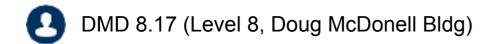


# COMP90038 Algorithms and Complexity

Lecturentps: Apalysise of Algorithms (with thanks to Harald Søndergaard)

#### **Toby Murray**











- Measure input size by natural number n
- Measure execution time as number of basic operations performed Assignment Project Exam Help
- Time complexity https://www.eatententen.number of basic operations Ads Wellhardion of n
- How to compare different t(n)?
  - Asymptotic growth rate
  - $O(g(n)), \Omega(g(n)), \Theta(g(n))$



Measure input size by natural number n

• Mea	Problem	Size Measure	Basic Operation
ope	Search in a list of <i>n</i> items sign	gnment Project Exam	Key comparison Help
· Tim		Matrix size https://powcoder.com (rows x columns)	
bas	Compute an	Add WeChat powcod	ler Float multiplication
• Hov	Graph problem	Number of nodes and edges	Visiting a node

- Asymptotic growth rate
- $O(g(n)), \Omega(g(n)), \Theta(g(n))$

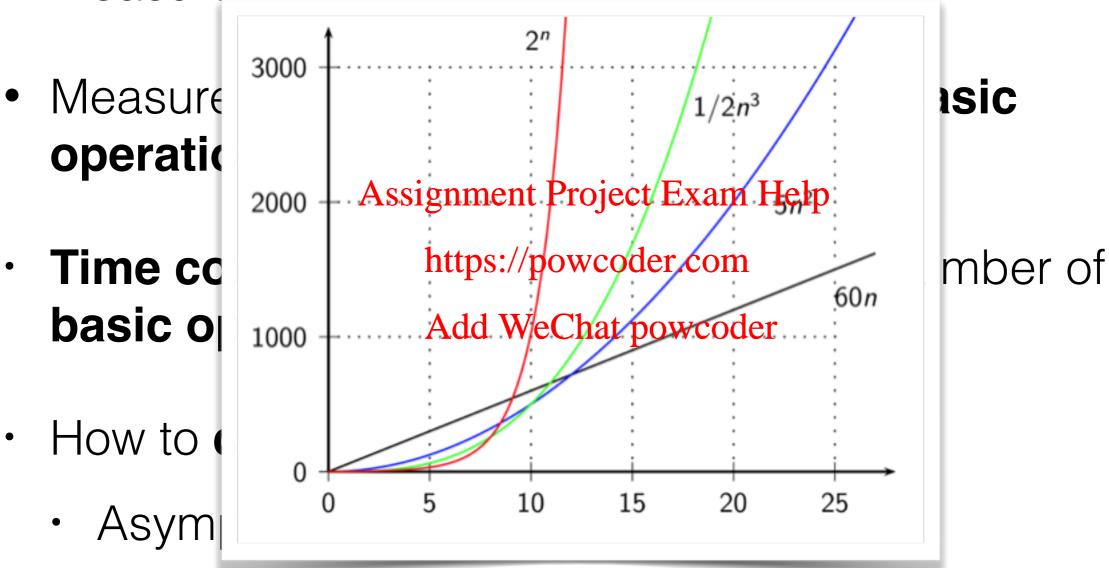


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#### Last Time: Time Complexity



Measure input size by natural number n



•  $O(g(n)), \Omega(g(n)), \Theta(g(n))$ 



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$$f(n) < g(n)$$
 iff  $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$ 



Measure input size by natural number n

- $t(n) \in O(g(n))$  Measure basic Don't care operation  $c \cdot g(n)$ ssignment Project Exam  $Help_{t(n)}$ https://powcoder.com lumber of Time con basic ope Add WeChat powcoder How to co  $\inf_{n \to \infty} \lim_{g(n)} \frac{f(n)}{g(n)} = 0$ Asympt
  - $O(g(n)), \Omega(g(n)), \Theta(g(n))$

#### Last Time: Time Complexity



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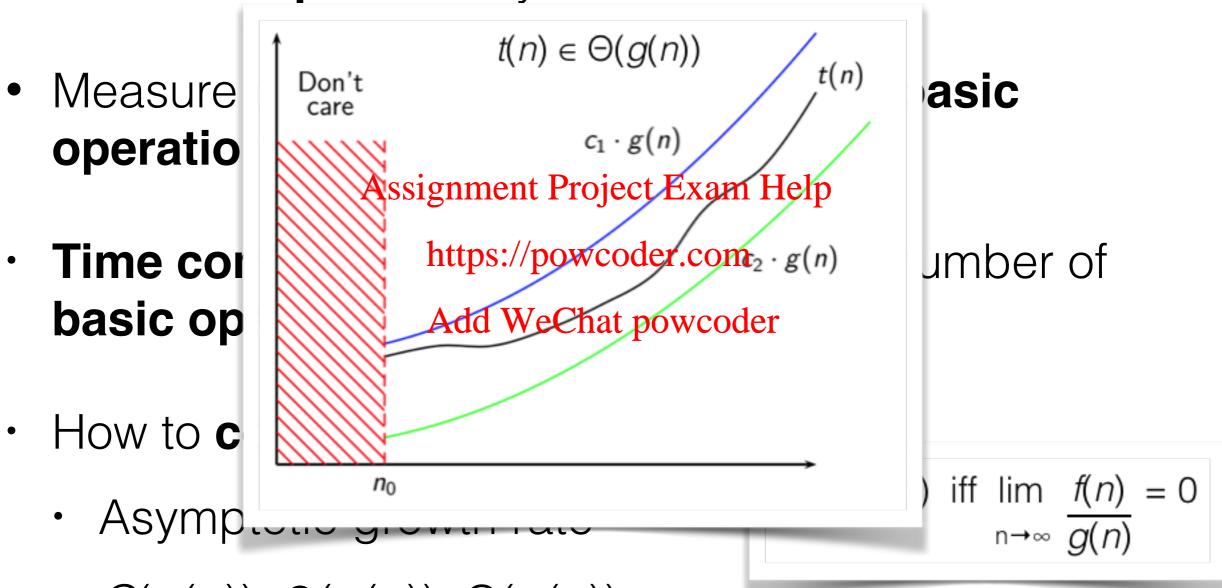
Assignment Project Exam Help

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 $O(g(n)), \Omega(g(n)), \Theta(g(n))$ 

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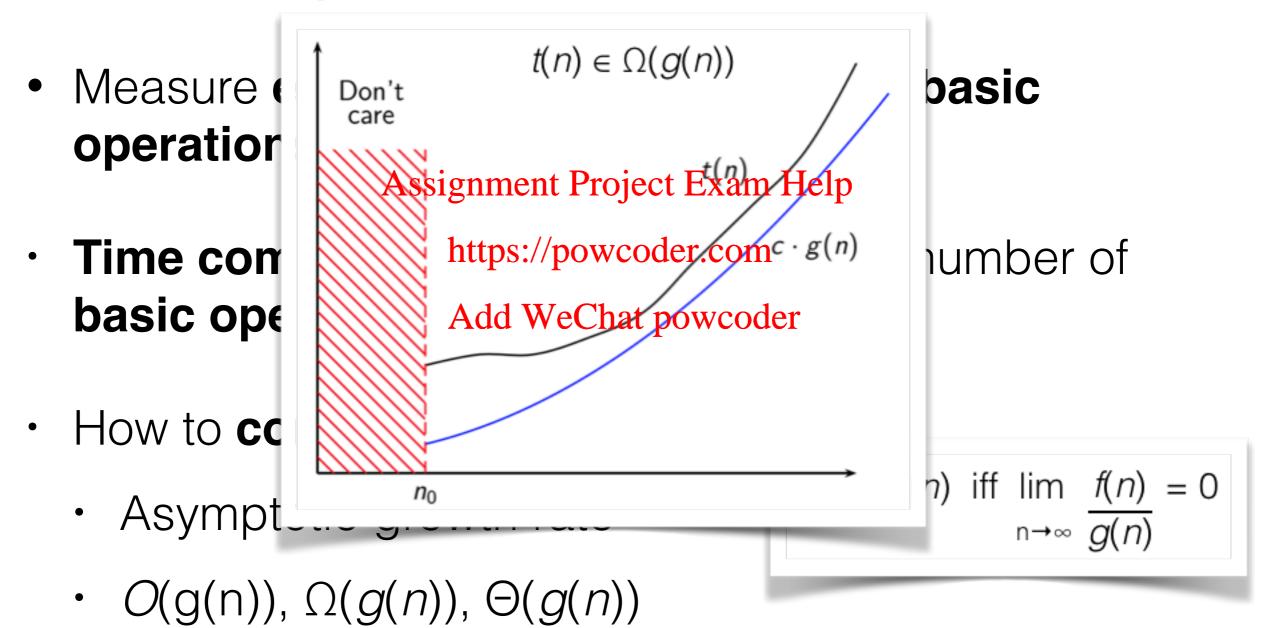
Assignment Project Exam Help

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$$f(n) < g(n)$$
 iff  $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$ 

#### Establishing Growth Rate



• In the last lecture we proved  $t(n) \in O(g(n))$  for some cases of t and g, using the definition of O directly:

$$n > n_0 \Rightarrow t(n) < c \cdot g(n)$$
 for some  $c$  and  $n_0$ .

