COMP9318: Data Warehousing and Data Mining

— L6: Association Rule Mining —

- Problem definition and preliminaries

What Is Association Mining?

- Association rule mining:
  - Finding frequent patterns, associations, correlations, or causal structures among sets of items or objects in transaction databases, relational databases, and other information repositories.
  - Frequent pattern: pattern (set of items, sequence, etc.) that occurs frequently in a database [AIS93]
- Motivation: finding regularities in data
  - What products were often purchased together? — Beer and diapers?!
  - What are the subsequent purchases after buying a PC?
  - What kinds of DNA are sensitive to this new drug?
  - Can we automatically classify web documents?

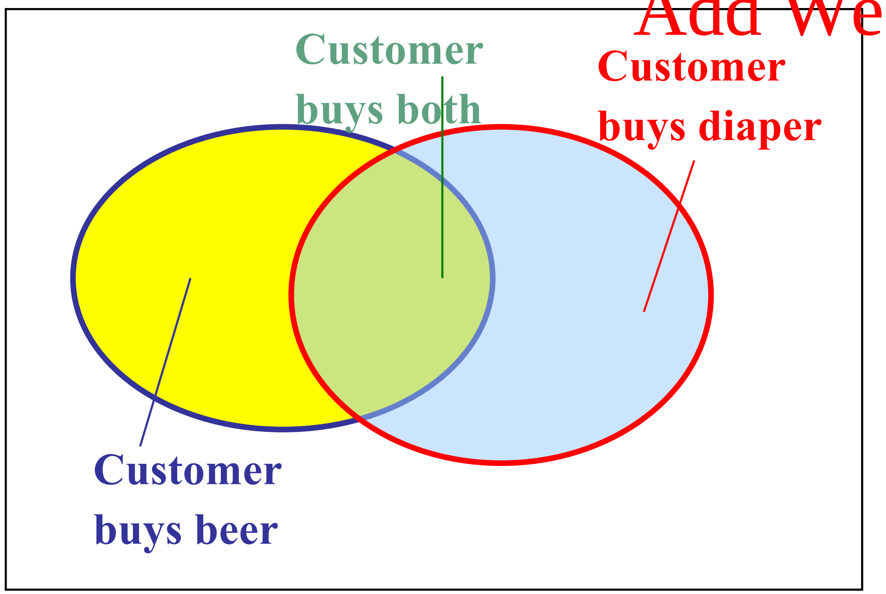
Why Is Frequent Pattern or Assoiciation Mining an Essential Task in Data Mining?

- Foundation for many essential data mining tasks
  - Association, correlation, causality
  - Sequential patterns, temporal or cyclic association, partial periodicity, spatial and multimedia association
  - Associative classification, cluster analysis, iceberg cube, fascicles (semantic data compression)
- Broad applications
  - Basket data analysis, cross-marketing, catalog design, sale campaign analysis
  - **Web log** (click stream) **analysis**, DNA sequence analysis, etc.

c.f., google's spelling suggestion

Basic Concepts: Frequent Patterns and Association Rules

Transaction-id	Items bought
10	{ A, B, C }
20	{ A, C }
30	{ A, D }
40	{ B, E, F }



- Itemset $X=\{x_1, \ldots, x_k\}$
  - **Shorthand**: $x_1\ x_2\ \ldots\ x_k$
- Find all the rules $X \rightarrow Y$ with min confidence and support
  - support, *s*, probability that a transaction contains $X \cup Y$
  - confidence, *c*, conditional probability that a transaction having *X* also contains *Y*.

Let min_support = 50%,
min_conf = 70%:

frequent itemset

$sup(AC) = 2$

association rule

$\cancel{A \Rightarrow C\ (50\%, 66.7\%)}$

$C \Rightarrow A\ (50\%, 100\%)$

Mining Association Rules—an Example

Transaction-id	Items bought
10	A, B, C
20	A, C
30	A, D
40	B, E, F

Min. support 50%
Min. confidence 50%

Frequent pattern	Support
{A}	75%
{B}	50%
{C}	50%
{A, C}	50%

For rule $A \rightarrow C$:

support = support($\{A\} \cup \{C\}$) = 50%

confidence = support($\{A\} \cup \{C\}$)/support($\{A\}$) = 66.6%

major computation challenge: calculate the support of itemsets
$\leftarrow$ The ***frequent itemset mining*** problem

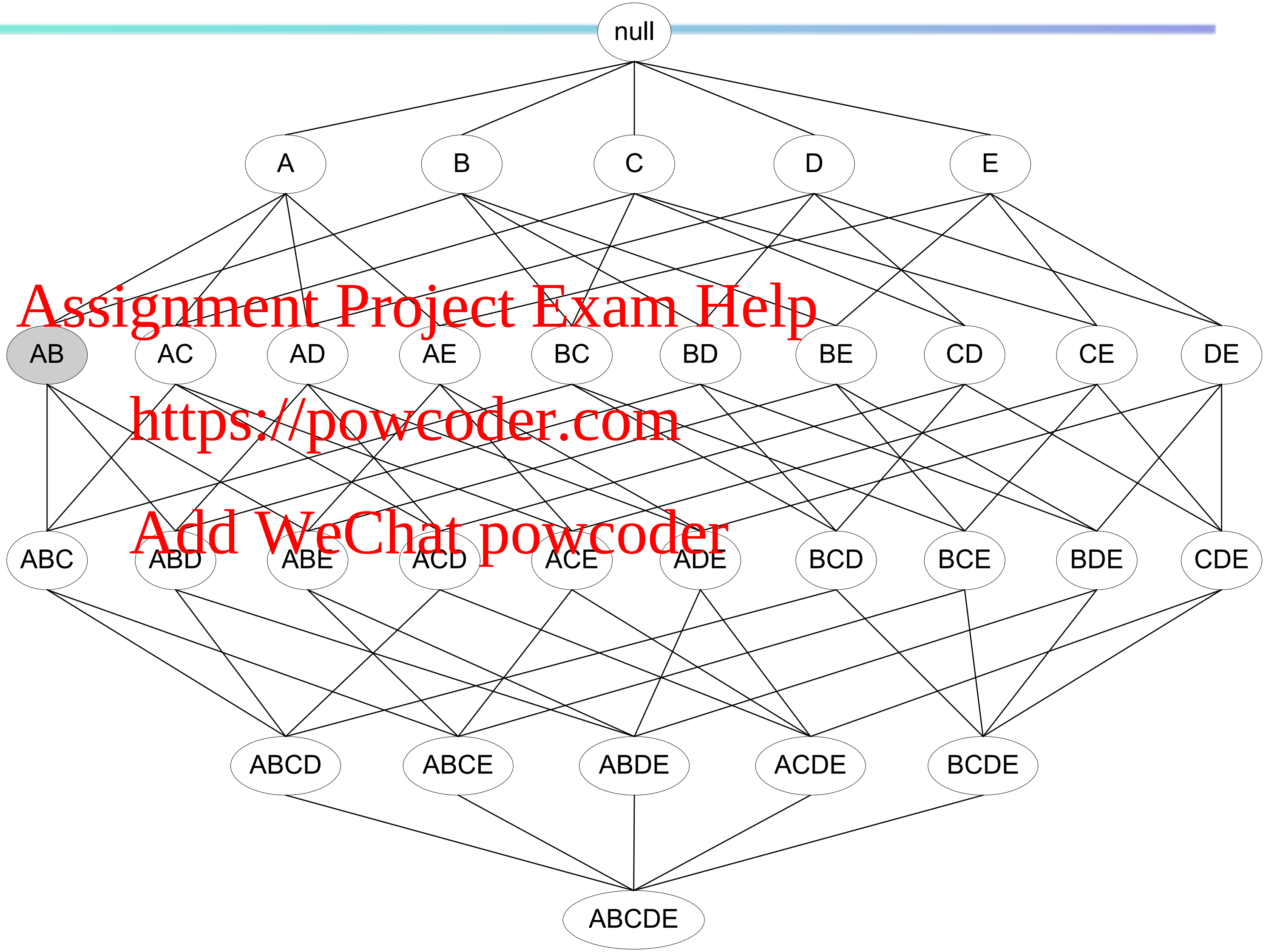
- Algorithms for scalable mining of (single-dimensional Boolean) association rules in transactional databases

