COMP3141

Software System Design and Implementation

Static Assurance with Types

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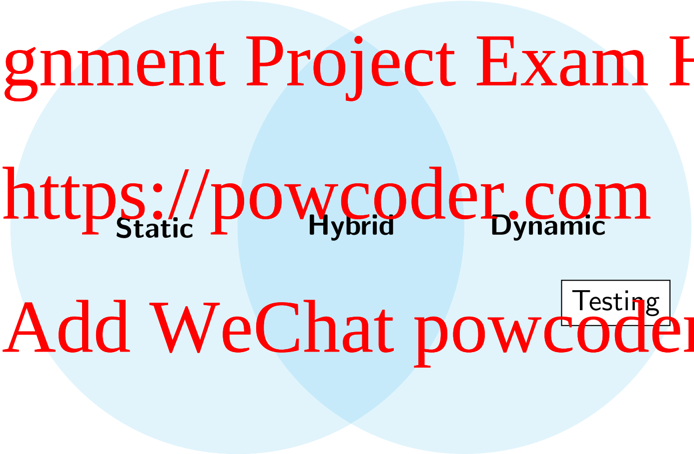
Methods of Assurance



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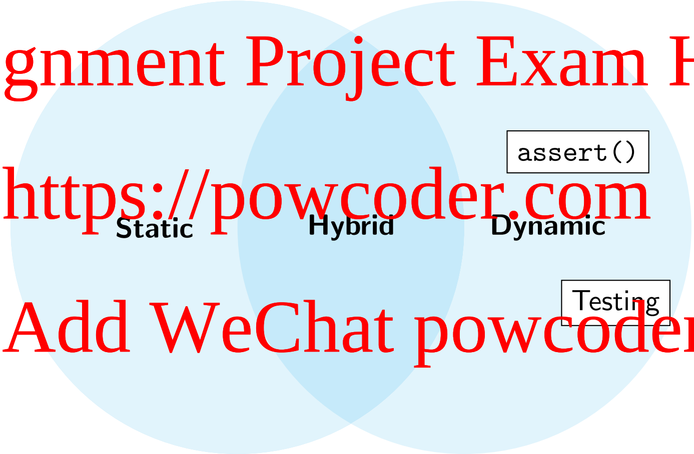

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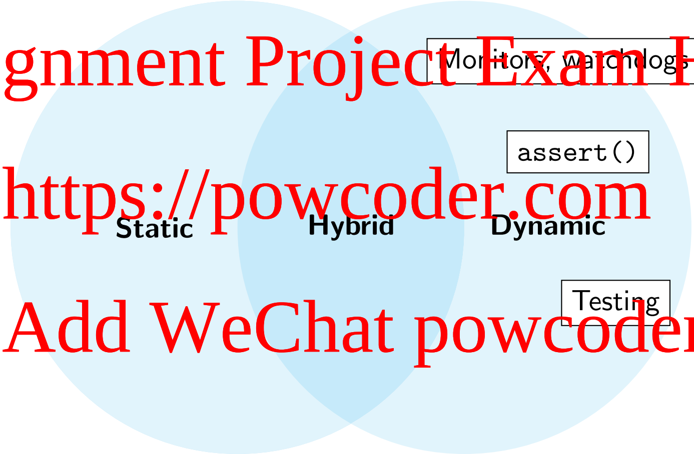

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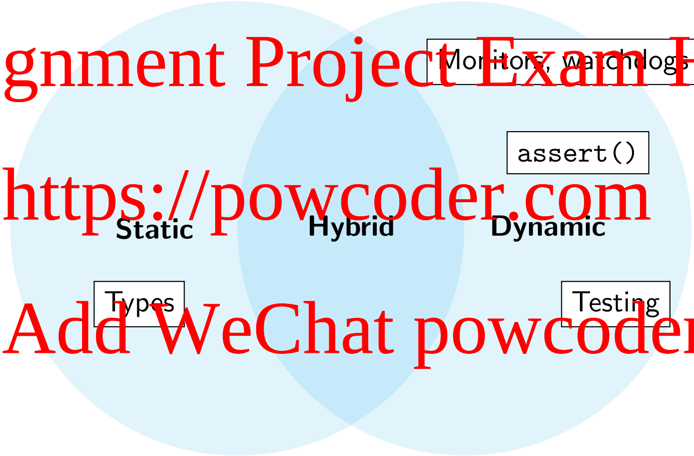