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A more common approach uses
 Add WeChat powcoder

• 
$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \begin{cases} 0 & f \text{ grows asymptotically slower than } g \\ c & f \text{ and } g \text{ have same order of growth} \\ \infty & f \text{ grows asymptotically faster than } g \end{cases}$$

• Use this to show that  $1000n \in O(n^2)$ 



$$\lim_{n\to\infty} \frac{1000n}{n^2}$$

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Assignment Project Exam Help

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$$\lim_{n\to\infty} \frac{1000n}{n^2}$$

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Assignment Project Exam Help

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$$\lim_{n \to \infty} \frac{1000n}{n^2}$$
Assignment Project Exam Help https://powcoder.com
$$= \text{Add WeChat powcoder}$$

So 1000*n* grows asymptotically slower than n<sup>2</sup>



$$\lim_{n \to \infty} \frac{1000n}{n^2}$$
Assignment Project Exam Help https://powcoder.com
$$= \text{Add WeChat powcoder}$$

So 1000*n* grows asymptotically slower than n<sup>2</sup>

Thus  $1000n \in O(n^2)$ 



What this tells us about how f(n) relates to O(g(n)),  $\Omega(g(n))$  and O(g(n))

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• 
$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \begin{cases} 0 & \text{httpg/pws-asymptotically slower than } g \\ c & f \text{ and } g \text{ have same order of growth} \\ \infty & f \text{ grows asymptotically faster than } g \end{cases}$$



## What this tells us about how f(n) relates to O(g(n)), $\Omega(g(n))$ and O(g(n))

#### Assignment Project Exam Help

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$$f(n) \in O(g(n))$$



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$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \begin{cases} 0 & \text{https://powcoder.com} \\ c & f \text{ and } g \text{ have same order of growth} \\ \infty & f \text{ grows asymptotically faster than } g \end{cases}$$



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$$\lim_{h \to \infty} \frac{t(n)}{g(n)} = \lim_{h \to \infty} \frac{t'(n)}{g'(n)}$$

where t' and g' are the derivatives of t and g

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https://powcoder.com



$$\lim_{h \to \infty} \frac{t(n)}{g(n)} = \lim_{h \to \infty} \frac{t'(n)}{g'(n)}$$

where t' and g' are the derivatives of t and g

• For example, show that  $\log_2$  n grows slower than  $\sqrt{n}$  https://powcoder.com



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• For example, show that  $\log_2$  n grows slower than  $\sqrt{n}$  https://powcoder.com

$$\lim_{n\to\infty}\frac{\log_2 n}{\sqrt{n}}$$



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$$\lim_{n \to \infty} \frac{\log_2 n}{\sqrt{n}} = \lim_{n \to \infty} \frac{\operatorname{Add} \log_2 n}{\frac{1}{2\sqrt{n}}}$$



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$$= 2 \log_e 2 \lim_{n \to \infty} \frac{\sqrt{n}}{n}$$



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$$\lim_{n \to \infty} \frac{\log_2 n}{\sqrt{n}} = \lim_{n \to \infty} \frac{\frac{1}{n} \frac{1}{n} \frac{1}{n} - \frac{1}{n}}{\frac{1}{2\sqrt{n}}} = \lim_{n \to \infty} \left( (\log_e 2) \frac{1}{n} \cdot 2\sqrt{n} \right)$$

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$$= 2\log_e 2 \lim_{n \to \infty} \frac{\sqrt{n}}{n} = 2\log_e 2 \lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0$$

## Finding Largest Element in an Array



the array)

```
(where n is length of
function MaxElement(A[\cdot], n)
    max \leftarrow A[0]
    for i \leftarrow 1 to n-1 do
        if A[i] > \max_{\text{Assignment Project Exam Help}}
             max \leftarrow A[i]
https://powcoder.com
    return max
                         Add WeChat powcoder
```

A: 3 23 12 42 6 69 18 2 3 5 6

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2

3

5

6

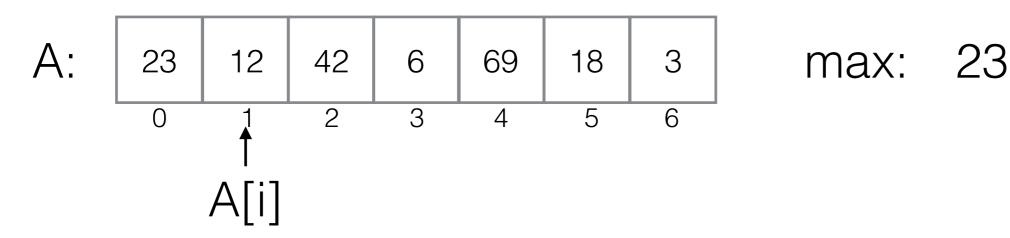
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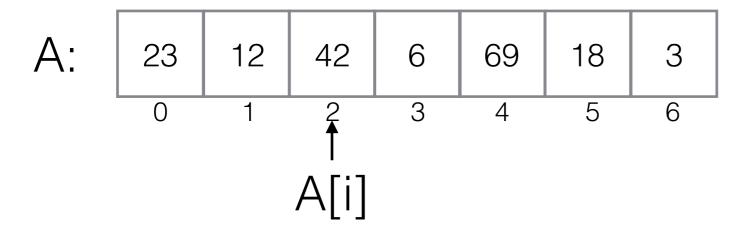


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23 max:

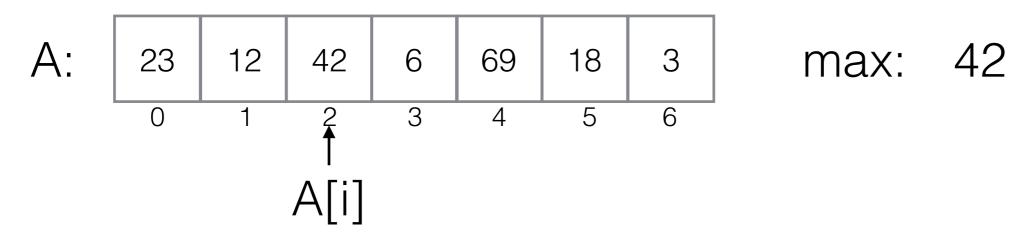
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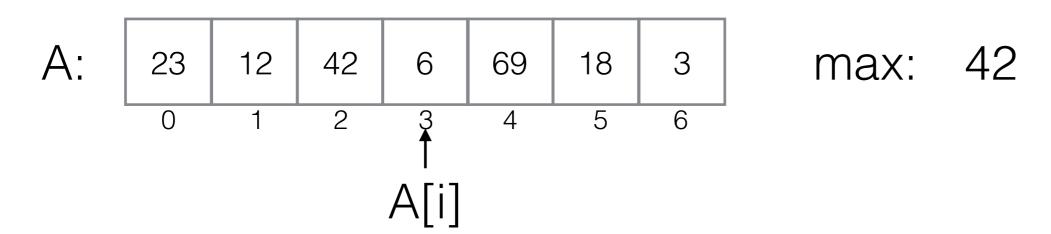
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https://powcoder.com

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Add WeChat powcoder
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## Finding Largest Element in an Array



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A: 3 23 12 42 6 69 18 3 2 6

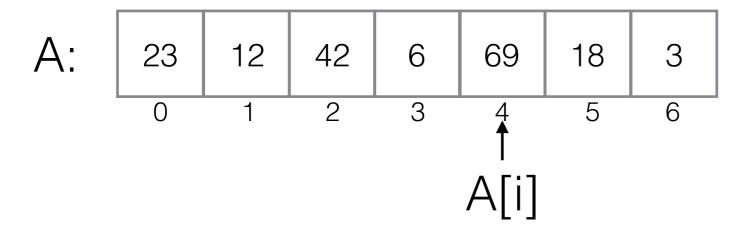
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## Finding Largest Element in an Array



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max:

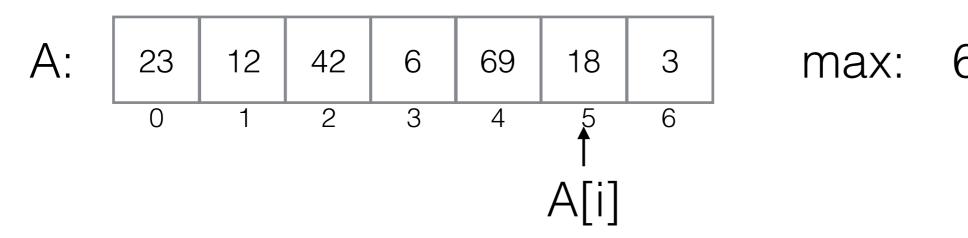
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## Finding Largest Element in an Array

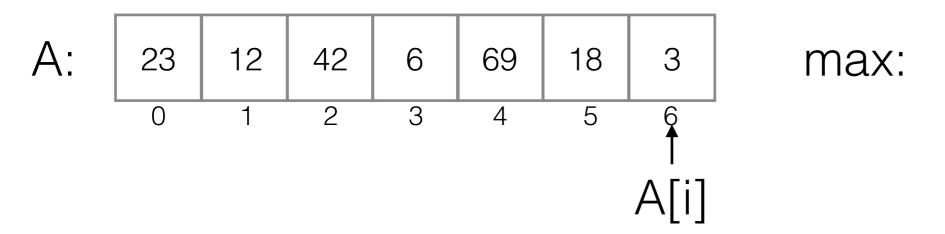


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69

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## Finding Largest Element in an Array

```
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https://powcoder.com

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Add WeChat powcoder
```

(where *n* is length of the array)

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Finding Largest Element in an Array
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https://powcoder.com length of the array
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https://powcoder.com length of the array
```

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Basic operation: comparison "A[i] > max"

## Finding Largest Element in an Array

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THE UNIVERSITY OF MELBOURNE
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https://powcoder.com/length of the array

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Add WeChat powcoder
```

Add WeChat powcoder

Basic operation: comparison "A[i] > max"

Count the number of basic operations executed for an array of size *n*:

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Finding Largest Element in an Array
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## Finding Largest Element in an Array



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```

Add WeChat powcoder
Basic operation: comparison "A[i] > max"

Count the number of basic operations executed for an array of size *n*:

$$C(n) = \sum_{i=1}^{n-1} 1$$

## Finding Largest Element in an Array



**function** MaxElement  $(A[\cdot], n)$  (where n is length of  $max \leftarrow A[0]$  the array) **for**  $i \leftarrow 1$  to n-1 **do if**  $A[i] > \max_{\mathbf{max}} \mathbf{then}_{\mathbf{Project}} \mathbf{Exam} \mathbf{He}_{\mathbf{S}}$  ize of input, n:  $max \leftarrow A[i]$ https://powcoder.com length of the array **return** max

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Basic operation: comparison "A[i] > max"

Count the number of basic operations executed for an array of size *n*:

$$C(n) = \sum_{i=1}^{n-1} 1 = ((n-1) - 1 + 1)$$

## Finding Largest Element in an Array



function MaxElement 
$$(A[\cdot], n)$$
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for  $i \leftarrow 1$  to  $n-1$  do

if  $A[i] > \max$  then Project Exam Helpize of input,  $n$ :

 $max \leftarrow A[i]$ 

https://powcoder.com length of the array

return max

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Basic operation: c

for an array of size *n*:

