$L = \{a^q \mid q \text{ a prime number}\}$

Demon picks pumping length p

I pick a^n where $n > p$ & n is a prime

Demon has to pick $y = a^k$ where $0 < k \leq p$.

I choose $i > 1$ exact value is deferred

New string is $y = a^{n+(i-1)k}$

so pick $i = n+1$ $y = a^{n+nk} = a^{n(k+1)}$

$n(k+1)$ is definitely not a prime number.

$L = \{a^n b^m \mid n \neq m\}$ hard to do with pumping

& $\overline{L}$ is a mess

but $L \cap a^* b^* = \{a^n b^n \mid n \geq 0\}$ and we know

this is not regular

$L = \{a^i b^j \mid i > j\}$

Demon picks p

I pick $a^p b^{p-1}$

Demon is forced to pick $y = a^k$, $0 < k \leq p$.

I choose $i = 0$

    The new string is $a^{p-k} b^{p-1}$

     $p-k$ is <u>not</u> strictly greater than $p-1$.

$\Sigma = \{0, 1, +, =\}$

   $L = \{x+y=z \mid x, y, z \in \{0,1\}^* \text{ & the equation is vali}$

Demon picks p

I pick $\underbrace{11\cdots 1}_{p} + 0 = \underbrace{11\cdots 1}_{p}$

Now demon has to pick y in the leading block of 1

I can choose any $i \neq 1$ to win.

$L = \{a^i b^j \mid \gcd(i,j) = 1\}$. We will show L is not regular
$$\overline{L} = \frac{1}{2}/a^i$$

Demon chooses p

I choose $q > p+1$ & such that q is a prime number.

My chosen string is $a^q b^q \in L$.

Demon is forced to choose $y = a^k$ $0 < k \le p$.

Here I pump "down" i.e. choose $i = 0$ so now
$$xz = a^{q-k} b^q$$

Note $q - k > (p+1) - k > 0$ in fact > 1

So $\gcd(q-k, q) = 1$ hence

$xz \in L$. ... not regular.

$\Sigma = \{a, b, \#, c\}$

$L = \{ x \# c y \mid x, y \in \{(a+b)^*, \#_a(x) = \#_b(y)\}$

Demon chooses p

I choose $a^p c b^p$

Demon is forced to choose $y = a^k$ $0 < k \le p$

I choose $i = 2$
$$x y^2 z = a^{p+k} c b^p \notin L.$$

$\Sigma = \{0, 1\}$

Given $X \subseteq \mathbb{N}$ we define

unary$(X) = \{1^n \mid n \in X\}$

binary$(X) = \{w \in \Sigma^* \mid w$ interpreted as a binary number $\in X\}$.

If binary(X) is regular does it mean unary(X) is regular?

NO! Consider $X = \{2^n \mid n \ge 0\}$.

binary$(X) = 0^* 1 0^*$

unary$(X) = \{1^n \mid n = 2^m$ for some $m\}$

$$\Sigma = \{a, b\}$$
$$\{w \in \Sigma^* \mid \#_a(\omega) \neq \#_b(\omega)\}$$

Demon picks
I pick $\quad a^p b^{p!}$ $p+p$
Demon is forced to pick $y = a^k; p \geq k > 0$.
I pick $\quad i = (p!/k) + 1$.

$$xy^i z = a^{p+(i-1)k} b^{p!+p}$$
$$= a^{p+p!} b^{p!+p} \notin L.$$

However, this is a clever arithmetic trick.
Here is a simpler way:
$\quad \{a^n b^n \mid n \geq 0\}$ is not regular.
$L \cap$

Hence L is not regular so L is not regular.

If $S \subseteq \mathbb{N}$ we define unary $(S) = \{1^n \mid n \in S\}$ &
binary $(S) = \{ \quad \}$
If binary (S) is regular does unary (S) have to be regular

No! $\quad S = \{2^n \mid n \geq 1\}$

binary $(S) = 100^*$ so clearly regular.
unary (S) is not regular
Demon picks p
I pick 1^{2^p}
Demon picks x, y, z s.t $|xy| \leq p$, $|y| > 0$ & $xyz = 1^{2^p}$
so $\quad k := |y| \leq p < 2^p$
I Pick $i = 2 \qquad xy^2z = 1^{2^p+k}$
$$2^p < 2^p + k \leq 2^p + p < 2^p + 2^p = 2^{p+1}$$
so the $\underset{\wedge \text{ length of the}}{\text{new}}$ string is __strictly__ between two
consecutive powers of 2 & hence cannot be a
power of 2.