COMP9318: Data Warehousing and Data Mining

— L2: Data Warehousing and OLAP —

- Why and What are Data Warehouses?

Data Analysis Problems

- The same data found in many different systems
  - Example: customer data across different departments
  - The same concept is defined differently
- Heterogeneous sources
  - Relational DBMS, OnLine Transaction Processing (OLTP)
  - Unstructured data in files (e.g., MS Excel) and documents (e.g., MS Word)

Data Analysis Problems (Cont’d)

- Data is suited for operational systems
  - Accounting,billing,etc.
  - Do not support analysis across business functions
- Data quality is bad
  - Missing data, imprecise data, different use of systems
- Data are “volatile”
  - Data deleted in operational systems (6months)
  - Data change over time – no historical information

Solution: Data Warehouse

- Defined in many different ways, but not rigorously.
  - A decision support database that is maintained separately from the organization's operational database
  - Support information processing by providing a solid platform of consolidated, historical data for analysis.
- "A data warehouse is a subject-oriented, integrated, time-variant, and nonvolatile collection of data in support of management's decision-making process."—W. H. Inmon
- Data warehousing:
  - The process of constructing and using data warehouses

Data Warehouse—Subject-Oriented

- Organized around major subjects, such as customer, product, sales.
- Focusing on the modeling and analysis of data for decision makers, not on daily operations or transaction processing.
- Provide a simple and concise view around particular subject issues by excluding data that are not useful in the decision support process.

Data Warehouse—Integrated

- Constructed by integrating multiple, heterogeneous data sources
  - relational databases, flat files, on-line transaction records
- Data cleaning and data integration techniques are applied.
  - Ensure consistency in naming conventions, encoding structures, attribute measures, etc. among different data sources
    - E.g., Hotel price: currency, tax, breakfast covered, etc.
  - When data is moved to the warehouse, it is converted.

Data Warehouse—Time Variant

- The time horizon for the data warehouse is significantly longer than that of operational systems.
  - Operational database: current value data.

  - Data warehouse data: provide information from a historical perspective (e.g., past 5-10 years)

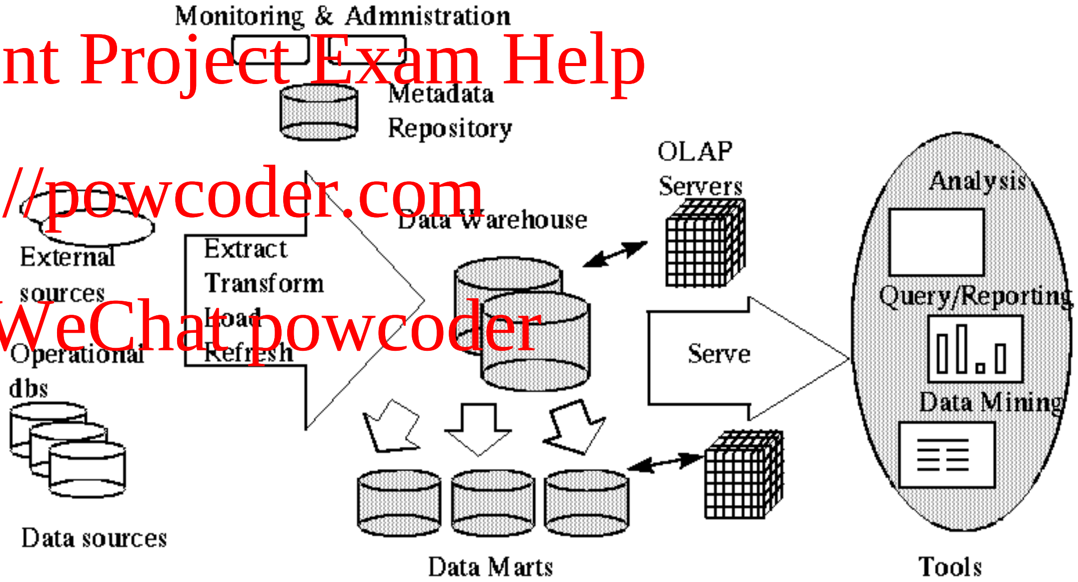
- Every key structure in the data warehouse
  - Contains an element of time, explicitly or implicitly
  - But the key of operational data may or may not contain “time element”.

Data Warehouse—Non-Volatile

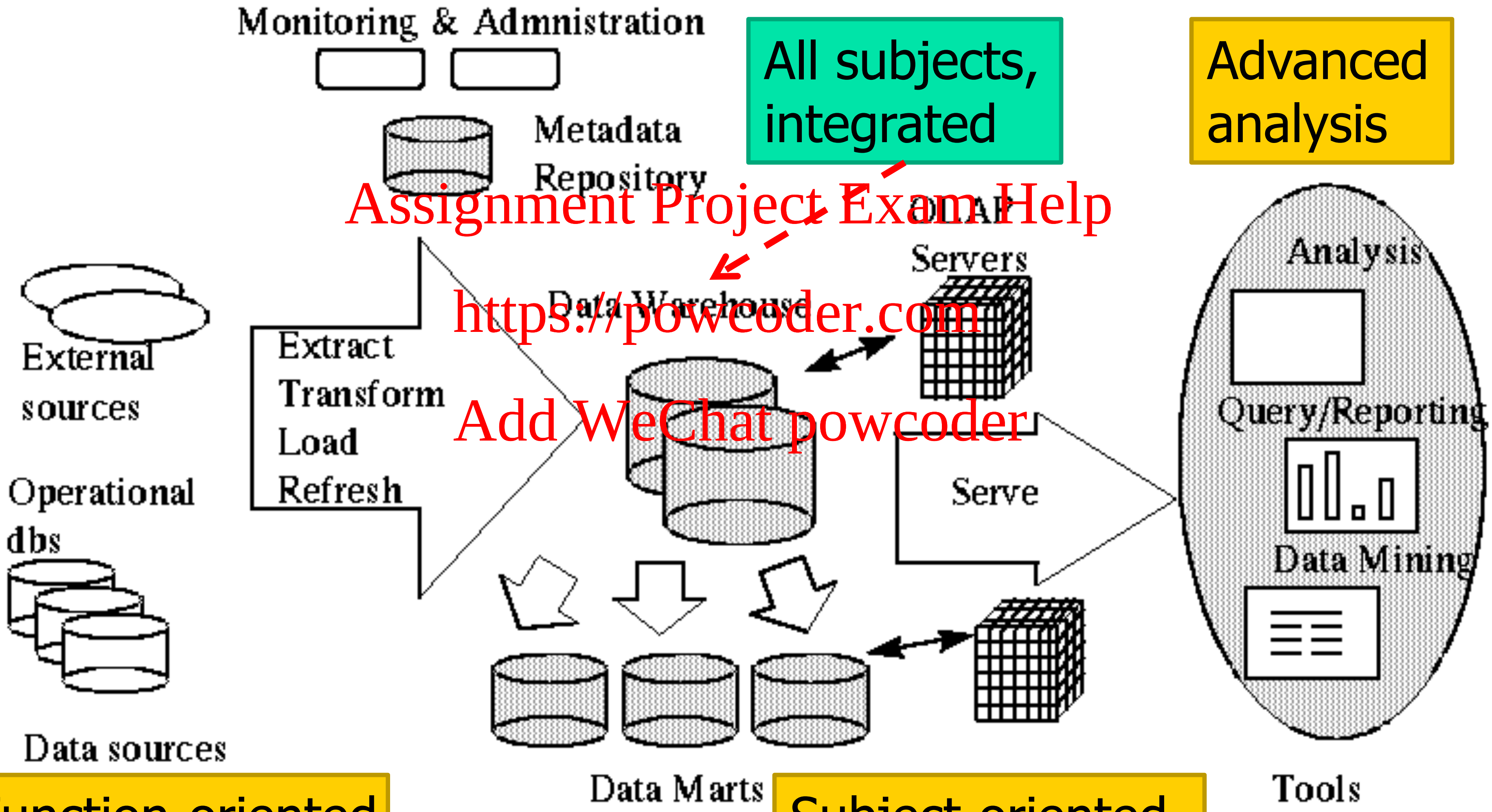
1. A physically separate store of data transformed from the operational environment.
2. Operational update of data does not occur in the data warehouse environment.
   - Does not require transaction processing, recovery, and concurrency control mechanisms
   - Requires only two operations in data accessing:
     - *initial loading of data* and *access of data*.

Data Warehouse Architecture

- Extract data from operational data sources
  - clean, transform
- Bulk load/refresh
  - warehouse is offline
- OLAP-server provides multidimensional view
- Multidimensional-olap (Essbase, oracle express)
- Relational-olap (Redbrick, Informix, Sybase, SQL server)



Data Warehouse Architecture



All subjects, integrated

Advanced analysis

Function-oriented systems

Subject-oriented systems

Why Separate Data Warehouse?

- High performance for both systems
  - DBMS— tuned for OLTP: access methods, indexing, concurrency control, recovery
  - Warehouse—tuned for OLAP: complex OLAP queries, multidimensional view, consolidation.
- Different functions and different data:
  - missing data: Decision support requires historical data which operational DBs do not typically maintain
  - data consolidation: DS requires consolidation (aggregation, summarization) of data from heterogeneous sources
  - data quality: different sources typically use inconsistent data representations, codes and formats which have to be reconciled

Why OLAP Servers?

- Different workload:
  - OLTP (on-line transaction processing)
    - Major task of traditional relational DBMS
    - Day-to-day operations: purchasing, inventory, banking, manufacturing, payroll, registration, accounting, etc.
  - OLAP (on-line analytical processing)
    - Major task of data warehouse system
    - Data analysis and decision making
- Queries hard/infeasible for OLTP, e.g.,
  - Which **week** we have the largest sales?
  - Does the sales of **dairy products** increase over time?
  - Generate a **spread sheet** of total sales by state and by year.
- Difficult to represent these queries by using SQL ← Why?

OLTP vs. OLAP

	OLTP	**OLAP**
users	clerk, IT professional	knowledge worker
function	day to day operations	decision support
DB design	application-oriented	subject-oriented
data	current, up-to-date detailed, flat relational isolated	historical, summarized, multidimensional integrated, consolidated
usage	repetitive	ad-hoc
access	read/write index/hash on prim. key	lots of scans
unit of work	short, simple transaction	complex query
# records accessed	tens	millions
#users	thousands	hundreds
DB size	100MB-GB	100GB-TB
metric	transaction throughput	query throughput, response

Comparisons

	Databases	Data Warehouses
Purpose	Many purposes; Flexible and general	One purpose: Data analysis
Conceptual Model	ER	Multidimensional
Logical Model	(Normalized) Relational Model	(Denormalized) Star schema / Data cube/cuboids
Physical Model	Relational Tables	ROLAP: Relational tables MOLAP: Multidimensional arrays
Query Language	SQL (hard for analytical queries)	MDX (easier for analytical queries)
Query Processing	B+-tree/hash indexes, Multiple join optimization, Materialized views	Bitmap/Join indexes, Star join, Materialized data cube

- The Multidimensional Model

The Multidimensional Model

- A data warehouse is based on a multidimensional data model which views data in the form of a data cube, which is a multidimensional generalization of 2D spread sheet.
- Key concepts:
  - **Facts**: the subject it models
    - Typically transactions in this course; other types includes snapshots, etc.
    - Measures: numbers that can be aggregated
    - Dimensions: context of the measure
  - Hierarchies:
    - Provide contexts of different granularities (aka. grains)
- Goals for dimensional modeling:
  - Surround facts with as much relevant context (dimensions) as possible ← Why?

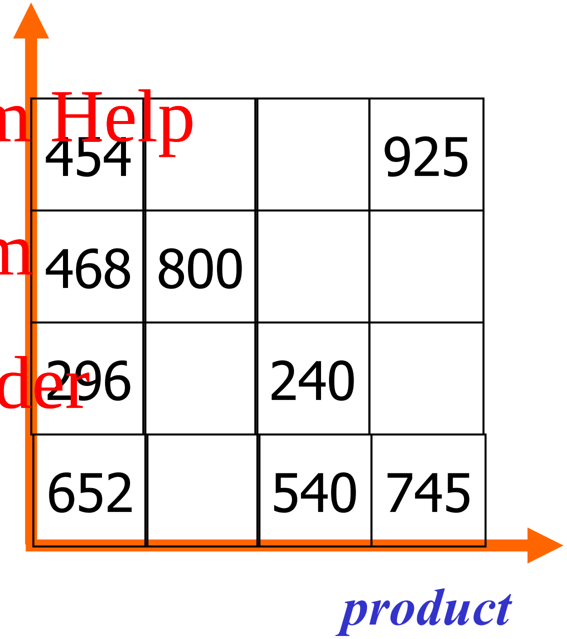
Supermarket Example

- Subject: analyze total sales and profits
- Fact: Each Sales **Transaction**
  - Measure: Dollars_Sold, Amount_Sold, Cost
  - Calculated Measure: Profit
- Dimensions:
  - Store
  - Product
  - Time

Visualizing the Cubes

- A valid instance of the model is a data cube

total Sales		product			
		p1	p2	p3	p4
city	NY	$454	-	-	$925
	LA	$468	$800	-	-
	SD	$296	-	$240	-
	SF	$652	-	$540	$745



454			925
468	800		
296		240	
652		540	745

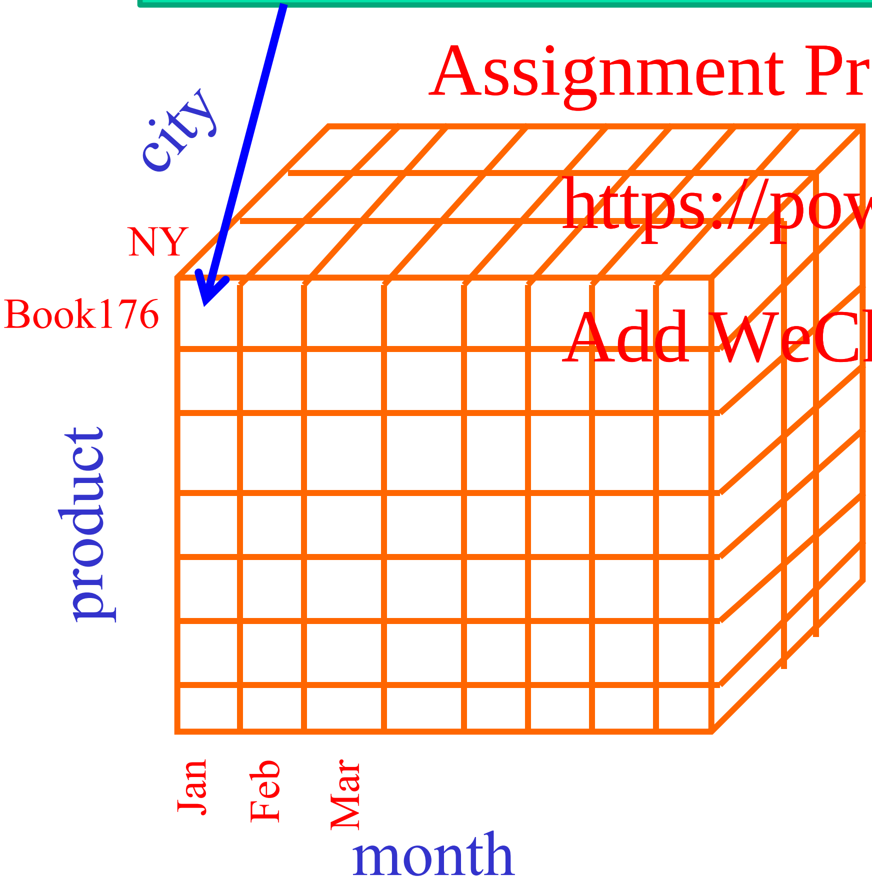
Concepts: cell, fact (=non-empty cell), measure, dimensions

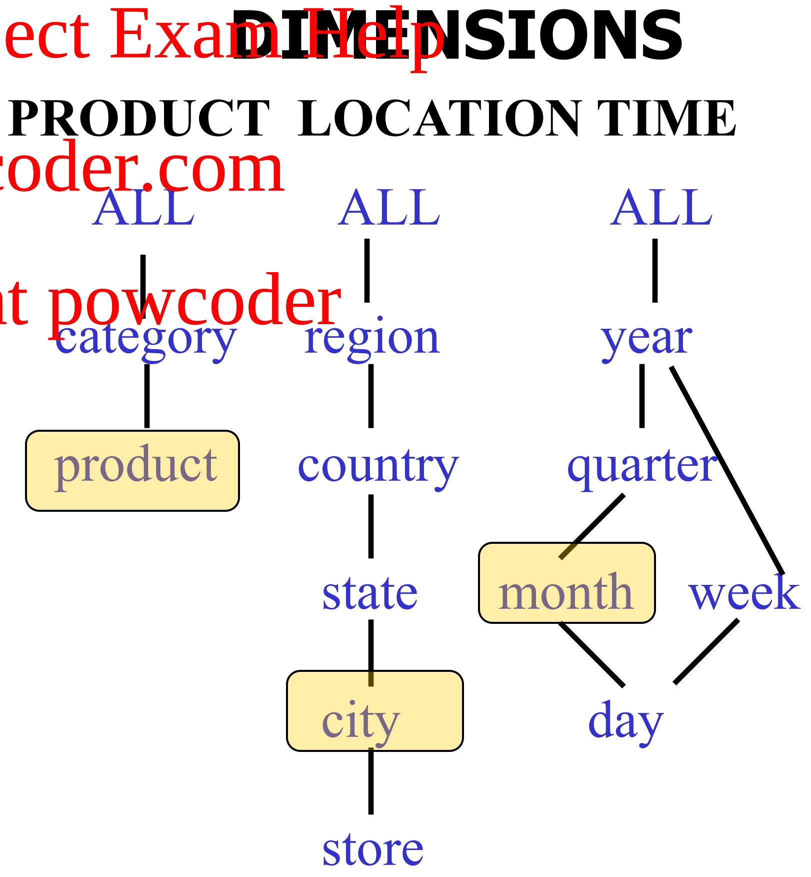
Q: How to generalize it to 3D?