Basic operation: c Count the number of basic 
$$\sum_{i=l}^{u} 1 = \underbrace{1+1+\dots+1}_{u-l+1 \text{ times}} = u-l+1$$

$$C(n) = \sum_{i=1}^{n-1} 1 = ((n-1) - 1 + 1)$$

## Finding Largest Element in an Array



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Basic operation: c

Count the number of basic  $\sum_{i=l}^{n} 1 = \underbrace{1 + 1 + \dots + 1}_{u-l+1 \text{ times}} = u - l + 1$ u-l+1 times

for an array of size *n*:

$$C(n) = \sum_{i=1}^{n-1} 1 = ((n-1) - 1 + 1) = n - 1$$

## Finding Largest Element in an Array



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Basic operation: c

Count the number of basic  $\sum_{i=l}^{n} 1 = \underbrace{1 + 1 + \dots + 1}_{u-l+1 \text{ times}} = u - l + 1$ u-l+1 times

for an array of size *n*:

$$C(n) = \sum_{i=1}^{n-1} 1 = ((n-1) - 1 + 1) = n - 1 \in \Theta(n)$$



```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = ninent Penject Exam Help

min \leftarrow jhttps://powcoder.com

swap A[i] and A[min]eChat powcoder
```

23	12	42	6	69	18	3
0	1	2	3	4	5	6



```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

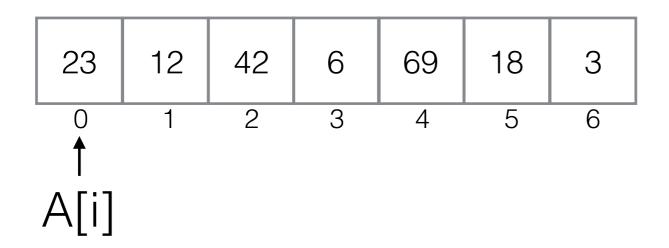
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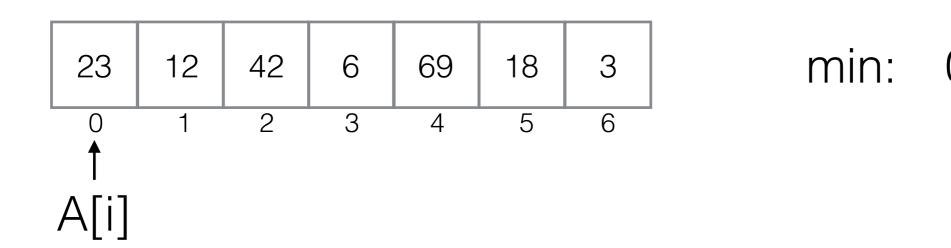
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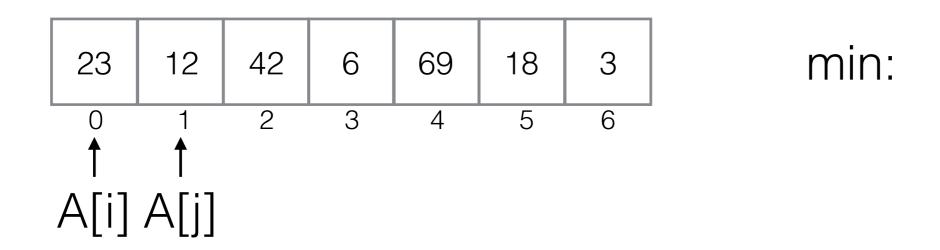
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min \leftarrow j https://powcoder.com

swap A[i] and A[min] eChat powcoder
```





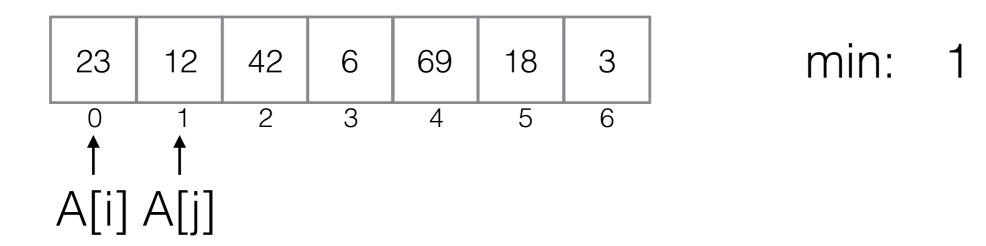
```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = A[j
```





```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

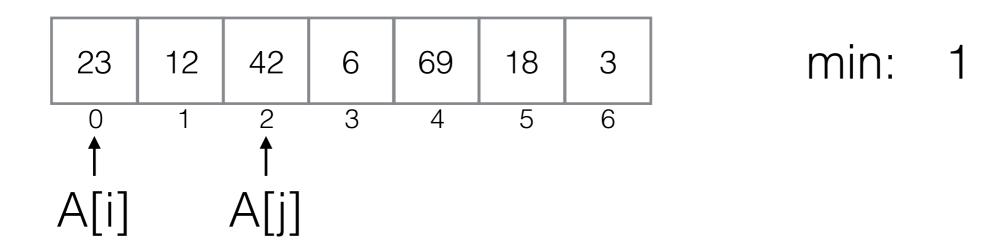
min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = ninent People ct Exam Help

min \leftarrow j https://powcoder.com

swap A[i] and A[min] eChat powcoder
```





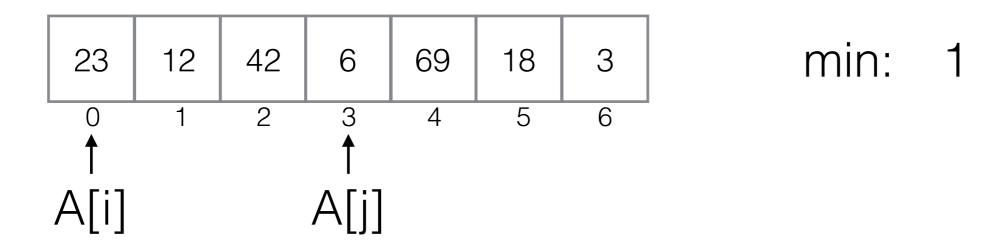
```
function SelSort(A[\cdot], n)

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for j \leftarrow i+1 to n-1 do

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```





```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

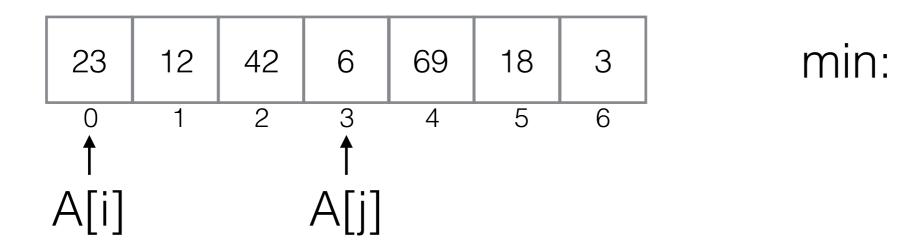
min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = ninent Penject Exam Help

min \leftarrow jhttps://powcoder.com

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```





```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

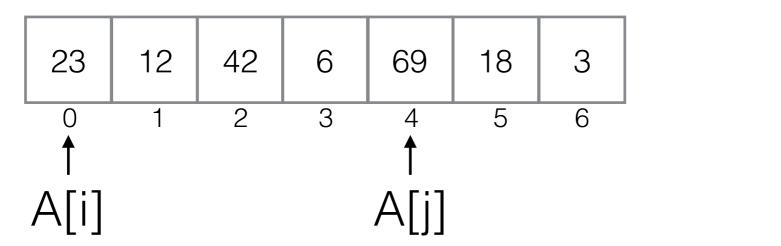
min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = ninent Penject Exam Help

min \leftarrow jhttps://powcoder.com

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```





```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

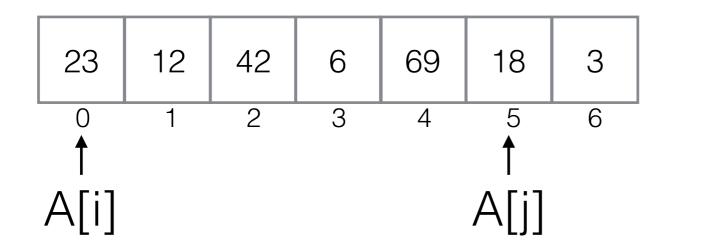
min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = ninent Penject Exam Help

min \leftarrow jhttps://powcoder.com

swap A[i] and A[min]eChat powcoder
```





```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

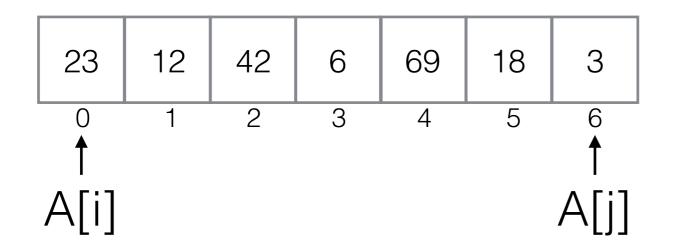
min \leftarrow i

for j \leftarrow i+1 to n-1 do

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min \leftarrow jhttps://powcoder.com

swap A[i] and A[min]eChat powcoder
```





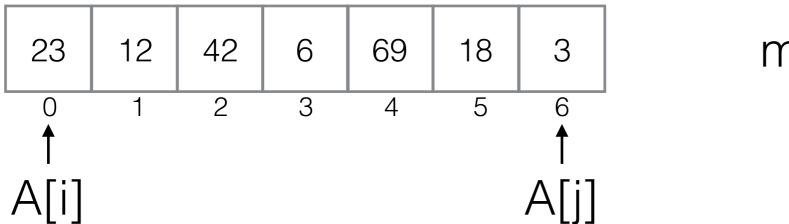
```
function SelSort(A[\cdot], n)

