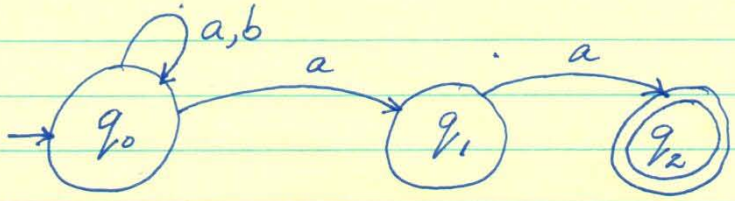
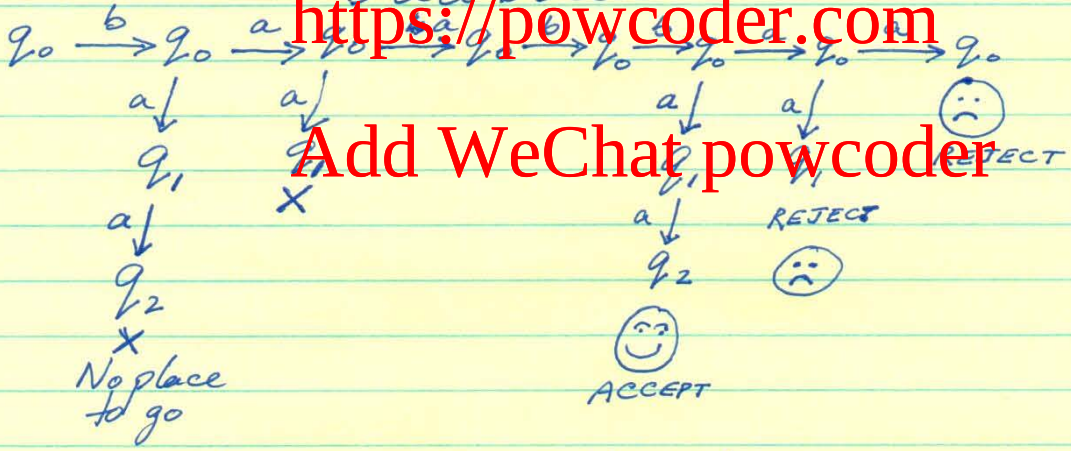
NFAs

How do we design a machine to accept all words ending in "aa"? Not very hard to make a DFA (do it!) but what if we had a machine that could guess?



Hey! Two transitions labelled "a" coming out of q_0. Nothing labelled with a "b" coming out of q_1 & nothing at all coming out of q_2. NFA = nondeterministic = guessing + backtracking + reject if there is no move.



It does not matter that some paths jammed or that some got to a reject state. at least one (more is OK) path must get to an accept state.

We can *always* construct a DFA that recognizes the same language as an NFA. In fact this can be done *algorithmically* and is a very important piece of software. The DFA one gets may be very complicated. Key point: You can use nondeterminism as a *design aid* & then use the algorithm to convert the NFA into an equivalent DFA.

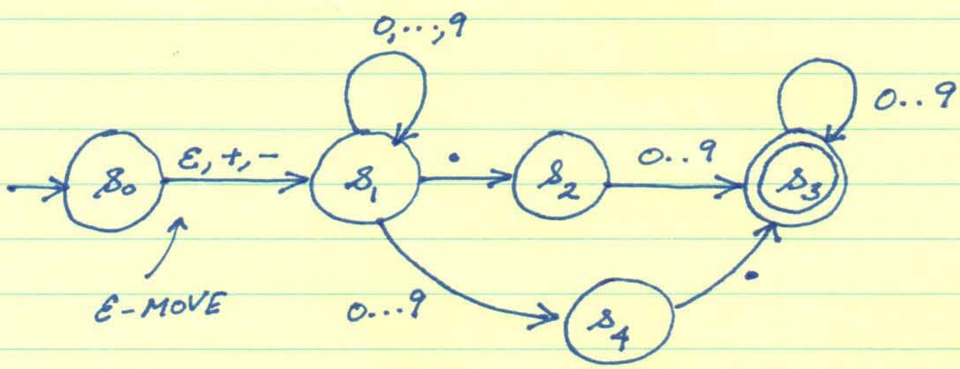
Another extension: NFA+ε moves: this *can* jump *without* reading a letter.

EXAMPLE A decimal number has (i) an *optional* + or − sign (ii) a sequence of digits (iii) a decimal point (iv) another sequence of digits.

Either (ii) or (iv) can be empty but not both

$\Sigma_1 = \{ \cdot, +, -, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$

307.3 [illegible] .1, . . .



At s_1 it guesses whether you want to put digits after the decimal. If it thinks that there will be no digits after the • it will go to s_4 but it forces at least one digit before the •.

Formal Def of NFA

Σ : alphabet is fixed

An NFA has 4 things

(1) Q : a finite set of states

(2) $Q_0 \subseteq Q$: a set of start states

(3) $F \subseteq Q$: a set of accept states

(4) A transition function

$$\Delta : Q \times \Sigma \longrightarrow 2^Q$$

(state, letter) $\longmapsto$ set of possible next states

We write 2^Q for the set of all subsets of Q.

Note $\phi \in 2^\phi$ so Δ could say that there is no next state in the projection.

NFA+ε : same (1), (2) & (3) but

(4') $\Delta : Q \times (\Sigma \cup \{\varepsilon\}) \longrightarrow 2^Q$

It is nice to write $s \xrightarrow{a} s'$ instead of $s' \in \Delta(s,a)$.

For NFA without ε:

We define Δ^* by induction $Q \times \Sigma^* \longrightarrow 2^Q$

$$\Delta^*(q, \varepsilon) = \{q\}$$

$a \in \Sigma$. $\quad \Delta^*(q,a) = \Delta(q,a)$

$w \in \Sigma^*, a \in \Sigma \quad \Delta^*(q, wa) = \bigcup_{q' \in \Delta^*(q,w)} \Delta(q', a)$

If N is an NFA

$$L(N) = \{ w \in \Sigma^* \mid \exists\, q \in Q_0,\ F \cap \Delta^*(q,w) \neq \phi \}$$

One of the places where the machine may end up is an accept state.