3D Cube and Hierarchies

Concepts: hierarchy (a tree of dimension values), level

Sales of book176 in NY in Jan can be found in this cell





DIMENSIONS

PRODUCT LOCATION TIME

PRODUCT: ALL – category – product

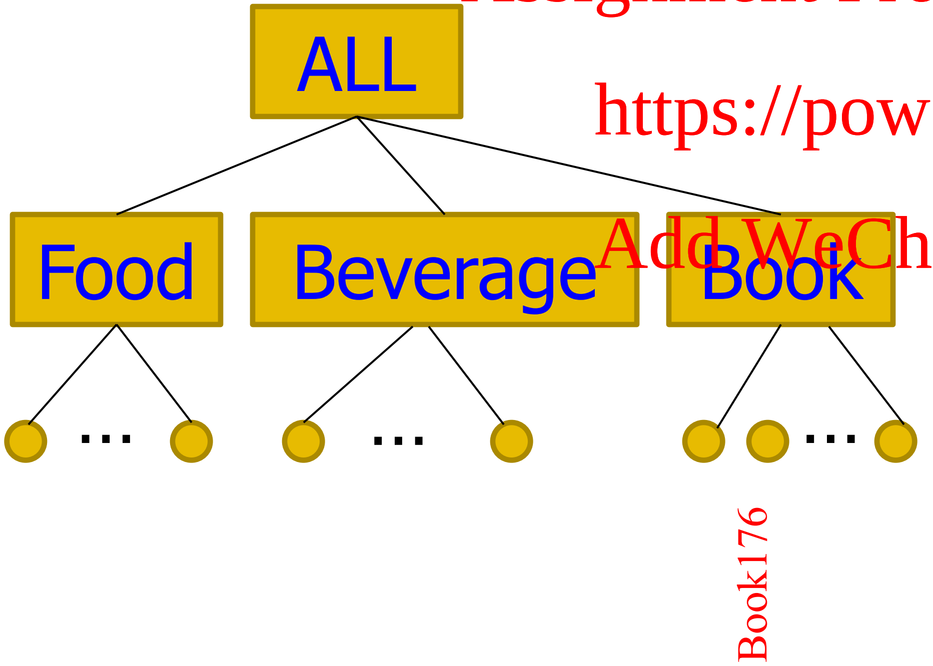
LOCATION: ALL – region – country – state – city – store

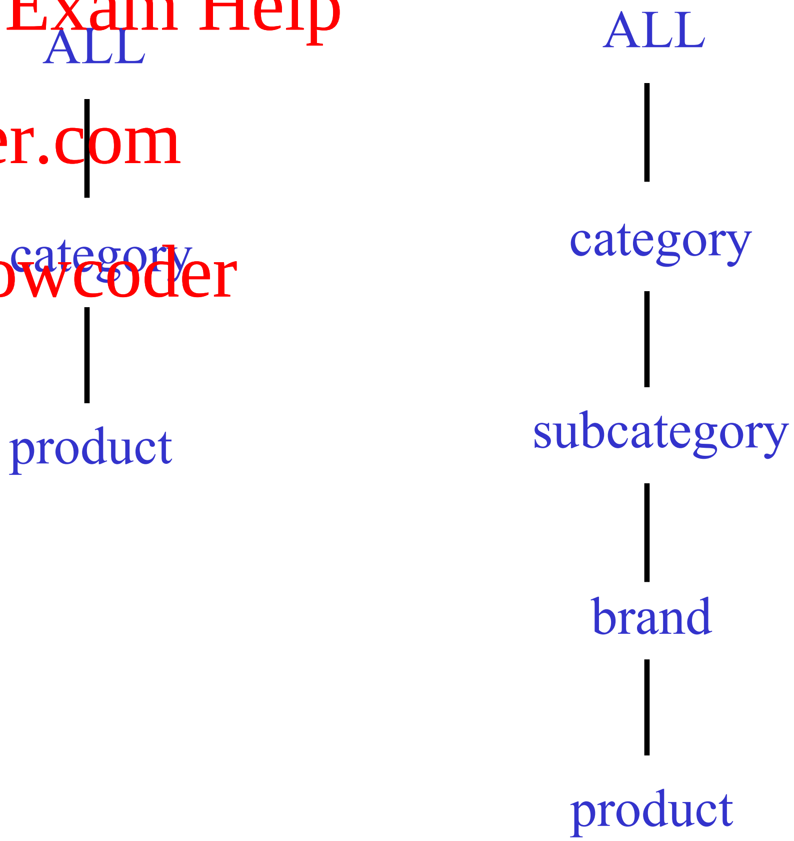
TIME: ALL – year – quarter – month – day; year – week – day

Hierarchies

Concepts: hierarchy (a tree of dimension values), level

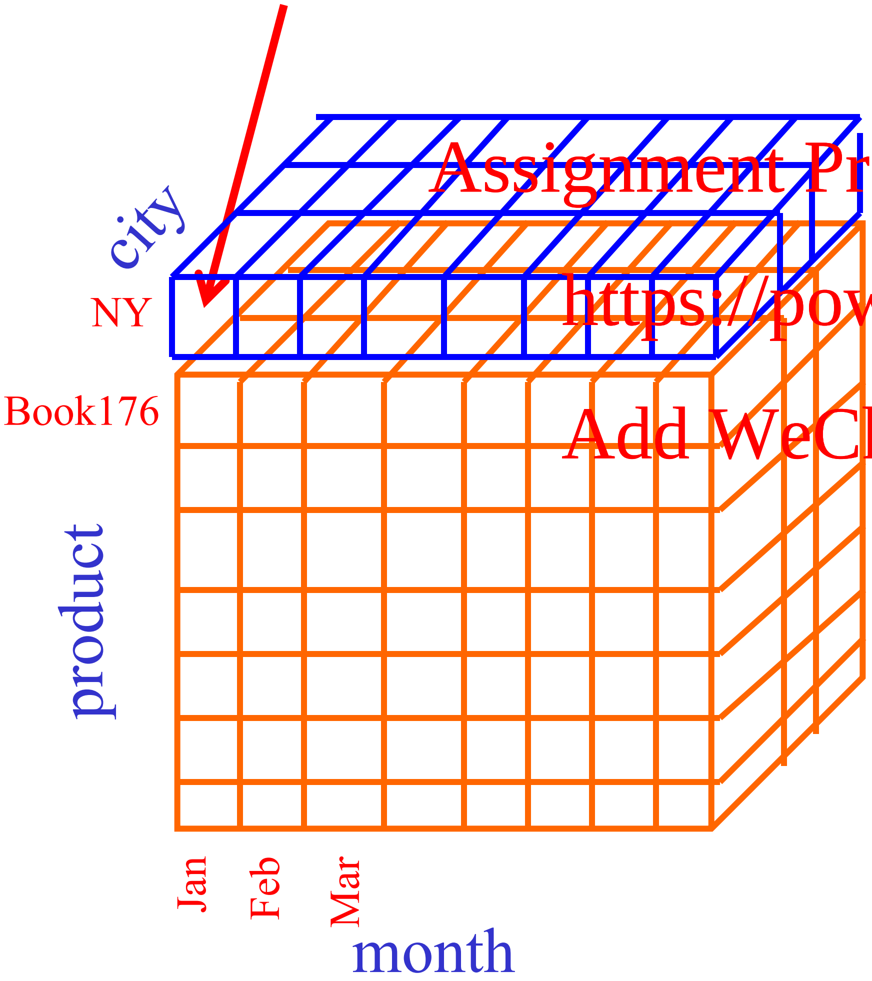
Which design is better? Why?