Association Rule Mining Algorithms

Candidate Generation & Verification

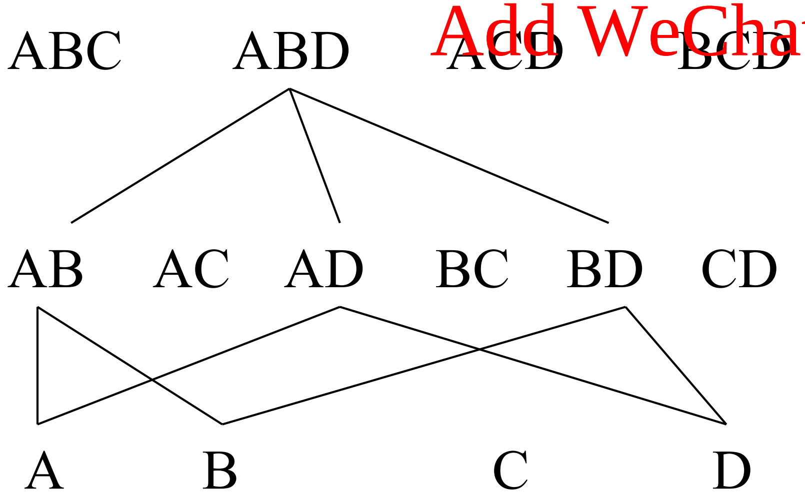
- Naïve algorithm
  - Enumerate all possible itemsets and check their support against *min_sup*
  - Generate all association rules and check their confidence against *min_conf*
- The Apriori property
  - Apriori Algorithm
  - FP-growth Algorithm

All Candidate Itemsets for {A, B, C, D, E}



Apriori Property

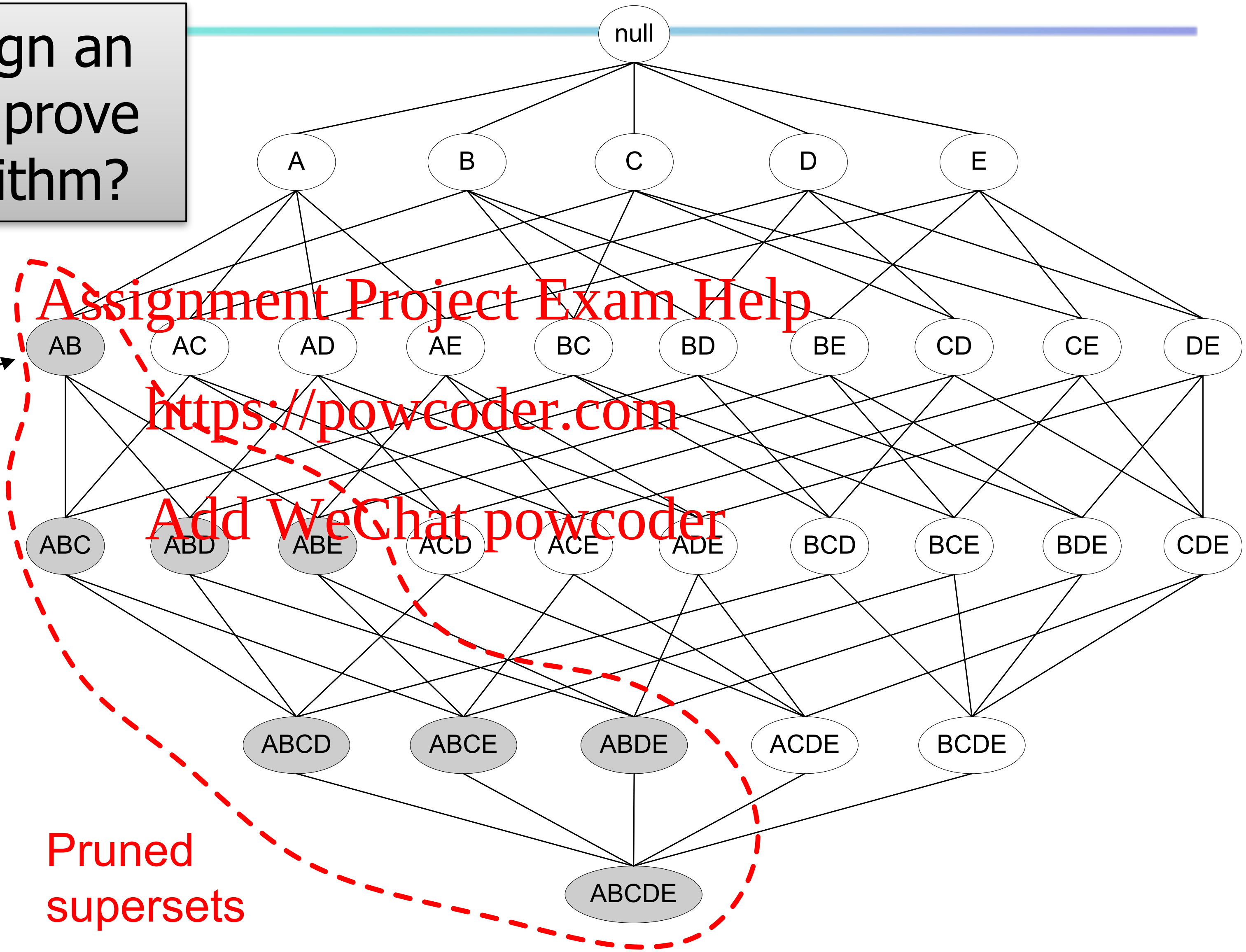
- A *frequent* (used to be called *large*) *itemset* is an itemset whose support is ≥ min_sup.
- Apriori property (downward closure): any subsets of a frequent itemset are also frequent itemsets
- Aka the anti-monotone property of support



"any supersets of an infrequent itemset are also infrequent itemsets"

Illustrating Apriori Principle

Q: How to design an algorithm to improve the naïve algorithm?



Found to be Infrequent

Pruned supersets

Apriori: A Candidate Generation-and-test Approach

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not be generated/tested!
- Algorithm [Agrawal & Srikant 1994]
  1. $C_k \leftarrow$ Perform level-wise candidate generation (from singleton itemsets)
  2. $L_k \leftarrow$ Verify C_k against L_k
  3. $C_{k+1} \leftarrow$ generated from L_k
  4. Goto 2 if C_{k+1} is not empty

The Apriori Algorithm

- Pseudo-code:

C_k: Candidate itemset of size k

L_k : frequent itemset of size k


```
L_1 = {frequent items};
for (k = 1; L_k !=∅; k++) do begin
    C_k+1 = candidates generated from L_k;
    for each transaction t in database do begin
        increment the count of all candidates in C_k+1
        that are contained in t
    end
    L_k+1 = candidates in C_k+1 with min_support
end
return ∪_k L_k;
```

The Apriori Algorithm—An Example

minsup = 50%

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3rd scan

L_3

Itemset	sup
{B, C, E}	2

Important Details of Apriori

1. How to generate candidates?
   - Step 1: self-joining L_k (what's the join condition? why?)
   - Step 2: pruning
2. How to count supports of candidates?

