COMP2610 / COMP6261 Information Theory

Lecture 1: Introduction

Robert C. Williamson

Research School of Computer Science
The Australian National University

July 23, 2018

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- **Engineering** (your telephone for example)
- **Computing** (What is that computers do? *They process information*)

References for the curious . . . for interest only!

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What Is Information? (1)

According to a dictionary definition, information can mean

1. Facts provided or learned about something or someone:
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Important!

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- Usually unhelpful to ask "What is?" questions! — "essentialism".
- Better to ask what happens to it? "Grothendieck's Relative method"

What is Information? (2)

In this course: information in the context of *communication* (includes information storage).

- Explicitly include uncertainty — indeed, rather than deriving information from probability theory, one can start with information and derive probability theory from that!

- Claude Shannon (1948): "Amount of unexpected data a message contains"
  - A theory of information **transmission**

What is Information? (3)

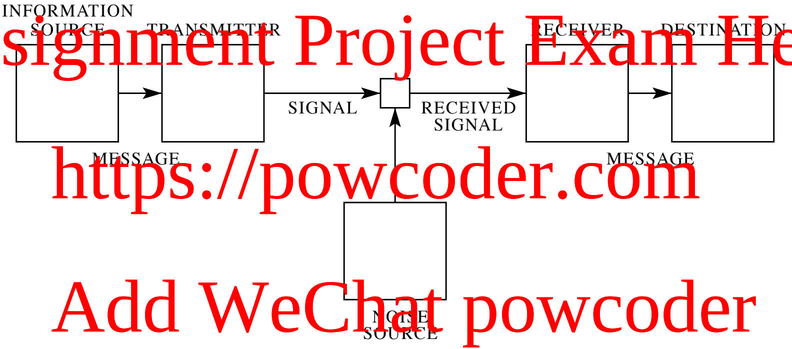

Fig. 1 — Schematic diagram of a general communication system.

From Claude Shannon, A Mathematical Theory of Communication, *Bell System Technical Journal* (1948).

What Is Information? (4)

Information is a message that is *uncertain* to receivers:

- If we receive something that we already knew with absolute certainty then it is non-informative

- Uncertainty is crucial in measuring information content

- We will deal with uncertainty using probability theory

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Information Theory

Information theory is the study of the fundamental *limits* and *potential* of the **representation** and **transmission** of information.

Examples

Example 1: What Number Am I Thinking of?

- I have in mind a number that is between 1 and 20
- You are allowed to ask me one question at a time
- I can only answer yes/no
- Your goal is to figure out the number **as quickly as possible**
- What strategy would you follow?

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Your strategy + my answers = a code for each number

Some variants:

- What if you knew I never chose prime numbers?
- What if you knew I was twice as likely to pick numbers more than 10?
- What if you knew I only ever chose one of 7 or 13?

Example 2: How Much Is Information Worth?

Simplified Version of “Deal or No Deal”

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How much would you pay to know:

1. Exactly which case contains the money?
2. Whether the case holding the money is numbered less than 8?
3. ... is less than 12?
4. Which range out of 0–3, 4–7, 8–11, or 12–15 the money case is in?

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Key Question:

- Can we use these ideas to *quantify* information?

Example 3: Redundancy and Compression

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Can you read this sentence without any vowels?

Written English (and other languages) has much redundancy:

- Approximately 1 bit of information per letter

- Naïvely there should be almost 5 bits per letter

(For the moment think of “bit” as “number of yes/no questions”)

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Key Question:

- How much redundancy can we *safely* remove?
  (Note: “rd” could be “read”, “red”, “road”, etc.)

Example 4: Error Correction

Hmauns hvae the aitliby to cerroct for eorrrs in txet and iegmas.

Key Question:

- How much noise is it possible to correct for and how?

A Summary of the History of Information Theory

1920s : Nyquist & Hartley at Bell Labs

1940 : Turing and Good at Bletchley Park (WWII)

1942 : Hedy Lamarr and George Antheil

1948 : Claude Shannon: "A Mathematical Theory of Communication"

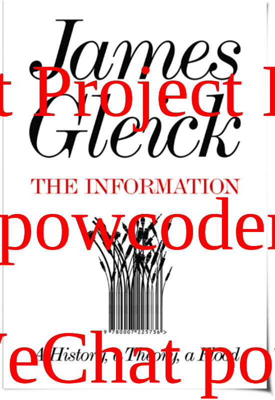
1951 : Huffman Coding

1958 : Peter Elias: "Two Famous Papers"

1970 : "Coding is Dead"

1970- : Revival with advent of digital computing
CDs, DVDs, MP3s, Digital TV, Mobiles, Internet, Deep-space comms (Voyager), ...

