Quiz 1: Answer and discussion

Prakash Panangaden

17th January 2021

Question 1 We have a set of cards. Each card has a face that is coloured RED or GREEN but never both. On the opposite face of the card a positive integer is printed. These statements are not in doubt. How we are told that if a card has a red face then the integer on the other side is even. We do not know whether to believe this statement. We see 4 cards on the table, we can only see one face of each card but, if we want, we can turn over any of the cards and see the other face. Now we see the following: the first card is red, the second has the number 6, the third has a green face and the fourth has the number 5. In order to test the claim that if a card has a red face the other face contains an even number which cards *must* we turn over.

Solution We must turn over the red card to see if it has an even number and we must turn over the card with the 5 to make sure the other side is not red. There is no reason to turn over the card with a 6, so what if the other side is green? I did **not** say even number *if and only if* red. Similarly, there is no reason to turn over the green card. So what if it happens to have an even number. This question tested whether you understood the difference between implication and bi-implication or two-way implication.

Question 2 In this question we are considering quantified statements about positive integers:

(a) $\forall n\ \exists m\ m = n + 1$

(b) $\exists m\ \forall n\ m = n + 1$

(c) $\forall n\ \forall m\ m = n + 1$

(d) $\exists m\ \exists n\ m = n + 1$

Which statements are true?

Solution (a) and (d).

Question 3 We define a binary relation on the set of students as follows: student A is related to student B if they have the same first name. The last name is irrelevant. Which of the following statements are correct?

(a) This is an equivalence relation but not a partial order.

(b) It is both an equivalence relation and a partial order.

(c) It is a partial order but not an equivalence relation.

(d) It is neither an equivalence relation nor a partial order.

Solution (a) only.

Question 4 We define a binary relation on the set of positive integers as follows: We say n is related to m if the last digit in the decimal notation for n is less than or equal to the last digit in the decimal notation for m. Which of the following statements are true?

(a) This is an equivalence relation but not a partial order.

(b) It is a partial order but not an equivalence relation.

(c) It is neither a partial order nor an equivalence relation.

(d) It is both an equivalence relation and a partial order.

Solution (c) only.

Question 5 Consider the positive prime numbers with the usual numerical order relation. Which of the following statements is true? There is only one correct answer.

(a) This set with the given ordering is both well-founded and totally ordered.

(b) This set with the given ordering is not well-founded because it does not include 0, nor is it totally ordered.

(c) This set is not totally ordered because you cannot compare prime numbers.

(d) This set is totally ordered but not well founded.

Solution (a).