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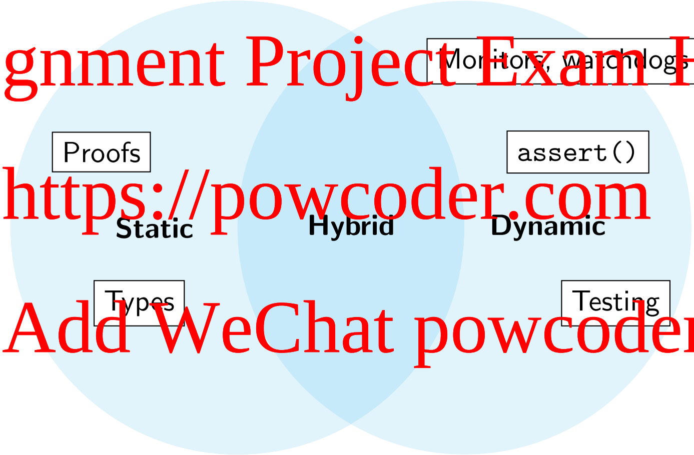

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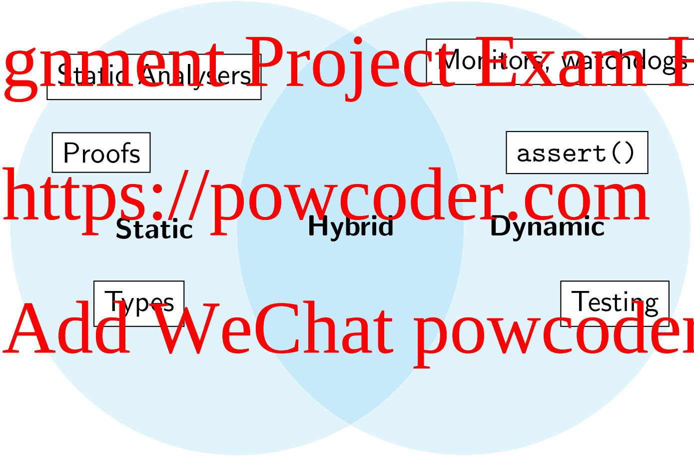

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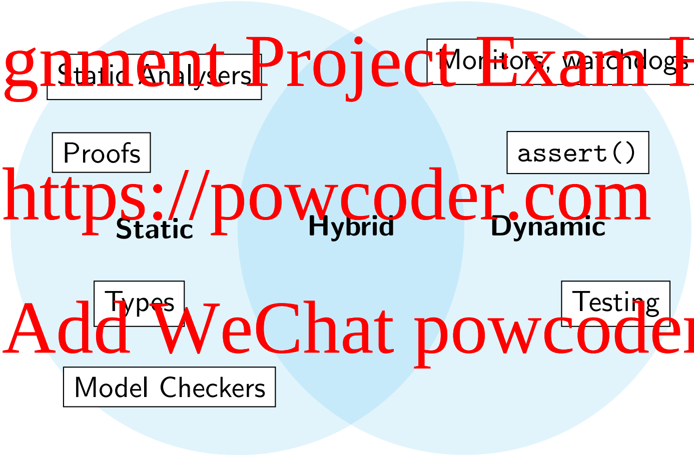

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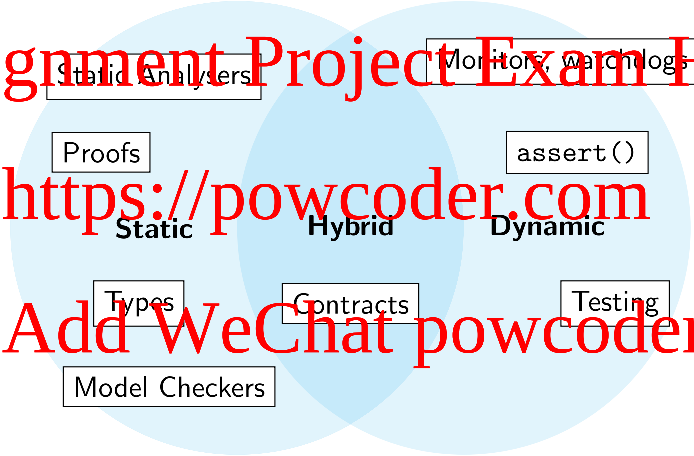

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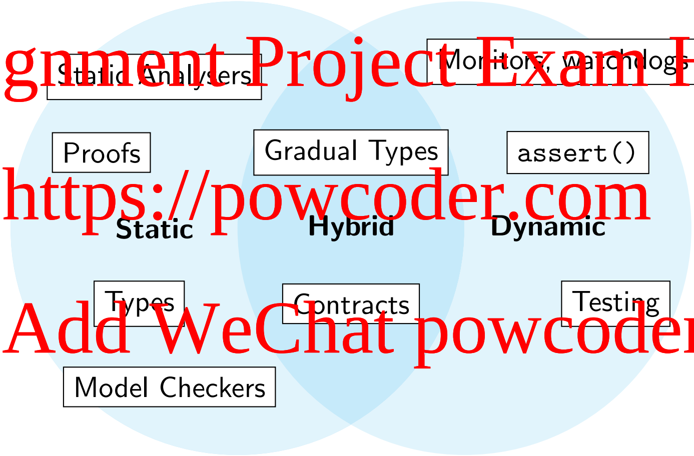

Static means of assurance analyse a program without running it.

