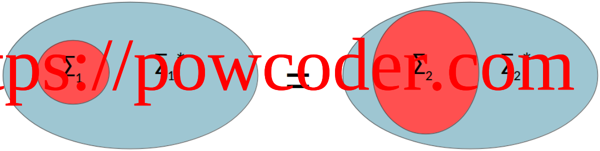
- We build the closure for the set of attributes and check:
  1. $(AD)^+ = (ACD)^+ = (ACDE)^+ = ACDE$ and $CE \subseteq (AD)^+$, hence $\Sigma \models AD \rightarrow CE$.
  2. $(BD)^+ = (BCD)^+ = (BCDE)^+ = BCDE$ and $AC \not\subseteq (BD)^+$, hence $\Sigma \not\models BD \rightarrow AC$.

Equivalence of Functional Dependencies

- Σ_1 and Σ_2 are **equivalent** if $\Sigma_1^* = \Sigma_2^*$.



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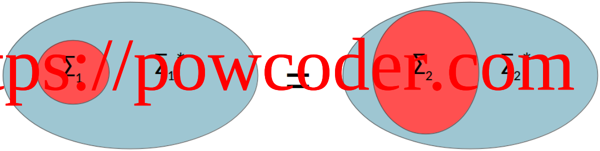

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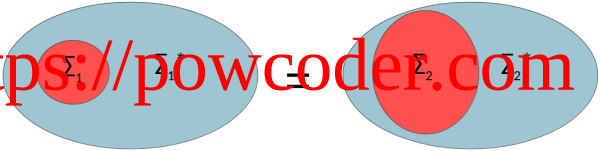

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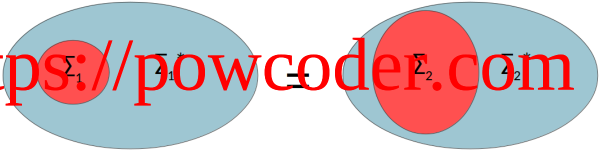

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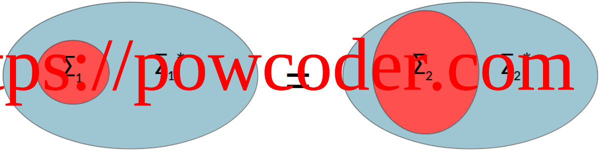

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- If $\Sigma_1 \models \Sigma_2$ and $\Sigma_2 \models \Sigma_1$, are Σ_1 and Σ_2 equivalent? Yes.

- **Questions:** Can we find the **minimal** one among equivalent sets of FDs?

Minimal Cover – The Hard Part!

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Minimal Cover – The Hard Part!

- Let Σ be a set of FDs. A **minimal cover** Σ_m of Σ is a set of FDs such that

   1. Σ_m is equivalent to Σ, i.e., start with $\Sigma_m = \Sigma$;

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3. **Determinant:** each FD has as few attributes on the left hand side as possible, i.e., for each FD $X \rightarrow A$ in Σ_m, check each attribute B of X to see if we can replace $X \rightarrow A$ with $(X - B) \rightarrow A$ in Σ_m;

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4. Remove a FD from Σ_m if it is redundant.

Minimal Cover - Examples

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     - check whether $\Sigma^* = \Sigma_1^*$?

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     - $\Sigma = \{B \rightarrow A, D \rightarrow A, \mathbf{AB} \rightarrow \mathbf{D}\}$, $\Sigma_1 = \{B \rightarrow A, D \rightarrow A, \mathbf{A} \rightarrow \mathbf{D}\}$
     - check whether $\Sigma^* = \Sigma_1^*$? (we have $\Sigma_1 \models \Sigma$, but $\Sigma \models \Sigma_1$?)
     - check $\Sigma \models \mathbf{A} \rightarrow \mathbf{D}$?

       If $\Sigma \models \mathbf{A} \rightarrow \mathbf{D}$, then $\Sigma \models \Sigma_1$ and $\Sigma_1 \models \Sigma$, indicating $\Sigma^* = \Sigma_1^*$.