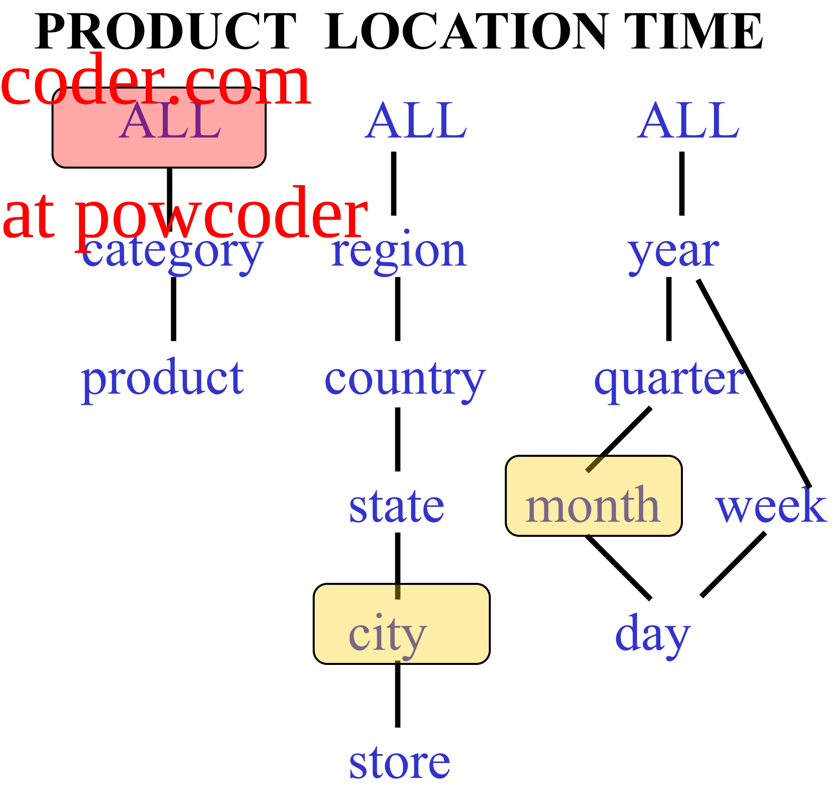


The (city, moth) Cuboid

Sales of ALL_PROD in NY in Jan

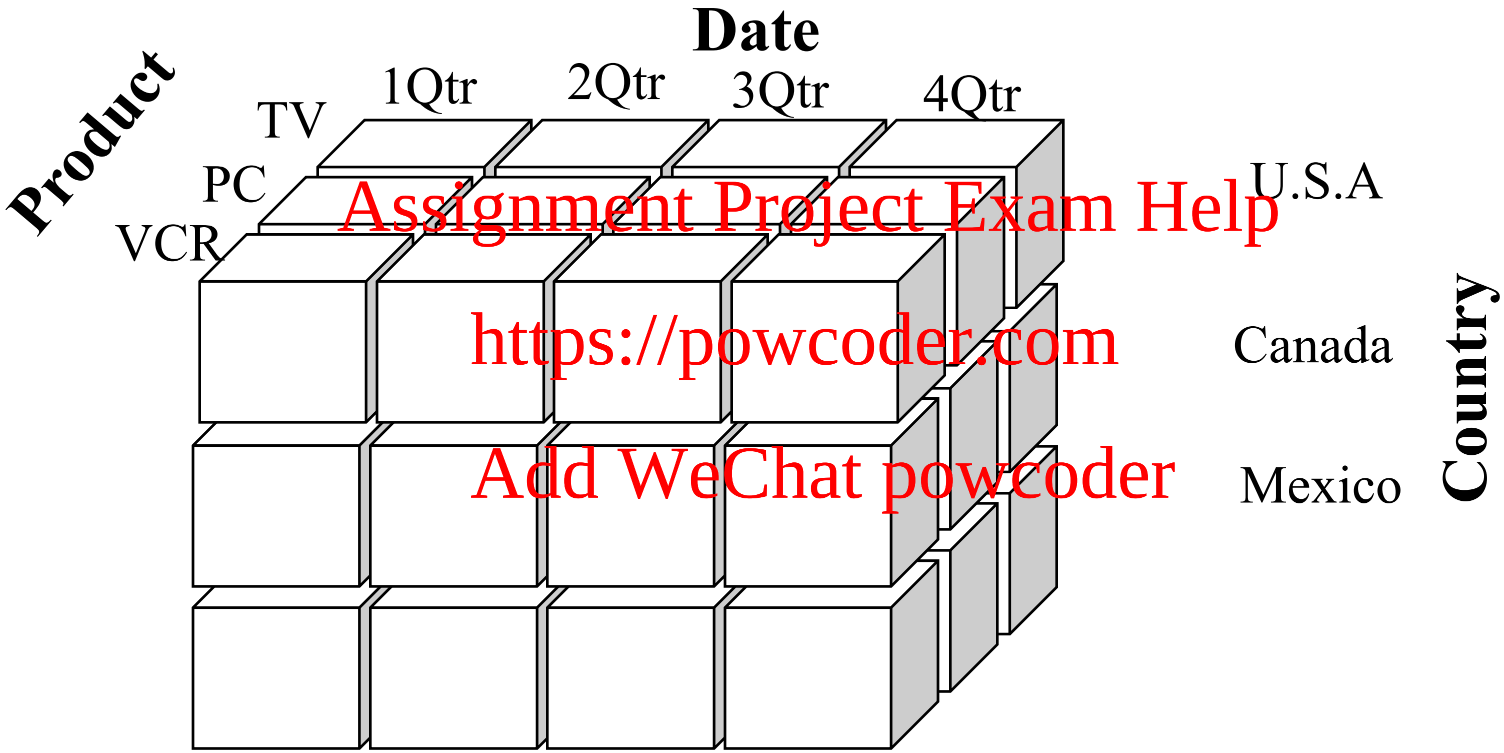


DIMENSIONS



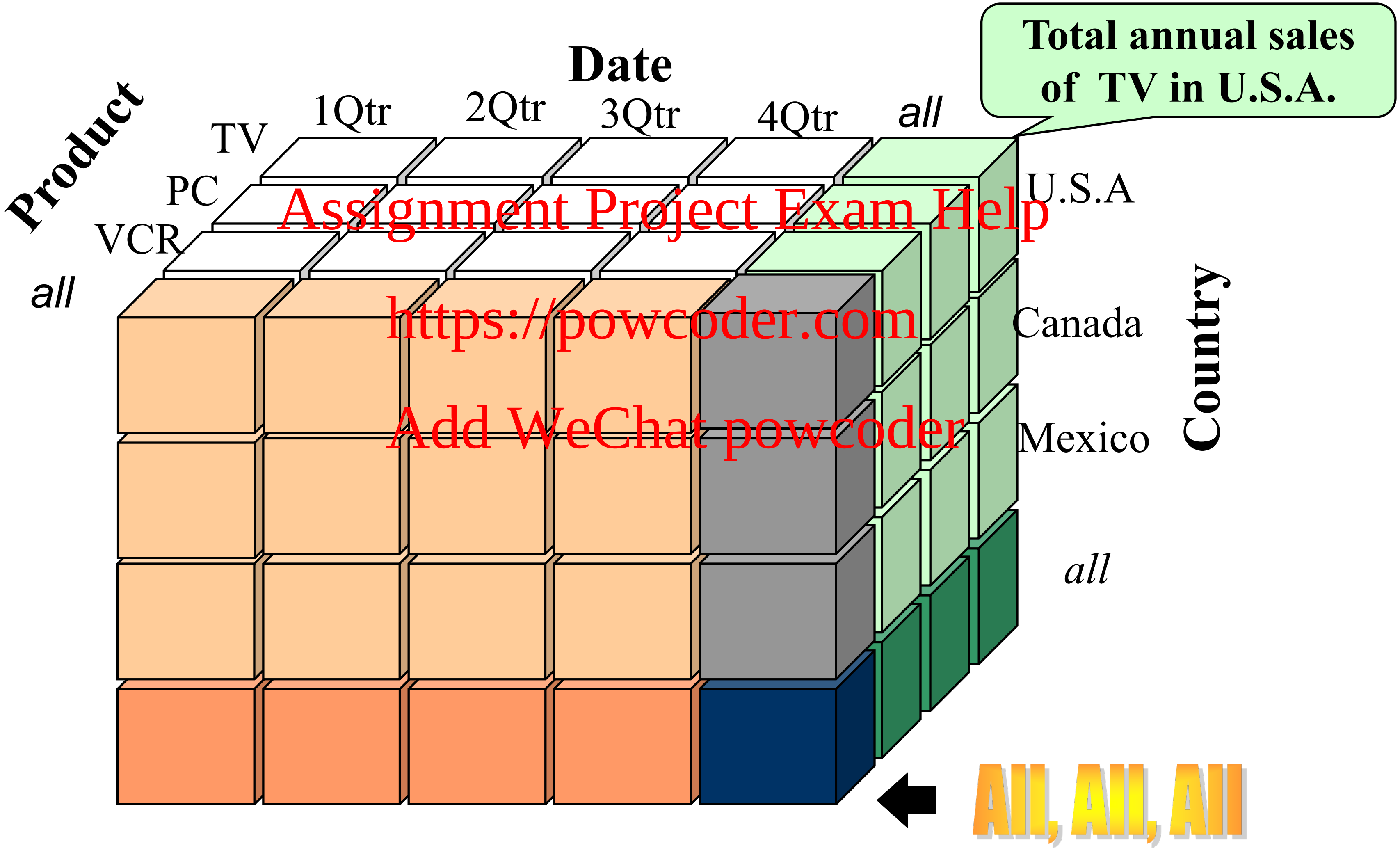
All the Cuboids

Assume: no other non-ALL levels on all dimensions.

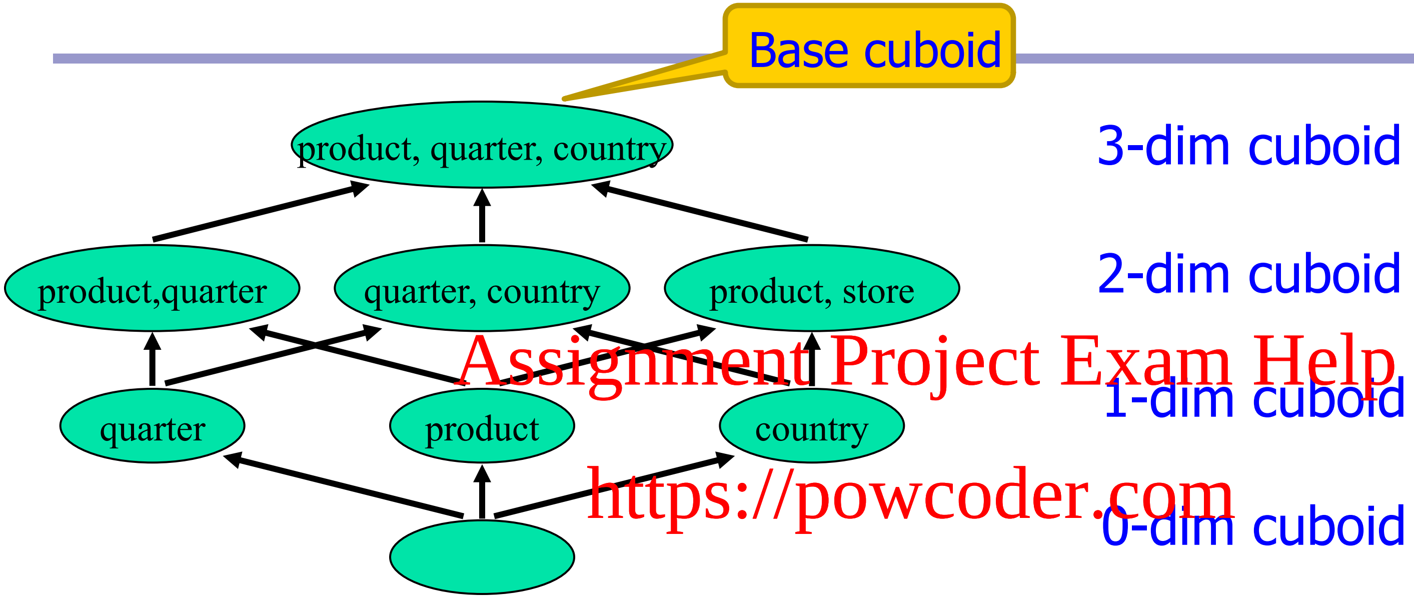


All the Cuboids /2

Assume: no other non-ALL levels on all dimensions.



Lattice of the cuboids



- n-dim cube can be represented as (D_1, D_2, …, D_d), where D_i is the set of allowed values on the i-th dimension.
  - if D_i = L_i (a particular level), then Di = all descendant dimension values of L_i.
  - ALL can be omitted and hence reduces the effective dimensionality
- A complete cube of d-dimensions consists of $\prod_{i=1}^{d}(n_i+1)$ cuboids, where n_i is the number of levels (excluding ALL) on i-th dimension.
  - They collectively form a lattice.

Properties of Operations

- All operations are closed under the multidimensional model
  - i.e., both input and output of an operation is a cube
- So that they can be composed

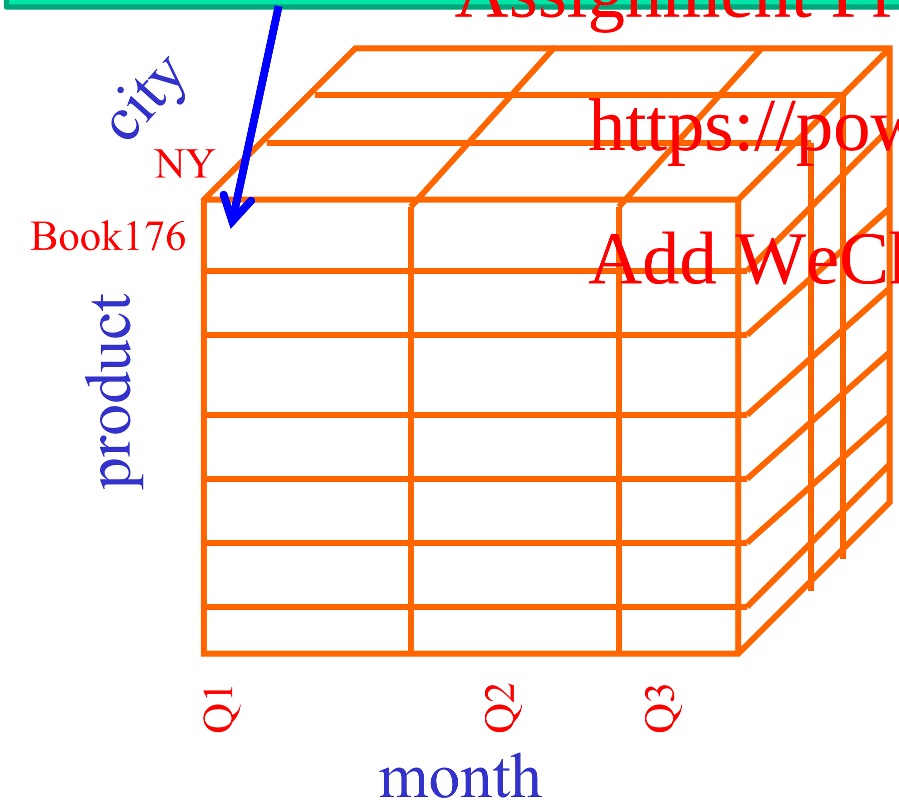
Q: What's the analogy in the Relational Model?

Common OLAP Operations

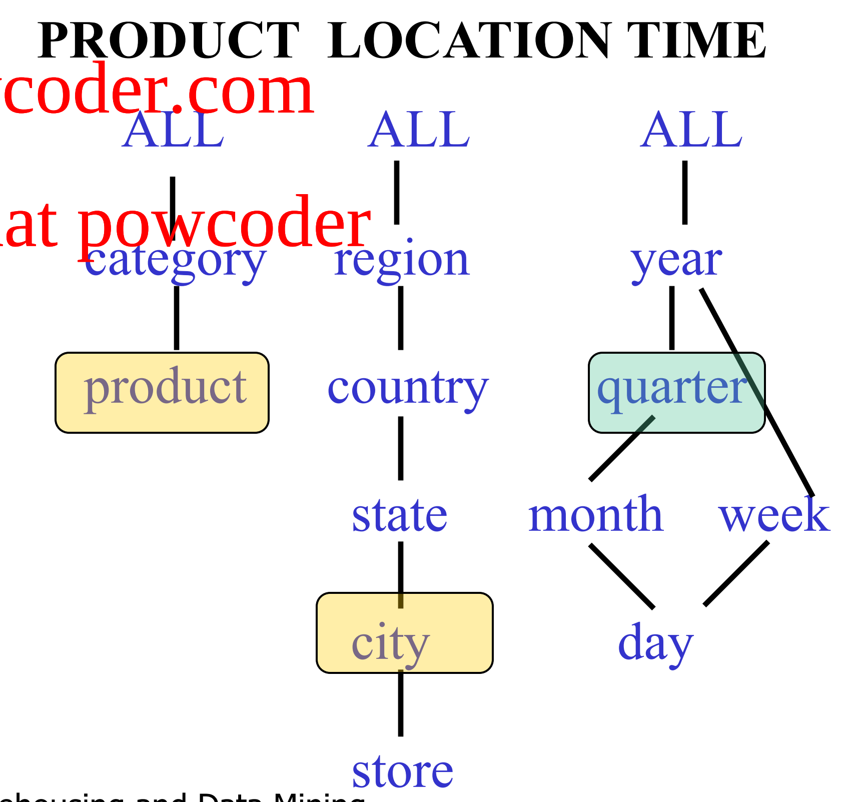
Q: what should be its value?