Static vs. Dynamic

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An exhaustive check is a check that is able to analyse all possible executions of a program.

- **However**, some properties cannot be checked statically in general (halting problem), or are intractable to feasibly check statically (state space explosion).

- Dynamic checks cannot be exhaustive, but can be used to check some properties where static methods are unsuitable.

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Types

Because types are integrated into the compiler, they cannot diverge from the source code. This means that type signatures are a kind of machine-checked documentation for your code.

Types

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- They are checked automatically by the compiler.
- They can be extended to encompass properties and proof systems with very high expressivity (covered next week).
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This week, we'll look at techniques to encode various correctness conditions inside Haskell's type system.

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Definition

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- We can use this parameter to track what data invariants have been established about a value.
- We can use this parameter to track information about the representation (e.g. units of measure).
- We can use this parameter to enforce an ordering of operations performed on these values (*type state*).

Validation

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data UG -- empty type
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data StudentID x = SID Int
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We can define a smart constructor that specialises the type parameter:

```haskell
sid :: Int -> Either (StudentID UG)
                     (StudentID PG)
```

(Recalling the following definition of Either)

```haskell
data Either a b = Left a | Right b
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(Recalling the following definition of Either)


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```

And then define functions:

```haskell
enrolInCOMP3141 :: StudentID UG -> IO ()
lookupTranscript :: StudentID x -> IO String
```

Units of Measure

In 1999, software confusing units of measure (pounds and newtons) caused a mars orbiter to burn up on atmospheric entry.


```haskell
data Kilometres
data Miles
data Value x = U Int
sydneyToMelbourne = (U 877 :: Value Kilometres)
losAngelesToSanFran = (U 383 :: Value Miles)
```

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In addition to tagging values, we can also enforce constraints on units:

```
data Square a
area :: Value m -> Value m -> Value (Square m)
area (U x) (U y) = U (x * y)
```

Note the arguments to area must have the same units.

Type State

Example

A `Socket` can either be ready to recieve data, or busy. If the socket is busy, the user must first use the `wait` operation, which blocks until the socket is ready. If the socket is ready, the user can use the `send` operation to send string data, which will make the socket busy again.

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```haskell
data Busy
data Ready
newtype Socket s = Socket ...

wait :: Socket Busy -> IO (Socket Ready)

send :: Socket Ready -> String -> IO (Socket Busy)
```

What assumptions are we making here?

Linearity and Type State

The previous code assumed that we didn't re-use old Sockets:

```
send2 :: Socket Ready -> String -> String
      -> IO (Socket Busy)
send2 s x y = do s'   <- send s x
                 s''  <- wait s'
                 s''' <- send s'' y
                 pure s'''
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But we can just re-use old values to send without waiting:


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Linear type systems can solve this, but not in Haskell (yet).

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Recall

Haskell types themselves have types, called kinds. Can we make the kind of our tag types more precise than $*$?

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Haskell types themselves have types, called kinds. Can we make the kind of our tag types more precise than $*$?

The `DataKinds` language extension lets us use data types as kinds:

```
{-# LANGUAGE DataKinds, KindSignatures #-}
data Stream = UG | PG
data StudentID (x :: Stream) = SID Int
-- rest as before
```

Motivation: Evaluation


```haskell
data Expr = BConst Bool
          | IConst Int
          | Times Expr Expr
          | Less Expr Expr
          | And Expr Expr
          | If Expr Expr Expr
data Value = BVal Bool | IVal Int
```

Example

Define an expression evaluator:

```haskell
eval :: Expr -> Value
```

Motivation: Partiality

Unfortunately the `eval` function is partial, undefined for input expressions that are not well-typed, like:

```
And (ICons 3) (BConst True)
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With any partial function, we can make it total by either expanding the co-domain (e.g. with a `Maybe` type), or constraining the domain.

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Recall

With any partial function, we can make it total by either expanding the co-domain (e.g. with a `Maybe` type), or constraining the domain.