Minimal Cover - Examples

- Given the set of FDs $\Sigma = \{B \rightarrow A, D \rightarrow A, AB \rightarrow D\}$, we can compute the minimal cover of Σ as follows:
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     - check $\Sigma \models \mathbf{A \rightarrow D}$?

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       If $\Sigma \not\models \mathbf{A \rightarrow D}$, then $\Sigma^* \neq \Sigma_1^*$.

Minimal Cover - Examples

- Given the set of FDs $\Sigma = \{B \to A, D \to A, AB \to D\}$, we can compute the minimal cover of Σ as follows:
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  2. check whether all the FDs in Σ have only one attribute on the right hand side (look good);
  3. check if $AB \to D$ can be replaced by $A \to D$?
     - $\Sigma = \{B \to A, D \to A, \mathbf{AB \to D}\}$, $\Sigma_1 = \{B \to A, D \to A, \mathbf{A \to D}\}$
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       If $\Sigma \not\models \mathbf{A \to D}$, then $\Sigma^* \neq \Sigma_1^*$.
     - $\Sigma \not\models \mathbf{A \to D}$ because $D \not\subseteq (A)^+$.

     No. $AB \to D$ cannot be replaced by $A \to D$.

Minimal Cover - Examples

- Given the set of FDs $\Sigma = \{B \rightarrow A, D \rightarrow A, AB \rightarrow D\}$, we can compute the minimal cover of Σ as follows:
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Minimal Cover - Examples

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Minimal Cover - Examples

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      If $\Sigma \not\models \mathbf{B \rightarrow D}$, then $\Sigma^* \neq \Sigma_2^*$
    - $\Sigma \models \mathbf{B \rightarrow D}$ because $D \subseteq (B)^+$.

   Yes. $AB \rightarrow D$ can be replaced by $B \rightarrow D$.

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  4. look for a redundant FD in $\{B \rightarrow A, D \rightarrow A, B \rightarrow D\}$
     - check whether $B \rightarrow A$ is redundant?

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  4. look for a redundant FD in $\{B \rightarrow A, D \rightarrow A, B \rightarrow D\}$
     - check whether $B \rightarrow A$ is redundant?
     - $B \rightarrow A$ is redundant because $\{D \rightarrow A, B \rightarrow D\} \models B \rightarrow A$;

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3. $AB \rightarrow D$ can be replaced by $B \rightarrow D$;
4. look for a redundant FD in $\{B \rightarrow A, D \rightarrow A, B \rightarrow D\}$
   - check whether $B \rightarrow A$ is redundant?
   - $B \rightarrow A$ is redundant because $\{D \rightarrow A, B \rightarrow D\} \models B \rightarrow A$;

Therefore, the minimal cover of Σ is $\{D \rightarrow A, B \rightarrow D\}$.

Minimal Cover

- **Theorem:**

The minimal cover of a set of functional dependencies Σ always exists but is not necessarily unique.

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Σ has two different minimal covers:

- $\Sigma_1 = \{A \rightarrow B, B \rightarrow C, C \rightarrow A\}$
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  Σ has two different minimal covers:

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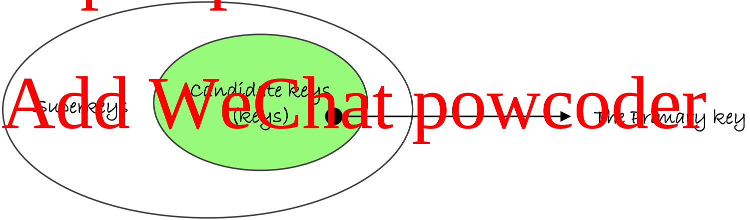
- The algorithm in the previous slide can find one, but not all minimal covers of a set of functional dependencies Σ.

Finding Keys

- Given a set Σ of FDs on a relation *R*, the question is:

How can we find all the (candidate) keys of *R*?



Finding Keys

- **Fact**: A key K of R always defines a FD $K \rightarrow R$.

[3] It extends Algorithm 15.2(a) in [Elmasri & Navathe, 7th edition, pp. 542], or Algorithm 2(a) or in Algorithm 2(a) in [Elmasri & Navathe, 6th edition pp. 558] to finding all keys of R

Finding Keys

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- **Algorithm**[3]:

Input: a set Σ of FDs on R.

