The pumping lemma for context-free languages

Tuesday, March 9, 2021 11:21 AM

Context-free languages

$S \to aSb \mid \varepsilon$

$L(G) = \{a^n b^n \mid n \geq 0\}$

$(ab)^*$ is a regular language with equal # of a's & b's.

$\{a^n b^n c^n \mid n \geq 0\}$ is NOT context-free.

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Recall every CFL has a CFG in Chomsky Normal form.

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$A \to BC$

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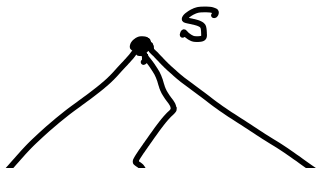
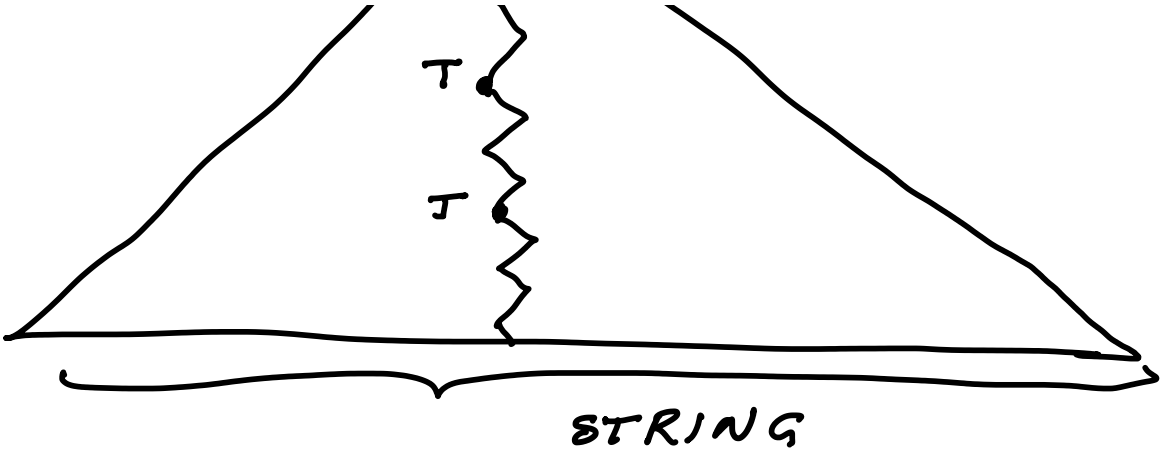
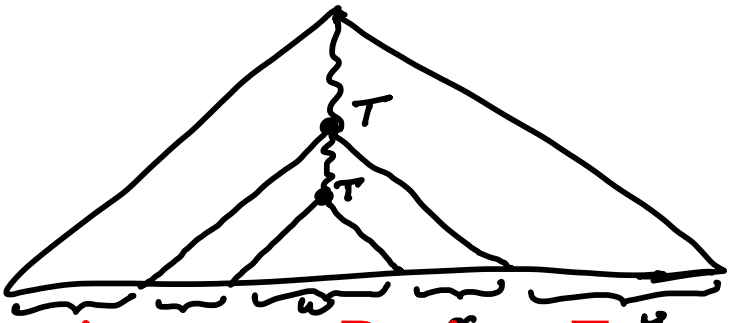
$A \to a$

When we are using a CNF grammar the parse tree is binary.

A grammar has only finitely many non-terminals. So if we generate a sufficient deep tree we must start repeating the non-terminals.







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$S \xrightarrow{*} uvwxy \qquad T \xrightarrow{*} vwx \qquad T \xrightarrow{*} w$

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V, x cannot both be empty (CNF)

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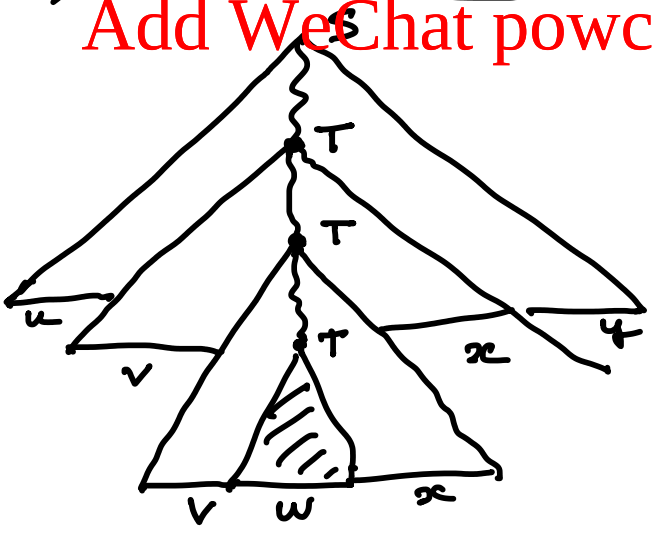
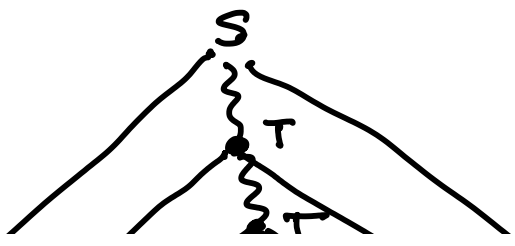
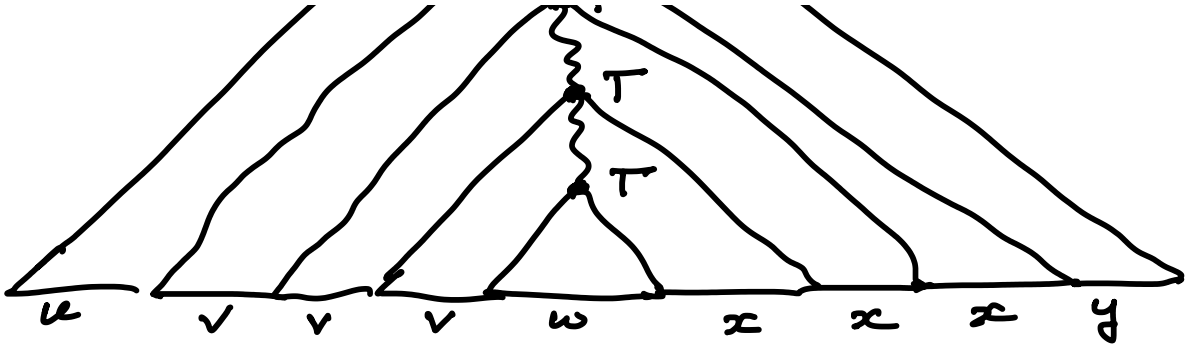