Can we use phantom types to constrain the domain of `eval` to only accept well-typed expressions?

Attempt: Phantom Types

Let's try adding a phantom parameter to `Expr`, and defining typed constructors with precise types:

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```
data Expr t = ...
bConst :: Bool -> Expr Bool
bConst = BConst
iConst :: Int -> Expr Int
iConst = IConst
times :: Expr Int -> Expr Int -> Expr Int
times = Times
less :: Expr Int -> Expr Int -> Expr Bool
less = Less
and :: Expr Bool -> Expr Bool -> Expr Bool
and = And
if'  :: Expr Bool -> Expr a -> Expr a -> Expr a
if' = If
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This makes invalid expressions into type errors (yay!):

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Bad News

Inside `eval`, the Haskell type checker cannot be sure that we used our typed constructors, so in e.g. the `IConst` case


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eval :: Expr t -> t
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We are unable to tell that the type `t` is definitely `Int`.

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Phantom types aren't strong enough!

GADTs

Generalised Algebraic Datatypes (*GADTs*) is an extension to Haskell that, among other things, allows data types to be specified by writing the types of their constructors:


```
{-# LANGUAGE GADTs, KindSignatures #-}
-- Unary natural numbers, e.g. 3 is S (S (S Z))
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When combined with the *type indexing* trick of phantom types, this becomes very powerful!

Expressions as a GADT

```
data Expr :: * -> * where
    BConst :: Bool -> Expr Bool
    IConst :: Int -> Expr Int
    Times :: Expr Int -> Expr Int -> Expr Int
    Less :: Expr Int -> Expr Int -> Expr Bool
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GHC now knows that if we have `IConst`, the type `t` must be `Int`.

Lists

We could define our own list type using GADT syntax as follows:

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data List (a :: *) :: * where
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hd (Cons x xs) = x
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We will constrain the domain of these functions by tracking the length of the list on the type level.

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As before, define a natural number kind to use on the type level:

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Now `hd` and `tl` can be total:

```
hd :: Vec a (S n) -> a
hd (Cons x xs) = x
tl :: Vec a (S n) -> Vec a n
tl (Cons x xs) = xs
```

Vectors, continued

Our map for vectors is as follows:

```
mapVec :: (a -> b) -> Vec a n -> Vec b n
mapVec f Nil = Nil
mapVec f (Cons x xs) = Cons (f x) (mapVec f xs)
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```


Properties

Using this type, it's impossible to write a mapVec function that changes the length of the vector.

Properties are verified by the compiler!

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The benefits of this extra static checking are obvious, however:

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Pragmatism

We should use type-based encodings only when the assurance advantages outweigh the clarity disadvantages.
The typical use case for these richly-typed structures is to eliminate partial functions from our code base.
If we never use partial list functions, length-indexed vectors are not particularly useful.

Appending Vectors

Example (Problem)

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appendV :: Vec a m -> Vec a n -> Vec a ???
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This function is not applicable to type-level `Nat`s, though.

$\Rightarrow$ we need a type level function.

Type Families

Type level functions, also called *type families*, are defined in Haskell like so:

```haskell
{-# LANGUAGE TypeFamilies #-}
type family Plus (x :: Nat) (y :: Nat) :: Nat where
  Plus Z     y = y
  Plus (S x) y = S (Plus x y)
```


We can use our type family to define appendV:

```haskell
appendV :: Vec a m -> Vec a n -> Vec a (Plus m n)
appendV Nil         ys = ys
appendV (Cons x xs) ys = Cons x (appendV xs ys)
```

Recursion

If we had implemented `Plus` by recursing on the second argument instead of the first:

```
{-# LANGUAGE TypeFamilies #-}
type family Plus' (x :: Nat) (y :: Nat) :: Nat where
  Plus' x Z     = x
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```

Why?

Answer

Consider the `Nil` case. We know `m = Z`, and must show that our desired return type `Plus' Z n` equals our given return type `n`, but that fact is not immediately apparent from the equations.

Type-driven development

- This lecture is only a taste of the full power of type-based specifications.
- Languages supporting dependent types (Idris, Agda) completely merge the type and value level languages, and support machine-checked proofs about programs.
- Haskell is also gaining more of these typing features all the time.

Next week: Fancy theory about types!

- Deep connections between types, logic and proof.
- Algebraic type structure for generic algorithms and refactoring.
- Using polymorphic types to infer properties for free.

Homework

1. Assignment 2 is released. Due on **7th August, 9 AM**.
2. The last programming exercise has been released, due next week.
3. This week's quiz is also up, due in Friday of Week 9.