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- if $X^+ = R$, then X is a superkey.

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  Input: a set Σ of FDs on R.

  Output: the set of all keys of R.

  - for every subset X of the relation R, compute its closure X^+
  - if $X^+ = R$, then X is a superkey.
  - if no proper subset Y of X with $Y^+ = R$, then X is a key.

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Finding Keys

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- **Algorithm**[3]:

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  Output: the set of all keys of R.
  - for every subset X of the relation R, compute its closure X^+
  - if $X^+ = R$, then X is a superkey.
  - if no proper subset Y of X with $Y^+ = R$, then X is a key.

- A **prime attribute** is an attribute occurring in a key, and a **non-prime attribute** is an attribute that is not a prime attribute.

[3] It extends Algorithm 15.2(a) in [Elmasri & Navathe, 7th edition, pp. 542], or Algorithm 2(a) or in Algorithm 2(a) in [Elmasri & Navathe, 6th edition pp. 558] to finding all keys of R

Exercises - Keys and Minimal Cover

- Consider RENTAL = {CustID, CustName, PropertyNo, DateStart, Owner} and the following set Σ of FDs:
  - $\{\text{CustID}\} \rightarrow \{\text{CustName}\}$
  - $\{\text{PropertyNo, StartDate}\} \rightarrow \{\text{CustID}\}$
  - $\{\text{PropertyNo, CustID}\} \rightarrow \{\text{StartDate}\}$
  - $\{\text{CustID, StartDate}\} \rightarrow \{\text{PropertyNo}\}$
  - $\{\text{Owner}\} \rightarrow \{\text{PropertyNo}\}$

- **Questions:**
  1. What are the keys of RENTAL?
  2. What is a minimal cover of Σ?

Exercises - Keys and Minimal Cover

- Consider RENTAL={CustID, CustName, PropertyNo, DateStart, Owner} and its FDs in the abbreviated form as
  - R={C, N, P, D, O}, and
  - $\Sigma = \{C \rightarrow N, PD \rightarrow C, CP \rightarrow D, OD \rightarrow P, O \rightarrow P\}$

- What are the keys of RENTAL?

Exercises - Keys and Minimal Cover

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- What are the keys of RENTAL?
- **Solution**: Check $(X)^+$ for every subset of {C, N, P, D, O}.
  - O never appears in the dependent of any FD, O must be part of each key.

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- What are the keys of RENTAL?
- **Solution**: Check $(X)^+$ for every subset of {C, N, P, D, O}.
  - O never appears in the dependent of any FD, O must be part of each key.
  - $(O)^+ = OP$
  - $(CO)^+ = CPNDO$, $(DO)^+ = CPNDO$ ...
  - Thus, {CustID, Owner} and {Owner, DateStart} are the keys.

Exercises - Keys and Minimal Cover

- Consider RENTAL={CustID, CustName, PropertyNo, DateStart, Owner} and its FDs in the abbreviated form as
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- What is a minimal cover of Σ?

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  - R={C, N, P, D, O}, and
  - $\Sigma = \{C \rightarrow N, PD \rightarrow C, CP \rightarrow D, CD \rightarrow P, O \rightarrow P\}$

- What is a minimal cover of Σ?
- **Solution**:
  1. start from Σ

Exercises - Keys and Minimal Cover

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  - R={C, N, P, D, O}, and
  - $\Sigma = \{C \rightarrow$ N, PD $\rightarrow$ C, CP $\rightarrow$ D, CD $\rightarrow$ P, O $\rightarrow$ P$\}$

- What is a minimal cover of Σ?
- **Solution**:
  1. start from Σ;
  2. check whether all the FDs in Σ have only one attribute on the right hand side (look good);

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- Consider RENTAL={CustID, CustName, PropertyNo, DateStart, Owner} and its FDs in the abbreviated form as
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- What is a minimal cover of Σ?
- **Solution**:

