

COMP302: Programming Languages and Paradigms

Assignment Project Exam Help

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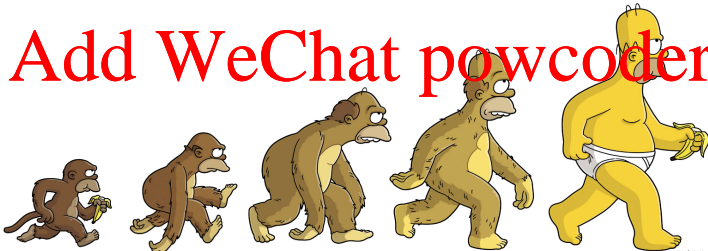
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School of Computer Science
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MACHINE

ASSEMBLY

PROCEDURAL

OBJECT ORIENTED

FUNCTIONAL

Can't see the forest for the trees

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Really ?

Inductive definition of a binary tree

- The empty binary tree `Empty` is a binary tree
- If `l` and `r` are binary trees and `v` is a value of type `α`, then `Node(v, l, r)` is a binary tree.
- Nothing else is a binary tree.

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How to define a recursive data type for trees in OCaml?

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How to define a recursive data type for trees in OCaml?

```
1 type 'a tree =  
2   Empty  
3 | Node of 'a * 'a tree * 'a tree
```

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How to define a recursive data type for trees in OCaml?

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Let's do some programming with trees!

How to prove it?

Step 2: How to reason inductively about trees?

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How to prove it?

Step 2: How to reason inductively about trees?

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Analyze their structure

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How to prove it?

Step 2: How to reason inductively about trees?

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The recipe ...

To prove a property $P(t)$ holds about a binary tree t

Base Case: $t = \text{Empty}$

Show $P(\text{Empty})$ holds

Step Case: $t = \text{Node}(x, l, r)$

IH $P(l)$

Assume the property P holds
for trees smaller than t .

IH $P(r)$

Show $P(\text{Node}(x, l, r))$ holds

Show the property P holds for
the tree t .

Let's prove something!

```
1 let rec insert ((x,d) as e) t = match t with
2   | Empty                -> Node(e, Empty, Empty)
3   | Node((y,d'), l, r) ->
4     if x = y then Node(e, l, r)
5     else (if x < y then Node((y,d'), insert e l, r)
6           else Node((y,d'), l, insert e r))
```

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Theorem: For all trees t , keys x , and data dx ,

$$\text{lookup } x \text{ (insert } (x, dx) t) \implies^* \text{Some } dx$$

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```
1 let rec lookup x t = match t with
2   | Empty                -> None
3   | Node ((y,d), l, r) ->
4     if x = y then Some(d)
5     else (if x < y then lookup x l
6           else lookup x r)
```

Remember the slice of cake?

Step 1. Define a set of cake slices
recursively.



is cake.



are cake, then both of them
put together is still cake:



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Give an OCaml data type definition for cake!