More on the History of Information Theory



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Information Theory and the Digital Age

by Aftab, Cheung, Kim, Thakkar, and Yeddanapudi.

`http://web.mit.edu/6.933/www/Fall2001/Shannon2.pdf`

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2. A Brief History
3. Course Overview
4. Logistics and Expectations
5. What's Next

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COMP2610/COMP6261 (Information Theory)

We will study the fundamental limits and potential of the *representation* and *transmission* of information.

- Mathematical Foundations
- Probabilistic Inference

- Coding and Compression

- Communication
- Kolmogorov Complexity (Guest Lecture)

Learning Outcomes

From `https://wattlecourses.anu.edu.au/course/view.php?id=25550`:

1. Understand and apply **fundamental concepts** in information theory such as probability, entropy, information content and their inter-relationships
2. Understand the principles of **data compression**
3. Compute **entropy** and **mutual information** of random variables
4. Implement and analyse basic **coding** and **compression algorithms**
5. Understand the relationship of information theoretical principles and **Bayesian inference** in data modelling and pattern recognition
6. Understand some key **theorems** and **inequalities** that quantify essential limitations on compression, communication and inference
7. Know the basic concepts regarding **communications over noisy channels**

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Course Overview

See Wattle site (authoritative)

- Lectures: 23 $\times$ 1 hour (two lectures per week); one public holiday

- By me, except one guest lecture by Marcus Hutter (Aside: about me).
- Tutorials: Starting week 2; schedule up shortly.

- Assignments: 3 (0% (optional), 20%, 20% each) (0% explained below)

- Final Exam (60%) Hurdle assessment. You have to pass the exam to pass the course. (New this year!)
- **Late Submission Policy: late submissions get zero marks — 100% penalty.**

Expectations

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- Learning mathematical material is hard and cannot be delegated or outsourced. "There is no royal road to geometry." Don't kid yourself!

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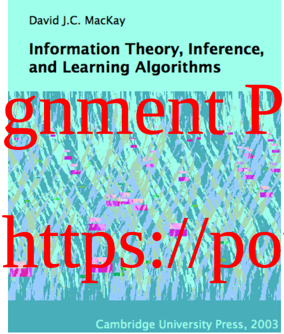
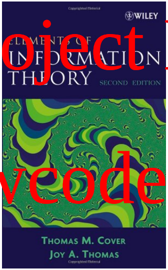
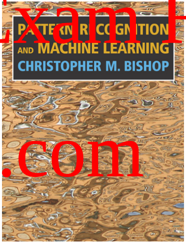
- In a nutshell: The secret of success is *deliberate practice.*

Textbook







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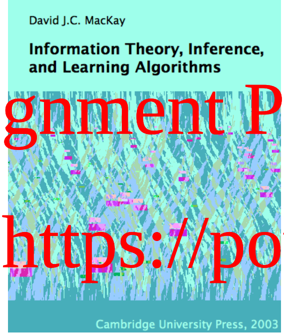

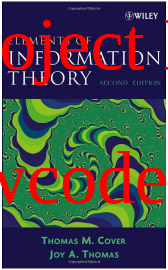

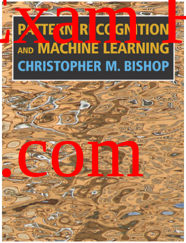

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For an alternative take – David MacKay's Lectures:
http://www.inference.phy.cam.ac.uk/itprnn_lectures/

Consultation & Other Issues

Consultation:

- Best way to contact the course lecturers and tutors is via **email**

  comp2610@anu.edu.au

- If you **really** need to meet in person, send an email request first

- Email response times may vary but consider 1 day as a fast reply and up to three days as a normal response time

- **Technical questions**: encouraged to post on Wattle's public forum

- Request for clarifying assignment: **must be** posted on Wattle

What's Next?

- If you are not comfortable about your probability and algebra skills, start today on improving them

- Get a copy of the text and start purusing it

- Sign up to a tutorial (will open tomorrow, time announced tomorrow)