COMP30026 Models of Computation

Regular Languages

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Lecture 15

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Semester 2, 2020

This Lecture is Being Recorded

DFAs vs NFAs

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The class of languages recognised by NFAs is exactly the class of regular languages.

Theorem: Every NFA has an equivalent DFA.

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The proof rests on the so-called subset construction.

Given NFA N, we construct DFA M, each of whose states corresponds to a set of N-states.

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If N has k states then M may have up to 2^k states (but it will often have far fewer than that).

DFAs vs NFAs

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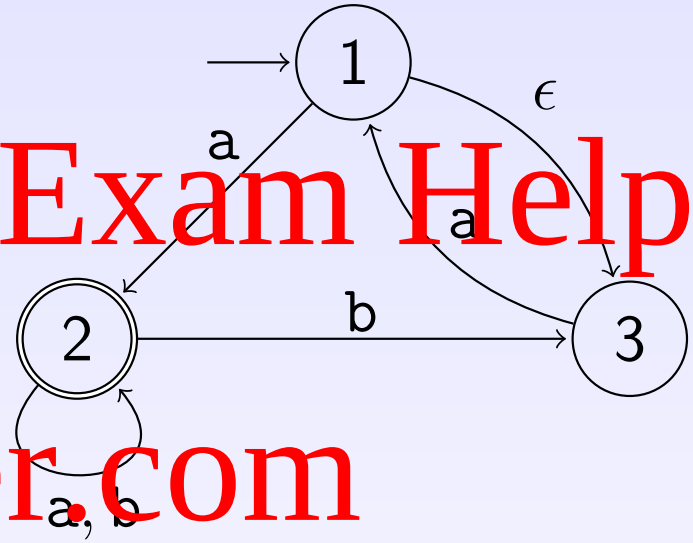
Consider the NFA on the right. We can systematically construct an equivalent DFA from the NFA.



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The DFA's start state is $\{1, 3\}$.
From $\{1, 3\}$ a takes us to $\{1, 2, 3\}$.
From $\{1, 2, 3\}$, a takes us back to $\{1, 2, 3\}$, b takes us to $\{2, 3\}$.

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Any state S which contains an accept state from the NFA will be an accept state for the DFA. Here we mark accept states with a star.

	a	b
A=$\{1,3\}$	B*	–
B*=$\{1,2,3\}$	B*	C*
C*=$\{2,3\}$	B*	C*

DFAs vs NFAs

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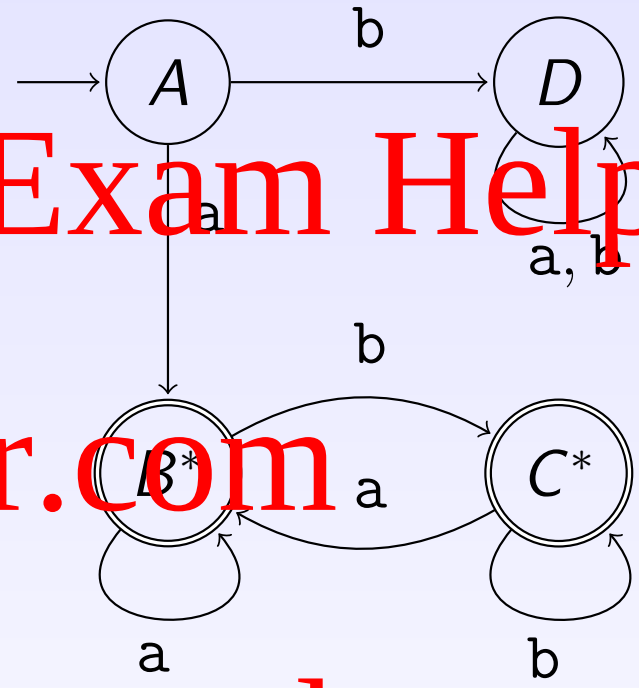
Here is the equivalent DFA that we derive.



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Any state S which contains an accept state from the NFA (in this case the NFA has just one, namely state 2) becomes an accept state for the DFA.

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We add (dead) state D that corresponds to the empty set.

	a	b
A=$\{1,3\}$	B*	D
B*=$\{1,2,3\}$	B*	C*
C*=$\{2,3\}$	B*	C*
D=$\emptyset$	D	D

More Formally . . .

Let $N = (Q, \Sigma, \delta, q_0, F)$. Let $\xrightarrow{*}_\epsilon$ be the reflexive transitive closure of $\rightarrow_\epsilon$, which in turn is defined by $s \rightarrow_\epsilon s'$ iff $s' \in \delta(s, \epsilon)$.