Example of Candidate-generation

- $L_3=\{abc, abd, acd, ace, bcd\}$
- Self-joining: L_3*L_3
  - *abcd* from *abc* and *abd*
  - *acde* from *acd* and *ace*
- Pruning:
  - *acde* is removed because *ade* is not in L_3
- $C_4=\{abcd\}$

Generating Candidates in SQL

- Suppose the items in L_{k-1} are listed in an order
- Step 1: self-joining L_{k-1}

insert into $\boldsymbol{C_k}$

select $\boldsymbol{p.item_1, p.item_2, \ldots, p.item_{k-1}, q.item_{k-1}}$

from $\boldsymbol{L_{k-1}\ p, L_{k-1}\ q}$

where $\boldsymbol{p.item_1 = q.item_1, \ldots, p.item_{k-2} = q.item_{k-2}, p.item_{k-1} < q.item_{k-1}}$

- Step 2: pruning

forall ***itemsets c in*** $\boldsymbol{C_k}$ do

    forall ***(k-1)-subsets s of c*** do

        if *(s is not in* L_{k-1}*)* **then delete** *c* **from** C_k

Derive rules from frequent itemsets

- Frequent itemsets != association rules
- One more step is required to find association rules

- For each frequent itemset *X*,

  For each proper nonempty subset *A* of *X*,

  - Let *B* = X - *A*
  - A → B is an association rule if
    - Confidence (A → B) ≥ *min_conf*,
      where support (A → B) = support (AB), and
      confidence (A → B) = support (AB) / support (A)

Example – deriving rules from frequent itemsets

- Suppose 234 is frequent, with supp=50%
  - Proper nonempty subsets: 23, 24, 34, 2, 3, 4, with supp=50%, 50%, 75%, 75%, 75%, 75% respectively
  - These generate these association rules:
    - 23 => 4, confidence=100%
    - 24 => 3, confidence=100%
    - 34 => 2, confidence=67% = (N*50%)/(N*75%)
    - 2 => 34, confidence=67%
    - 3 => 24, confidence=67%
    - 4 => 23, confidence=67%
    - All rules have support = 50%

Q: is there any optimization (e.g., pruning) for this step?

Deriving rules

- To recap, in order to obtain A → B, we need to have Support(AB) and Support(A)
- This step is not as time-consuming as frequent itemsets generation
  - Why?
- It's also easy to speedup using techniques such as parallel processing.
  - How?
- Do we really need candidate generation for deriving association rules?
  - Frequent-Pattern Growth (FP-Tree)

Bottleneck of Frequent-pattern Mining

- Multiple database scans are costly
- Mining long patterns needs many passes of scanning and generates lots of candidates
  - To find frequent itemset $i_1 i_2 \ldots i_{100}$
    - # of scans: 100
    - # of Candidates: $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1$
- Bottleneck: candidate-generation-and-test

Can we avoid candidate generation ***altogether****?*

- FP-growth

No Pain, No Gain

	Java	Lisp	Scheme	Python	Ruby
Alice	X				X
Bob				X	X
Charlie	X			X	X
Dora		X	X		

minsup = 1

- Apriori:
  - L1 = {J, L, S, P, R}
  - C2 = all the $(^{5}{}_{2})$ combinations
    - Most of C2 do not contribute to the result
    - There is no way to tell because

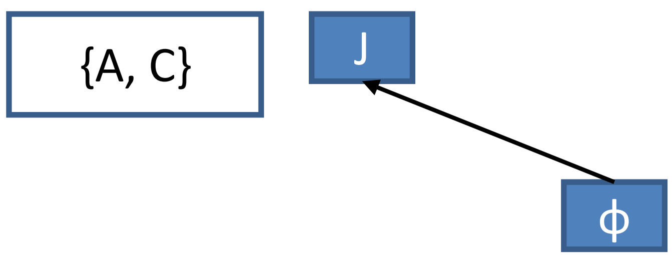
No Pain, No Gain

	Java	Lisp	Scheme	Python	Ruby
Alice	X				X
Bob				X	X
Charlie	X			X	X
Dora		X	X		

minsup = 1

Ideas:
- Keep the support set for each frequent itemset
- DFS

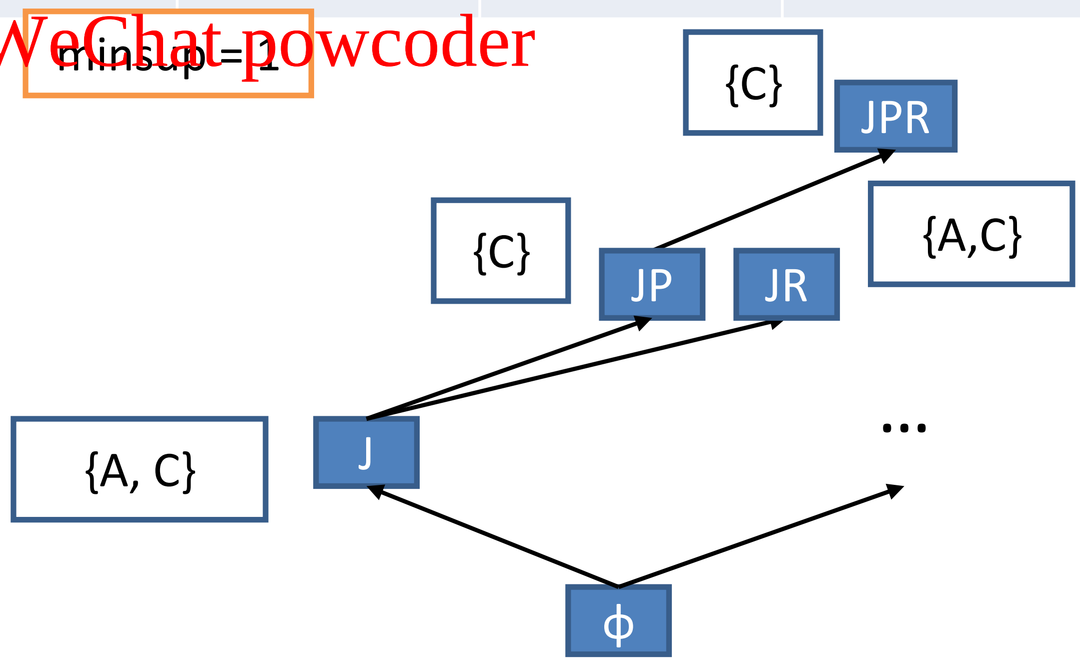
J ➔ JL?
J ➔ ???
Only need to look at support set for J



No Pain, No Gain

	Java	Lisp	Scheme	Python	Ruby
Alice	X				X
Bob				X	X
Charlie	X			X	X
Dora		X	X		

Ideas:
- Keep the support set for each frequent itemset
- DFS



Notations and Invariants

- CondiditonalDB:
  - $DB|p = \{t \in DB \mid t \text{ contains itemset } p\}$
  - $DB = DB|\varnothing$ (i.e., conditioned on nothing)
  - Shorthand: $DB|px = DB|(p \cup x)$
- SupportSet($p \cup x$, DB) = SupportSet(x, DB|p)
  - $\{x \mid x \bmod 6 = 0 \wedge x \in [100]\} =$
    $\{x \mid x \bmod 3 = 0 \wedge x \in even([100])\}$
- A FP-tree is equivalent to a DB|p
  - One can be converted to another
  - Next, we illustrate the alg using conditionalDB

FP-tree Essential Idea /1

- Recursive algorithm again!
- Freq**Itemsets**(DB|p):
  - X = Find**Locally**Frequent**Items**(DB|p)

    $\boxed{\text{output } \{ (x\ p) \mid x \in X \}}$

  - Foreach x in X
    - DB*|px = GetConditionalDB⁺(DB*|p, x)
    -
    - Freq**Itemsets**(DB*|px)

easy task, as only items (not itemsets) are needed

all frequent itemsets in DB|p belong to one of the following categories:

patterns ~ $x_i p$
patterns ~ ★px_1
patterns ~ ★px_2
patterns ~ ★px_i
patterns ~ ★px_n

obtained via recursion

No Pain, No Gain

DB|J

	Java	Lisp	Scheme	Python	Ruby
Alice	X				X
Charlie	X			X	X

minsup = 1

- Freq**Itemsets**(DB|J):
  - {**P**, **R**} ← Find**Locally**Frequent**Items**(DB|J)
  - Output {JP, JR}
  - Get DB*|J**P**; Freq**Itemsets**(DB*|J**P**)
  - Get DB*|J**R**; Freq**Itemsets**(DB*|J**R**)
  - // Guaranteed no other frequent itemset in DB|J

FP-tree Essential Idea /2

- Freq**Itemsets**(DB|p):
  - If boundary condition, then …
  - X = Find**Locally**Frequent**Items**(DB|p)
  - [optional] DB*|p = PruneDB(DB|p, X)

    output { (x p) | x ∈ X }
  - Foreach x in X
    - DB*|px = GetConditionalDB⁺(DB*|p, x)
    - [optional] if DB*|px is degenerated, then powerset(DB*|px)
    - Freq**Itemsets**(DB*|px)

Also output each item in X (appended with the conditional pattern)

Remove items not in X; potentially reduce # of transactions (∅ or dup). Improves the efficiency.

