

12–14 August 2020

## Plan

This is the week when you need to get through Grok modules 2 and 3, if you have not already done that. Don't fall behind! We will often provide more exercises than can possibly be covered in a tutorial. That is so that those who want more practice can have that. Exercises that say "drill" will tend to cover old ground, rather than introduce new ideas.

## The exercises

6. If any good questions or thoughts came up as you worked with Grok, now is a good time to share them. We have a question about list types. What is the type of `f` defined below? Is it well-typed? Did somebody forget the square brackets in the last equation? Explain the function's behaviour in English.

```
f [] = 0
f [x] = x
f y = 42
```

7. For each of the following pairs, indicate whether the two formulas have the same truth table.
- |   |   |
|---|---|
| (a) $\neg P \Rightarrow Q$ and $P \Rightarrow \neg Q$                       | (e) $P \Rightarrow (Q \Rightarrow R)$ and $Q \Rightarrow (P \Rightarrow R)$   |
| (b) $\neg P \Rightarrow Q$ and $Q \Rightarrow \neg P$                       | (f) $P \Rightarrow (Q \Rightarrow R)$ and $(P \Rightarrow Q) \Rightarrow R$   |
| (c) $\neg P \Rightarrow Q$ and $\neg Q \Rightarrow P$                       | (g) $(P \wedge Q) \Rightarrow R$ and $P \Rightarrow (Q \Rightarrow R)$        |
| (d) $(P \Rightarrow Q) \Rightarrow P$ and $P \Rightarrow (P \Rightarrow Q)$ | (h) $P \vee Q \Rightarrow R$ and $(P \Rightarrow R) \wedge (Q \Rightarrow R)$ |
8. Find a formula that is equivalent to  $(P \wedge \neg Q) \vee P$  but simpler, that is, using fewer symbols.
9. Recall that  $\oplus$  is the "exclusive or" connective. Show that  $(P \oplus Q) \oplus Q$  is equivalent to  $P$ .
10. Show that  $P \Leftrightarrow (Q \Leftrightarrow R) \equiv (P \Leftrightarrow Q) \Leftrightarrow R$ . This tells us that we could instead write

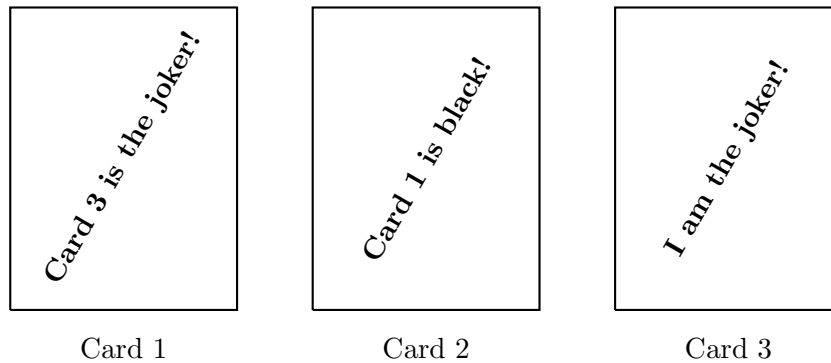
$$P \Leftrightarrow Q \Leftrightarrow R \tag{1}$$

without introducing any ambiguity. Mind you, that may not be such a good idea, because many people (incorrectly) tend to read " $P \Leftrightarrow Q \Leftrightarrow R$ " as

$$P, Q, \text{ and } R \text{ all have the same truth value} \tag{2}$$

Show that (1) and (2) are incomparable, that is, neither is a logical consequence of the other.

11. Three playing cards lie face down on a table. One is red, one is black, and one is the joker. On the back of each card is written a sentence:



The red card has a true sentence written on its back and the black card has a false sentence. Which card is red, which is black, and which is the joker?

12. Consider the formula  $P \Rightarrow \neg P$ . Is that a contradiction? Can a proposition imply its own negation?
13. Let  $F$  and  $G$  be propositional formulas. What is the difference between ' $F \equiv G$ ' and ' $F \Leftrightarrow G$ ' — do we really need both? Show that  $F \equiv G$  iff  $F \Leftrightarrow G$  is valid.
14. By negating a satisfiable proposition, can you get a tautology? A satisfiable proposition? A contradiction? Illustrate your affirmative answers.
15. (Drill.) Recall that  $\Leftrightarrow$  is the bimplication connective. Show that  $(P \Leftrightarrow Q) \equiv (\neg P \Leftrightarrow \neg Q)$ .
16. (Drill.) Is this claim correct:  $(P \wedge Q) \Leftrightarrow P$  is logically equivalent to  $(P \vee Q) \Leftrightarrow Q$ ? That is, do we have
- $$((P \wedge Q) \Leftrightarrow P) \equiv ((P \vee Q) \Leftrightarrow Q) ?$$
17. (Drill.) Find a formula equivalent to  $P \Leftrightarrow (P \wedge Q)$  but simpler, that is, using fewer symbols.
18. (Drill.) For each of the following propositional formulas, determine whether it is satisfiable, and if it is, whether it is a tautology:
- $P \Leftrightarrow ((P \Rightarrow Q) \Rightarrow P)$
  - $(P \Rightarrow \neg Q) \wedge ((P \vee Q) \Rightarrow P)$
  - $((P \Rightarrow Q) \Rightarrow Q) \wedge (Q \oplus (P \Rightarrow Q))$