- **Roll-up**: move up the hierarchy

Sales of book176 in NY in Q1 here



DIMENSIONS



PRODUCT LOCATION TIME

ALL ALL ALL

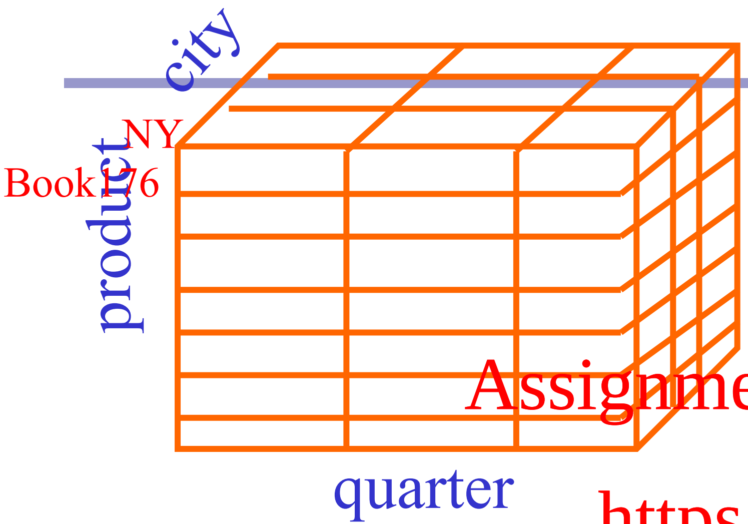
category region year

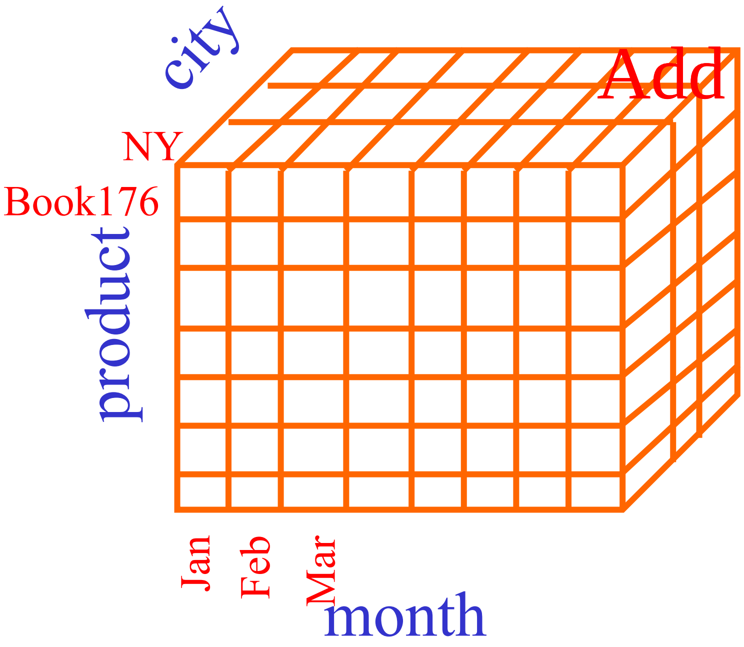
product country quarter

state month week

city day

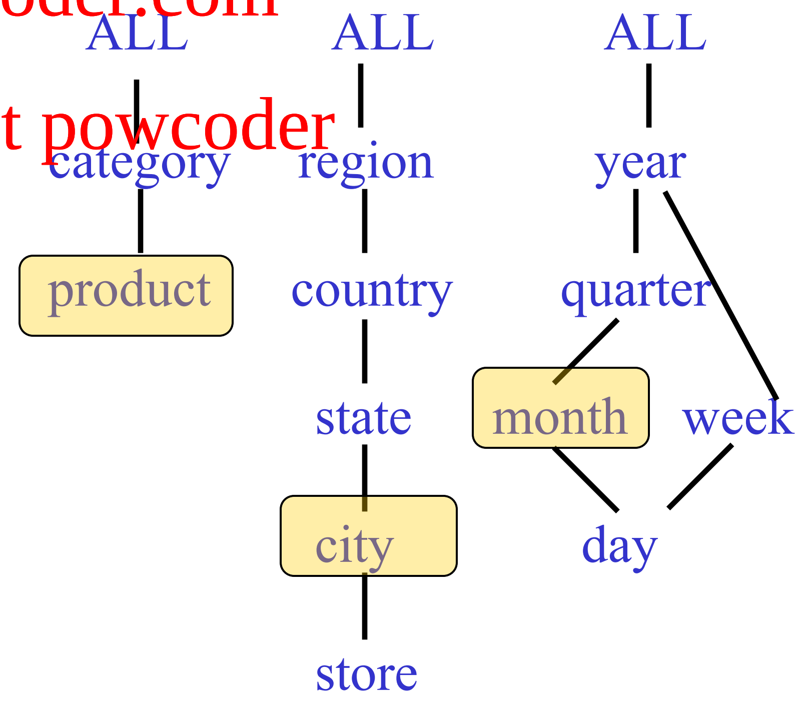
store





DIMENSIONS

PRODUCT LOCATION TIME



ALL — category — product

ALL — region — country — state — city — store

ALL — year — quarter — month — day; year — week — day

Data Cube Measures: Three Categories

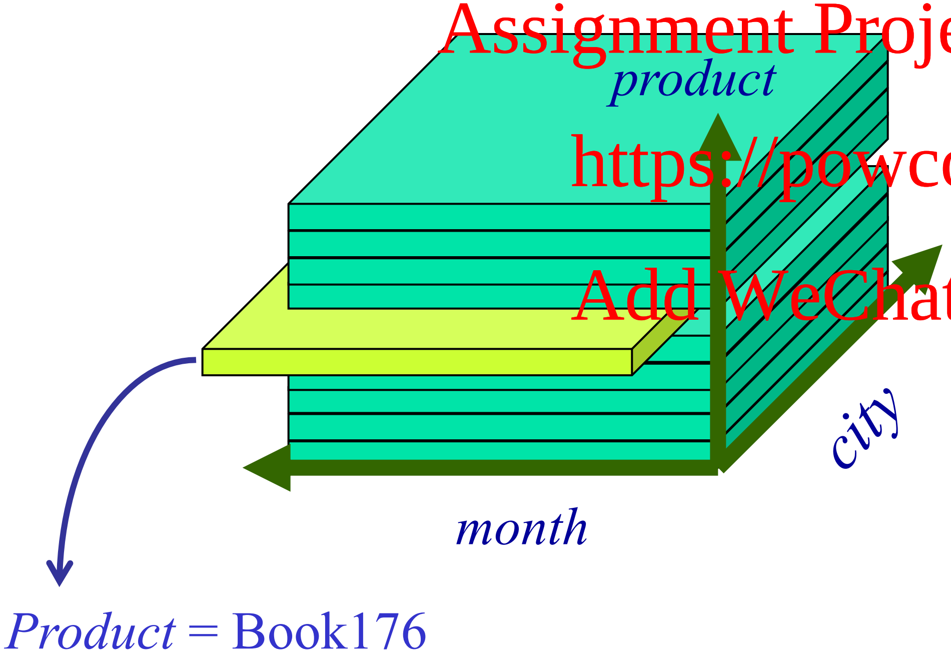
- Distributive: if the result derived by applying the function to *n* aggregate values is the same as that derived by applying the function on all the data without partitioning
  - E.g., count(), sum(), min(), max()
- Algebraic: if it can be computed by an algebraic function with *M* arguments (where *M* is a bounded integer), each of which is obtained by applying a distributive aggregate function
  - E.g., avg(), min_N(), standard_deviation()
- Holistic: if there is no constant bound on the storage size needed to describe a subaggregate.
  - E.g., median(), mode(), rank()

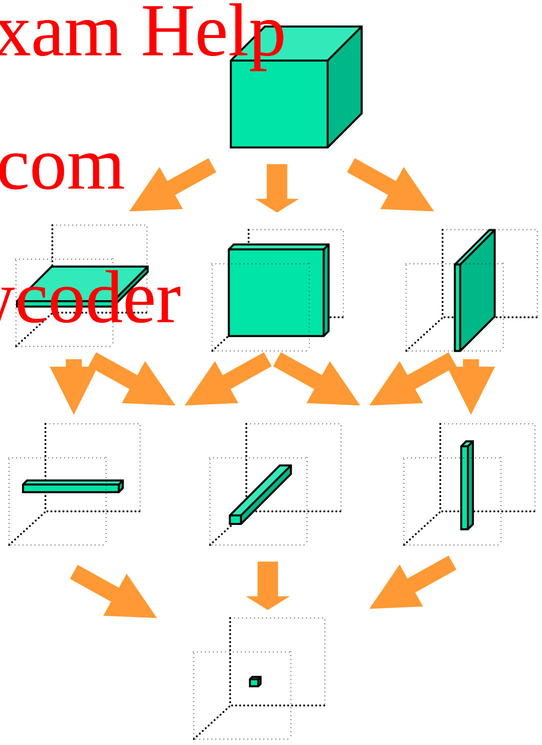
Common OLAP Operations

- **Drill-down:** move down the hierarchy
  - more fine-grained aggregation

Slice and Dice Queries

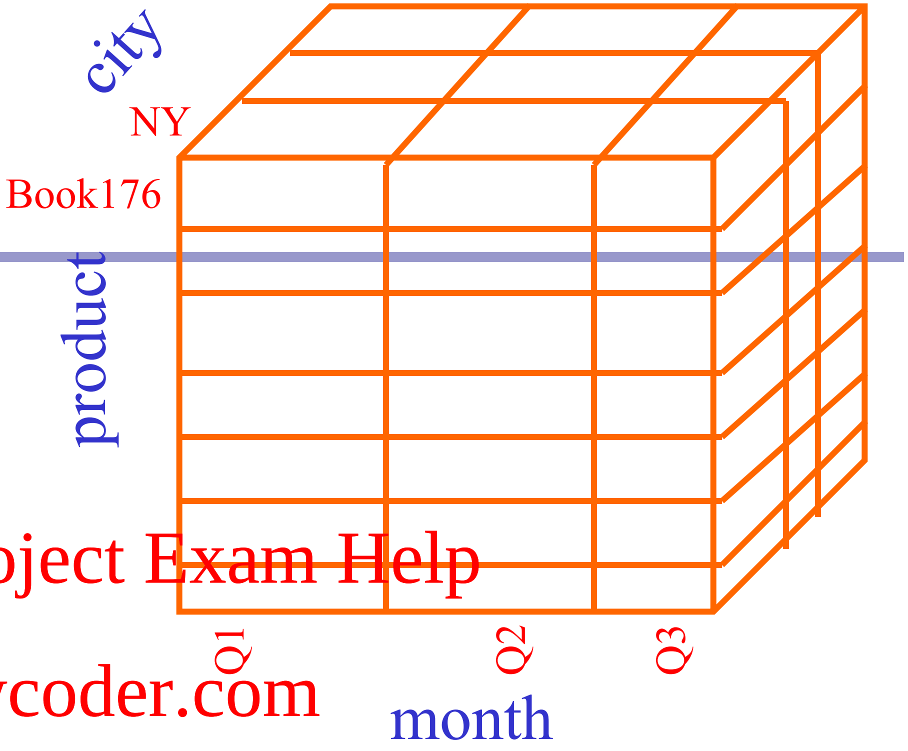
- Slice and Dice: select and project on one or more dimension values





The output cube has smaller dimensionality than the input cube

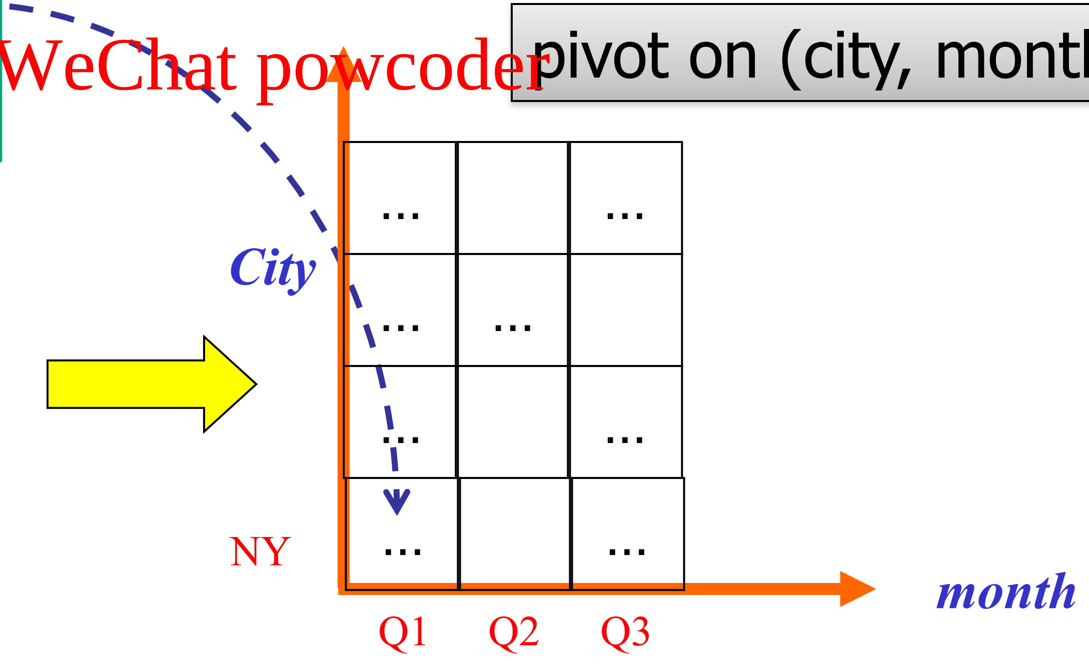
Pivoting



- Pivoting: aggregate on selected dimensions
  - usually 2 dims (cross-tabulation)

Sales (of all products) in NY in Q1 =sum(???)

pivot on (city, month)



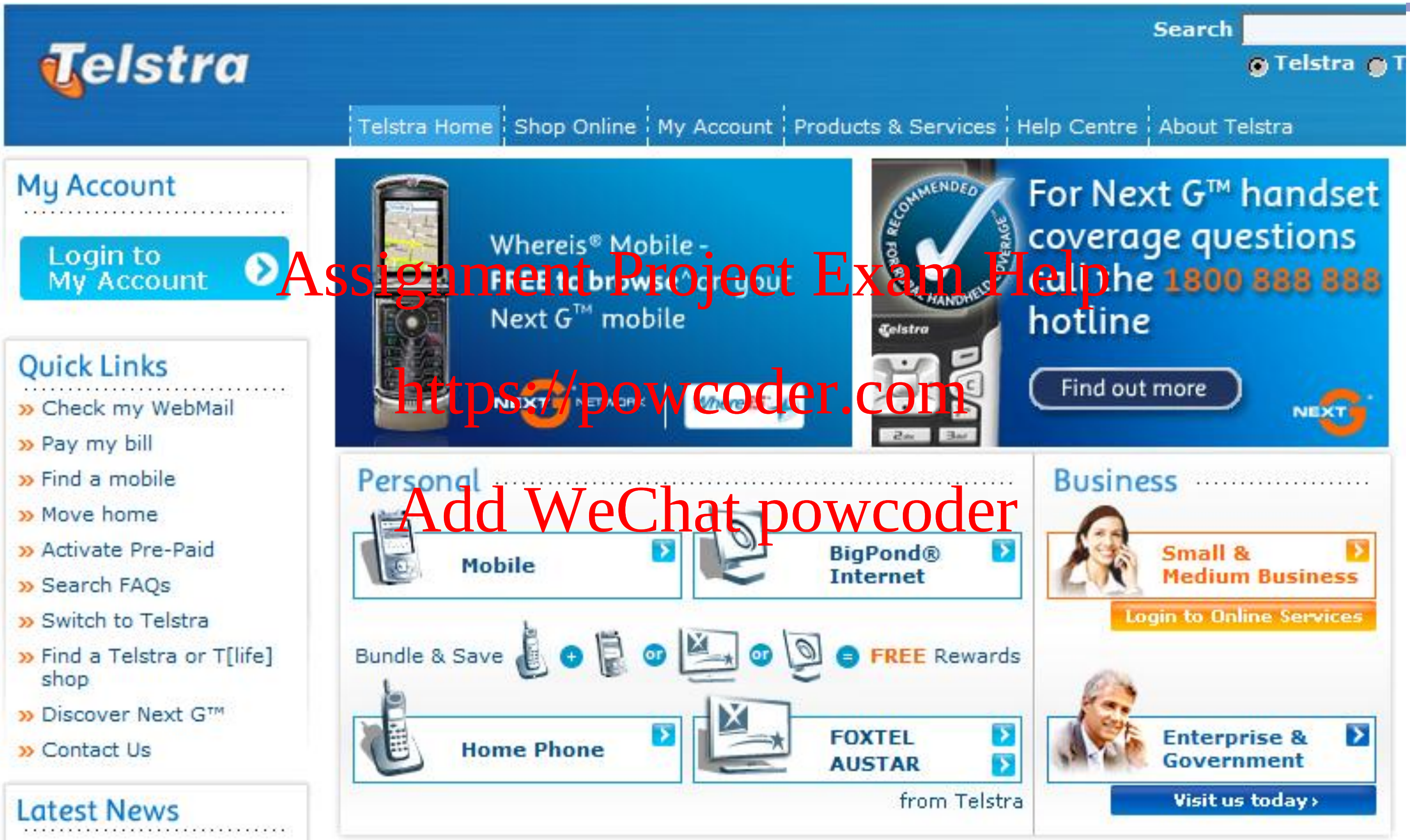
City	Q1	Q2	Q3
	…		…
	…	…	
	…		…
NY	…		…

A Reflective Pause

- Let’s review the definition of data cubes again.

- **Key message**:
  - Disentangle the “object” from its “representation” or “implementation”

Modeling Exercise 1: Monthly Phone Service Billing



Theme: analyze the income/revenue of Telstra

Solution

- FACT

- MEASURE

- DIMENSIONS

- The Logical Model

Logical Models

- Two main approaches:
  - Using relational DB technology:
    - Star schema, Snowflake schema, Fact constellation
  - Using multidimensional technology:
    - Just as multidimensional data cube

