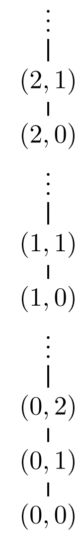
The University of Melbourne
School of Computing and Information Systems
COMP30026 Models of Computation

Selected Problem Set Solutions, Week 12



P12.1 See the Week 12 lecture slides.

P12.2 See the Week 12 lecture slides.

P12.3 (b) is not well-founded, as we can have infinite strictly decreasing sequences in $\mathbb{Q}$, such as $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$. But (a), (c) and (d) are all well-founded. For (d) it may help to look at the Hasse diagram for $\mathbb{N} \times \mathbb{N}$ ordered by $\prec$ (shown here in the margin).

P12.4

```
hailstone :: Integer -> Int
hailstone 0 = 0
hailstone 1 = 0
hailstone n
  | even n = hailstone (n `div` 2) + 1
  | otherwise = hailstone (3*n+1) + 1
```


P12.5 Assume $\mathcal{B}$ is countable. Then we can enumerate $\mathcal{B}$:

Element								
b_1	$\boxed{0}$	0	1	0	1	1	1	...
b_2	1	$\boxed{0}$	1	1	1	0	1	...
b_3	1	0	$\boxed{1}$	1	1	0	1	...
b_4	0	1	0	$\boxed{0}$	1	0	0	...
$\vdots$								

However, the infinite sequence which has

$$i\text{'th bit} = \begin{cases} 0 & \text{if the } i\text{th bit of } b_i \text{ is } 1 \\ 1 & \text{if the } i\text{th bit of } b_i \text{ is } 0 \end{cases}$$

is different from each of the b_i. Hence no enumeration can exist, and $\mathcal{B}$ is uncountable. This should not be surprising, because the set $\mathcal{B}$ is really the same as (or is isomorphic to) $\mathbb{N} \to \Sigma$.