COMP4418: Knowledge Representation and Reasoning

Nonmonotonic Reasoning

Maurice Pagnucco

ARC Centre of Excellence for Autonomous Systems and National ICT Australia
School of Computer Science and Engineering
The University of New South Wales
Sydney, NSW, 2052

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Nonmonotonic Reasoning

- Suppose you are told "Tweety is a bird"
- What conclusions would you draw?
- Now, consider being further informed that "Tweety is an emu"

- What conclusions would you draw now? Do they differ from the conclusions that you would draw without this information? In what way(s)?

- Nonmonotonic reasoning is an attempt to capture a form of *commonsense* reasoning

1. Nonmonotonicity

2. Closed World Assumption
3. Predicate Completion

4. Circumscription
5. Default Logic

6. Nonmonotonic Consequence
   - KLM Systems

Nonmonotonic Reasoning

- In classical logic the more facts (premises) we have, the more conclusions we can draw
- This property is known as *Monotonicity*

$$\text{If } \Delta \subseteq \Gamma, \text{ then } Cn(\Delta) \subseteq Cn(\Gamma)$$

(where Cn denotes classical consequence)

- However, the previous example shows that we often do not reason in this manner

- Might a nonmonotonic logic—one that does not satisfy the Monotonicity property—provide a more effective way of reasoning?

Why Nonmonotonicity?

- Problems with the classical approach to consequence
  - It is usually not possible to write down all we would like to say about a domain
  - Inferences in classical logic simply make implicit knowledge explicit; we would also like to reason with tentative statements
  - Sometimes we would like to represent knowledge about something that is not *entirely* true or false; uncertain knowledge
- Nonmonotonic reasoning is concerned with getting around these shortcomings

Makinson's Classification

Makinson has suggested the following classification of nonmonotonic logics:

- Additional background assumptions
- Restricting the set of valuations
- Additional rules

David Makinson, *Bridges from Classical to Nonmonotonic Logic*, Texts in Computing, Volume 5, King's College Publications, 2005.

Nonmonotonicity

- Classical logic satisfies the following property
- Monotonicity: If $\Delta \subseteq \Gamma$, then $Cn(\Delta) \subseteq Cn(\Gamma)$ (equivalently, $\Gamma \vdash \phi$ implies $\Gamma \cup \Delta \vdash \phi$)
- However, we often draw conclusions based on 'what is normally the case' or 'true by default'
- More information can lead us to retract previous conclusions
- We shall adopt the following notation
  - $\vdash$ classical consequence relation
  - $\mid\!\sim$ nonmonotonic consequence relation

Consequence Operation *Cn*

Other properties of consequence operation *Cn*:

Inclusion $\Delta \subseteq Cn(\Delta)$

Cumulative Transitivity $\Delta \subseteq \Gamma \subseteq Cn(\Delta)$ implies $Cn(\Gamma) \subseteq Cn(\Delta)$

Compactness If $\phi \in Cn(\Delta)$ then there is a finite $\Delta' \subseteq \Delta$ such that $\phi \in Cn(\Delta')$

Disjunction in the Premises

$$Cn(\Delta \cup \{a\}) \cap Cn(\Delta \cup \{b\}) \subseteq Cn(\Delta \cup \{a \vee b\})$$

Note: $\Delta \vdash \phi$ iff $\phi \in Cn(\Delta)$
alternatively: $Cn(\Delta) = \{\phi : \Delta \vdash \phi\}$

Example

Suppose I tell you ‘Tweety is a bird’
You might conclude ‘Tweety flies’
I then tell you ‘Tweety is an emu’
You conclude ‘Tweety does not fly’

$bird(Tweety) \mid\sim flies(Tweety)$
$bird(Tweety) \wedge emu(Tweety) \mid\sim \neg flies(Tweety)$

The Closed World Assumption

- A *complete* theory is one in which for every ground atom in the language, either the atom or its negation appears in the theory

- The *closed world assumption* (CWA) completes a base (non-closed) set of formulae by including the negation of a ground atom whenever the atom does not follow from the base

- In other words, if we have no evidence as to the truth of (ground atom) P, we assume that it is false
- Given a base set of formulae Δ we first calculate the *assumption* set

  $\neg P \in \Delta_{asm}$ iff for ground atom P, $\Delta \not\vdash P$
- $CWA(\Delta) = Cn\{\Delta \cup \Delta_{asm}\}$

Example

$$\Delta = \{P(a), P(b), P(a) \rightarrow Q(a)\}$$
$$\Delta_{asm} = \{\neg Q(b)\}$$

Theorem: The CWA applied to a consistent set of formulae Δ is inconsistent iff there are positive ground literals $L_1, \ldots, L_n$ such that $\Delta \models L_1 \vee \ldots \vee L_n$ but $\Delta \not\models L_i$ for $i = 1, \ldots, n$.