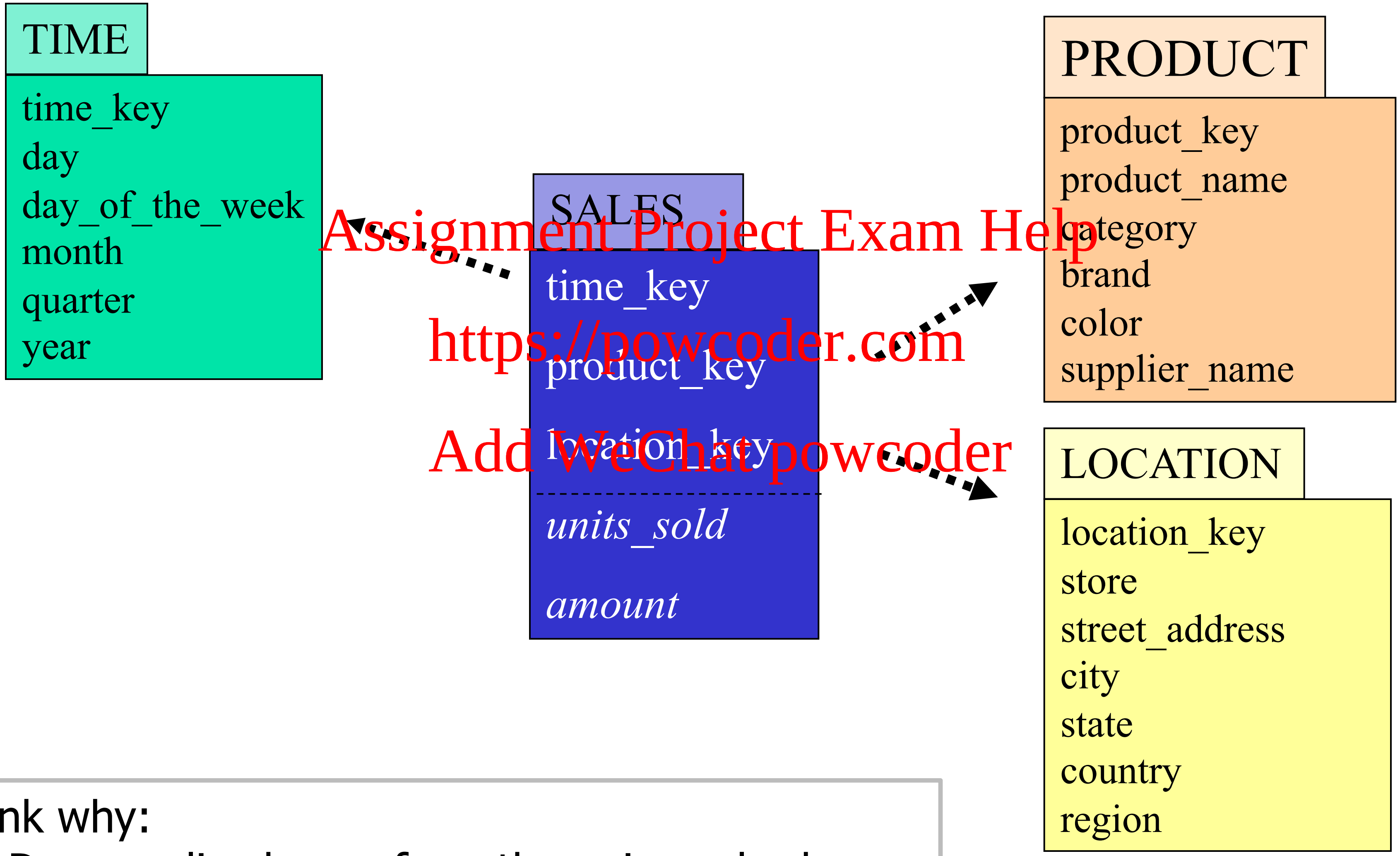
Universal Schema → Star Schema

- Many data warehouses adopt a star schema to represent the multidimensional model
- Each dimension is represented by a dimension-table
  - LOCATION(location_key, store, street_address, city, state, country, region)
  - dimension tables are not normalized
- Transactions are described through a fact-table
  - each tuple consists of a logical pointer to each of the dimension-tables (foreign-key) and a list of measures (e.g. sales $$$)

The universal schema for supermarket

Store	City	State	Prod	Brand	Category	$Sold	#Sold	Cost
S136	Syd	NSW	76Ha	Nestle	Biscuit	40	10	18
S173	Melb	Vic	76Ha	Nestle	Biscuit	20	5	11

The Star Schema



TIME

time_key
day
day_of_the_week
month
quarter
year

SALES

time_key
product_key
location_key

units_sold
amount

PRODUCT

product_key
product_name
category
brand
color
supplier_name

LOCATION

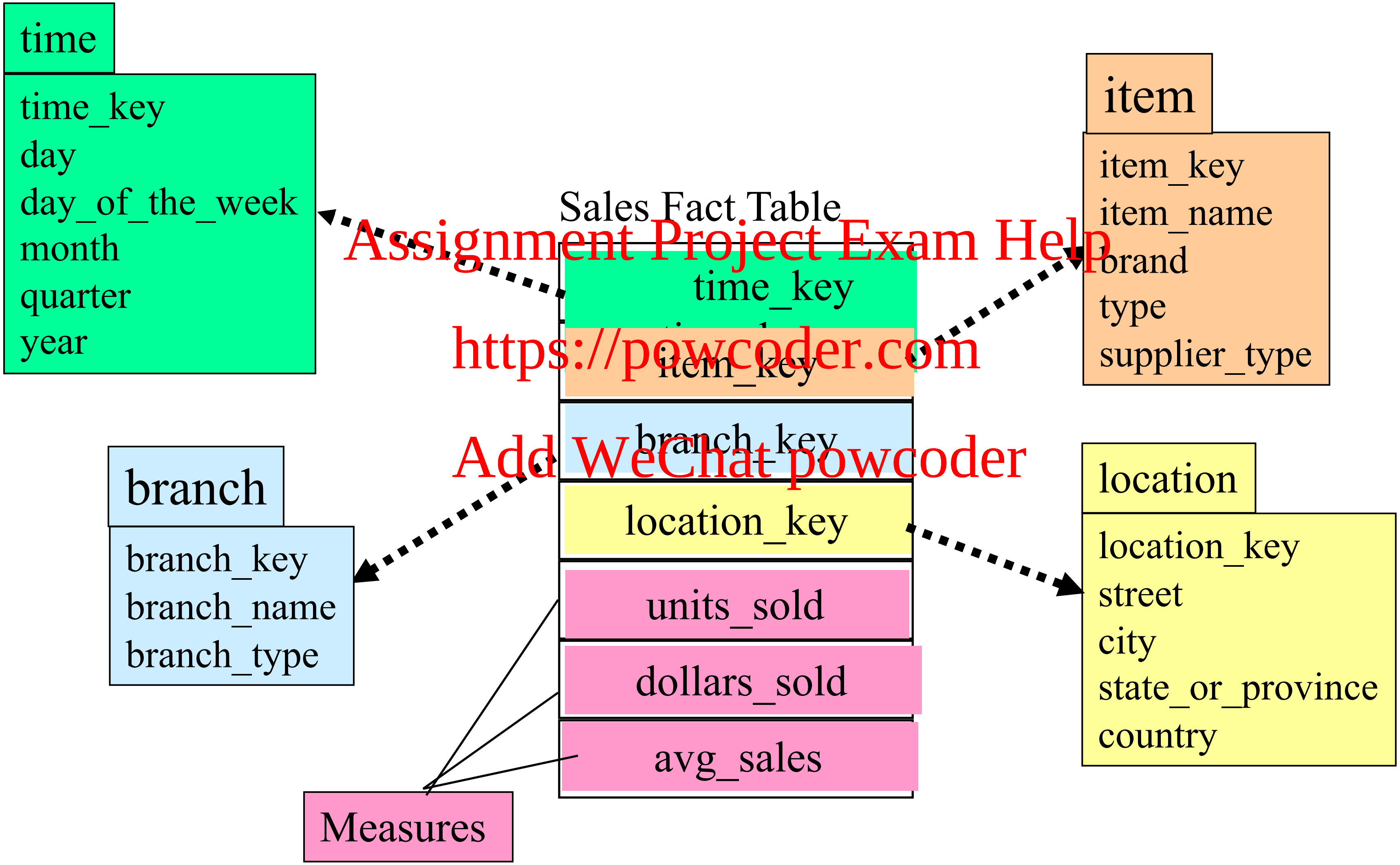
location_key
store
street_address
city
state
country
region

Think why:
(1) Denormalized once from the universal schema
(2) Controlled redundancy

Typical Models for Data Warehouses

- Modeling data warehouses: dimensions & measures
  - Star schema: A fact table in the middle connected to a set of dimension tables
  - Snowflake schema: A refinement of star schema where some dimensional hierarchy is normalized into a set of smaller dimension tables, forming a shape similar to snowflake
  - Fact constellations: Multiple fact tables share dimension tables, viewed as a collection of stars, therefore called galaxy schema or fact constellation

Example of Star Schema



time
time_key
day
day_of_the_week
month
quarter
year

Sales Fact Table
time_key
item_key
branch_key
location_key
units_sold
dollars_sold
avg_sales

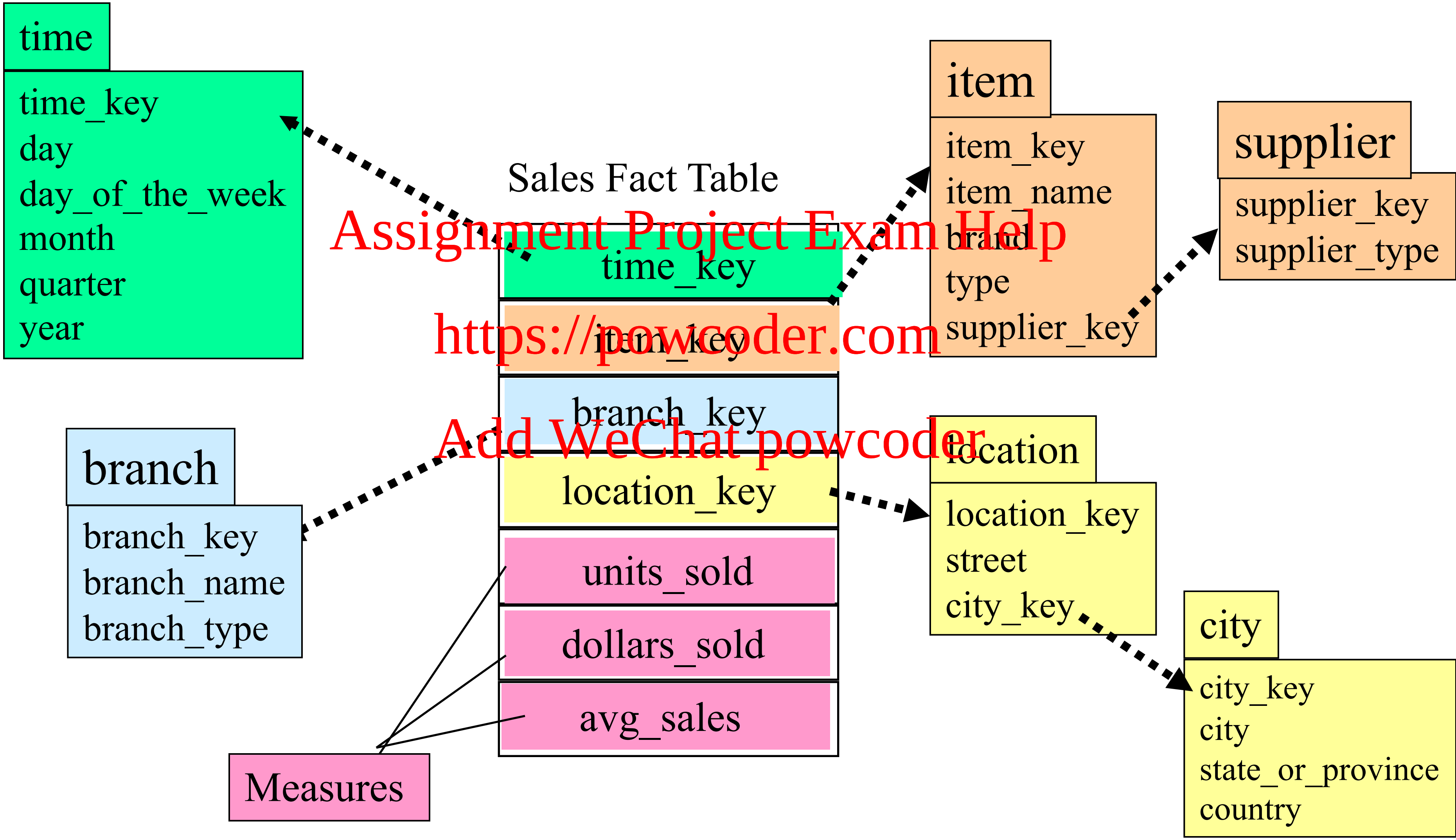
item
item_key
item_name
brand
type
supplier_type

branch
branch_key
branch_name
branch_type

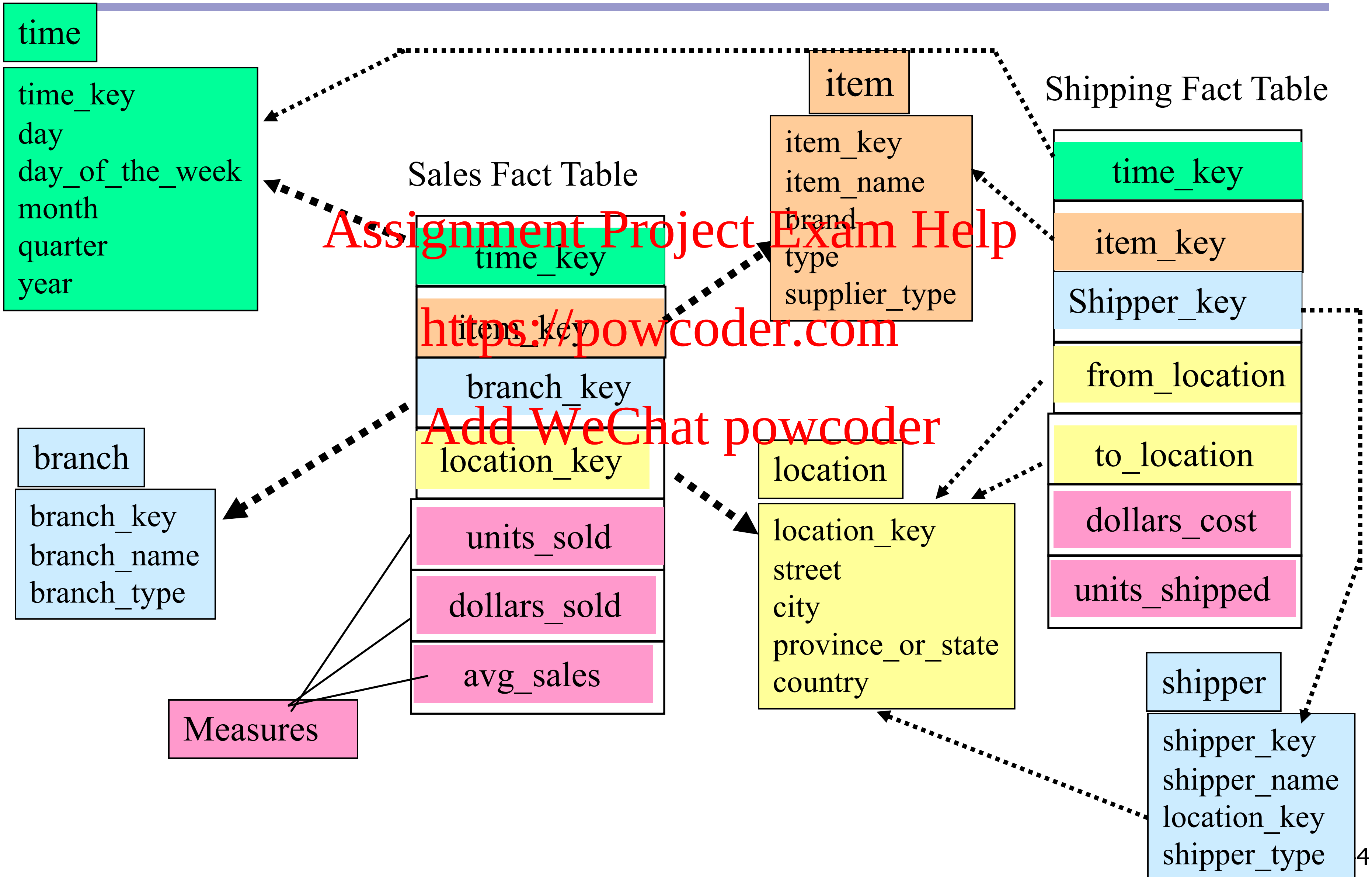
location
location_key
street
city
state_or_province
country

Measures

Example of Snowflake Schema



Example of Fact Constellation



time
time_key
day
day_of_the_week
month
quarter
year

Sales Fact Table
time_key
item_key
branch_key
location_key
units_sold
dollars_sold
avg_sales

Measures

branch
branch_key
branch_name
branch_type

item
item_key
item_name
brand
type
supplier_type

location
location_key
street
city
province_or_state
country

Shipping Fact Table
time_key
item_key
Shipper_key
from_location
to_location
dollars_cost
units_shipped

shipper
shipper_key
shipper_name
location_key
shipper_type

Advantages of Star Schema

- Facts and dimensions are clearly depicted
  - dimension tables are relatively static, data is loaded (append mostly) into fact table(s)
  - easy to comprehend (and write queries)