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```





```
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for i \leftarrow 0 to n-2 do

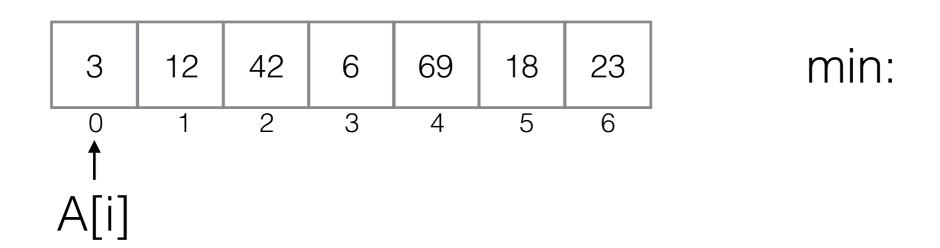
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for j \leftarrow i+1 to n-1 do

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min \leftarrow jhttps://powcoder.com

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```





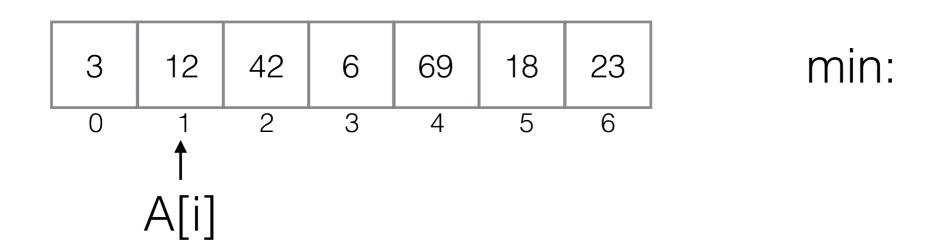
```
function SelSort(A[\cdot], n)

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```





```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

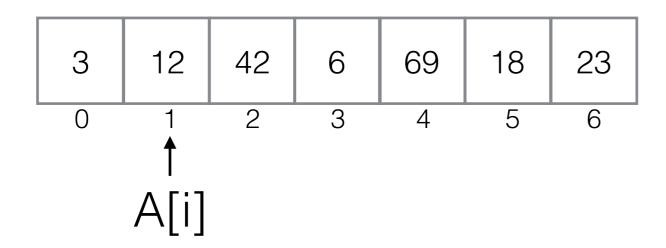
min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = ninent Penject Exam Help

min \leftarrow jhttps://powcoder.com

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```





```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

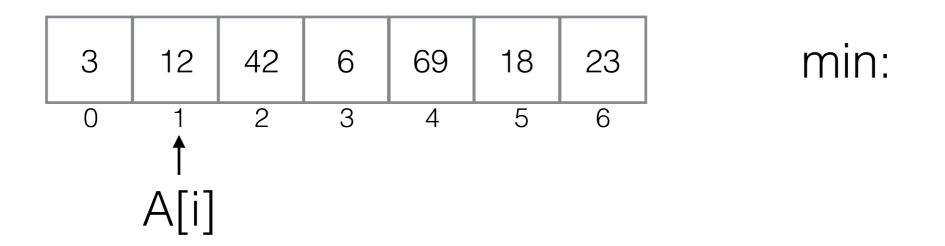
min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = ninent Penject Exam Help

min \leftarrow jhttps://powcoder.com

swap A[i] and A[min]eChat powcoder
```





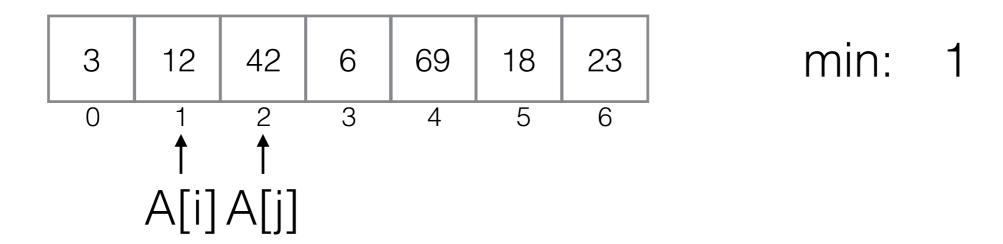
```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = A[j
```





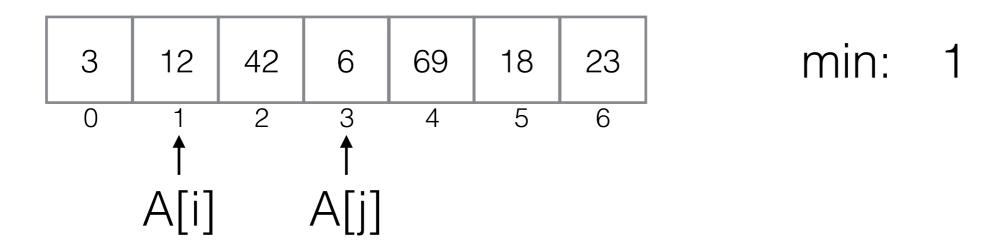
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function SelSort(A[\cdot], n)

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```





```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

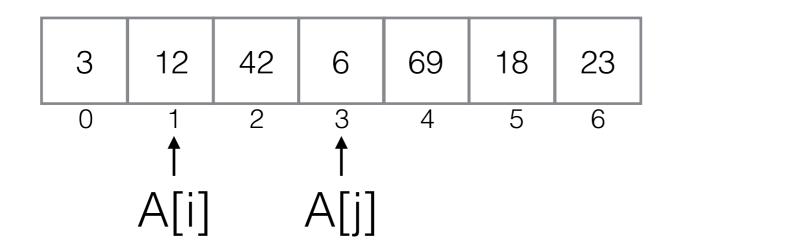
min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = ninent Penject Exam Help

min \leftarrow jhttps://powcoder.com

swap A[i] and A[min]eChat powcoder
```



min: 3



```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

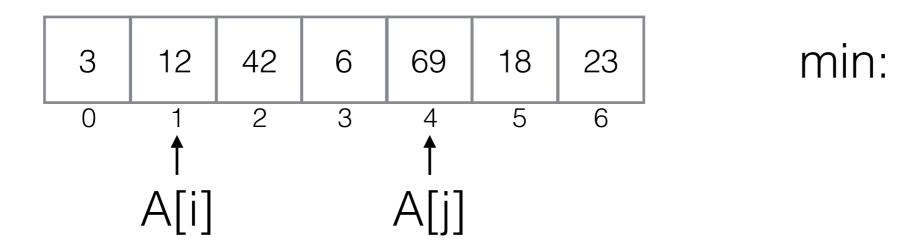
min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = ninent Penject Exam Help

min \leftarrow jhttps://powcoder.com

swap A[i] and A[min] = Chat powcoder
```





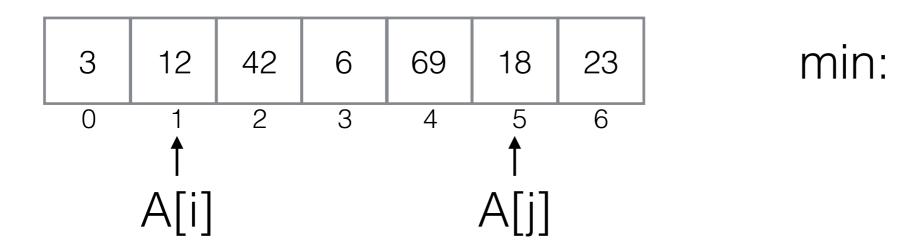
```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = A[j
```





```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

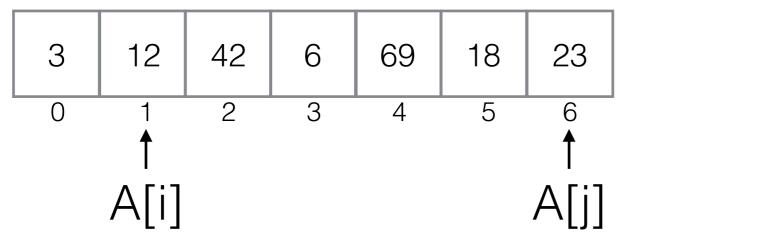
min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[j] = ninent Penject Exam Help

min \leftarrow jhttps://powcoder.com

swap A[i] and A[min]eChat powcoder
```



min: 3



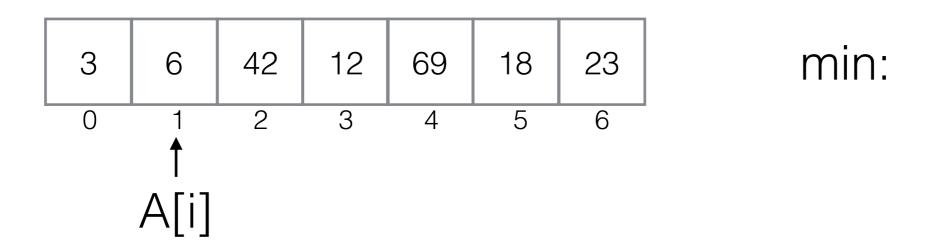
```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

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for j \leftarrow i+1 to n-1 do

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```





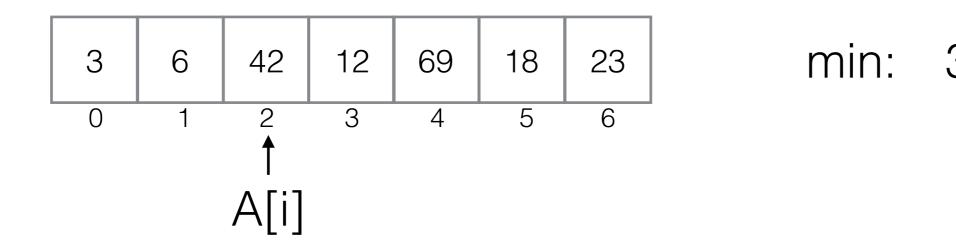
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```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[min] = Regiect Exam Helpasic operation:

min <math>\leftarrow j https://powcoder.comparison A[j] < A[min]

swap A[i] and A[min] eChat powcoder
```



```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

min \leftarrow i

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min <math>\leftarrow j https://powcoder.comparison A[j] < A[min]

swap A[i] and A[min] eChat powcoder
```

$$C(n) =$$



```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[min] the project Exam Helpasic operation:

min \leftarrow j https://powcoder.comparison A[j] < A[min]

swap A[i] and A[min] eChat powcoder
```

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1$$



```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[min] = Regiect Exam Helpasic operation:

<math>min \leftarrow j https://powcoder.comparison A[j] < A[min]

swap A[i] and A[min] eChat powcoder
```

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-i)$$



```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < AA[min] therefore Exam Helpasic operation:

min \leftarrow j https://powcoder.comparison A[j] < A[min] swap A[i] and A[min] eChat powcoder
```

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-i) = \sum_{i=0}^{n-2} (n-1) - \sum_{i=0}^{n-2} i$$



```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[min] = Exam Helpasic operation:

