COMP302: Programming Languages and Paradigms

Prof. Brigitte Pientka (Sec 01)
bpientka@cs.mcgill.ca

Francisco Ferreira (Sec 02)
fferre8@cs.mcgill.ca

School of Computer Science
McGill University

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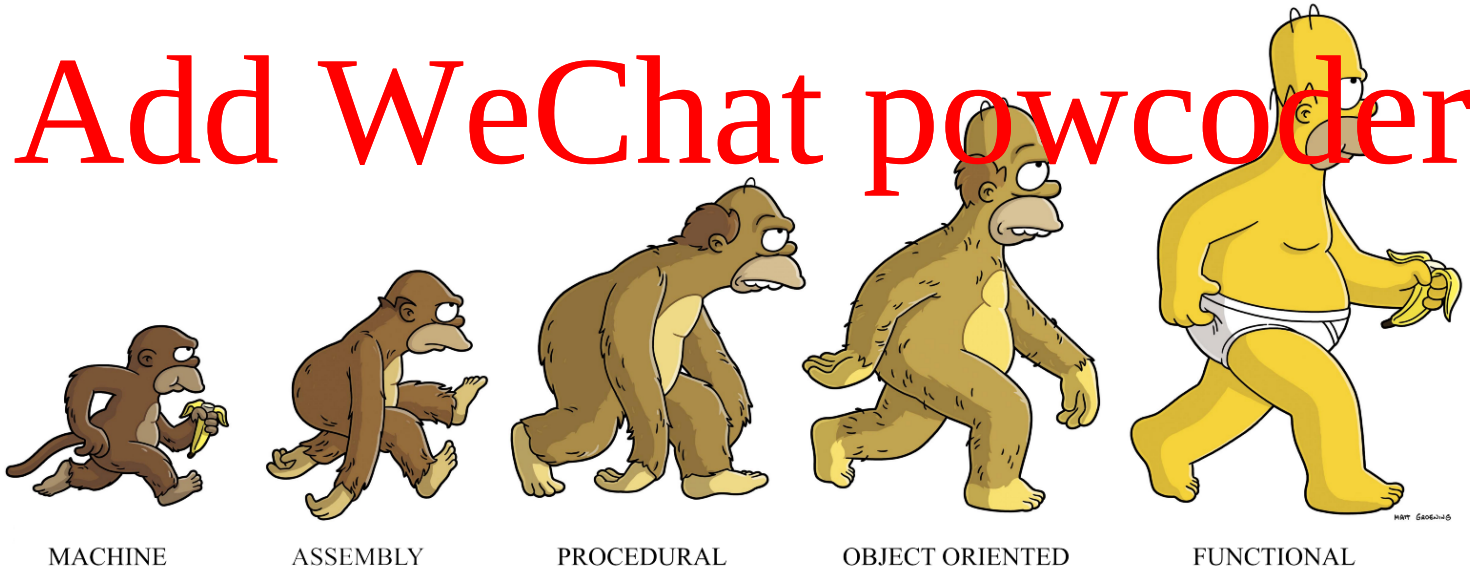
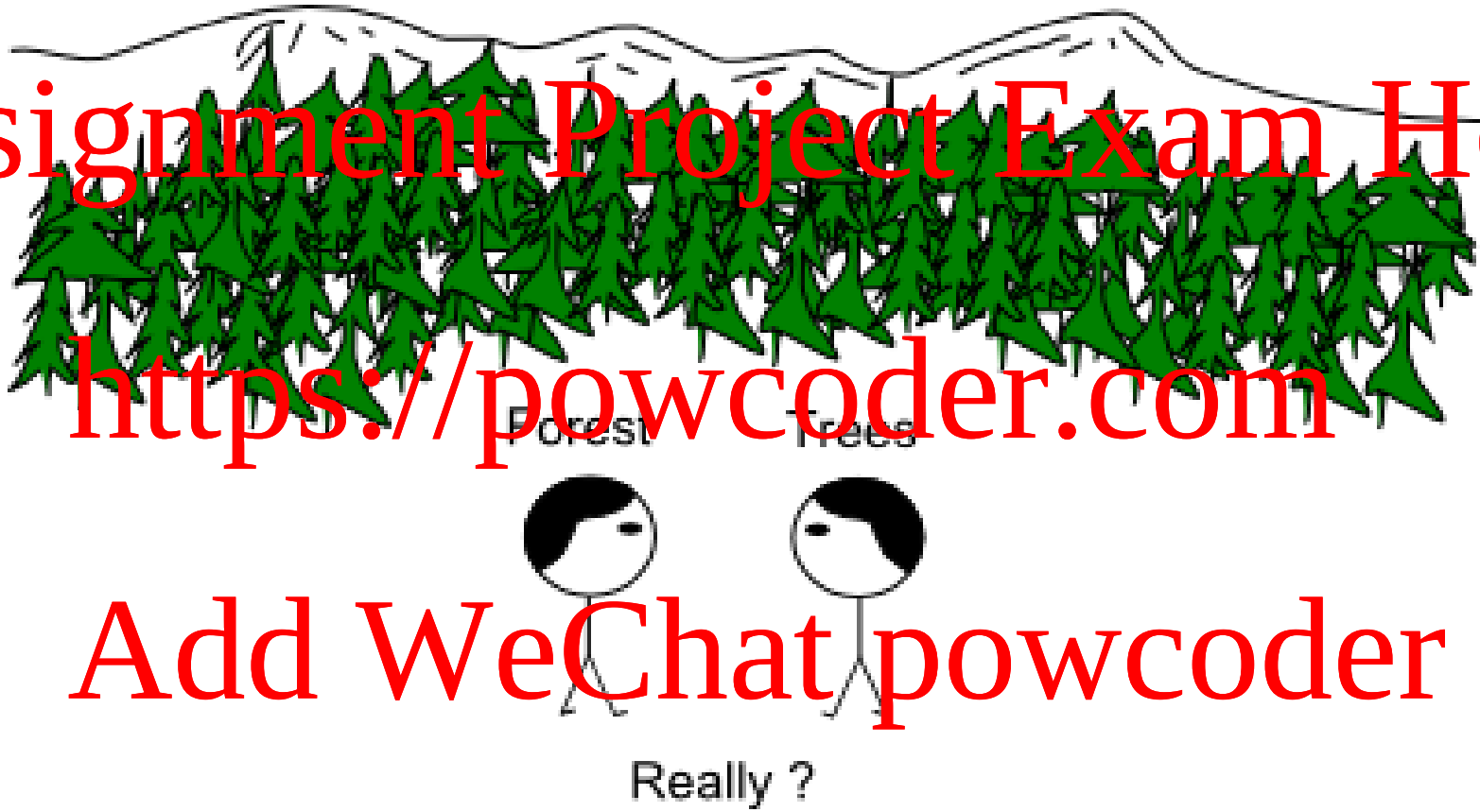


