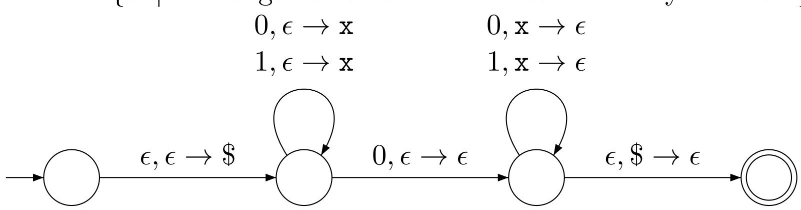
THE UNIVERSITY OF MELBOURNE
SCHOOL OF COMPUTING AND INFORMATION SYSTEMS
COMP30026 MODELS OF COMPUTATION

# Selected Problem Set Solutions, Week 10

P10.1 Here is a PDA for $\{w \mid \text{the length of } w \text{ is odd and its middle symbol is } \mathtt{0}\}$




P10.3 For the case $v \neq \epsilon$ we define

$$\delta((q_p, q_d), v, x) = \left\{ \ ((r_p, r_d), y) \ \middle| \ (r_p, y) \in \delta_P(q_p, v, x) \land r_d = \delta_D(q_d, v) \ \right\}$$

But we must also allow transitions that don't consume input, so:

$$\delta((q_p, q_d), \epsilon, x) = \left\{ \ ((r_p, q_d), y) \ \middle| \ (r_p, y) \in \delta_P(q_p, \epsilon, x) \ \right\}$$



P10.5 We are looking at the context-free grammar $G$:

$$\begin{array}{lcl} S & \to & A\ B\ A \\ A & \to & \mathtt{a}\ A \mid \epsilon \\ B & \to & \mathtt{b}\ B \mid \epsilon \end{array}$$



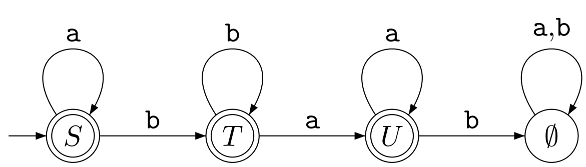
(a) The grammar is ambiguous. For example, a has two different leftmost derivations:



$$\begin{array}{l} S \Rightarrow A\ B\ A \Rightarrow B\ A \Rightarrow A \Rightarrow \mathtt{a}\ A \Rightarrow \mathtt{a} \\ S \Rightarrow A\ B\ A \Rightarrow \mathtt{a}\ A\ B\ A \Rightarrow \mathtt{a}\ B\ A \Rightarrow \mathtt{a}\ A \Rightarrow \mathtt{a} \end{array}$$

(b) $L(G) = \mathtt{a}^*\mathtt{b}^*\mathtt{a}^*$.

(c) To find an unambiguous equivalent context-free grammar it helps to build a DFA for $\mathtt{a}^*\mathtt{b}^*\mathtt{a}^*$. (If this is too hard, we can always construct an NFA, which is easy, and then translate the NFA to a DFA using the subset construction method, which is also easy.) Below is the DFA we end up with. The states are named $S$, $T$, and $U$ to suggest how they can be made to correspond to variables in a context-free grammar. The DFA translates easily to the grammar on the right. The resulting grammar is a so-called *regular* grammar, and it is easy to see that it is unambiguous—there is never a choice of rule to use.




$$\begin{array}{lcl} S & \to & \epsilon \mid \mathtt{a}\ S \mid \mathtt{b}\ T \\ T & \to & \epsilon \mid \mathtt{b}\ T \mid \mathtt{a}\ U \\ U & \to & \epsilon \mid \mathtt{a}\ U \end{array}$$

P10.6 Here is a context-free grammar that will do the job ($S$ is the start symbol):

$$\begin{array}{lcl} S & \to & \epsilon \mid \mathtt{a}\ A \\ A & \to & \mathtt{a}\ A \mid \mathtt{b}\ B \\ B & \to & \epsilon \mid \mathtt{a}\ A \mid \mathtt{b}\ B \end{array}$$