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Let $E(S)$ be the 'ϵ-closure' of $S \subseteq Q$, that is, S together with all states reachable from states in S, using only ϵ steps:

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$$E(S) = \bigcup_{s \in S} \{s' \in Q \mid s \xrightarrow{*}_\epsilon s'\}$$

We construct $M = (\mathcal{P}(Q), \Sigma, \delta', q_0', F')$ as follows:

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- $q_0' = E(\{q_0\})$.
- $\delta'(S, v) = \bigcup_{s \in S} E(\delta(s, v))$.
- $F' = \{S \subseteq Q \mid S \cap F \neq \emptyset\}$.

Note: This construction may include unreachable states.

Closure Results for Regular Languages

Theorem: The class of regular languages is closed under union.

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Proof: Let A and B be regular languages, with recognisers M_A and M_B. An NFA that recognises $A \cup B$ is easily constructed:

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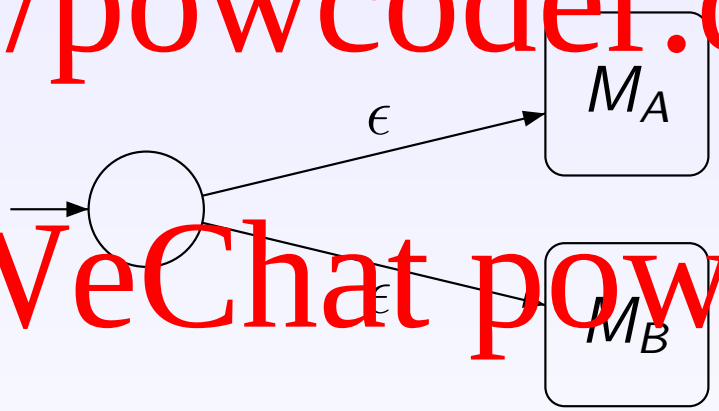

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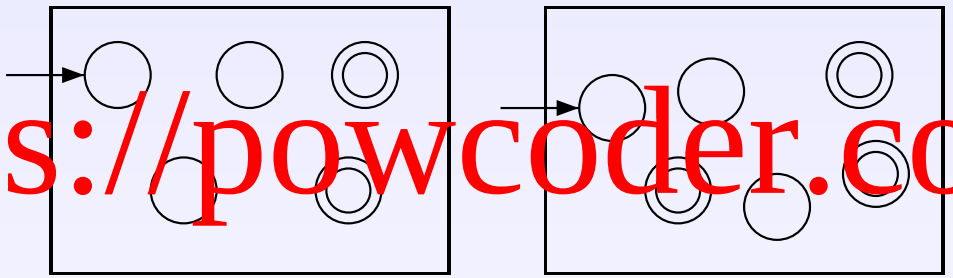
The ϵ-transitions go to the start states of M_A and M_B.

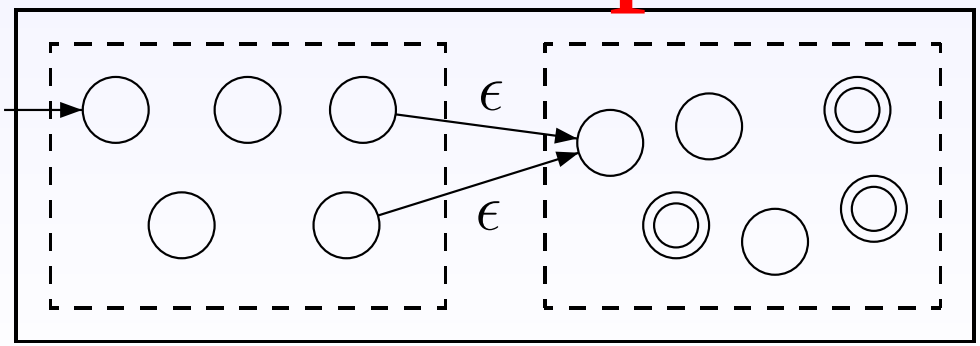
Closure Results for Regular Languages

Theorem: The class of regular languages is closed under $\circ$.

Proof: Let A and B be regular languages with these recognisers:



From these we can easily construct an NFA that recognises $A \circ B$:



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The Last Construction, Formally

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Let recognisers for A and B be these DFAs, respectively:

- $M_A = (Q, \Sigma, \delta, q_0, F)$
- $M_B = (Q', \Sigma, \delta', q_0', F')$

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Then a recogniser for $A \circ B$ is the NFA $(Q \cup Q', \Sigma, \delta'', q_0, F')$, where

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$$\delta''(q, v) = \begin{cases} \{\delta'(q, v)\} & \text{if } q \in Q' \text{ and } v \in \Sigma \\ \{\delta(q, v)\} & \text{if } q \in Q \text{ and } v \in \Sigma \\ \{q_0'\} & \text{if } q \in F \text{ and } v = \epsilon \end{cases}$$

Closure Results for Regular Languages

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Theorem: The class of regular languages is closed under Kleene star.