Can't see the forest for the trees



Inductive definition of a binary tree

- The empty binary tree `Empty` is a binary tree
- If `l` and `r` are binary trees and `v` is a value of type `'a`, then `Node(v, l, r)` is a binary tree.
- Nothing else is a binary tree.

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  Empty
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Let's do some programming with trees!

How to prove it?

Step 2: How to reason inductively about trees?

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- Analyze their structure

How to prove it?

Step 2: How to reason inductively about trees?

Analyze their structure

The recipe ...

To prove a property `P(t)` holds about a binary tree `t`

Base Case: `t = Empty`

Show `P(Empty)` holds

Step Case: `t = Node(x, l, r)`

IH `P(l)`

IH `P(r)`

Assume the property `P` holds for trees smaller than `t`.

Show `P(Node(x, l, r)` holds

Show the property `P` holds for the tree `t`.

Let's prove something!

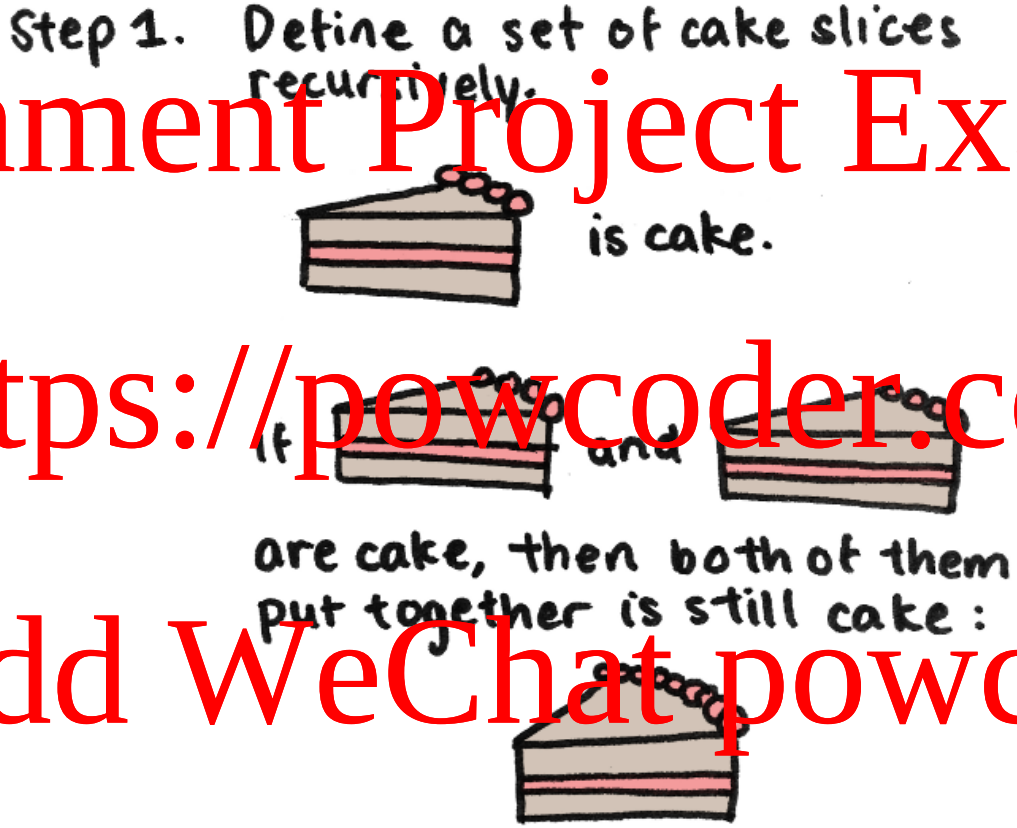
```
let rec insert ((x,d) as e) t = match t with
  | Empty                      -> Node(e, Empty, Empty)
  | Node((y,d'), l, r) ->
    if x = y then  Node(e, l, r)
    else  (if x < y then Node((y,d'), insert e l, r)
           else  Node((y,d'), l, insert e r))
```


Theorem: For all trees `t`, keys `x`, and data `dx`,

`lookup x (insert (x, dx) t)` $\Longrightarrow^*$ `Some dx`


```
let rec lookup x t = match t with
  | Empty                  -> None
  | Node ((y,d), l, r) ->
  if x = y then  Some(d)
  else (if x < y then lookup x l
        else lookup x r)
```

Remember the slice of cake?



Give an OCaml data type definition for cake!