1. start from Σ;
2. check whether all the FDs in Σ have only one attribute on the right hand side (look good);
3. determine if PD $\rightarrow$ C, CP $\rightarrow$ D and CD $\rightarrow$ P have any redundant attribute on the left hand side (look good);

Exercises - Keys and Minimal Cover

- Consider RENTAL={CustID, CustName, PropertyNo, DateStart, Owner} and its FDs in the abbreviated form as
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- **Solution**:

  1. start from Σ;
  2. check whether all the FDs in Σ have only one attribute on the right hand side (look good);
  3. determine if PD → C, CP → D and CD → P have any redundant attribute on the left hand side (look good);
  4. look for a redundant FD in Σ (none of FDs in Σ are redundant);

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- **Solution**:
  1. start from Σ;
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  3. determine if $PD \rightarrow C$, $CP \rightarrow D$ and $CD \rightarrow P$ have any redundant attribute on the left hand side (look good);
  4. look for a redundant FD in Σ (none of FDs in Σ are redundant);

Therefore, Σ is a minimal cover itself.

Accommodation Database

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {??}

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  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {??}
- We have some requirements on BOOKING:

R1 A booking can be made for one day only.
R2 A guest can make several bookings in a hotel for different days.
R3 A guest cannot make two or more bookings in the same hotel for the same day.
R4 A guest can make two or more bookings in different hotels for the same day.
R5 A room in any hotel can only be booked by one guest on the same date, i.e., no *double-booking*.

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

  R1 A booking can be made for one day only.

How to Identify FDs?

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- Which functional dependency does the following requirement imply?

R1 A booking can be made for one day only.

$\hookrightarrow$ {guestNo, hotelNo, roomNo} $\rightarrow$ {date}?

How to Identify FDs?

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  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

R1 A booking can be made for one day only.

$\hookrightarrow$ {guestNo, hotelNo, roomNo} $\rightarrow$ {date}? **No**

How to Identify FDs?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

R1 A booking can be made for one day only.

↪ {guestNo, hotelNo, roomNo} → {date}? **No**

guestNo	hotelNo	roomNo	Date
001	H1	R101	28/08/2020
001	H1	R101	29/08/2020

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

R2 A guest can make several bookings in a hotel for different days.

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How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

R2 A guest can make several bookings in a hotel for different days.

None

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {??}
- Which functional dependency does the following requirement imply?

  R3 A guest cannot make two or more bookings in the same hotel for the same day.

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {??}

- Which functional dependency does the following requirement imply?

  R3 A guest cannot make two or more bookings in the same hotel for the same day.

  $\hookrightarrow$ {guestNo, hotelNo, date} $\rightarrow$ {roomNo}?

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {??}
- Which functional dependency does the following requirement imply?

R3 A guest cannot make two or more bookings in the same hotel for the same day.

$\hookrightarrow$ {guestNo, hotelNo, date} $\rightarrow$ {roomNo}? **Yes**

guestNo	hotelNo	roomNo	Date
001	H1	R101	29/08/2020
001	H1	R102 ×	29/08/2020

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

R4 A guest can make two or more bookings in different hotels for the same day.

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

R4 A guest can make two or more bookings in different hotels for the same day.

None

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {??}
- Which functional dependency does the following requirement imply?

R5 A room in any hotel can only be booked by one guest on the same date, i.e., no *double-booking*.

$\hookrightarrow$ {hotelNo, date, roomNo} $\rightarrow$ {guestNo}

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

R5 A room in any hotel can only be booked by one guest on the same date, i.e., no *double-booking*.

$\hookrightarrow$ {hotelNo, date, roomNo} $\rightarrow$ {guestNo} **Yes**

guestNo	hotelNo	roomNo	Date
001	H1	R101	29/08/2020
002 ×	H1	R101	29/08/2020

How to Find Candidate Keys?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- FDs on BOOKING
  - {guestNo, hotelNo, date} $\rightarrow$ {roomNo} by **R3**
  - {hotelNo, date, roomNo} $\rightarrow$ {guestNo} by **R5**

How to Find Candidate Keys?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- FDs on BOOKING
  - {guestNo, hotelNo, date} $\rightarrow$ {roomNo} by **R3**
  - {hotelNo, date, roomNo} $\rightarrow$ {guestNo} by **R5**

- Candidate keys on BOOKING
  - {guestNo, hotelNo, date}
  - {hotelNo, date, roomNo}

Australian National University

How to Identify FDs?