- Note that in the example above we limited our attention to the object constants that appeared in Δ however the language could contain other constants. This is known as the *Domain Closure Assumption* (DCA).
- Another common assumption is the *Unique-Names Assumption* (UNA).

If two ground terms can't be proved equal, assume that they are not.

Predicate Completion

Idea: The only objects that satisfy a predicate are those that must

- For example, suppose we have $P(a)$. Can view this as

  $$\forall x.\ x = a \rightarrow P(x)$$

  the *if*-half of a definition
- Can add the *only if* part:

  $$\forall x.\ P(x) \rightarrow x = a$$

- Giving:

  $$\forall x.\ P(x) \leftrightarrow x = a$$

Predicate Completion

- **Definition:** A clause is *solitary* in a predicate P if whenever the clause contains a postive instance of P, it contains only one instance of P.

  - For example, $Q(a) \lor P(a) \lor \neg P(b)$ is not solitary in P
    $Q(a) \lor R(a) \lor P(b)$ is solitary in P

- Completion of a predicate is only defined for sets of clauses solitary in that predicate

Predicate Completion

- Each clause can be written:
  $\forall y.\ Q_1 \wedge \ldots \wedge Q_m \rightarrow P(t)$ (P not contained in Q_i)
  $\forall y.\ \forall x.\ (x = t) \wedge Q_1 \wedge \ldots \wedge Q_m \rightarrow P(x)$
  $\forall x.(\forall y.\ (x = t) \wedge Q_1 \wedge \ldots \wedge Q_m \rightarrow P(x))$ (normal form of clause)
- Doing this to every clause gives us a set of clauses of the form:
  $\forall x.\ E_1 \rightarrow P(x)$
  $\vdots$
  $\forall x.\ E_n \rightarrow P(x)$
- Grouping these together we get:
  $\forall x.\ E_1 \vee \ldots \vee E_n \rightarrow P(x)$
- Completion becomes: $\forall x.\ P(x) \leftrightarrow E_1 \vee \ldots \vee E_n$
  and we can add this to the original set of formulae

Example

- Suppose $\Delta = \{\forall x.\ Emu(x) \rightarrow Bird(x),$
  $Bird(Tweety),$
  $\neg Emu(Tweety)\}$

- We can write this as
  $$\forall x.\ (Emu(x) \vee x = Tweety) \rightarrow Bird(x)$$
- Predicate completion of P in Δ becomes
  $$\Delta \cup \{\forall x.\ Bird(x) \rightarrow Emu(x) \vee x = Tweety\}$$

Circumscription

- **Idea:** Make extension of predicate as small as possible
- Example:

$$\forall x.Bird(x) \wedge \neg Ab(x) \rightarrow Flies(x)$$
$$Bird(Tweety), Bird(Sam), Tweety \neq Sam,$$
$$\neg Flies(Sam)$$

- Want to be able to conclude *Flies*(*Tweety*) but $\neg$*Flies*(*Sam*)

- Accept interpretations where *Ab* predicate is as "small" as possible
- That is, we *minimise abnormality*

Circumscription

- Given interpretations $\mathbf{I}_1 = \langle D, I_1 \rangle$, $\mathbf{I}_2 = \langle D, I_2 \rangle$, $\mathbf{I}_1 \leq \mathbf{I}_2$ iff for every predicate $P \in \mathbf{P}$, $I_1[P] \subseteq I_2[P]$.
- $\Gamma \models_{circ} \phi$ iff for every interpretation $\mathbf{I}$ such that $\mathbf{I} \models \Gamma$, either $\mathbf{I} \models \phi$ or there is a $\mathbf{I}' < \mathbf{I}$ and $\mathbf{I}' \models \Gamma$.