<math>min \leftarrow j

min \leftarrow j

min \leftarrow j

swap A[i] and A[min] = C

for A[i] = A[min]

A[i] = A[min]

A[i] = A[min]
```

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-i) = \sum_{i=0}^{n-2} (n-1) - \sum_{i=0}^{n-2} i$$
$$= (n-1)^2 - \frac{(n-2)(n-1)}{2}$$



```
function SelSort(A[\cdot], n)

for i \leftarrow 0 to n-2 do

min \leftarrow i

for j \leftarrow i+1 to n-1 do

if A[j] < A[min] = Exam Helpasic operation:

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swap A[i] and A[min] = C

for A[i] = A[min]

A[i] = A[min]

A[i] = A[min]
```

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-i) = \sum_{i=0}^{n-2} (n-1) - \sum_{i=0}^{n-2} i$$
$$= (n-1)^2 - \frac{(n-2)(n-1)}{2} = \frac{n(n-1)}{2} \in \Theta(n^2)$$

```
function MatrixMult(A[\cdot,\cdot], B[\cdot,\cdot], n) \triangleright For n \times n matrices
    for i \leftarrow 0 to n-1 do
         for j \leftarrow 0 to n-1 do
              C[i,j] \leftarrow 0.0
              for k \leftarrow Oatsignment Reject Exam Help
                  C[i,j] \leftarrow C[i,j] + A[i,k]_{om}B[k,j]
    return C
                               Add WeChat powcoder
```

```
function MatrixMult(A[\cdot,\cdot], B[\cdot,\cdot], n) \triangleright For n \times n matrices
    for i \leftarrow 0 to n-1 do
                                                                               i: 0
         for j \leftarrow 0 to n-1 do
              C[i,j] \leftarrow 0.0
              for k \leftarrow Oatsignment Reject Exam Help
                   C[i,j] \leftarrow C[j,j] \rightarrow A[j,k]_{oin}B[k,j]
    return C
                                Add WeChat powcoder
```

function MATRIXMULT(
$$A[\cdot,\cdot],B[\cdot,\cdot],n$$
)  $ightharpoonup$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$ 

for  $k \leftarrow 0$  to general Respect Exam Help

 $C[i,j] \leftarrow \text{Int}[i,j]$  by  $C[i,j]$ 

return  $C$ 

Add WeChat powcoder

$$\begin{bmatrix}
5 & 7 & 8 & 2 \\
3 & 4 & 9 & 6
\end{bmatrix}$$

function MATRIXMULT
$$(A[\cdot,\cdot],B[\cdot,\cdot],n)$$
  $ightharpoonup$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$  i: 0

for  $k \leftarrow 0$  to general Respect Exam Help

 $C[i,j] \leftarrow \text{Int}[i,j]$  powed for  $k$  boin  $k$  [ $k$ ,  $k$ ]

return  $k$  Add WeChat powcoder

$$\begin{bmatrix}
5 & 7 & 8 & 2 & 0 & 0 \\
3 & 4 & 9 & 6 & 0 & 0
\end{bmatrix}$$

function MATRIXMULT(
$$A[\cdot,\cdot],B[\cdot,\cdot],n$$
)  $ightharpoonup$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$ 

for  $k \leftarrow 0$  to general religional religional religionary  $E[k,j]$ 

return  $C$ 

Add WeChat powcoder

$$\begin{bmatrix}
5 & 7 & 8 & 2 & 0 \\
3 & 4 & 9 & 6
\end{bmatrix}$$
A

B

 $C$ 

function MATRIXMULT(
$$A[\cdot,\cdot],B[\cdot,\cdot],n$$
)  $ightharpoonup$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$ 

for  $k \leftarrow 0$  Atsignment Respect Exam Help

 $C[i,j] \leftarrow \text{Int}[j,j]$  bow Older Roin  $B[k,j]$ 

return  $C$ 

Add WeChat powcoder

$$\begin{bmatrix}
5 & 7 & 8 & 2 & 40 \\
3 & 4 & 9 & 6 & 40
\end{bmatrix}$$

A

B

 $C$ 

function MATRIXMULT(
$$A[\cdot,\cdot],B[\cdot,\cdot],n$$
)  $ightharpoonup$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$ 

for  $k \leftarrow 0$  to gent Exam Help

 $C[i,j] \leftarrow P[i,j] \rightarrow Add$  WeChat powcoder

[5 7 8 2 40

A B C

function MATRIXMULT(
$$A[\cdot,\cdot],B[\cdot,\cdot],n$$
)  $ightharpoonup$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$ 

for  $k \leftarrow 0$  Atsignment Respect Exam Help

 $C[i,j] \leftarrow \text{Int}[j,j]$  bow Older Roin  $B[k,j]$ 

return  $C$ 

Add WeChat powcoder

$$\begin{bmatrix}
5 & 7 & 8 & 2 & 103 \\
3 & 4 & 9 & 6
\end{bmatrix}$$

A

B

C

function MATRIXMULT
$$(A[\cdot,\cdot],B[\cdot,\cdot],n)$$
  $ightharpoonup$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$  i: 0

for  $k \leftarrow 0$  to general respect Exam Help

 $C[i,j] \leftarrow \text{Int}[i,j]$  powed for  $k \in J$  powed for  $k \in J$  for  $k \in J$  and  $k \in J$  for  $k \in$ 

function MATRIXMULT
$$(A[\cdot,\cdot],B[\cdot,\cdot],n)$$
  $\triangleright$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$ 

for  $k \leftarrow 0$  to grament Respect Exam Help

 $C[i,j] \leftarrow \inf[j:j] - \inf[j:j] - \inf[j:j] = 0.0$ 

return  $C$ 

Add WeChat powcoder

$$\begin{bmatrix}
5 & 7 & 8 & 2 & 103 & 0 \\
3 & 4 & 9 & 6 & 103 & 0
\end{bmatrix}$$

В

function MATRIXMULT
$$(A[\cdot,\cdot],B[\cdot,\cdot],n)$$
  $ightharpoonup$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$ 

for  $k \leftarrow 0$  to general Respect Exam Help

 $C[i,j] \leftarrow \text{Int}[i,j]$  powed for  $k \in 0$ 

return  $C$ 

Add WeChat powcoder

$$\begin{bmatrix}
5 & 7 & 8 & 2 & 103 & 0 \\
3 & 4 & 9 & 6 & 3
\end{bmatrix}$$

A

B

C

function MATRIXMULT
$$(A[\cdot,\cdot],B[\cdot,\cdot],n)$$
  $ightharpoonup$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$ 

for  $k \leftarrow 0$  to general Respect Exam Help

 $C[i,j] \leftarrow \text{Int}[i,j]$  powed for  $k \in 0$ 

return  $C$ 

Add WeChat powcoder

$$\begin{bmatrix}
5 & 7 & 8 & 2 & 103 & 10 \\
3 & 4 & 9 & 6
\end{bmatrix}$$

A

B

C

function MATRIXMULT(
$$A[\cdot, \cdot], B[\cdot, \cdot], n$$
)  $\triangleright$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$ 

for  $k \leftarrow 0$  Atsignment Respect Exam Help

 $C[i,j] \leftarrow \text{Int}[j,j]$   $\Rightarrow \text{Wolder Roin} B[k,j]$ 

return  $C$ 

Add WeChat powcoder

$$\begin{bmatrix}
5 & 7 & 8 & 2 & 103 & 10 \\
3 & 4 & 9 & 6 & 3
\end{bmatrix}$$

A

B

 $C$ 

function MATRIXMULT(
$$A[\cdot, \cdot], B[\cdot, \cdot], n$$
)  $\triangleright$  For  $n \times n$  matrices for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

 $C[i,j] \leftarrow 0.0$ 

for  $k \leftarrow 0$  Atsignment Respect Exam Help

 $C[i,j] \leftarrow \text{Int}[j,j]$   $\Rightarrow \text{Wolder Roin} B[k,j]$ 

return  $C$ 

Add WeChat powcoder

$$\begin{bmatrix}
5 & 7 & 8 & 2 & 103 & 52 \\
3 & 4 & 9 & 6 & 3
\end{bmatrix}$$

A

B

C

```
function MATRIXMULT(A[\cdot,\cdot],B[\cdot,\cdot],n) \triangleright For n \times n matrices for i \leftarrow 0 to n-1 do

for j \leftarrow 0 to n-1 do

C[i,j] \leftarrow 0.0

for k \leftarrow 0Atsignment Reject Exam Help

C[i,j] \leftarrow C[i,j] \leftarrow C[i,j] \rightarrow A[i,j]

return C

Add WeChat powcoder
```

```
function MATRIXMULT(A[\cdot,\cdot],B[\cdot,\cdot],n) 
ightharpoonup For n\times n matrices for i\leftarrow 0 to n-1 do

for j\leftarrow 0 to n-1 do

C[i,j]\leftarrow 0.0
for k\leftarrow 0Ateignment Respect Exam Help
C[i,j]\leftarrow \inf[j:j] \rightarrow A[j:k] \sin B[k,j]
return C
Add WeChat powcoder
```

```
function MATRIXMULT(A[\cdot,\cdot],B[\cdot,\cdot],n) 
ightharpoonup For n\times n matrices for i\leftarrow 0 to n-1 do

for j\leftarrow 0 to n-1 do

C[i,j]\leftarrow 0.0
for k\leftarrow 0Ateignment Respect Exam Help
C[i,j]\leftarrow \inf[j:j] \rightarrow A[j:k] \sin B[k,j]
return C
Add WeChat powcoder
```

$$M(n) =$$