Also gets rid of items already processed before x ➔ *avoid duplicates*

Grayed items are for illustration purpose only.

Lv 1 Recursion

- minsup = 3

F C A D G I M P
A B C F L M O
B F H J O W
B C K S P
A F C E L P M N

DB

F C A M P
F C A B M
F B
C B P
F C A M P

DB*

X = {F, C, A, B, M, P}

Output: F, C, A, B, M, P

DB*|P

F C A M P
C B P
F C A M P

DB*|M (sans P)

DB*|B (sans MP)

DB*|A (sans BMP)

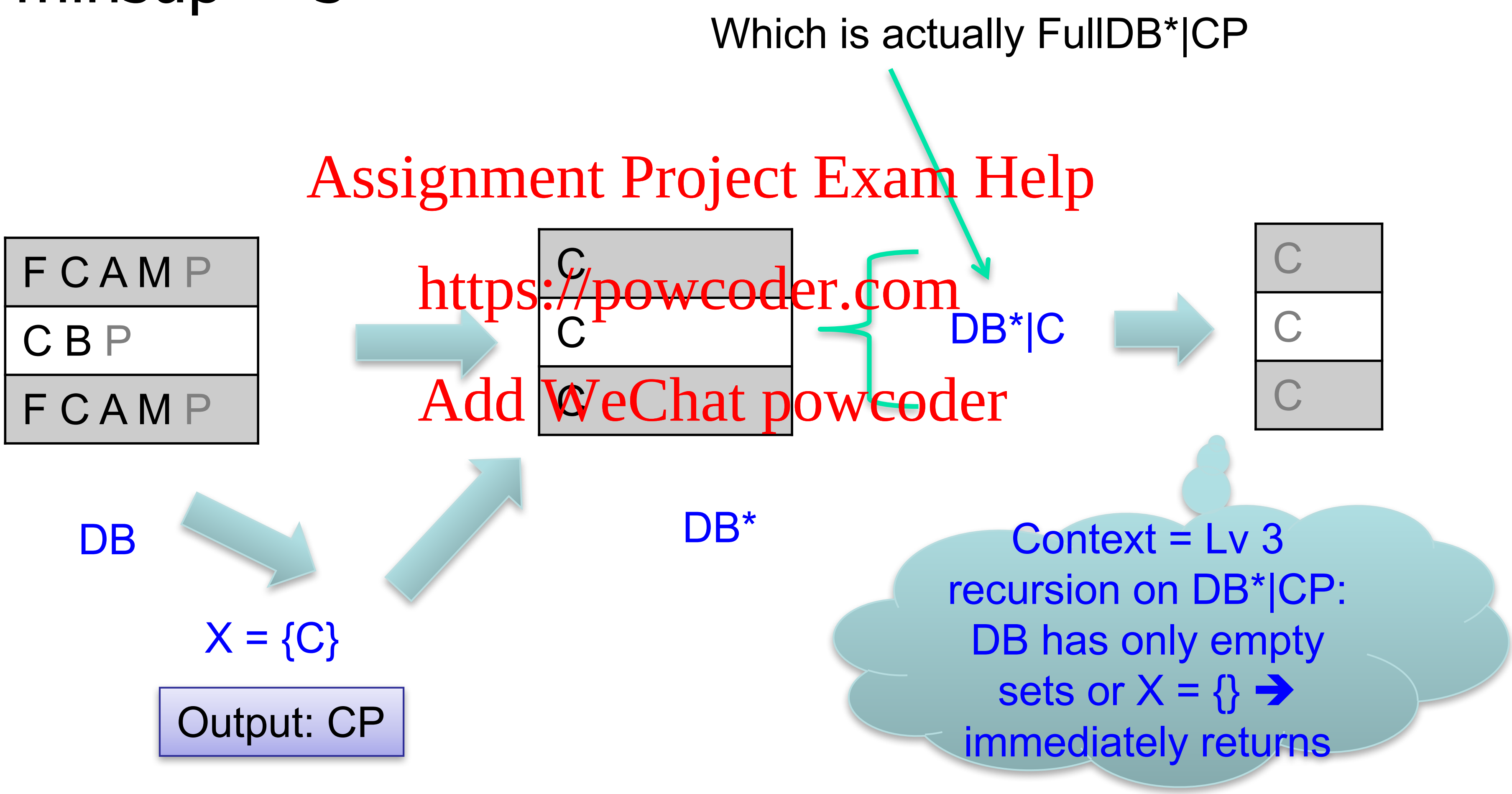
DB*|C (sans ABMP)

DB*|F (sans CABMP)

