Multi-agent Decision Making

COMP 4418 – Assignment 3

Due 25 Nov. 2020, 15:00
Total Marks: 50
Late Penalty: 10 marks per day
Worth: 15% of the course

Question 1 (10 marks) Consider a resource allocation setting in which indivisible items are to be allocated to agents, and agents have positive and additive utilities over the items. Prove or disprove the following:

1. If an allocation is proportional, it is envy-free.

2. If an allocation satisfies MmS fairness, it is proportional.

3. If an allocation is envy-free, it is Pareto optimal.

Question 2 (10 marks) Consider a Shapley-Scarf housing market with a set of agents $N = \{0, 1, 2, 3, 4\}$, a set of items $O = \{o_0, o_1, o_2, o_3, o_4\}$, and an endowment function $\omega : N \to 2^O$ such that $\omega(i) = \{o_i\}$. The preferences of the agents are as follows from left to right in decreasing order of preference.

$$0 : o_0, o_4, o_2, o_1, o_3$$
$$1 : o_0, o_2, o_4, o_1, o_3$$
$$2 : o_3, o_0, o_2, o_4, o_1$$
$$3 : o_0, o_2, o_3, o_1, o_4$$
$$4 : o_3, o_2, o_1, o_4, o_0$$

Find the outcome of the TTC (top trading cycles) algorithm. Can agent 4 misreport her preference to get a more preferred allocation? Prove or disprove that the outcome is individually rational.

Question 3 (10 marks) Consider Shapley-Scarf housing markets in which we are only allowed to obtain allocations in which at most two agents are a part of a trading cycle and each agent can be a part of at most of one trading cycle. Discuss at least three axiomatic properties that you consider to be desirable for the problem and explain why. Design a polynomial-time algorithm for the problem and prove it satisfies two of these properties.

Question 4 (10 marks) Consider the following market with 10 students

$$N = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

and three schools $C = \{c_0, c_1, c_2\}$. All the schools have the same capacity 4. The preferences of the students are as follows from left to right in decreasing order of preference.

$$\succ_0: c_0, c_2, c_1$$
$$\succ_1: c_0, c_2, c_1$$
$$\succ_2: c_0, c_2, c_1$$
$$\succ_3: c_0, c_1, c_2$$
$$\succ_4: c_2, c_0, c_1$$
$$\succ_5: c_2, c_0, c_1$$
$$\succ_6: c_2, c_0, c_1$$
$$\succ_7: c_2, c_0, c_1$$
$$\succ_8: c_0, c_1, c_2$$
$$\succ_9: c_2, c_1, c_0$$

The priorities of the schools are as follows from left to right in decreasing order of priority.

$$\succ_{c_0}: 9, 4, 1, 5, 3, 8, 6, 7, 2, 0$$
$$\succ_{c_1}: 5, 4, 3, 0, 6, 2, 7, 1, 9, 8$$
$$\succ_{c_2}: 2, 6, 9, 7, 3, 5, 4, 8, 1, 0$$

Find the outcome that rule of the student-proposing deferred acceptance algorithm and explain how you found the matching. Prove or disprove that the resultant matching for the example is Pareto optimal for the students.

Question 5 (10 marks) Consider a resource allocation setting in which agents have additive positive utilities over indivisible items. Consider the *sequential allocation* rule in which the policy of turns is $(1, 2, \ldots, n)^*$ where the policy $(1, 2, \ldots, n)^*$ indicates the turns $1, 2, \ldots, n$ repeat. Prove or disprove that rule

1. returns a Pareto optimal allocation.

2. returns a proportional allocation.

3. returns an EF1 allocation.

SUBMISSION

- You will need to answer the questions in a file named `assn3.pdf`. Submit using the command:

```
give cs4418 assn3 assn3.pdf
```

- Your answers are to be submitted in a single PDF file.

- The deadline for this submission is 25. Nov 2020, 15:00

Academic Honesty and Plagiarism

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- the submission of work derived from another person, or jointly written with someone else will, at the very least, result in automatic failure for COMP4418 with a mark of zero;

- allowing another student to copy from you will, at the very least, result in a mark of zero for your own assignment; and

- severe or second offences will result in automatic failure, exclusion from the University, and possibly other academic discipline.

- students are not allowed to derive solutions together as a group during such discussions. Students are also warned not to send solution fragments of the assignments to each other in any form (e.g. as email or listings).

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