```
function MATRIXMULT(A[\cdot,\cdot],B[\cdot,\cdot],n) 
ightharpoonup For n\times n matrices for i\leftarrow 0 to n-1 do for j\leftarrow 0 to n-1 do C[i,j]\leftarrow 0.0 for k\leftarrow 0 to given Places Exam Help C[i,j]\leftarrow \inf_{x\in J}\int_{\text{pow}}A[i,k]\sin B[k,j] return C Add WeChat powcoder
```

$$M(n) = 1$$

function MATRIXMULT
$$(A[\cdot,\cdot],B[\cdot,\cdot],n)$$
  $ightharpoonup$  For  $n\times n$  matrices for  $i\leftarrow 0$  to  $n-1$  do

for  $j\leftarrow 0$  to  $n-1$  do

$$C[i,j]\leftarrow 0.0$$
for  $k\leftarrow 0$ Atsignment Roject Exam Help
$$C[i,j]\leftarrow \inf[j:j] \text{pow} \text{Coder} \text{Roin} B[k,j]$$
return  $C$ 
Add WeChat powcoder

$$M(n) = \sum_{k=0}^{n-1} 1$$

function MATRIXMULT
$$(A[\cdot,\cdot],B[\cdot,\cdot],n)$$
  $ightharpoonup$  For  $n\times n$  matrices for  $i\leftarrow 0$  to  $n-1$  do for  $j\leftarrow 0$  to  $n-1$  do  $C[i,j]\leftarrow 0.0$  for  $k\leftarrow 0$  to gent representation  $C[i,j]\leftarrow C[i,j]$  for  $C[i,j]\leftarrow C[i,j]$  for  $C[i,j]$  for

$$M(n) = \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

function MATRIXMULT
$$(A[\cdot,\cdot],B[\cdot,\cdot],n)$$
  $ightharpoonup$  For  $n\times n$  matrices for  $i\leftarrow 0$  to  $n-1$  do for  $j\leftarrow 0$  to  $n-1$  do  $C[i,j]\leftarrow 0.0$  for  $k\leftarrow 0$  to gent representation  $C[i,j]\leftarrow C[i,j]$  for  $C[i,j]\leftarrow C[i,j]$  for  $C[i,j]$  for

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

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$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \dots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

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$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (\text{Massign}) \text{ MenoProject Exam Help}$$
https://powcoder.com

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \dots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} ((\text{Massign})_{\text{men OProject } Exam^{-1} n^{-1} n^{-1} \\ \text{https://powcoder.coin}^{n-1} 0 j=0$$

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \dots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} ((n \cdot 1) + \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (n \cdot 1) + \sum_{i=0}^{n-1} (n$$

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \dots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} ((\text{Massighmen OP to jedt } \underbrace{\text{Exam Help}}_{\text{https://powcoder.com}} n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (n \cdot 1)$$

$$\sum_{i=l}^{u} ca_i = c \sum_{i=l}^{u} a_i$$

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \dots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$= \sum_{i=0}^{n-1} \left( n \cdot \sum_{j=0}^{n-1} 1 \right)^{\text{Add WeChat powcoder}}$$

$$\sum_{i=l}^{u} ca_i = c \sum_{i=l}^{u} a_i$$

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \dots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$=\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}((\text{Massign})\text{Project Exam})\text{Help} n = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1}(n\cdot 1)$$
https://powcoder.coin j=0

$$= \sum_{i=0}^{n-1} \left( n \cdot \sum_{j=0}^{n-1} 1 \right)^{\text{Add WeChat powcoder}} = \sum_{i=0}^{n-1} n^2$$

$$\sum_{i=l}^{u} ca_i = c \sum_{i=l}^{u} a_i$$

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \dots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$= \sum_{i=0}^{n-1} \left( n \cdot \sum_{j=0}^{n-1} 1 \right)^{\text{Add WeChat powcoder}}$$

$$= \sum_{i=0}^{n-1} n^2$$

$$\sum_{i=l}^{u} ca_i = c \sum_{i=l}^{u} a_i$$

$$= n^3$$

$$M(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$$

$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \dots + 1}_{u-l+1 \text{ times}} = u - l + 1$$

$$=\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}((\text{AssignmenOPhojedt}\,\text{Exam})\text{Help} n = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1}(n\cdot 1)$$
https://powcoder.coim = 0

$$= \sum_{i=0}^{n-1} \left( n \cdot \sum_{j=0}^{n-1} 1 \right)^{\text{Add WeChat powcoder}}$$

$$= \sum_{i=0}^{n-1} n^2$$

$$\sum_{i=l}^{u} ca_i = c \sum_{i=l}^{u} a_i$$

$$= n^3$$

$$\in \Theta(n^3)$$



function F(n)if n = 0 then return 1else return  $F(n-1) \cdot n$ 

Assignment Project Exam Help

https://powcoder.com



```
function F(n)
if n = 0 then return 1
else return F(n-1) \cdot n
```

Assignment Project Exam Help

F(5) https://powcoder.com



function 
$$F(n)$$
  
if  $n = 0$  then return  $1$   
else return  $F(n-1) \cdot n$ 

Assignment Project Exam Help

$$F(5) = F(4) \cdot 5$$

https://powcoder.com



function 
$$F(n)$$
  
if  $n = 0$  then return  $1$   
else return  $F(n-1) \cdot n$ 

Assignment Project Exam Help
$$F(5) = F(4) \cdot 5$$

$$= (F(3) \cdot 4) \cdot 5$$

$$= (F(3) \cdot 4) \cdot 5$$
Add WeChat powcoder



function 
$$F(n)$$
  
if  $n = 0$  then return  $1$   
else return  $F(n-1) \cdot n$ 

$$F(5) = F(4) \cdot 5$$
https://powcoder.com
$$= (F(3) \cdot 4) \cdot 5$$

$$= ((F(2) \cdot 3) \cdot 4) \cdot 5$$



function 
$$F(n)$$
  
if  $n = 0$  then return  $1$   
else return  $F(n-1) \cdot n$ 

$$F(5) = F(4) \cdot 5$$
https://powcoder.com
$$= (F(3) \cdot 4) \cdot 5$$

$$= ((F(2) \cdot 3) \cdot 4) \cdot 5$$

$$= (((F(1) \cdot 2) \cdot 3) \cdot 4) \cdot 5$$



function 
$$F(n)$$
  
if  $n = 0$  then return 1  
else return  $F(n-1) \cdot n$ 

$$F(5) = F(4) \cdot 5$$
https://powcoder.com
$$= (F(3) \cdot 4) \cdot 5$$

$$= ((F(2) \cdot 3) \cdot 4) \cdot 5$$

$$= (((F(1) \cdot 2) \cdot 3) \cdot 4) \cdot 5$$

$$= (((F(0) \cdot 1) \cdot 2) \cdot 3) \cdot 4) \cdot 5$$



function 
$$F(n)$$
  
if  $n = 0$  then return 1  
else return  $F(n-1) \cdot n$ 

Assignment Project Exam Help

$$F(5) = F(4) \cdot 5$$

$$= (F(3) \cdot 4) \cdot 5$$

$$= (F(2) \cdot 3) \cdot 4) \cdot 5$$

$$= ((F(2) \cdot 3) \cdot 4) \cdot 5$$

$$= (((F(1) \cdot 2) \cdot 3) \cdot 4) \cdot 5$$

$$= ((((F(0) \cdot 1) \cdot 2) \cdot 3) \cdot 4) \cdot 5$$

$$= ((((1 \cdot 1) \cdot 2) \cdot 3) \cdot 4) \cdot 5$$



function 
$$F(n)$$
  
if  $n = 0$  then return  $1$   
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Assignment Project Exam Help

$$F(5) = F(4) \cdot 5$$

$$= (F(3) \cdot 4) \cdot 5$$

$$= (F(2) \cdot 3) \cdot 4) \cdot 5$$

$$= ((F(2) \cdot 3) \cdot 4) \cdot 5$$

$$= (((F(1) \cdot 2) \cdot 3) \cdot 4) \cdot 5$$

$$= (((F(0) \cdot 1) \cdot 2) \cdot 3) \cdot 4) \cdot 5$$

$$= ((((1 \cdot 1) \cdot 2) \cdot 3) \cdot 4) \cdot 5$$

$$= (1 \cdot 1) \cdot 2 \cdot 3 \cdot 4 \cdot 5$$

$$= (1 \cdot 1) \cdot 2 \cdot 3 \cdot 4 \cdot 5$$

$$= (1 \cdot 1) \cdot 2 \cdot 3 \cdot 4 \cdot 5$$

$$= (1 \cdot 1) \cdot 2 \cdot 3 \cdot 4 \cdot 5$$

$$= (1 \cdot 1) \cdot 2 \cdot 3 \cdot 4 \cdot 5$$



function F(n)if n = 0 then return 1 else return  $F(n-1) \cdot n$ 

Assignment Project Exam Help

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function F(n)if n = 0 then return 1else return  $F(n-1) \cdot n$ 

Basic operation: multiplication

Assignment Project Exam Help

https://powcoder.com



function F(n)if n = 0 then return 1 else return  $F(n-1) \cdot n$ 

Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**) https://powcoder.com



function F(n)if n = 0 then return 1 else return  $F(n-1) \cdot n$ 

Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**) https://powcoder.com

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M(0) =



function F(n)if n = 0 then return 1 else return  $F(n-1) \cdot n$ 

Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**) https://powcoder.com

$$M(0) = 0$$



function 
$$F(n)$$
  
if  $n = 0$  then return 1  
else return  $F(n-1) \cdot n$ 

Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**) https://powcoder.com

$$M(0) = 0$$

$$M(n) =$$



function 
$$F(n)$$
  
if  $n = 0$  then return 1  
else return  $F(n-1) \cdot n$ 

Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**) https://powcoder.com

$$M(0) = 0$$

$$M(n) =$$



function 
$$F(n)$$
  
if  $n = 0$  then return 1  
else return  $F(n-1) \cdot n$ 

Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**) https://powcoder.com

$$M(0) = 0$$

$$M(n) = +1$$



function 
$$F(n)$$
  
if  $n = 0$  then return 1  
else return  $F(n-1) \cdot n$ 

Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**) https://powcoder.com

$$M(0) = 0$$

$$M(n) = M(n-1) + 1$$



function 
$$F(n)$$
  
if  $n = 0$  then return 1  
else return  $F(n-1) \cdot n$ 

Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**) https://powcoder.com

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$$M(0) = 0$$

$$M(n) = M(n-1) + 1$$

Need to express M(n) in **closed form** (i.e. non-recursively)



function 
$$F(n)$$
  
if  $n = 0$  then return 1  
else return  $F(n-1) \cdot n$ 

Basic operation: multiplication

We express the cost **recursively** (as a **recurrence relation**) https://powcoder.com

Add WeChat powcoder

$$M(0) = 0$$

$$M(n) = M(n-1) + 1$$

Need to express M(n) in **closed form** (i.e. non-recursively)

Try: "telescoping" aka "backward substitution"



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

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$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1)?

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https://powcoder.com



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is 
$$M(n-1)$$
?

$$M(n-1) = M((n-1)-1)+1$$

Assignment Project Exam Help

https://powcoder.com



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1)?

$$M(n-1) = M((n-1)-1)+1$$

Assignment Project Exam  $H_{\overline{elp}}M(n-2)+1$ 

https://powcoder.com



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is 
$$M(n-1)$$
?

$$M(n-1) = M((n-1)-1)+1$$

Assignment Project Exam  $H_{elp}M(n-2) + 1$ 

$$M(n) =$$

https://powcoder.com



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$

Assignment Project Exam  $H_{elp}M(n-2)+1$ 

$$M(n) = (M(n-2) + 1)_{\text{https://powcoder.com}}$$



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is 
$$M(n-1)$$
?

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$

Assignment Project Exam  $H_{elp}M(n-2)+1$ 

$$M(n) = (M(n-2) + \frac{1}{https:})$$
 by  $\frac{1}{plowcoder.com}$   
=  $M(n-2) + \frac{2}{https:}$  We Chat powcoder



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$

Assignment Project Exam  $H_{elp}M(n-2) + 1$ 

$$M(n) = (M(n-2) + \frac{1}{https:})$$
 powcoder.com  
=  $M(n-2) + \frac{2}{Add}$  WeChat powcoder  
=  $(M(n-3) + 1) + 2$ 



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$
  
Assignment Project Exam Help $M(n-2)+1$ 

$$M(n) = (M(n-2) + 1) + 1$$

$$= M(n-2) + 2 + 2 + 3 + 3$$

$$= M(n-3) + 1 + 2$$

$$= M(n-3) + 3$$



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$
  
Assignment Project Exam Help $M(n-2)+1$   
 $M(n) = (M(n-2)+1) + 1$ 
Move the power of the power o

$$= M(n-2) + 2 dd$$
 WeChat powcoder

$$= (M(n-3)+1)+2$$

$$= M(n-3) + 3$$

. . .



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$
  
Assignment Project Exam Help $M(n-2)+1$   
 $M(n) = (M(n-2) + 1) + 1$   
Assignment Project Exam Help $M(n-2)+1$ 

$$= M(n-2) + 2 dd WeChat powcoder$$
$$= (M(n-3) + 1) + 2$$

$$= M(n-3) + 3$$

• • •

$$= M(n-n) + n$$



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$

Assignment Project Exam Help  $M(n-2)+1$ 
 $M(n) = (M(n-2) + \frac{1}{ht}) + \frac{1}{ht}$  be well as  $M(n-2) + \frac{1}{ht}$ .



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$

Assignment Project Exam Help  $M(n-2)+1$ 
 $M(n) = (M(n-2) + \frac{1}{2}) + \frac{1}{2} + \frac{1}$ 

= n



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$
  
Assignment Project Exam Help $M(n-2)+1$ 

$$M(n) = (M(n-2) + \frac{1}{\text{https://powcoder.com}})$$

$$= M(n-2) + \frac{2}{\text{Add WeChat powcoder}}$$

$$= (M(n-3) + 1) + 2$$

$$= M(n-3) + 3$$
...
$$= M(n-n) + n$$

$$= M(0) + n$$

= n



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is 
$$M(n-1)$$
?

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$

Assignment Project Exam  $H_{elp}M(n-2) + 1$ 

$$M(n) = (M(n-2) + \frac{1}{\text{https://powcoder.com}})$$

$$= M(n-2) + Add \text{ WeChat powcoder}$$
Closed form:

$$= (M(n-3)+1)+2$$

$$= M(n-3) + 3$$

$$= M(n-n) + n$$

$$= M(0) + n$$

$$= n$$

$$M(n) = n$$



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$

Assignment Project Exam  $H_{elp}M(n-2) + 1$ 

$$M(n) = (M(n-2) + \frac{1}{\text{https://powcoder.com}})$$

$$= M(n-2) + Add \text{ WeChat powcoder}$$
Closed form:

$$= (M(n-3)+1)+2$$

$$= M(n-3) + 3$$

$$= M(n-n) + n$$

$$= M(0) + n$$

$$= n$$

$$M(n) = n$$

#### **Complexity:**



$$M(n) = M(n-1) + 1$$

$$M(0) = 0$$

What is 
$$M(n-1)$$
?

What is M(n-1)? 
$$M(n-1) = M((n-1)-1)+1$$

Assignment Project Exam  $H_{elp}M(n-2) + 1$ 

$$M(n) = (M(n-2) + 1)_{\text{https://powcoder.com}}$$

$$= M(n-2) + Add \text{ WeChat powcoder}$$
Closed form:

$$= (M(n-3)+1)+2$$

$$= M(n-3) + 3$$

$$= M(n-n) + n$$

$$= M(0) + n$$

$$= n$$

$$M(n) = n$$

#### Complexity:

$$M(n) \in \Theta(n)$$

## Binary Search in Sorted Array