F C A
F C A
F C A

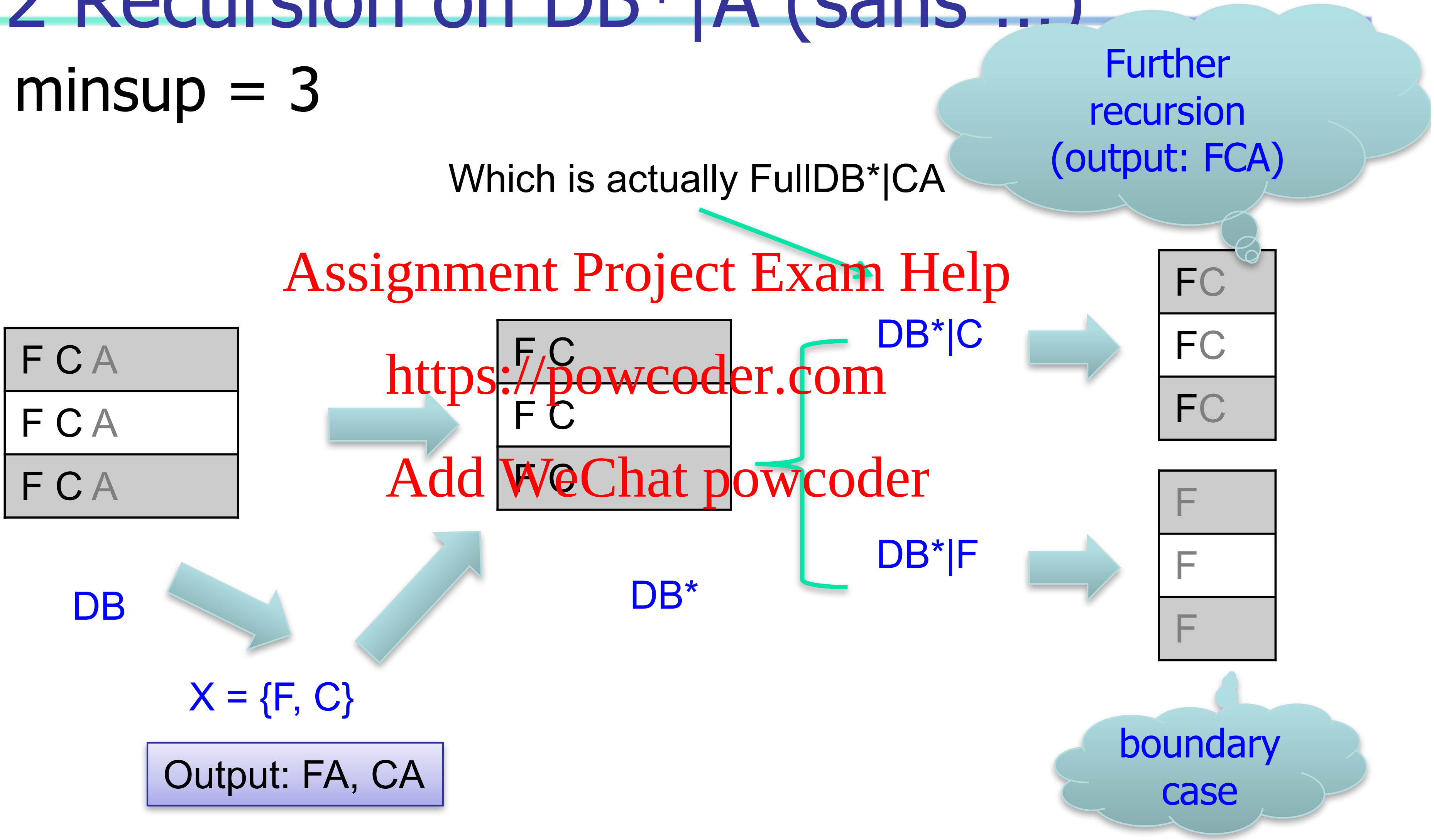
Lv 2 Recursion on DB*|P

- minsup = 3



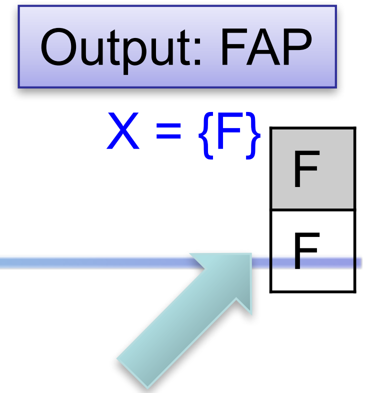
Lv 2 Recursion on DB*|A (sans ...)

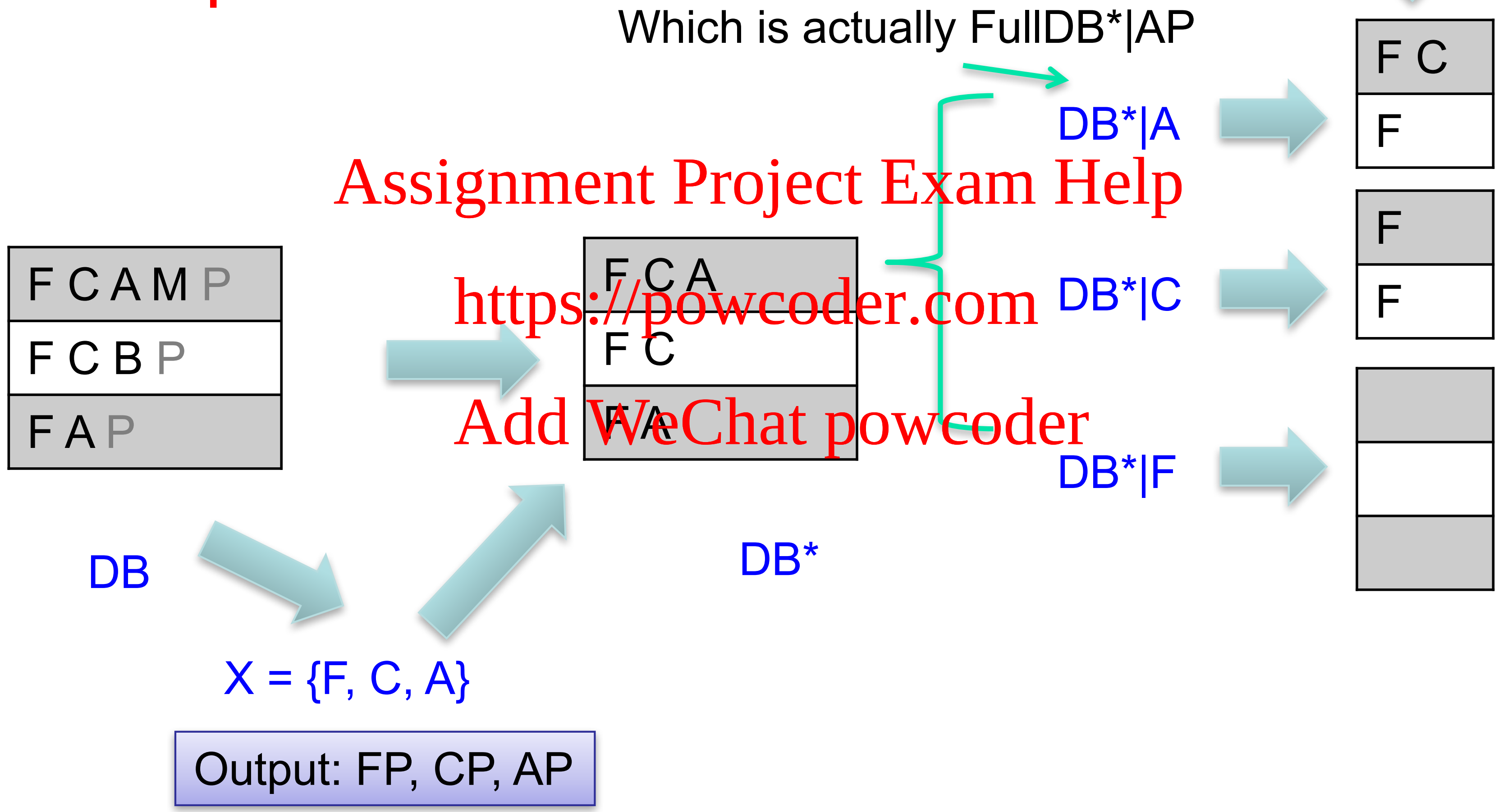
- minsup = 3



Different Example:
Lv 2 Recursion on DB*|P

- minsup = 2





I will give you back the FP-tree

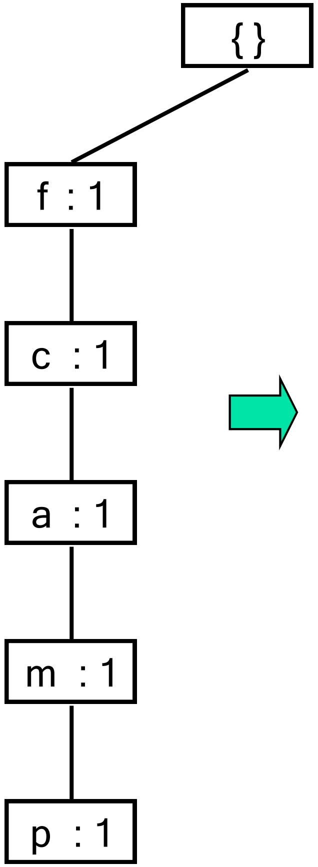
- An FP-tree tree of DB consists of:
  - A fixed **order** among items in DB
  - A prefix, **threaded** tree of **sorted** transactions in DB
  - Header table: (item, freq, ptr)
- When used in the algorithm, the input DB is always pruned (c.f., PruneDB())
  - Remove infequent items
  - Remove infrequent items in every transaction