“Find total sales per product-category in our stores in Europe”

```
SELECT PRODUCT.category, SUM(SALES.amount)
FROM     SALES, PRODUCT,LOCATION
WHERE    SALES.product_key = PRODUCT.product_key
AND      SALES.location_key = LOCATION.location_key
AND      LOCATION.region=“Europe”
GROUP BY PRODUCT.category
```

Operations: Slice (Loc.Region.Europe) + Pivot (Prod.category)

- Query Language

Query Language

- Two approaches:
  - Using relational DB technology: SQL (with extensions such as CUBE/PIVOT/UNPIVOT)
  - Using multidimensional technology: MDX

```
SELECT PRODUCT.category,
SUM(SALES.amount)
FROM     SALES, PRODUCT,LOCATION
WHERE    SALES.product_key =
PRODUCT.product_key
AND      SALES.location_key =
LOCATION.location_key
AND      LOCATION.region="Europe"
GROUP BY PRODUCT.category
```

```
SELECT
 {[PRODUCT].[category]} on ROWS,
 {[MEASURES].[amount]} on COLUMNS
FROM     [SALES]
WHERE  ([LOCATION].[region].[Europe])
```

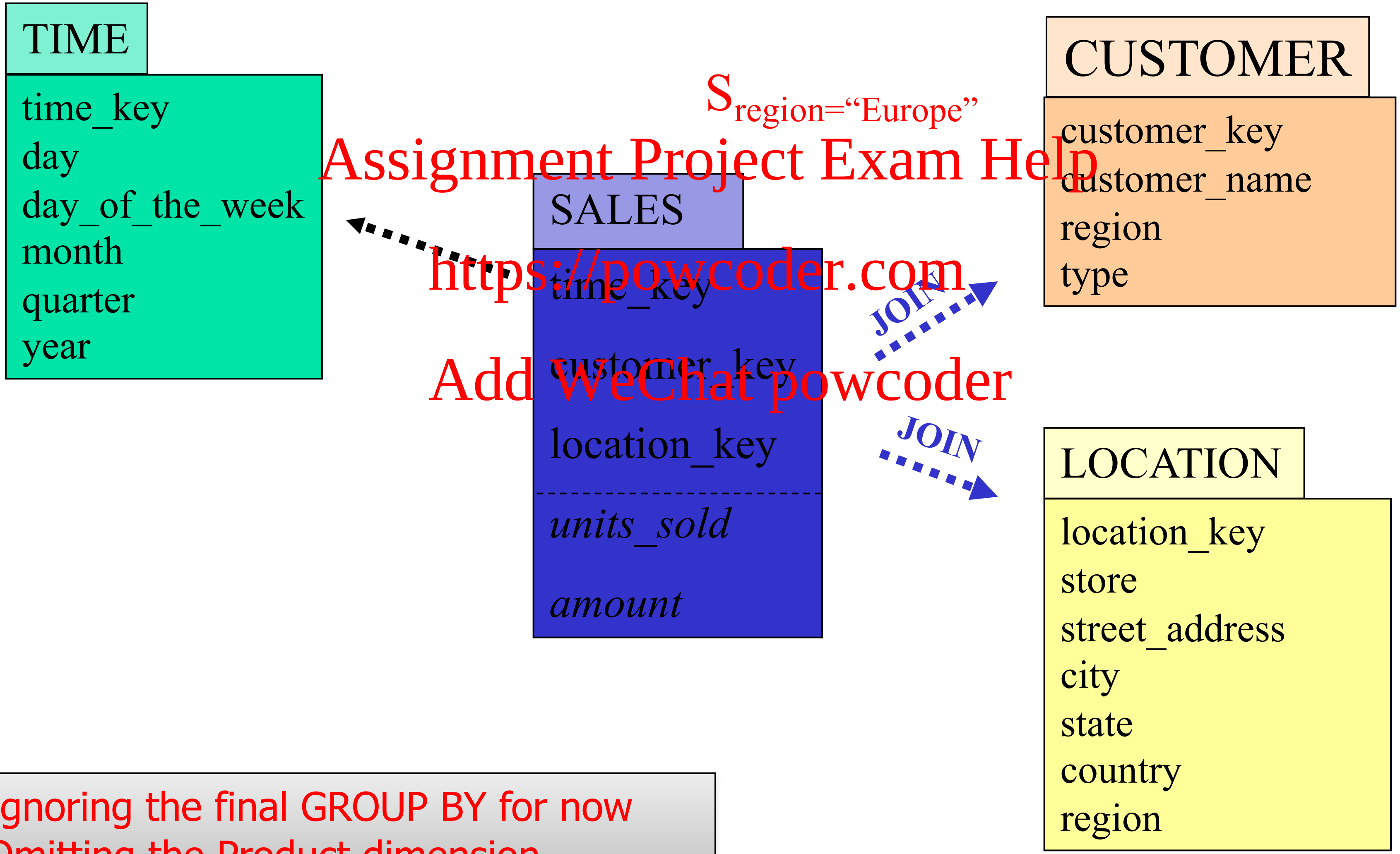
Operations: Slice (Loc.Region.Europe) + Pivot (Prod.category, Measures.amnt)

- Physical Model + Query Processing Techniques

Physical Model + Query Processing Techniques

- Two main approaches:
  - Using relational DB technology: ROLAP
  - Using multidimensional technology: MOLAP
- Hybrid: HOLAP
  - Base cuboid: ROLAP
  - Other cuboids: MOLAP

Q1: Selection on low-cardinality attributes



TIME

time_key
day
day_of_the_week
month
quarter
year

$S_{\text{region=“Europe”}}$

SALES

time_key
customer_key
location_key

units_sold
amount

CUSTOMER

customer_key
customer_name
region
type

LOCATION

location_key
store
street_address
city
state
country
region

- Ignoring the final GROUP BY for now
- Omitting the Product dimension

Chap 4.2 in [JPT10]

Indexing OLAP Data: Bitmap Index

(1) BI on dimension tables

- Index on an attribute (column) with low distinct values
- Each distinct values, v, is associated with a n-bit vector (n = #rows)
  - The *i*-th bit is set if the *i*-th row of the table has the value v for the indexed column
- Multiple BIs can be efficiently combined to enable optimized scan of the table

Custom

Cust	Region	Type
C1	Asia	Retail
C2	Europe	Dealer
C3	Asia	Dealer
C4	America	Retail
C5	Europe	Dealer

BI on Customer.Region

v	bitmap
Asia	1 0 1 0 0
Europe	0 1 0 0 1
America	0 0 0 1 0

https://docs.oracle.com/cd/B28359_01/server.111/b28313/indexes.htm#CIHGAFFF

Indexing OLAP Data: Bitmap Index /2

(1) Bitmap join index (BI on Fact Table Joined with Dimension tables)

- Conceptually, perform a join, map each dimension value to the bitmap of corresponding fact table rows.

```
-- ORACLE SYNTAX –

CREATE BITMAP INDEX sales_cust_region_bjix
 ON    sales(customer.cust_region)
FROM sales, customer
WHERE sales.cust_id = customers.cust_id;
```

Indexing OLAP Data: Bitmap Index /3

Sales

time	customer	loc	Sale
101	C1	100	1
173	C1	200	2
208	C2	100	3
863	C3	200	5
991	C1	100	8
1001	C2	200	13
1966	C4	100	21
2017	C5	200	34

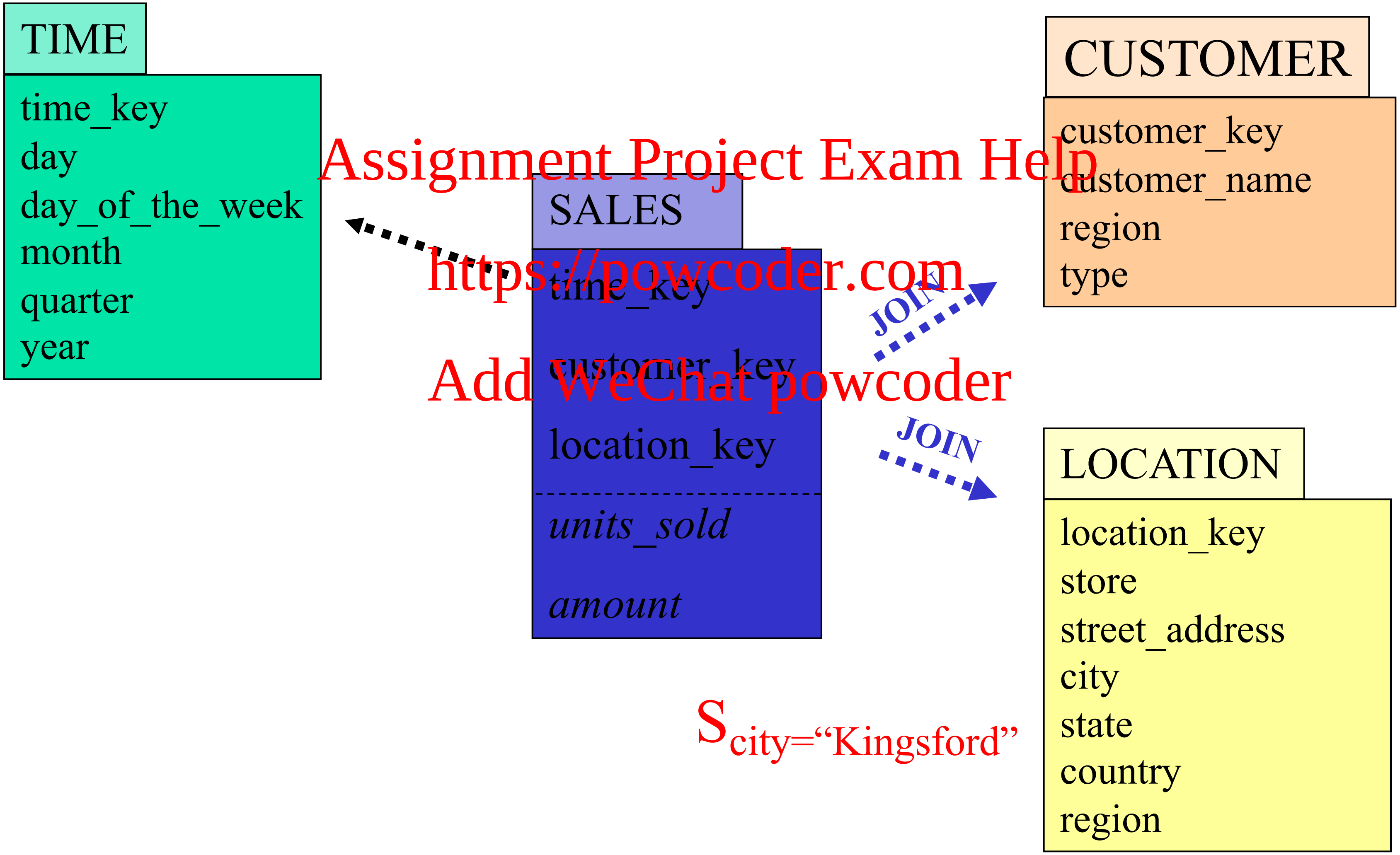
Customer

Cust	Region	Type
C1	Asia	Retail
C2	Europe	Dealer
C3	Asia	Dealer
C4	America	Retail
C5	Europe	Dealer

BI on Sales(Customer.Region)

v	bitmap
Asia	11011000
Europe	00100101
America	00000010

Q2: Selection on high-cardinality attributes



TIME

time_key
day
day_of_the_week
month
quarter
year

SALES

time_key
customer_key
location_key
units_sold
amount

CUSTOMER

customer_key
customer_name
region
type

LOCATION

location_key
store
street_address
city
state
country
region

JOIN

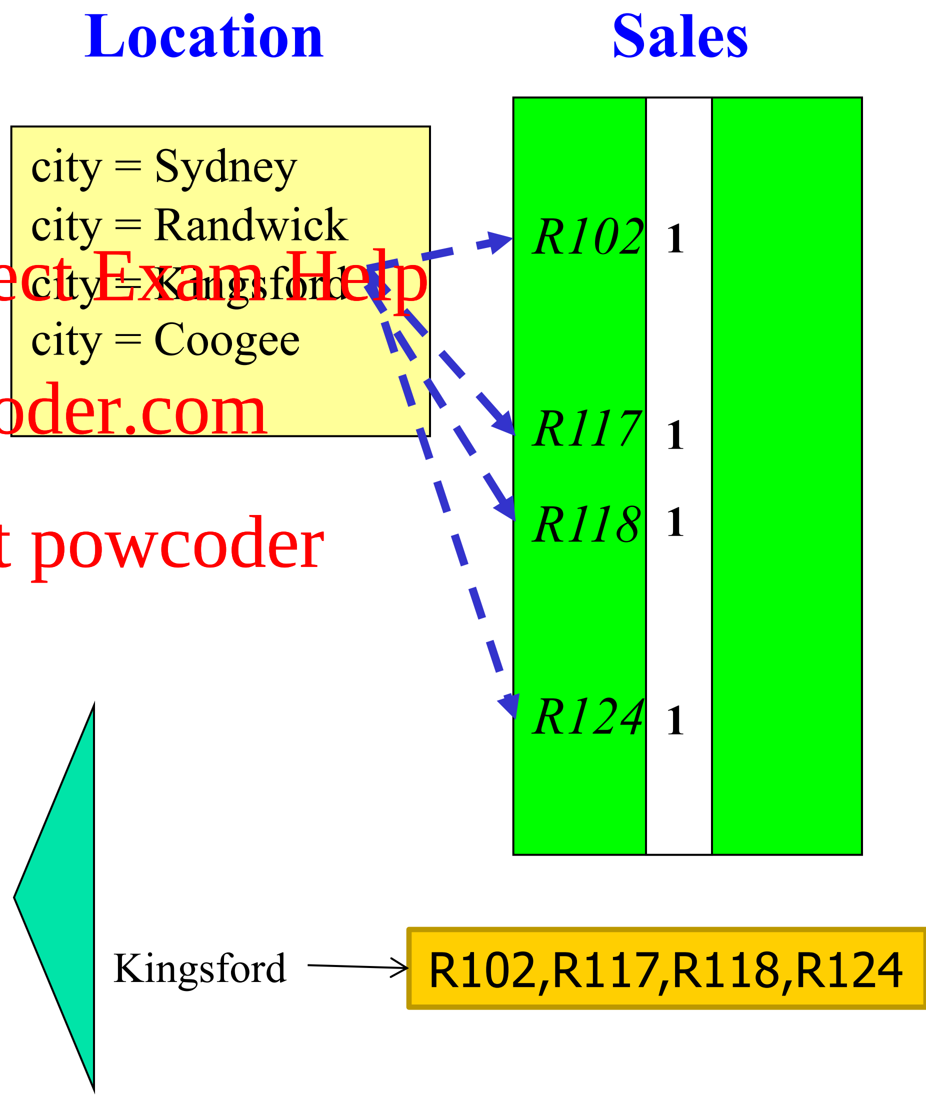
JOIN

$S_{city=\text{“Kingsford”}}$

Chap 4.3 in [JPT10]

Indexing OLAP Data: Join Indices

- Join index relates the values of the dimensions of a star schema to rows in the fact table.
  - a join index on *city* maintains for each distinct city a list of ROW-IDs of the tuples recording the sales in the city
- Join indices can span multiple dimensions OR
  - can be implemented as bitmap-indexes (per dimension)
  - use bit-op for multiple-joins