```
function BINSEARCH(A[\cdot], lo, hi, key)

if lo > hi then return -1

mid \leftarrow lo + (hi - lo)/2

if A[mid] = key then return mid

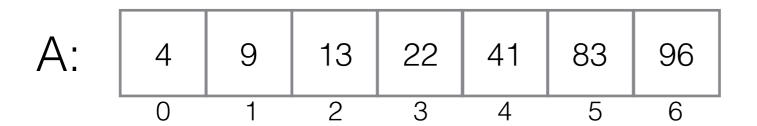
else

Assignment Project Exam Help

if A[mid] > keyhtthe powcoder.com

return BINSEARCH(A, lo, mid - 1, key)

else return BINSEARCH(A, mid + 1, hi, key)
```



## Binary Search in Sorted Array



```
function BINSEARCH(A[\cdot], lo, hi, key)

if lo > hi then return -1

mid \leftarrow lo + (hi - lo)/2

if A[mid] = key then return mid

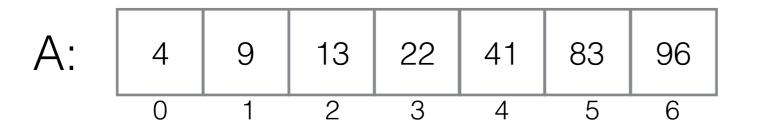
else

Assignment Project Exam Help

if A[mid] > keyhttherpowcoder.com

return BINSEARCH(A, lo, mid -1, key)

else return BINSEARCH(A, mid +1, hi, key)
```



## Binary Search in Sorted Array



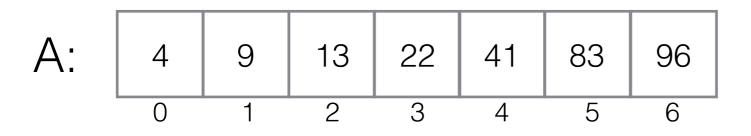
lo: 0

function BINSEARCH( $A[\cdot]$ , lo, hi, key)

if lo > hi then return -1  $mid \leftarrow lo + (hi - lo)/2$ if A[mid] = key then return midelse

Assignment Project Exam Help

if A[mid] > keyhttpenpowcoder.comreturn BinSearch(A, lowedder - 1, key)else return BinSearch(A, mid + 1, hi, key)



## Binary Search in Sorted Array



```
function BINSEARCH(A[\cdot], lo, hi, key)

if lo > hi then return -1

mid \leftarrow lo + (hi - lo)/2

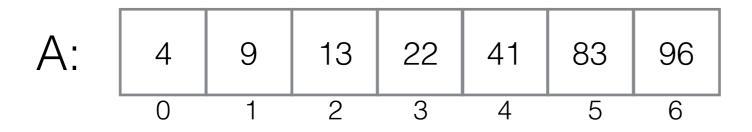
if A[mid] = key then return mid

else

Assignment Project Exam Help
```

hi: 6

if A[mid] > keyhtthenpowcoder.comreturn BINSEARCH(A, lowedder - 1, key)else return BINSEARCH(A, mid + 1, hi, key)



## Binary Search in Sorted Array



```
function BINSEARCH(A[\cdot], lo, hi, key)

if lo > hi then return -1

mid \leftarrow lo + (hi - lo)/2

if A[mid] = key then return mid

else

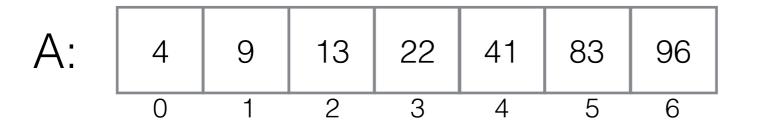
Assignment Project Exam Help
```

lo: 0

hi: 6

key: 41

if A[mid] > keyhttphenpowcoder.comreturn BINSEARCH(A, lowedder - 1, key)else return BINSEARCH(A, mid + 1, hi, key)



## Binary Search in Sorted Array



function BINSEARCH( $A[\cdot]$ , lo, hi, key)

if lo > hi then return -1  $mid \leftarrow lo + (hi - lo)/2$ if A[mid] = key then return midelse

Assignment Project Exam Help

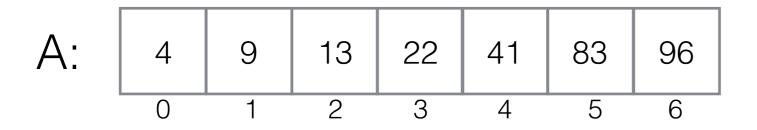
lo: 0

hi: 6

key: 41

mid: 3

if A[mid] > keyhtthe powcoder.comreturn BINSEARCH(A, lowedder, -1, key)else return BINSEARCH(A, mid + 1, hi, key)



## Binary Search in Sorted Array



function BINSEARCH( $A[\cdot]$ , lo, hi, key)

if lo > hi then return -1  $mid \leftarrow lo + (hi - lo)/2$ if A[mid] = key then return midelse

Assignment Project Exam Help

lo: 0

hi: 6

key: 41

mid: 3

if A[mid] > keyhttpsetpowcoder.com

return BINSEARCH(A, lowerder - 1, key) else return BINSEARCH(A, mid + 1, hi, key)

A: 4 9 13 22 41 83 96 0 1 2 3 4 5 6

BinSearch(A,0,6,41)

### Binary Search in Sorted Array