FP-tree Example

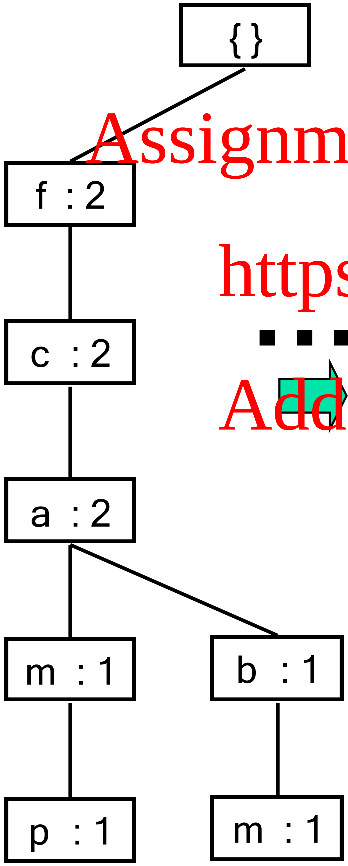
minsup = 3

TID	*Items bought*	*(ordered) frequent items*
100	{*f, a, c, d, g, i, m, p*}	{*f, c, a, m, p*}
200	{*a, b, c, f, l, m, o*}	{*f, c, a, b, m*}
300	{*b, f, h, j, o, w*}	{*f, b*}
400	{*b, c, k, s, p*}	{*c, b, p*}
500	{*a, f, c, e, l, p, m, n*}	{*f, c, a, m, p*}

TID	*Items bought*	*(ordered) frequent items*
100	*{f, a, c, d, g, i, m, p}*	*{f, c, a, m, p}*
200	*{a, b, c, f, l, m, o}*	*{f, c, a, b, m}*
300	*{b, f, h, j, o, w}*	*{f, b}*
400	*{b, c, k, s, p}*	*{c, b, p}*
500	*{a, f, c, e, l, p, m, n}*	*{f, c, a, m, p}*

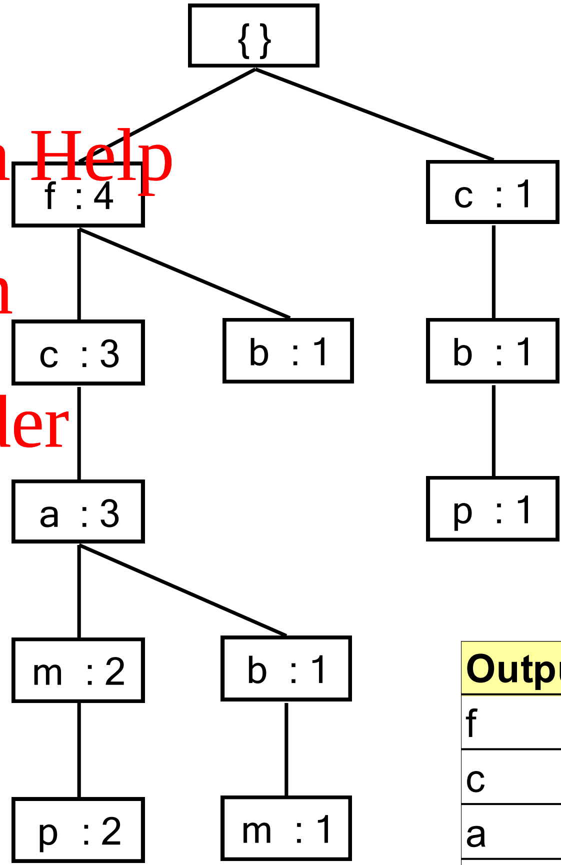


Insert t_1



Insert t_2

Item	**freq**	**head**
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

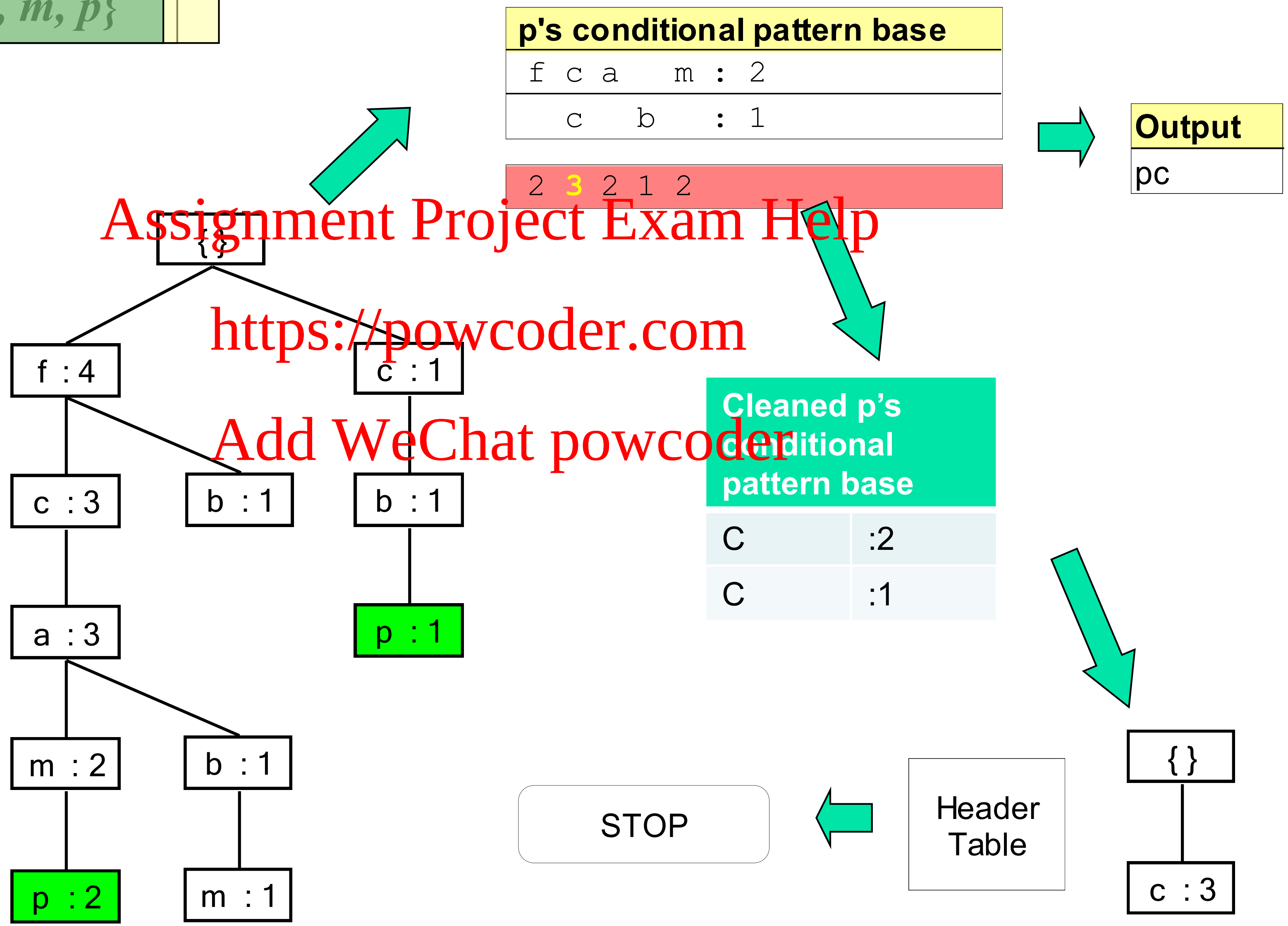


Insert all t_i

Output
f
c
a
b
m
p

TID	frequent items
100	{f, c, a, m, p}
200	{f, c, a, b, m}
300	{f, b}
400	{c, b, p}
500	{f, c, a, m, p}

