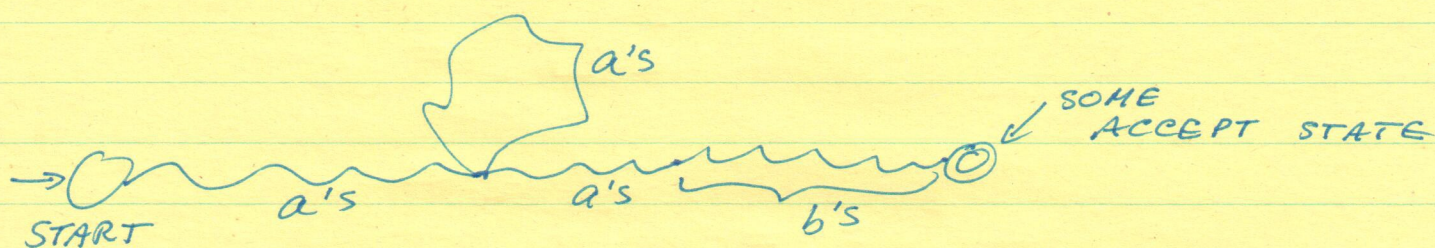# PUMPING LEMMA

No DFA can accept $L = \{a^n b^n \mid n \geq 0\}$ because no DFA can count how many a's there are.

<u>Idea</u>   Suppose we have a putative recognizer for L; this is is a DFA. Suppose it has n states. Choose a string of the form $a^m b^m$ where $m > n$.

SOME ACCEPT STATE

START   a's   a's   b's

Sub... ...twice as it reads the a's. So there is a loop of size, say, k driven by a's. B... ...able to recognize

$a^{m+k} b^m$: going around the loop twice
$a^{m+2k} b^m$: going around the loop 3 times
$\vdots$

also $a^{m-k} b^m$: skipping the loop

FORMAL STATEMENT: If L is a regular language
$\exists\, p \in \mathbb{N}$ $p > 0$ s.t. $\forall\, w \in L$ with $|w| \geq p$
$\exists\, x, y, z \in \Sigma^*$ s.t. $w = xyz$ & $|xy| \leq p$ & $|y| > 0$
$\forall\, i \in \mathbb{N}$ $xy^i z \in L$

L regular $\Rightarrow$ L can be pumped
$\Updownarrow$

L cannot be pumped $\Rightarrow$ L is not regular

NOTE

L can be pumped does <u>NOT</u> imply
L is regular

CONTRAPOSITIVE : Suppose $L \subseteq \Sigma^*$ is a language
~~then~~ s.t. $\forall p > 0 \; \exists w \in L$ with $|w| \geq p$ s.t.

$\quad \forall x, y, z \in \Sigma^*$ s.t. $\omega = xyz$ & $|xy| \leq p$ & $|y| > 0$

$\quad\quad \exists i \in \mathbb{N}$ s.t. $xy^i z \notin L$

then $\quad L$ is not regular.

How to cope with all those quantifiers?
$\quad\quad\quad$ G A M E S.

$\forall:$ $\quad$ Demon $\quad\quad\quad\quad$ $\exists:$ You

(1) $\quad$ Demon chooses p
(2) $\quad$ You choose $\quad\quad\quad w$ $\quad\quad$ with $|w| \geq p$
(3) $\quad$ Demon ~~Assignment Project Exam Help~~ tions above
(4) $\quad$ You choose i & ~~show~~ $xy^i z \notin L$

$\quad$ https://powcoder.com

Demon's choices are (1) symbolic to cover all

cases & (3) ~~Add WeChat powcoder~~ demonic

choices.

EXAMPLE $\quad\quad L = \{ a^n b^n \mid n \in \mathbb{N} \}$

(1) $\quad$ Demon chooses p
(2) $\quad$ I choose $\quad a^p b^p$
(3) $\quad$ Demon has to choose x, y, z with $|xy| \leq p$
$\quad\quad$ so x, y consist purely of a's & $y \neq \varepsilon$ since $|y| > 0$
(4) $\quad$ I choose $i = 2$ so the string $xy^i z$ is
$\quad\quad a^{p+i\cdot l} b^p$ where $l = |y| > 0$
$\quad\quad p + i \cdot l \neq p$ so this string is not in L.

Thus L is not regular