```
function BINSEARCH(A[\cdot], lo, hi, key)

if lo > hi then return -1

mid \leftarrow lo + (hi - lo)/2

if A[mid] = key then return mid

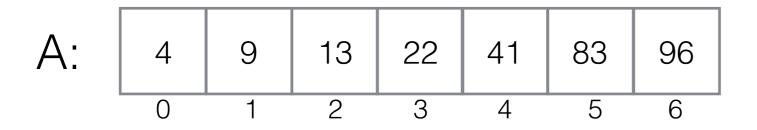
else

Assignment Project Exam Help

if A[mid] > keyhttpherpowcoder.com

return BINSEARCH(A_i lowerid -1, key)

else return BINSEARCH(A_i, mid + 1, hi, key)
```



## Binary Search in Sorted Array



```
function BINSEARCH(A[\cdot], lo, hi, key)

if lo > hi then return -1

mid \leftarrow lo + (hi - lo)/2

if A[mid] = key then return mid

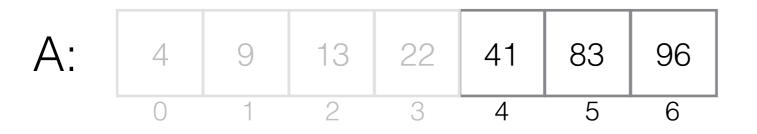
else

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if A[mid] > keyhttherpowcoder.com

return BINSEARCH(A, lo, mid -1, key)

else return BINSEARCH(A, mid +1, hi, key)
```



## Binary Search in Sorted Array



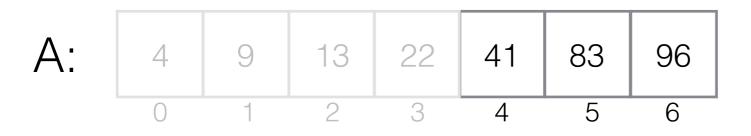
lo: 4

function BINSEARCH( $A[\cdot]$ , lo, hi, key)

if lo > hi then return -1  $mid \leftarrow lo + (hi - lo)/2$ if A[mid] = key then return midelse

Assignment Project Exam Help

if 
$$A[mid] > keyhtthempowcoder.com$$
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else return  $BINSEARCH(A, mid + 1, hi, key)$ 



## Binary Search in Sorted Array



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function BINSEARCH(A[\cdot], lo, hi, key)

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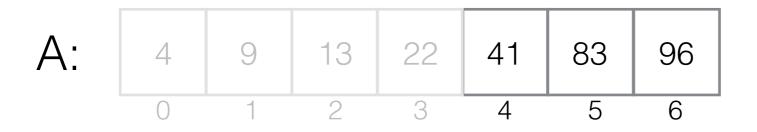
mid \leftarrow lo + (hi - lo)/2

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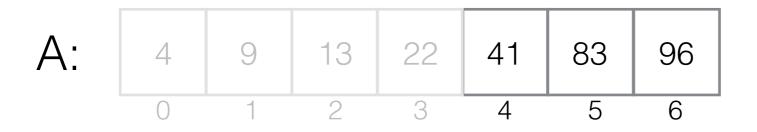
Assignment Project Exam Help
```

lo: 4

hi: 6

key: 41

```
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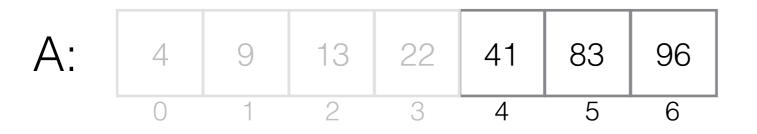
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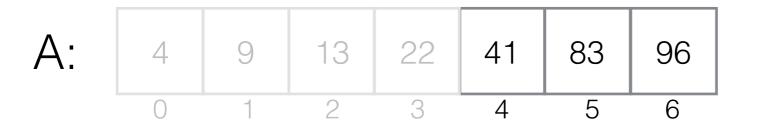
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Assignment Project Exam Help
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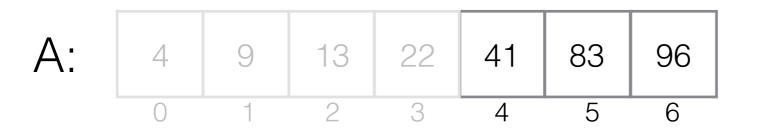
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Assignment Project Exam Help

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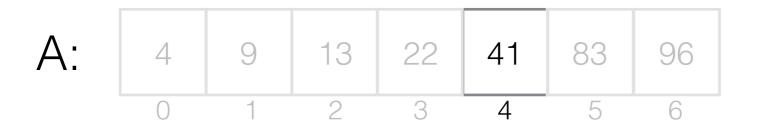
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Assignment Project Exam Help
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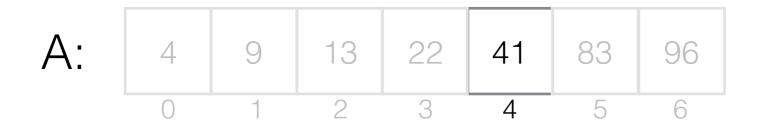
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Assignment Project Exam Help
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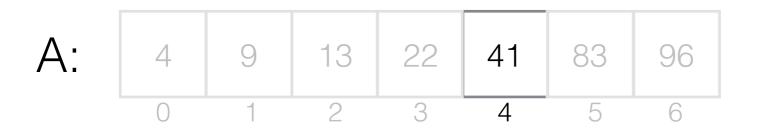
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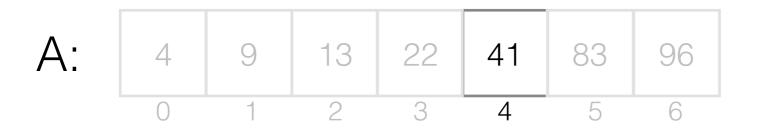
in the initial sequence of the powcoder.com

lo: 4

key: 41

mid: 4
```

return BINSEARCH(A, mid + 1, hi, key)
else return BINSEARCH(A, mid + 1, hi, key)



## Binary Search in Sorted Array



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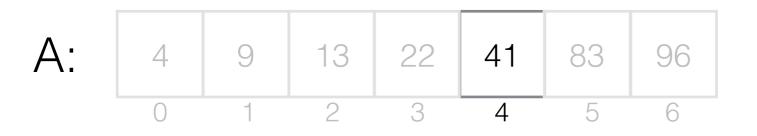
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Assignment Project Exam Help
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returns 4

### Binary Search in Sorted Array



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Assignment Project Exam Help

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Assignment Project Exam Help

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Basic operation: key comparison A[mid] = key

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$$C(0) =$$

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```

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$$C(0) = 0$$

#### Example:

### Binary Search in Sorted Array



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Basic operation: key comparison A[mid] = key

$$C(0) = 0 \qquad C(n) = +1$$

#### Example:

#### Binary Search in Sorted Array



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Assignment Project Exam Help

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Basic operation: key comparison A[mid] = key

$$C(0) = 0$$
  $C(n) = C(n/2) + 1$ 



### A **smoothness rule** allows us to assume that n is a power of 2

$$C(0) = 0$$

$$C(n) = C(n/2) + 1$$

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https://powcoder.com



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$$C(n) = C(n/2) \frac{1}{powcoder.com}$$
  
=  $(C(n/4) d + W) Chal powcoder$ 



### A **smoothness rule** allows us to assume that n is a power of 2

$$C(0) = 0$$

$$C(n) = C(n/2) + 1$$

$$C(n) = \frac{C(n/2)}{\text{https!//powcoder.com}}$$

$$= \frac{(C(n/4))}{\text{dd-W}} \text{Chal powcoder}$$

$$= \frac{((C(n/8) + 1) + 1)}{\text{chal powcoder}}$$



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$$\cdots$$

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$$= 1 + \log_2 n$$

$$C(n) \in \Theta(\log n)$$



In O,  $\Omega$ ,  $\Theta$ , expressions we can just write "log" for any logarithmic function no matter what the base is

Asymptotically, all toganithmic to the same, since

 $log_a^{https://p(dwgodb)}(log_b x)$ 



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```
function Euclid (m, n) while n \neq 0 do r \leftarrow m \mod n m \leftarrow n
```

function Euclid(m, n)
 if n = 0 then
 return m
 return Euclid(n, m mod n)

 $n \leftarrow r$  return m

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Running time is linear in size (in size (in bits)) of input, i.e. O(\log(m+n))
```



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r \leftarrow m \mod n function \text{Euclid}(m, n)

r \leftarrow m \mod n return m

m \leftarrow n return \text{Euclid}(n, m \mod n)

n \leftarrow r

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Running time is linear in size (in bits) of input, i.e.

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since the value of *m* (and *n*) is at least halved in every two iterations.



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Running time is linear in size (in bits) of input, i.e.

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Why? After two iterations, m becomes m mod n; also

$$1 < n < m \implies m \mod n < m/2$$



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Suppose

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Suppose

$$t_1(n) \in O(g_1(n))$$

$$t_2(n) \in O(g_2(n))$$

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$$t_1(n) + t_2(n) \in Q(\max\{g_1(n), g_2(n)\})$$

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Add WeChat powcoder 
$$c \cdot t_1(n) \in \mathcal{O}(g_1(n))$$



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Add WeChat powcoder 
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Add WeChat powcoder 
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$$t_1(n) \cdot t_2(n) \in O(g_1(n) \cdot g_2(n))$$



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Add WeChat powcoder 
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$$t_1(n) \cdot t_2(n) \in O(g_1(n) \cdot g_2(n))$$

(for nested loops: count number of times outer loop is executed, multiply by cost of inner loop)



From Stirling's formula:

$$n! \in O(n^{n+\frac{1}{2}})$$

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this is not the time complexity of an algorithm to compute n-factorial

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this is not the time complexity of an algorithm to compute n-factorial

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$$\sum_{i=0}^{n} i^{2} = \frac{n}{3} (n + \frac{1}{2})(n + 1)$$
https://powcoder.com<sup>2</sup>

$$\sum_{i=0}^{n} (2i + d\mathbf{q})$$
WeChat(powcqder

$$\sum_{i=1}^{n} 1/i = O(\log n)$$



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WeChat(powcqder

$$\sum_{i=1}^{n} 1/i = O(\log n)$$

See also Cormen's Appendix A or Levitin's Appendix A.

Levitin's Appendix B is a tutorial on recurrence relations.

#### The Road Ahead



• You'll get much more familiar with asymptotic analysis as we assignment Project Fram Help we meet in this course.

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 Next week we begin our study of algorithms by looking at brute force approaches