Item	freq	head
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

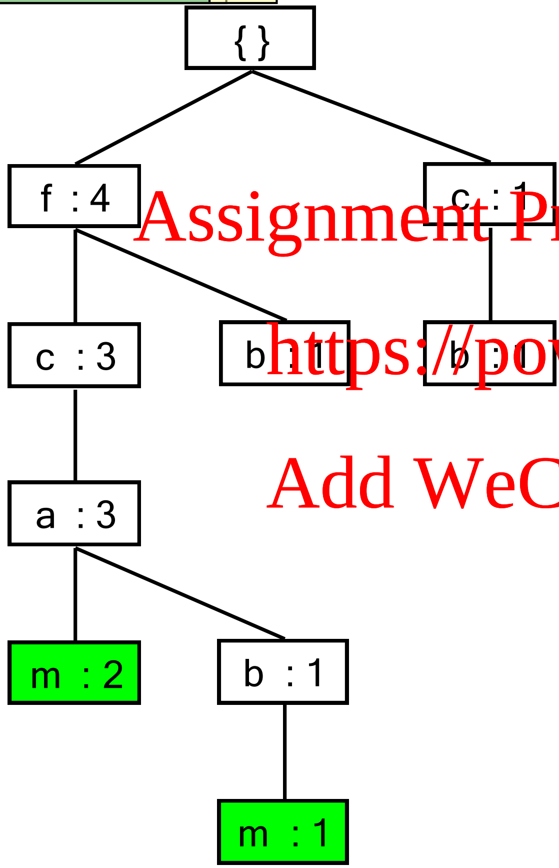


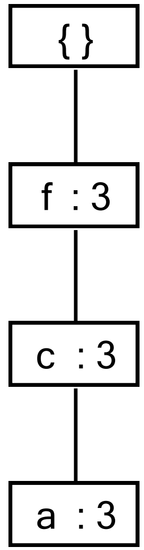
TID	frequent items
100	{f, c, a, m, p}
200	{f, c, a, b, m}
300	{f, b}
400	{c, b, p}
500	{f, c, a, m, p}