- ϕ is true in all minimal models
- Now consider

$$\forall x. Bird(x) \wedge \neg Ab(x) \rightarrow Flies(x)$$
$$\forall x. Emu(x) \rightarrow Bird(x) \wedge \neg Flies(x)$$
$$Bird(Tweety)$$

Reiter's Default Logic (1980)

- Add default rules of the form $\frac{\alpha:\beta}{\gamma}$
  - "If α can be proven and consistent to assume β, then conclude γ"
- Often consider *normal* default rules $\frac{\alpha:\beta}{\beta}$
- Example: $\frac{\mathit{bird}(x):\mathit{flies}(x)}{\mathit{flies}(x)}$
- Default theory $\langle D, W\rangle$
  D – set of defaults; W – set of facts
- *Extension* of default theory contains as many default conclusions as possible and must be consistent (and is closed under classical consequence Cn)
- Concluding whether formula ϕ follows from $\langle D, W\rangle$
  - Sceptical inference: ϕ occurs in *every* extension of $\langle D, W\rangle$
    Credulous inference: ϕ occurs in *some* extension of $\langle D, W\rangle$

Examples

- $W = \{\}$; $D = \{\frac{:p}{\neg p}\}$ – no extensions
- $W = \{p \vee r\}$; $D = \{\frac{p:q}{q}, \frac{r:q}{q}\}$ – one extension $\{p \vee r\}$
- $W = \{p \vee q\}$; $D = \{\frac{:\neg p}{\neg p}, \frac{:\neg q}{\neg q}\}$ – two extensions $\{\neg p, p \vee q\}$; $\{\neg q, p \vee q\}$
- $W = \{emu(Tweety), \forall x.emu(x) \rightarrow bird(x)\}$; $D = \{\frac{bird(x):flies(x)}{flies(x)}\}$ – one extension
- What if we add $\frac{emu(x):\neg flies(x)}{\neg flies(x)}$?
- Poole (1988) achieves a similar effect (but not quite as general) by changing the way the underlying logic is used rather than introducing a new element into the syntax

Default Theories—Properties

Observation: Every normal default theory (default rules are all normal) has an extension

Observation: If a normal default theory has several extensions, they are mutually inconsistent

Observation: A default theory has an inconsistent extension iff D is inconsistent

Theorem: (Semi-monotonicity)

Given two normal default theories $\langle D, W \rangle$ and $\langle D', W \rangle$ such that $D \subseteq D'$ then, for any extension $\mathcal{E}(D, W)$ there is an extension $\mathcal{E}(D', W)$ where $\mathcal{E}(D, W) \subseteq \mathcal{E}(D', W)$

(The addition of normal default rules does not lead to the retraction of consequences.)

Nonmonotonic Consequence

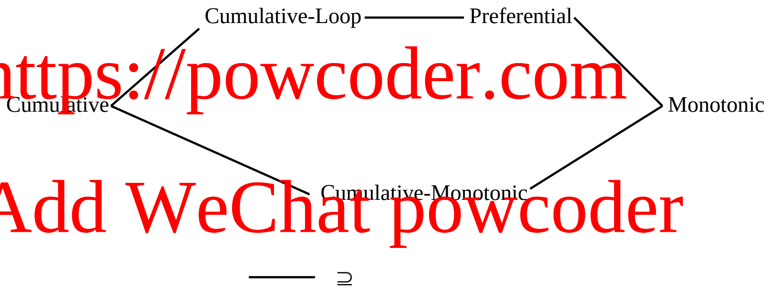
- Abstract study and analysis of nonmonotonic consequence relation $\mid\sim$ in terms of general properties Kraus, Lehmann and Magidor (1991)

- Some common properties include:
  - **Supraclassicality** If $\phi \vdash \psi$, then $\phi \mid\sim \psi$
  - **Left Logical Equivalence** If $\vdash \phi \leftrightarrow \psi$ and $\phi \mid\sim \chi$, then $\psi \mid\sim \chi$
  - **Right Weakening** If $\vdash \psi \rightarrow \chi$ and $\phi \mid\sim \psi$, then $\phi \mid\sim \chi$
  - **And** If $\phi \mid\sim \psi$ and $\phi \mid\sim \chi$, then $\phi \mid\sim \psi \wedge \chi$

- Plus many more!

KLM Systems

- Kraus, Lehman and Magidor (1991) study various classes of nonmonotonic consequence relations



- This has been extended since. A good reference for this line of work is Schlechta (1997)

Summary

- Nonmonotonic reasoning attempts to capture a form of commonsense reasoning
- Nonmonotonic reasoning often deals with inferences based on defaults or 'what is usually the case'
- Belief change and nonmonotonic reasoning: two sides of the same coin?
- Can introduce abstract study of nonmonotonic consequence relations in same way as we study classical consequence relations
- Similar links exist with conditionals
- One area where nonmonotonic reasoning is important is reasoning about action (dynamic systems)