Location

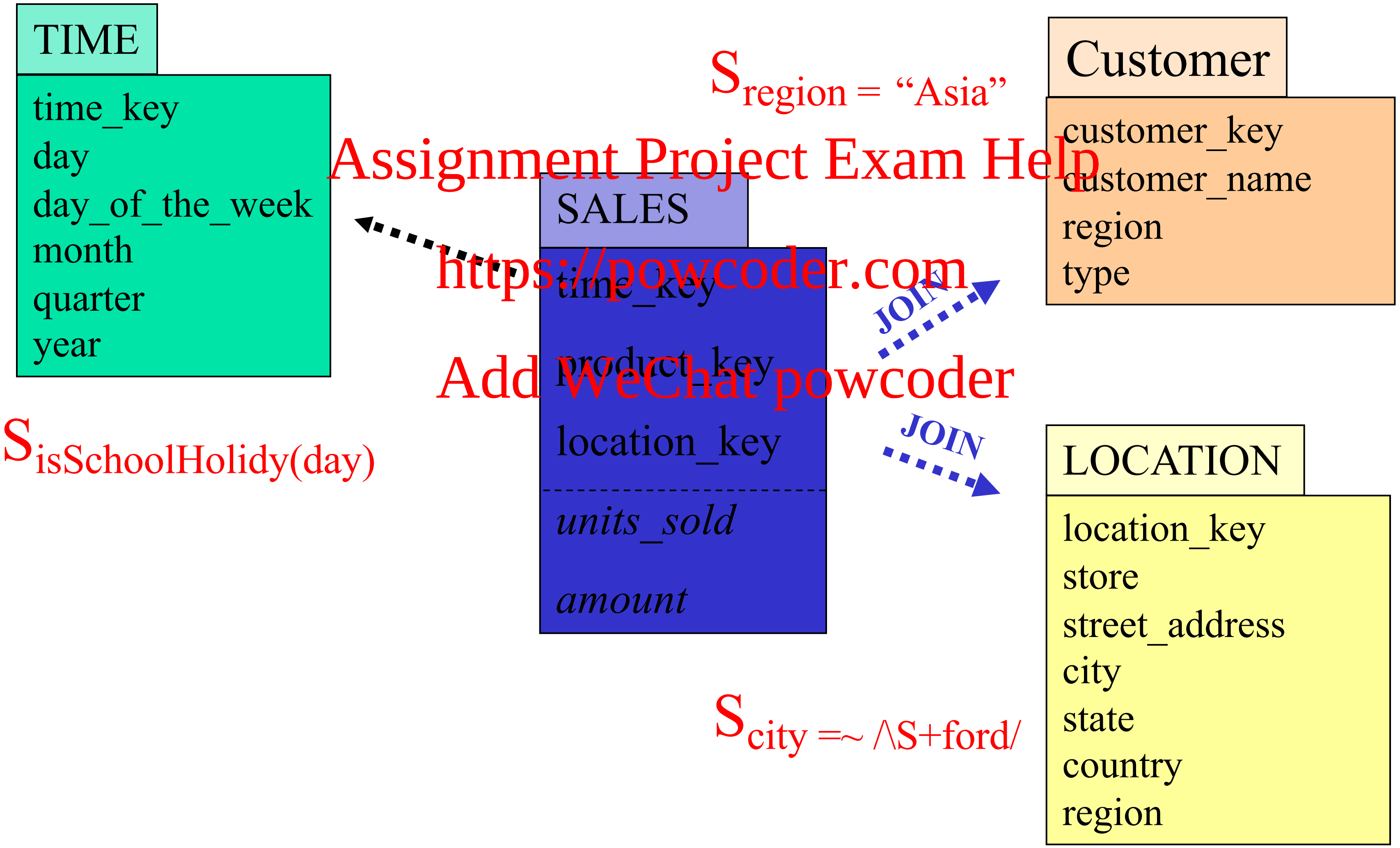
Sales

city = Sydney
city = Randwick
city = Kingsford
city = Coogee

R102	1
R117	1
R118	1
R124	1

Kingsford → R102,R117,R118,R124

Q3: Arbitrary selections on Dimensions



TIME

time_key
day
day_of_the_week
month
quarter
year

$S_{\text{isSchoolHolidy(day)}}$

SALES

time_key
product_key
location_key

units_sold
amount

$S_{\text{region = “Asia”}}$

Customer

customer_key
customer_name
region
type

JOIN

JOIN

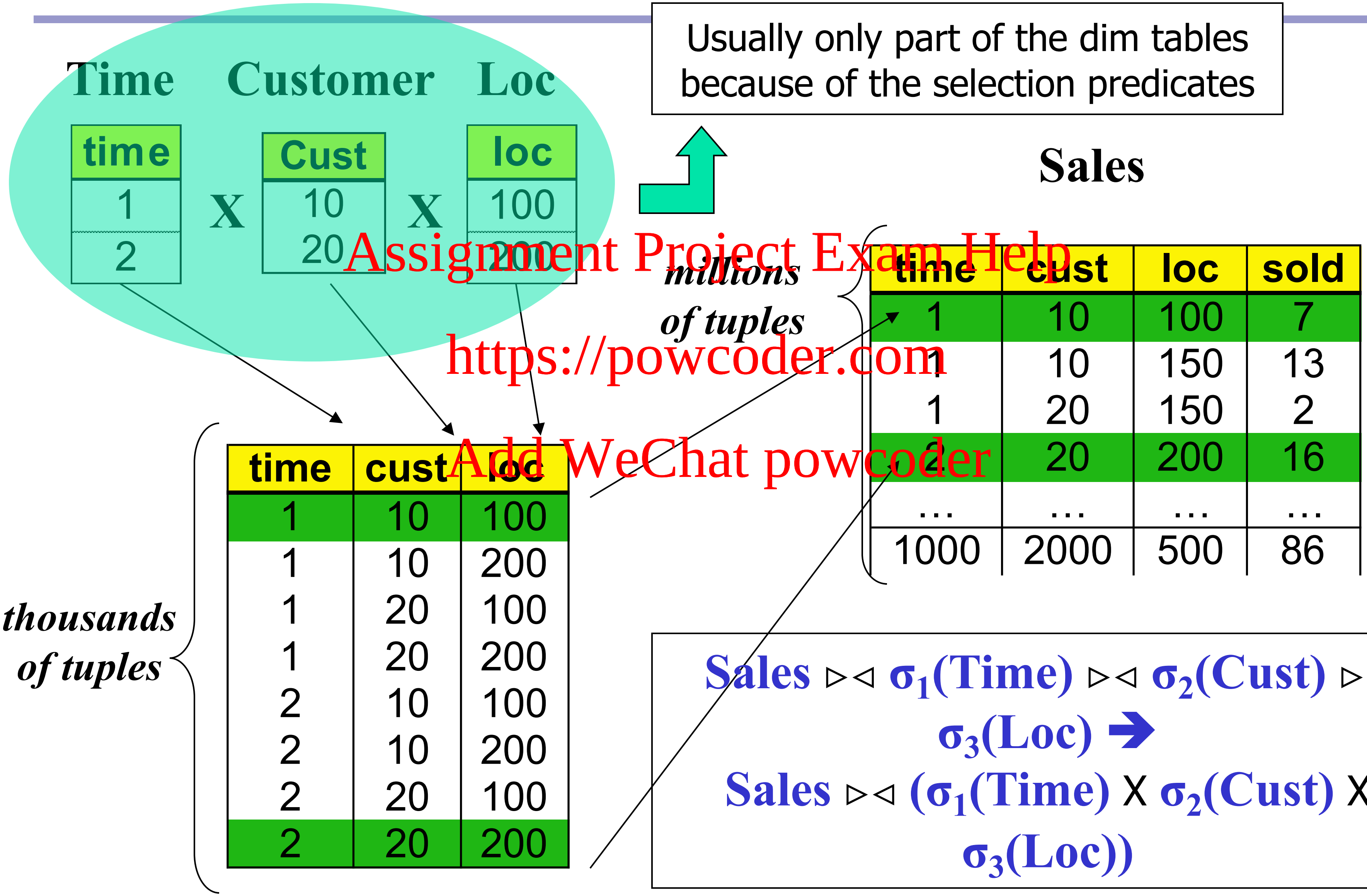
LOCATION

location_key
store
street_address
city
state
country
region

$S_{\text{city =~ /\S+ford/}}$

Chap 4.4 in [JPT10]

Star Query and Star Join (Cont.)



Usually only part of the dim tables because of the selection predicates

time
1
2

Cust
10
20

loc
100
200

time	cust	loc
1	10	100
1	10	200
1	20	100
1	20	200
2	10	100
2	10	200
2	20	100
2	20	200

Sales

time	cust	loc	sold
1	10	100	7
1	10	150	13
1	20	150	2
2	20	200	16
…	…	…	…
1000	2000	500	86

$$\text{Sales} \bowtie \sigma_1(\text{Time}) \bowtie \sigma_2(\text{Cust}) \bowtie \sigma_3(\text{Loc}) \Rightarrow$$

$$\text{Sales} \bowtie (\sigma_1(\text{Time}) \times \sigma_2(\text{Cust}) \times \sigma_3(\text{Loc}))$$

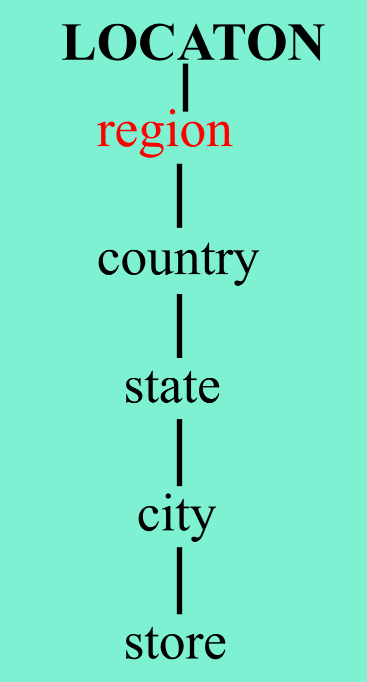
Q4: Coarse-grain Aggregations

- *"Find total sales per customer type in our stores in Europe"*
  - Join-index will prune ¾ of the data (uniform sales), but the remaining ¼ is still large (several millions transactions)
    - Index is unclustered
- High-level aggregations are expensive!!!!!

⇒Long Query Response Times

⇒Pre-computation is necessary

⇒Pre-computation is most beneficial



LOCATON

region

country

state

city

store

Cuboids = GROUP BYs

- Multidimensional aggregation = selection on corresponding cuboid

$$GB_{(type,\ city)}(\ Sales \bowtie \sigma_1(Time) \bowtie \sigma_2(Cust) \bowtie \sigma_3(Loc))\ )$$

σ_1 selects some Years, σ_2 selects some Brands, σ_3 selects some Cities,

$$GB_{(type,\ city)}(\ \sigma_{1'2'3'}(\mathbf{Cuboid}(\mathit{Year},\ \mathit{Type},\ \mathit{City}))\ )$$

- Materialize some/all of the cuboids
  - A complex decision involving cuboid sizes, query workload, and physical organization

Two Issues

- How to store the materialized cuboids?
- How to compute the cuboids efficiently?

CUBE BY in ROLAP

Sales		Product				
		1	2	3	4	ALL
Store	1	454	-	-	925	1379
	2	468	800	-	-	1268
	3	296	-	240	-	536
	4	652	-	540	745	1937
	ALL	1870	800	780	1670	5120

Store	Product_key	sum(amout)
1	1	454
1	4	925
2	1	468
2	2	800
3	1	296
3	3	240
4	1	625
4	3	240
4	4	745
1	ALL	1379
2	ALL	1268
3	ALL	536
4	ALL	1937
ALL	1	1870
ALL	2	800
ALL	3	780
ALL	4	1670
ALL	ALL	5120

4 Group-bys here:

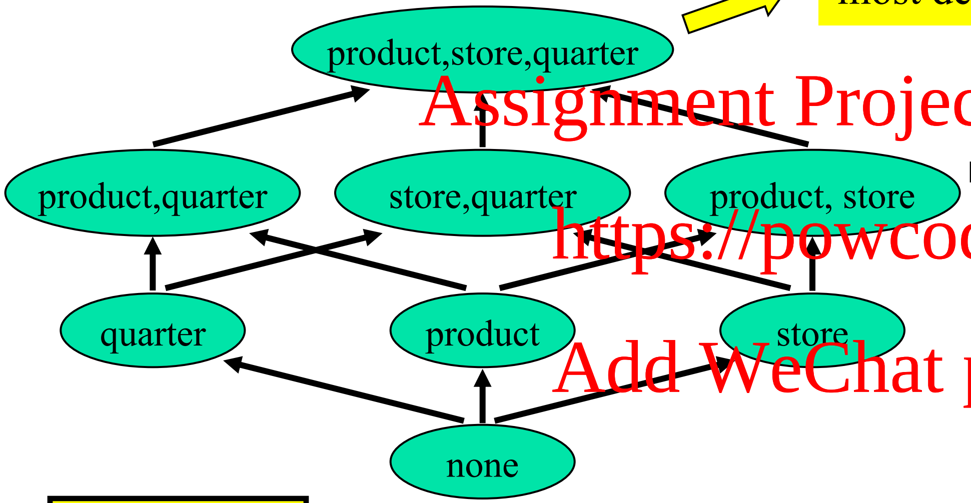
(store,product)
(store)
(product)
()

- Need to write 4 queries!!!
- Compute them independently

```
SELECT LOCATION.store, SALES.product_key, SUM (amount)
FROM SALES, LOCATION
WHERE SALES.location_key=LOCATION.location_key
CUBE BY SALES.product_key, LOCATION.store
```

Top-down Approach

- Model dependencies among the aggregates:



most detailed "view"

can be computed from *view* (product,store,quarter) by summing-up all quarterly sales

Cuboid AB

A	B	M
1	1	10
1	3	20
1	2	30
1	1	40
2	4	50

Cuboid A

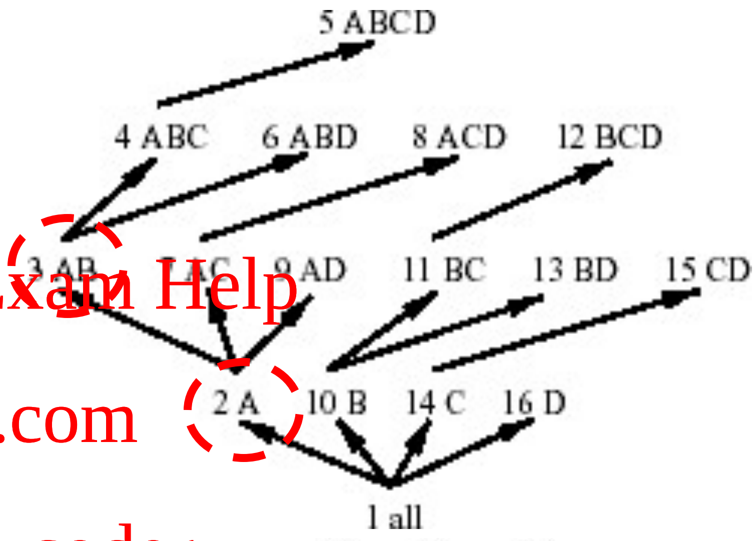
A	B	M
1	(all)	???
2	(all)	???

Cuboid B

A	B	M
(all)	1	???
(all)	2	???
(all)	3	???
(all)	4	???

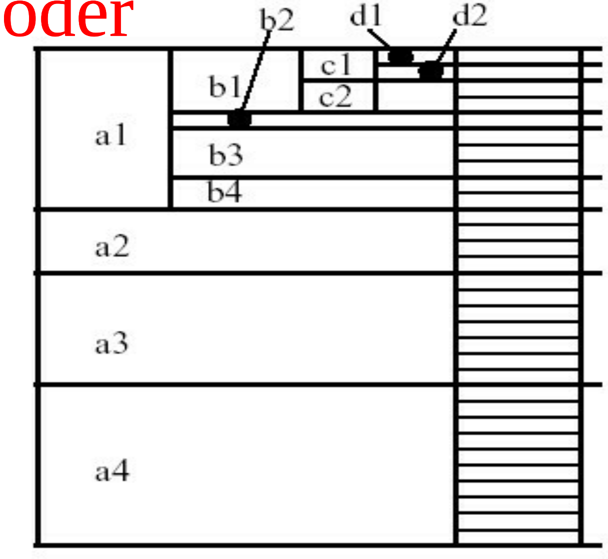
Bottom-Up Approach (BUC)

- BUC (Beyer & Ramakrishnan, SIGMOD'99)
- Ideas
  - Compute the cube from bottom up
  - Divide-and-conquer
- A simpler recursive version:
  - BUC-SR



A	B	...
1	1	...
1	3	...
1	2	...
1	1	...
2	...	...

