$L = \{a^q \mid q \text{ a prime number}\}$

Demon picks pumping length p

I pick a^n where $n > p$ & n is a prime

Demon has to pick $y = a^k$ where $0 < k \leq p$.

I choose $i > 1$ exact value is deferred

New string is $y = a^{n+(i-1)k}$

so pick $i = n+1$ $\quad y = a^{n+nk} = a^{n(k+1)}$

$n(k+1)$ is definitely not a prime number.

$L = \{a^n b^m \mid n \neq m\}$ hard to do with pumping

& $\overline{L}$ is a mess

but $L \cap a^* b^* = \{a^n b^n \mid n \geq 0\}$ and we know

this is not regular

$L = \{a^i b^j \mid i > j\}$

Demon picks p

I pick $a^p b^{p-1}$

Demon is forced to pick $y = a^k$, $0 < k \leq p$.

I choose $i = 0$

The new string is $a^{p-k} b^{p-1}$

$p-k$ is <u>not</u> strictly greater than $p-1$.

$\Sigma = \{0, 1, +, =\}$

$L = \{x+y=z \mid x, y, z \in \{0,1\}^*$ & the equation is vali

Demon picks p

I pick $\underbrace{11\cdots1}_{p} + 0 = \underbrace{11\cdots1}_{p}$

Now demon has to pick y in the leading block of 1'

I can choose any $i \neq 1$ to win.

$L = \{ a^i b^j \mid \gcd(i,j) = 1 \}$. We will show $\bar{L}$ is not regular

$\bar{L} = \Sigma^* / L$

Demon chooses p

I choose $q > p+1$ & such that q is a prime number.

My chosen string is $a^q b^q \in \bar{L}$.

Demon is forced to choose $y = a^k \quad 0 < k \leq p$.

I pump "down" i.e. choose $i = 0$ so now

$$xz = a^{q-k} b^q$$

Note $\quad q - k > (p+1) - k > 0$ in fact > 1

So $\gcd(q-k, q) = 1$ hence

$xz \notin \bar{L}$. Thus $\bar{L}$ is not regular so L is not regular.

$\Sigma = \{a, b, c\}$

$L = \{ x c y \mid x, y \in (a+b)^*, \#_a(x) = \#_b(y) \}$

Demon chooses p

I choose $\quad a^p c b^p$

Demon is forced to choose $y = a^k \quad 0 < k \leq p$

I choose $\quad i = 2$

$$x y^2 z = a^{p+k} c b^p \notin L.$$

$\Sigma = \{0, 1\}$

Given $X \subseteq \mathbb{N}$ we define

unary$(X) = \{ 1^n \mid n \in X \}$

binary$(X) = \{ w \in \Sigma^* \mid w$ interpreted as a binary number $\in X \}$.

If binary(X) is regular does it mean unary(X) is regular?

<u>NO!</u> Consider $X = \{ 2^n \mid n \geq 0 \}$.

$$\text{binary}(X) = 0^* 1 0^*$$

$$\text{unary}(X) = \{ 1^n \mid n = 2^m \text{ for some } m \}.$$

$\Sigma = \{a, b\}$

$\{ w \in \Sigma^* \mid \#_a(w) \neq \#_b(w) \}$

Demon picks p

I pick $a^p b^{p!+p}$

Demon is forced to pick $y = a^k$; $p \geq k > 0$.

I pick $i = (p!/k) + 1$.

$$xy^iz = a^{p+(i-1)k} b^{p!+p}$$
$$= a^{p+p!} b^{p!+p} \notin L.$$

However, this is a clever arithmetic trick.

Here is a simpler way:

$\{a^n b^n \mid n \geq 0\}$ is not regular.

$\bar{L} \cap a^*b^* = \{a^n b^n \mid n \geq 0\}$

Hence $\bar{L}$ is not regular so L is not regular.

If $S \subseteq \mathbb{N}$ we define unary$(S) = \{ 1^n \mid n \in S \}$ &

binary$(S) = \{ w \in \{0,1\}^* \mid w$ read as binary number $\in S\}$.

If binary(S) is regular does unary(S) have to be regular

<u>No!</u> $S = \{ 2^n \mid n \geq 1\}$

binary(S) = 100^* so clearly regular.

unary(S) is not regular

Demon picks p

I pick 1^{2^p}

Demon picks x, y, z s.t $|xy| \leq p$, $|y| > 0$ & $xyz = 1^{2^p}$

so $k := |y| \leq p < 2^p$

I Pick $i = 2$ $\quad xy^2z = 1^{2^p+k}$

$$2^p < 2^p + k \leq 2^p + p < 2^p + 2^p = 2^{p+1}$$

so the length of the new string is <u>strictly</u> between two consecutive powers of 2 & hence cannot be a power of 2.