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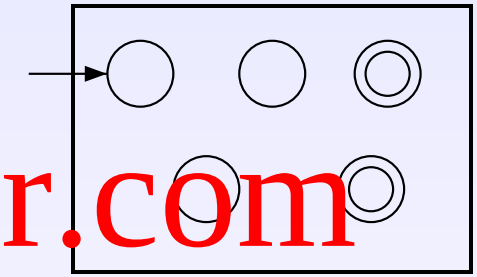
Proof: Let A be a regular language with the recogniser shown on the right.



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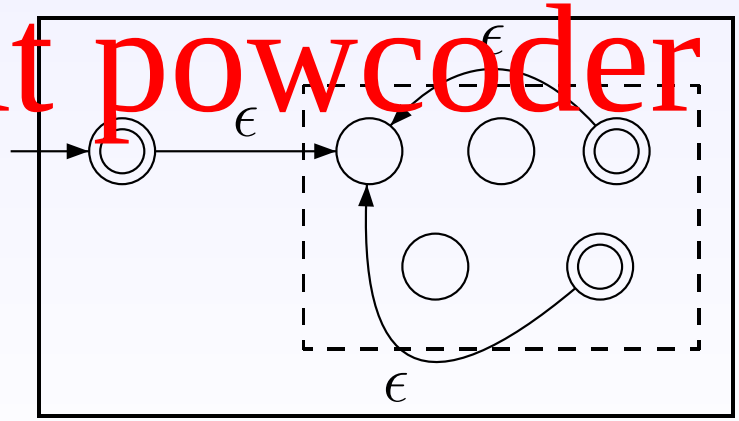
Here is how we construct an NFA to recognise A^*:



Closure Results for Regular Languages

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Regular languages have several other closure properties.

They are closed under

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- intersection,
- complement, A^c
- difference (this follows, as $A \setminus B = A \cap B^c$)
- reversal.

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Algorithms for Manipulating Automata

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For some of these closure results, we will use the tutorials to develop useful DFA manipulation algorithms.

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You will see, for example, how to systematically build DFAs for languages $A \cap B$, out of DFAs for A and B.

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Equivalence of DFAs

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We can always find a minimal DFA for a given regular language (by minimal we mean having the smallest possible number of states).

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Since a DFA has a unique start state and the transition function is total and deterministic, we can test two DFAs for equivalence by minimizing them.

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Minimizing DFAs

There is no guarantee that DFAs that are produced by the various algorithms, such as the subset construction method, will be minimal.

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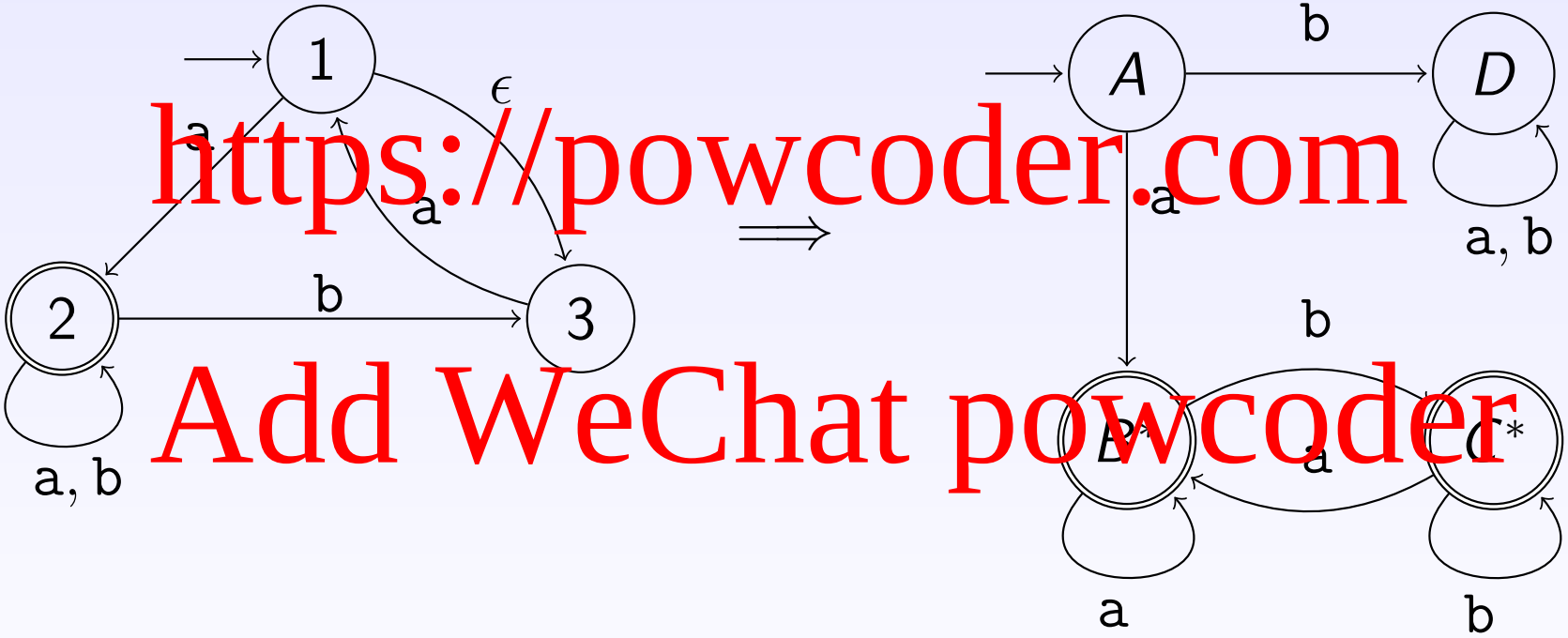

