COMP302: Programming Languages and Paradigms

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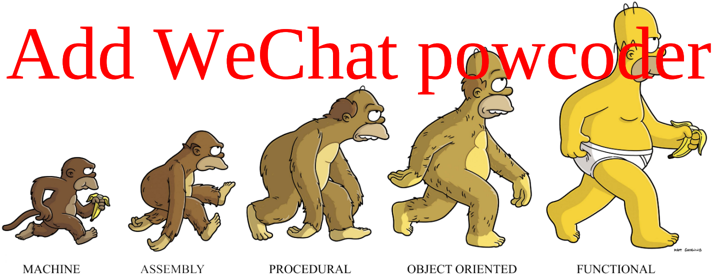
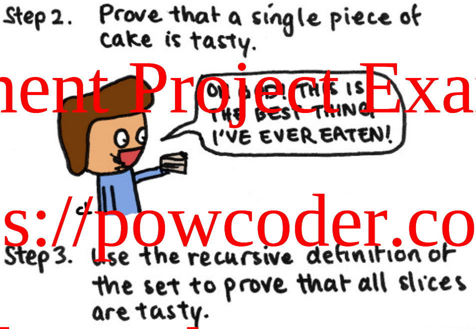
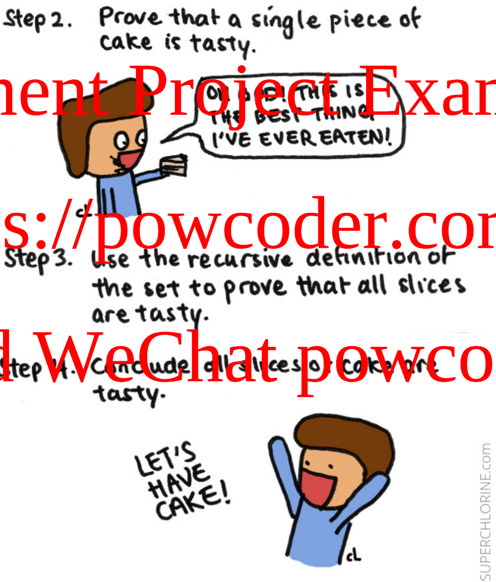


Functional Tidbit: Cake!

Functional Tidbit: Cake is tasty!


Step 2. Prove that a single piece of cake is tasty.
I'VE EVER EATEN!
Step 3. Use the recursive definition of the set to prove that all slices are tasty.


Step 2. Prove that a single piece of cake is tasty.
I'VE EVER EATEN!
Step 3. Use the recursive definition of the set to prove that all slices are tasty.
Step 4. Conclude all slices of cake are tasty.
LET'S HAVE CAKE!
SUPERCHLORINE.com

More on Structural Induction

Two programs: Do they compute the same value?

Program A (naive)

```
(* rev : 'a list -> 'a list *)
let rec rev l = match l with
  | [] -> []
  | x::l -> (rev l) @ [x]
```


Program B (tail-recursive)

```
(* rev' : 'a list -> 'a list *)
let rev' l =
  (* rev_tr : 'a list -> 'a list -> 'a list *)
  let rec rev_tr l acc = match l with
    | []   -> acc
    | h::t -> rev_tr t (h::acc)
  in
    rev_tr l []
```

Program A (naive)

```
(* rev : 'a list -> 'a list *)
let rec rev l = match l with
  | [] -> []
  | x::l -> (rev l) @ [x]
```


Theorem: For all lists l. rev l = rev' l

Program B (tail-recursive)

```
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let rev' l =
  (* rev_tr : 'a list -> 'a list -> 'a list *)
  let rec rev_tr l acc = match l with
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    rev_tr l []
```

What to prove? – Finding the invariant!

Program A (naive)

```
(* rev : 'a list -> 'a list *)
let rec rev l = match l with
  | [] -> []
  | x::l -> (rev l) @ [x]
```


What is the relationship between `l`, `acc` and `rev_tr l acc`?

Program B (tail-recursive)

```
 (* rev' : 'a list -> 'a list *)
 let rev l =
   (* rev_tr : 'a list -> 'a list -> 'a list *)
   let rec rev_tr l acc = match l with
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     | h::t -> rev_tr t (h::acc)
   in
     rev_tr l []

```

Program A (naive)

```
(* rev : 'a list -> 'a list *)
let rec rev l = match l with
  | [] -> []
  | x::l -> (rev l) @ [x]
```


For all `l`, `acc`, `(rev l) @ acc` $\Downarrow$ `v` and `rev_tr l acc` $\Downarrow$ `v`

Program B (tail-recursive)

```
(* rev' : 'a list -> 'a list *)
let rev' l =
   (* rev_tr : 'a list -> 'a list -> 'a list *)
   let rec rev_tr l acc = match l with
     | []   -> acc
     | h::t -> rev_tr t (h::acc)
   in
     rev_tr l []
```

Exercise

Theorem: For all `l`, `acc`,

`length (rev_tr l acc)` $\Downarrow$ `v` and `length l + length acc` $\Downarrow$ `v`

We often simply write instead:

For all `l`, `acc`,

`length (rev_tr l acc) = length l + length acc`

Can't see the forest for the trees

Inductive definition of a binary tree

- The empty binary tree `Empty` is a binary tree
- If `l` and `r` are binary trees and `v` is a value of type `'a` then `Node(v, l, r)` is a binary tree.
- Nothing else is a binary tree.

Inductive definition of a binary tree

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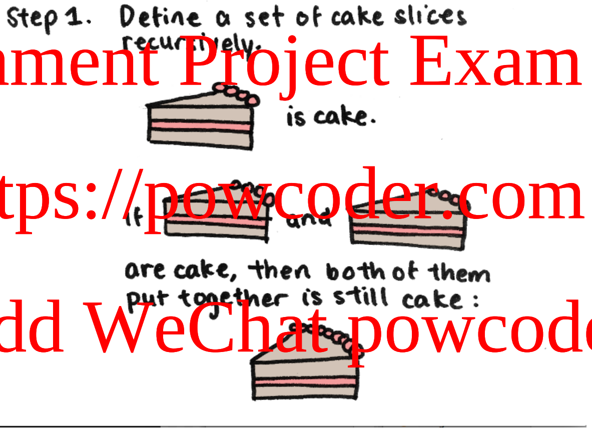
- Nothing else is a binary tree.

How can we define a data type that describes binary trees?

Example of a mutual recursive data type definition:

```
type 'a forest = Forest of ('a tree) list
and 'a tree = Empty | Node of 'a * 'a forest
```

Remember the slice of cake?



Give an OCaml data type definition for cake!