m's conditional pattern base
f c a : 2
f c a b : 1
3 3 3 1

Output
mf
mc
ma

Item	freq	head
f	4	
c	4	
a	3	
b	3	
m	3	





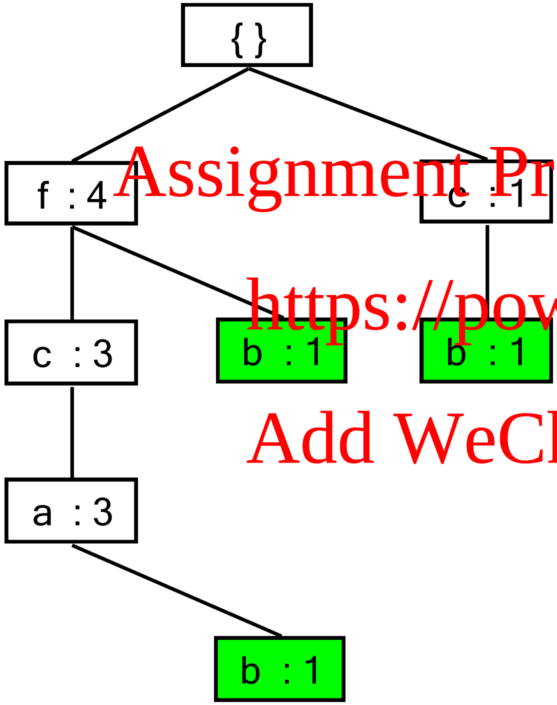
Header Table

gen_powerset

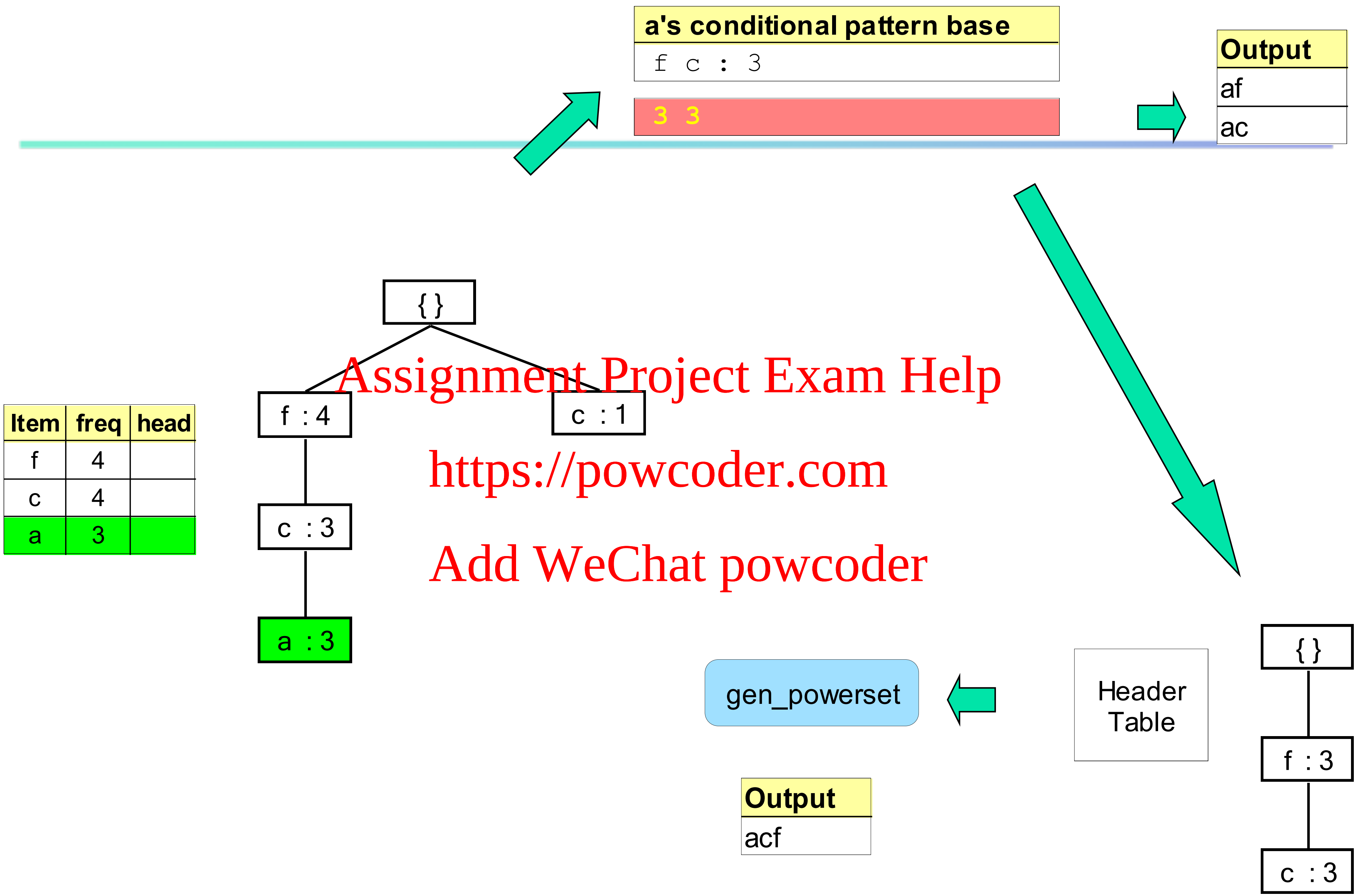
Output
mac
maf
mcf
macf

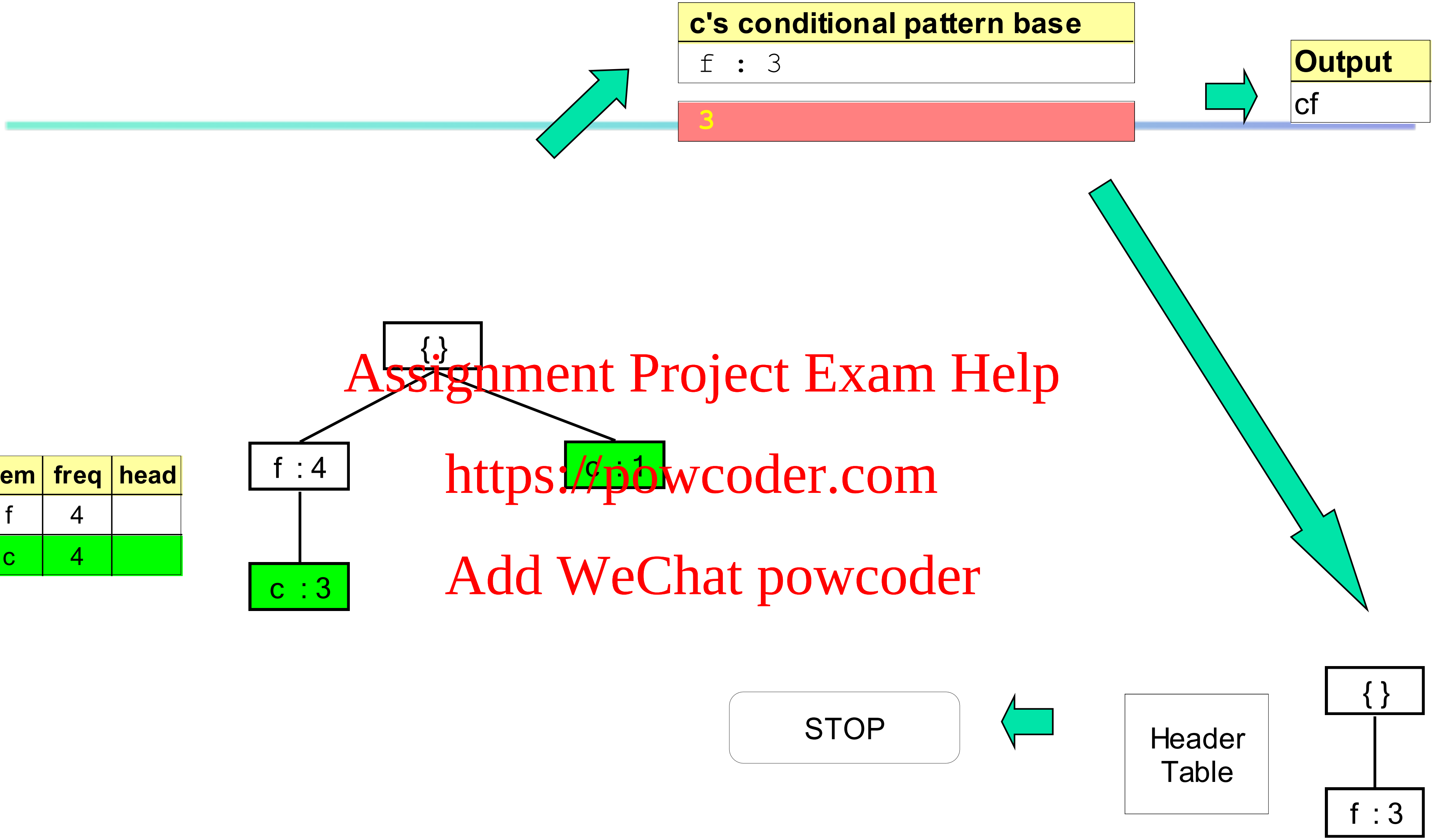
b's conditional pattern base
f c a : 1
f : 1
c : 1
2 2 1

Item	freq	head
f	4	
c	4	
a	3	
b	3	



STOP

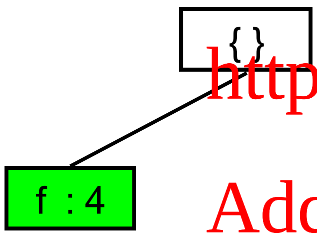

a's conditional pattern base
f c : 3
3 3
Output
af
ac
{ }
f : 4
c : 1
c : 3
a : 3
Item freq head
f 4
c 4
a 3
gen_powerset
Header Table
{ }
f : 3
c : 3
Output
acf



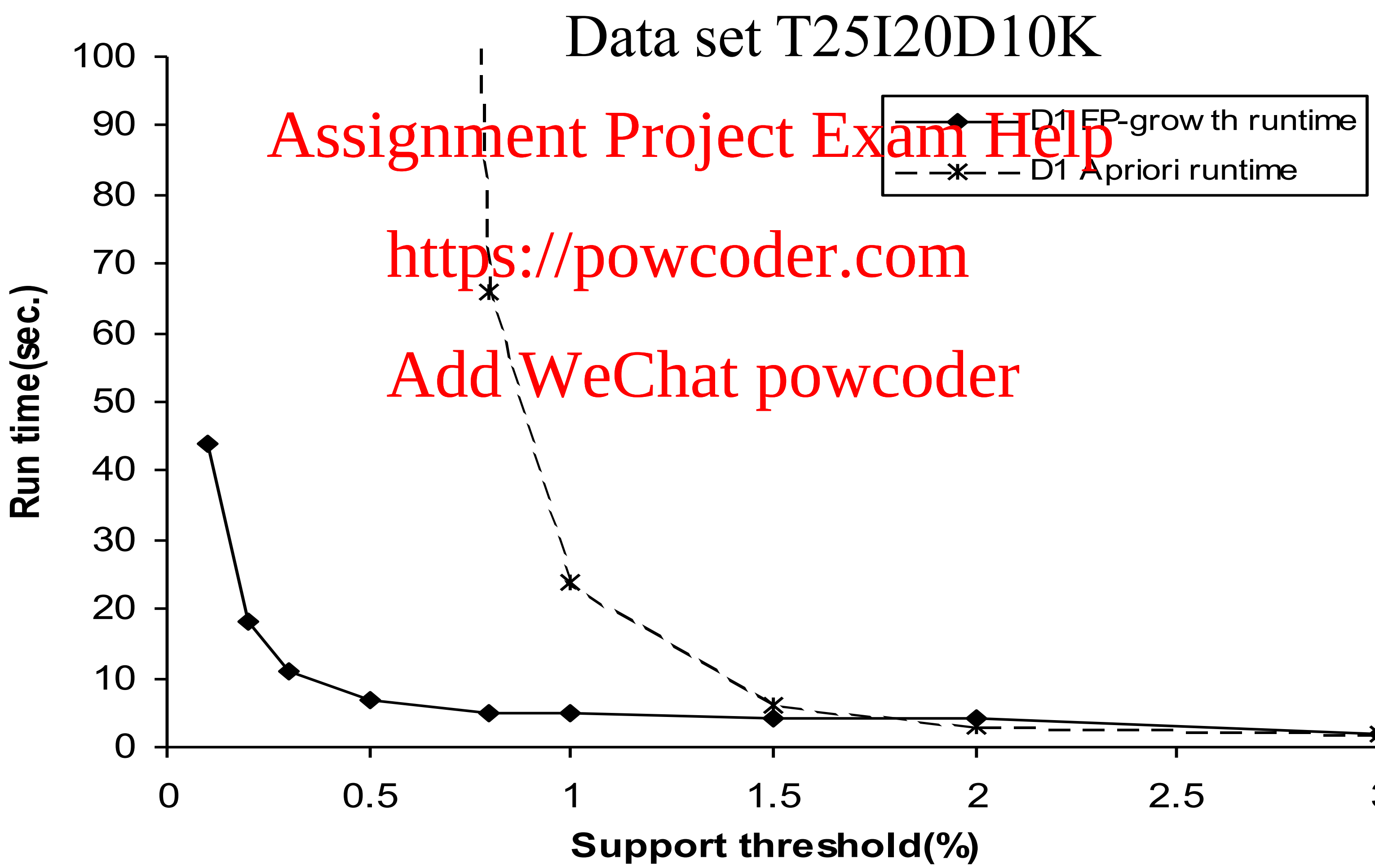
Item	freq	head
f	4	
c	4	

STOP

Item	freq	head
f	4	



FP-Growth vs. Apriori: Scalability With the Support Threshold



Why Is FP-Growth the Winner?

- Divide-and-conquer:
  - decompose both the mining task and DB according to the frequent patterns obtained so far
  - leads to focused search of smaller databases
- Other factors
  - no candidate generation, no candidate test
  - compressed database: FP-tree structure
  - no repeated scan of entire database
  - basic ops—counting local freq items and building sub FP-tree, no pattern search and matching