- Consider BOOKING(guestNo, hotelNo, date, roomNo) and the following changes:

R1 A booking can be made for one day only.
R2 A guest can make several bookings in a hotel for different days.
R3 ~~A guest cannot make two or more bookings in the same hotel for the same day.~~
R4 ~~A guest can make two or more bookings in different hotels for the same day.~~
R5 A room in any hotel can only be booked by one guest on the same date, i.e., no *double-booking*.
R6 A guest is not allowed to make more than one booking for the same day even in the different hotels.

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

  R6 A guest is not allowed to make more than one booking for the same day even in the different hotels.

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

R6 A guest is not allowed to make more than one booking for the same day even in the different hotels.

$\hookrightarrow$ {guestNo, date} $\rightarrow$ {hotelNo, roomNo}

How to Find Candidate Keys?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- FDs on BOOKING
  - {hotelNo, date, roomNo} $\rightarrow$ {guestNo} by **R5**
  - {guestNo, date} $\rightarrow$ {hotelNo, roomNo} by **R6**

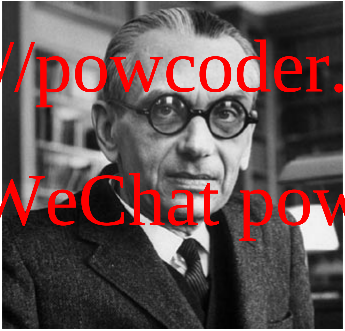
How to Find Candidate Keys?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- FDs on BOOKING
  - {hotelNo, date, roomNo} → {guestNo} by **R5**
  - {guestNo, date} → {hotelNo, roomNo} by **R6**

- Candidate keys on BOOKING
  - {hotelNo, date, roomNo}
  - {guestNo, date}

(credit cookie) Kurt Gödel and Incompleteness Theorem



Kurt Gödel (1906-1978)

Armstrong's Inference Rules

- **Two questions:**
  - Are all the FDs inferred using the Armstrong's inference rules correct?
    $\rightsquigarrow$ **soundness (you cannot prove anything that is wrong)**

  - Can we use the Armstrong's inference rules to infer all possible FDs?
    $\rightsquigarrow$ **completeness (you can prove anything that is right)**

- **Theorem** (W. W. Armstrong)
  - The Armstrong's inference rules are both **sound** and **complete**.

Hilbert's program (1920s)

- **Formulation of mathematics**: formalize all true mathematical statements
- **Completeness**: all true mathematical statements can be proved
- **Consistency**: no contradiction can be obtained in the formalism
- **Decidability**: decide the truth or falsity of any mathematical statement.

Hilbert's program (1920s)

- **Formulation of mathematics**: formalize all true mathematical statements
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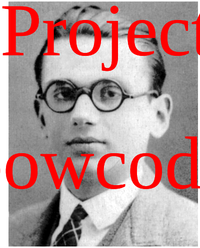
David Hilbert (1862-1943)

We must know. We will know.

Kurt Gödel and Incompleteness Theorem



Kurt Gödel
(1906-1978)

- **Theorem** (Kurt Gödel, 1931)
  For any computable axiomatic system that is powerful enough to describe the arithmetic of the natural numbers, **there will always be at least one true but unprovable statement**.

Kurt Gödel and Gödel Prize

Kurt Gödel
(1906-1978)

John von Neumann
(1903-1957)

- Kurt Gödel’s achievement in modern logic is singular and monumental – indeed it is more than a monument, it is a landmark which will remain visible far in space and time. — **John von Neumann**

Kurt Gödel and Gödel Prize

Kurt Gödel
(1906-1978)

John von Neumann
(1903-1957)

- Kurt Gödel's achievement in modern logic is singular and monumental – indeed it is more than a monument, it is a landmark which will remain visible far in space and time. — **John von Neumann**
- The **Gödel prize** became an annual prize for outstanding papers in the area of theoretical computer science since 1993.