$A = \{1, 3\}$, $B^* = \{1, 2, 3\}$, $C^* = \{2, 3\}$, and $D = \emptyset$.

Generating a Minimal DFA

The following algorithm takes an NFA and produces an equivalent minimal DFA. Of course the input can also be a DFA.

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1. Reverse the NFA;
2. Determinize the result;
3. Reverse again;
4. Determinize.

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To reverse an NFA A with initial state q_0 and accept states $F \neq \emptyset$:

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1. If $F = \{q\}$, let q be the accept state.
2. Otherwise add a new state q which becomes the only accept state; then, for each state in F, add an ϵ transition to q.
3. Reverse every transition in the resulting NFA, making q the initial state and q_0 the (only) accept state.

Minimization Example

Consider the NFA that we determinized two slides ago.
Here it is on the left, with its reversal on the right:

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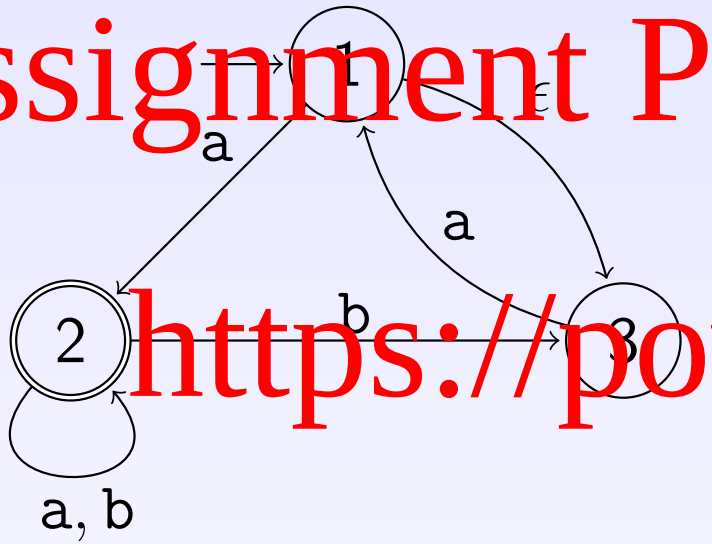

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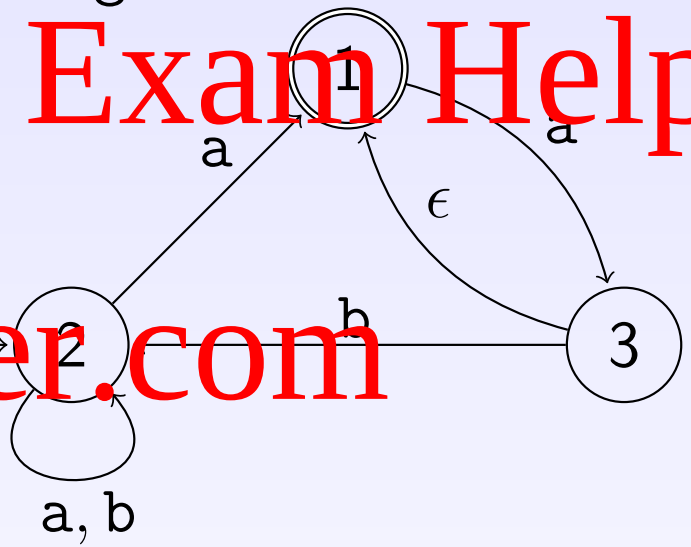