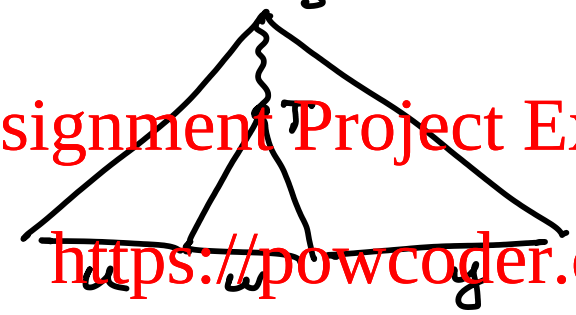
So $u\,v^2\,w\,x^2\,y \in L$





$u\,v\,v\,v\,w\,x\,x\,x\,y \in L$

$$\forall i \geq 0 \quad u v^i w x^i y \in L$$



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$u w y \in L$

Full statement of the pumping lemma:

$\forall$ CFLs L $\exists\ p > 0$

$\forall s \in L \quad |s| \geq p$

$\exists\ u, v, w, x, y \in \Sigma^*$ such that

- $s = u v w x y$
- $|vx| \neq 0$
- $|vwx| \leq p$

$\forall i \geq 0 \quad u v^i w x^i y \in L$

CFL ⇒ every string can be pumped

CONTRA POSITIVE:

if there is even one string that cannot be pumped

⇒ not a CFL

Fix L some language

If $\forall p > 0$

$\exists s \in L \text{ s.t. } |s| \geq p$

$\forall u, v, w, x, y \in \Sigma^* \text{ s.t.}$

$- s = uvwxy$

$- |vx| > 0$

$\rightarrow \boxed{- |vwx| \leq p}$

$\exists i \geq 0 \quad u v^i w x^i y \notin L$

then L is NOT a CFL.

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EXAMPLES

(1) $L = \{ a^n b^n a^n \mid n \geq 0 \}$

- Demon chooses $p > 0$

- I choose $a^b b^b a^b$

Have to analyze how the adversary may break up the string into u, v, w, x, y:

- v (or x) crosses a boundary between two blocks: $i = 2$, now a's and b's are out of order
- v, x both contain only a's then have to be in the same block: now choose $i = 2$ (or 3 (2?) the 3 blocks do not have the same length.
- same argument if both v, x contain only b's
- v contains only a's and x contains only b's: now one block is untouched by pumping: $i = 2$ and the block lengths cannot match

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EXAMPLE 2 Σ: arbitrary $|\Sigma| \geq 2$

$\rightarrow L = \{ww \mid w \in \Sigma^*\}$ $L = \{w w^{rev} \mid w \in \Sigma^*\}$

L is not context free

FACT: if L is a CFL & R is regular then $L \cap R$ is a CFL.

$\hat{L} = L \cap a^* b^* a^* b^*$

$\hat{L} = \{a^i b^j a^i b^j \mid i, j \geq 0\}$

D even picks $p > 0$

I pick $a^p b^p a^p b^p$

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- v or x straddles a block boundary: $i=2$ letters are out of order
- If v, x are in the same block: choose $i=2$. Either 1st & 3rd blocks are mismatched or 2nd & 4th are mismatched
- If v is in the first block, x is in the 2nd block: choose $i=2$, now 1st & 3rd no longer match / also 2nd & 4th blocks don't match

Note : v, x must be in the same block or in adjacent blocks.

$\overline{L}$ is context-free : we have seen a PDA already and there is a grammar in the supplementary notes.

CFL's are not closed under complement.

- $L_1 = \{a^n b^n c^m \mid n, m \geq 0\}$ $L_2 = \{a^m b^n c^n \mid n, m$

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