...



Understanding Recursion /1

- Powerful computing/problem-solving techniques
- Examples
  - Factorial:
    - f(n) = 1, if n = 1
    - f(n) = f(n-1) * n, if n ≥ 1
  - Quick sort:
    - Sort([x]) = [x]
    - Sort([x1, …, *pivot*, … xn]) = sort[ys] ++ sort[zs]), where

      ys = [x | x ***in*** xi, x ≤ *pivot*]

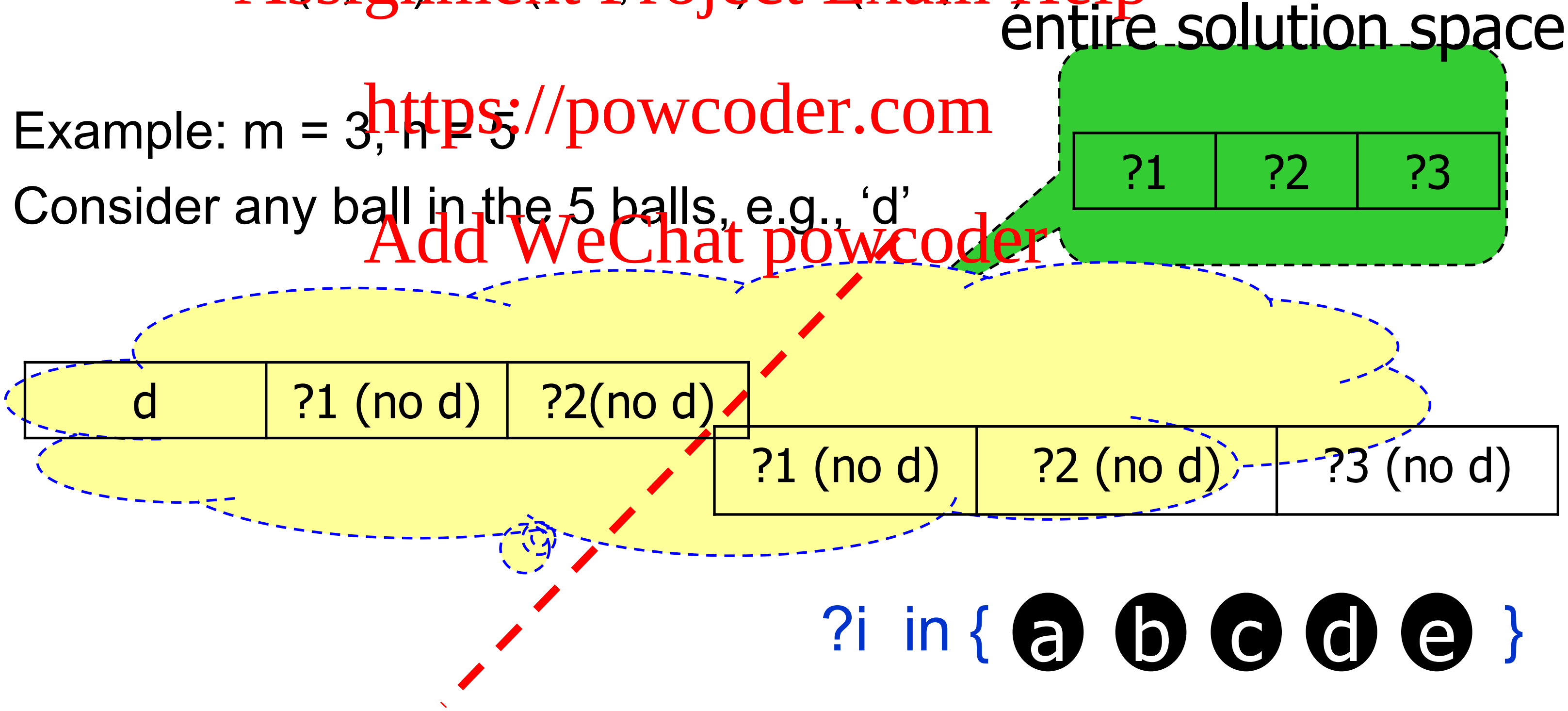
      zs = [x | x **<-** xi, x > *pivot*]

f(0) = 0! = ???

List comprehension in Haskell or python

Understanding Recursion /2

- Let C(n, m) be the number of ways to select m balls from n numbered balls
- Show that C(n, m) = C(n-1, m-1) + C(n-1, m)
- Example: m = 3, n = 5
- Consider any ball in the 5 balls, e.g., 'd'



entire solution space

?1 ?2 ?3

d | ?1 (no d) | ?2(no d)

?1 (no d) | ?2 (no d) | ?3 (no d)

?i in { a b c d e }

Key Points

- Sub-problems need to be "**smaller**", so that a simple/trivial boundary case can be reached
- Divide-and-conquer
  - There may be multiple ways the entire solution space can be divided into **disjoint** sub-spaces, each of which can be conquered **recursively**.

Geometric Intuition /1

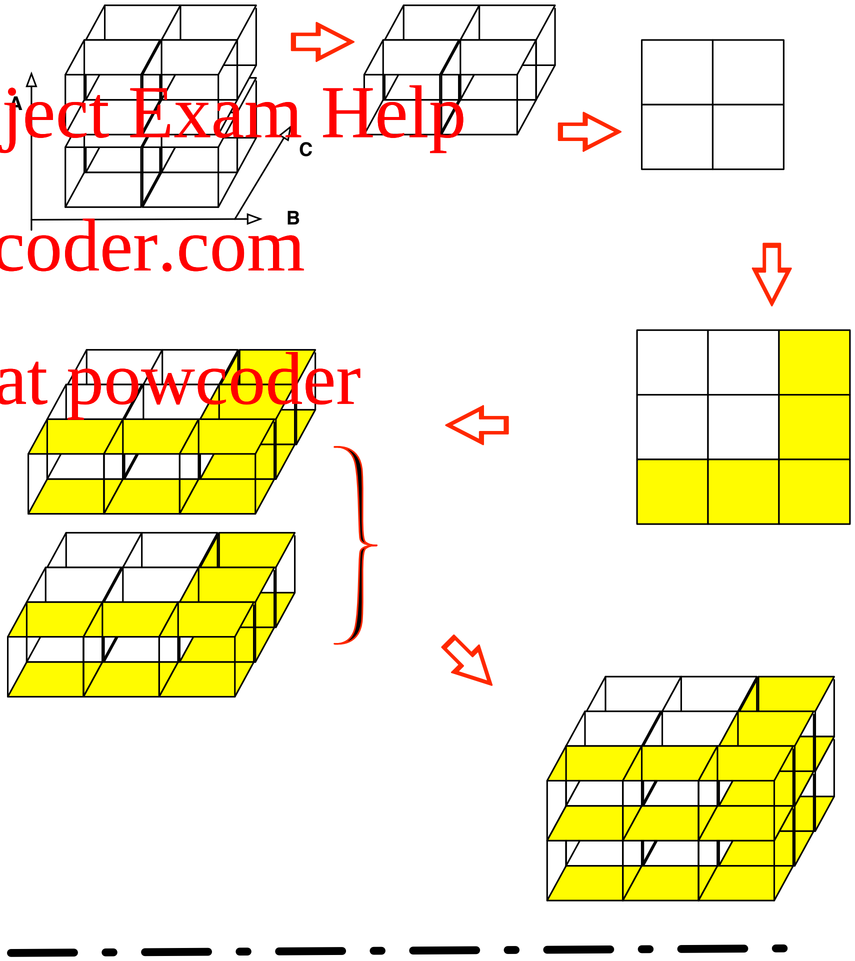
- Reduce Cube(in 2D) to Cube(in 1D)

	b1	b2	b3	
a1	M11	M12	M13	[Step 1]
a2	M21	M22	M23	[Step 1]
	[Step 2]	[Step 2]	[Step 2]	[Step 3]

	b1	b2	b3	*
[a1] ×	M11	M12	M13	[Step 1]
[a2] ×	M21	M22	M23	[Step 1]
[*] ×	[Step 2]	[Step 2]	[Step 2]	[Step 3]

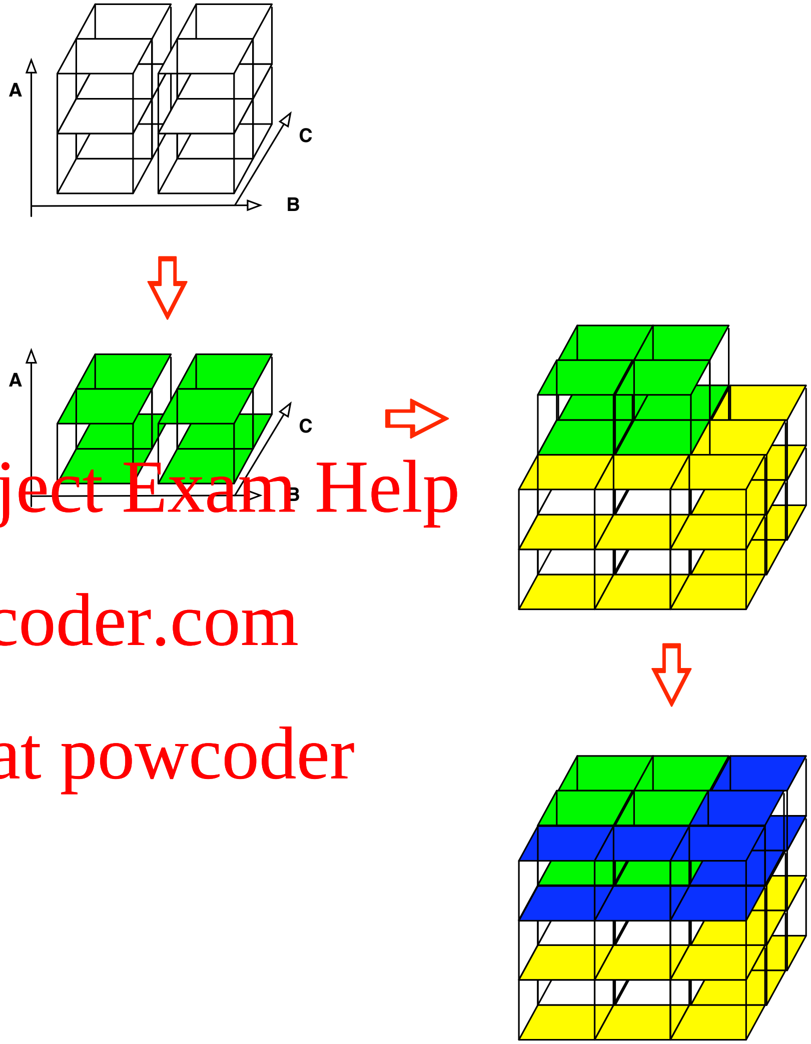
Geometric Intuition /2

- Reduce Cube(in 3D) to Cube(in 2D)



Geometric Intuition /3

- Reduce Cube(in 3D) to Cube(in 2D)



Algebraic Derivation

- How to compute *n*-dim cube on (*n*+1)-dim base cuboid (array)?
  - What do output tuples look like?

- How to compute (*n*+1)-dim cube on (*n*+1)-dim base cuboid (array)?
  - What else do we need?

[{r1-r5}, ABC]

[{r1-r5}, BC]

	A	B	C	M
r1	1	1	1	10
r2	1	1	2	20
r3	1	2	1	30
r4	1	3	1	40
r5	2	1	1	50

You need to fix the incomplete output part of the algorithm!

BUC-SR (Simple Recursion)*

- BUC-SR(data, dims)
  - If (dims is empty)
    - Output (sum(data))
  - Else
    - Dims = [dim1, rest_of_dims]
    - For each distinct value v of dim1
      - slice_v = slice of data on “dim1 = v”
      - BUC-SR(slice_v, rest_of_dims)
    - data’ = Project(data, rest_of_dims)
    - BUC-SR(data’, rest_of_dims)

Boundary case:
data is essentially a list of measure values

General case:
1)Slice on dim1. Call BUC-SR recursively for each slice

2)Project out dim1, and call BUC-SR on it recursively

Example

Input

Internal Output

Output

	1	2	3	*
1	30	30	40	100
2	50			50
*	80	30	40	150

[{r1-r5}, AB]

	A	B	M
r1	1	1	10
r2	1	1	20
r3	1	2	30
r4	1	3	40
r5	2	1	50

[{r1-r4}, B]

B	M
1	10
1	20
2	30
3	40

B	M
1	30
2	30
3	40
*	100

A	B	M
1	1	30
1	2	30
1	3	40
1	*	100

[{r5}, B]

B	M
1	50

B	M
1	50
*	50

A	B	M
2	1	50
2	*	50

[{r1'-r5'}, B]

B	M
1	**80**
2	30
3	40

B	M
1	80
2	30
3	40
*	150

A	B	M
*	1	80
*	2	30
*	3	40
*	*	150

Try a 3D-Cube by Yourself

[{r1-r5}, ABC]

	A	B	C	M
r1	1	1	1	10
r2	1	1	2	20
r3	1	2	1	30
r4	1	3	1	40
r5	2	1	1	50

MOLAP

- (Sparse) array-based multidimensional storage engine
- Pros:
  - small size (esp. for dense cubes)
  - fast in indexing and query processing
- Cons:
  - scalability
  - conversion from relational data

Multidimensional Array

Step 2: An **injective** map from cell to offset

f(time, item) = 4*time + item

time	item	dollars_sold
Q1	home entertainment	605
Q2	home entertainment	680
Q3	home entertainment	812
Q4	home entertainment	927
Q1	computer	825
Q2	computer	952
Q3	computer	1023
Q4	computer	1038
Q1	phone	14
Q2	phone	31
Q3	phone	30
Q4	phone	38
Q1	security	400
Q2	security	512
Q3	security	501
Q4	security	580

Step 1

Mappings

time	*value*
Q1	0
Q2	1
Q3	2
Q4	3

item	*value*
home entertainment	0
computer	1
phone	2
security	3

time	item	dollars_sold	offset
0	0	605	0
1	0	680	4
2	0	812	8
3	0	927	12
0	1	825	1
1	1	952	5
2	1	1023	9
3	1	1038	13
0	2	14	2
1	2	31	6
2	2	30	10
3	2	38	14
0	3	400	3
1	3	512	7
2	3	501	11
3	3	580	15

Multidimensional Array

Step 3: If dense, only need to store sorted slots

offset	dollars_sold
0	605
1	825
2	14
3	400
4	680
5	952
6	31
7	512
8	812
9	1023
10	30
11	501
12	927
13	1038
14	38
15	580

- Think: how to decode a slot?

Dense MD array
605
825
14
400
680
952
31
512
812
1023
30
501
927
1038
38
580

Step 2: An **injective** map from cell to offset

f(time, item) = 4*time + item

The Sparse Case

time	item	dollars_sold
Q1	home entertainment	605
/ same table but with many rows deleted to make it sparse */*		
Q4	security	580

Step 1

Mappings

time	*value*
Q1	0
Q2	1
Q3	2
Q4	3

item	*value*
home entertainment	0
computer	1
phone	2
security	3

time	item	dollars_sold	offset
0	0	605	0
/ same table but with many rows deleted to make it sparse */*			
3	3	580	15

Multidimensional Array

Choice 1

offset	dollars_sold
0	605
15	580

Choice 2

Dense MD array
605
-
-
-
-
-
-
-
-
-
-
-
-
-
-
580

- Think: how to decode a slot?
- Multidimensional array is typically sparse
  - Use sparse array (i.e., offset + value)
  - Could use chunk to further reduce the space
- Space usage:
  - (d+1)*n*4 vs 2*n*4
- HOLAP:
  - Store all non-base cuboid in MD array
  - Assign a value for ALL