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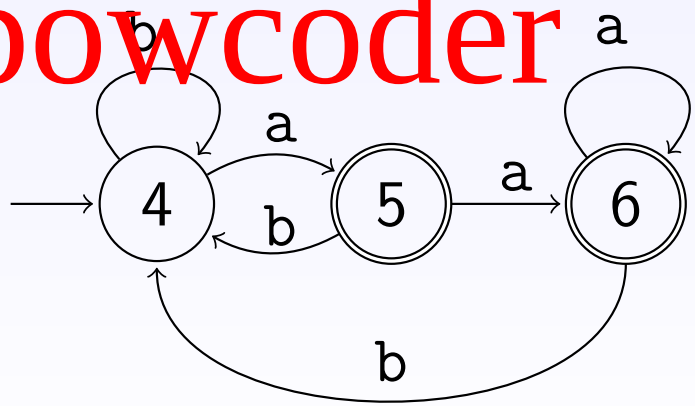
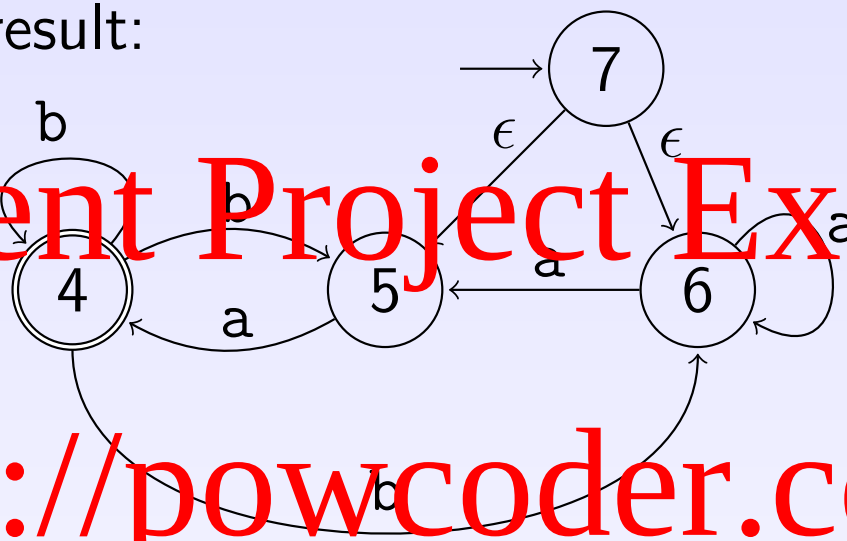
Making the reversed NFA deterministic.
We renamed the states to avoid later confusion;
4 corresponds to $\{2\}$, 5 to $\{1, 2\}$, and 6 to $\{1, 2, 3\}$.



Minimization Example

Now reversing the result:



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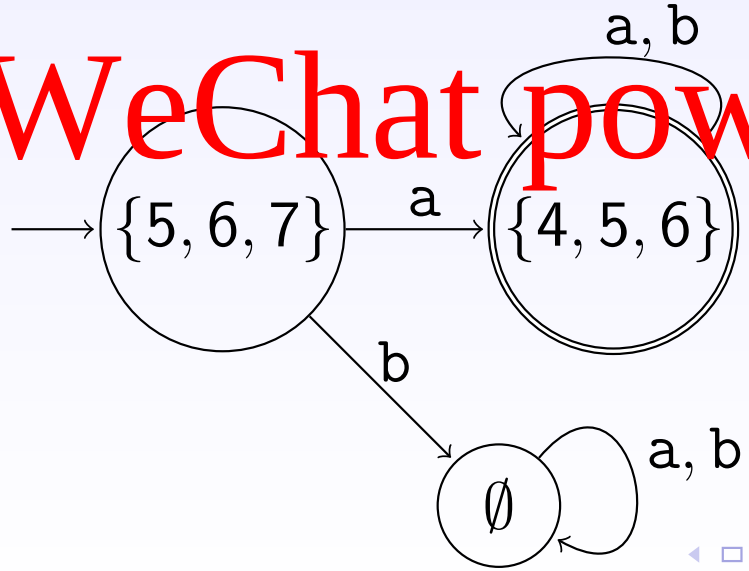
And finally making the result deterministic:



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Next Week

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We next look at regular expressions—another way to capture the regular languages.

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Also, we will use a technique called "pumping", for proving that a language is not regular and introduce context-free languages.

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