

COMP9334: Capacity Planning of Computer Systems and Networks

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Week 8B: Optimisation (1):
Linear programming

Three Weeks of Optimisation

- The lectures for next three weeks will focus on optimization methods for network related design and applications
- You will learn:
 - How to formulate optimization problems
 - Tools to solve optimization problems
- An introduction only, because optimization is a big topic
 - Emphasis is on applying optimization methods rather than the theory behind

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Motivation (1)

- A modern approach to managing computer networks is based on the concept of *software-defined networking*
- Two types of nodes:
 1. Simple packet switches
 2. Controllers
- A controller can control a number of simple packet switches but they must be placed in a strategic location in the network
- If the delay between the controller and a packet switch is too long, then it can degrade the network performance

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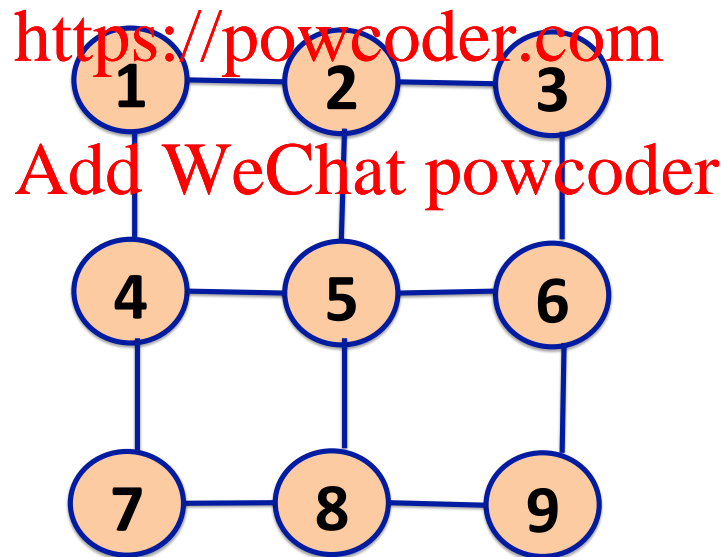
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Motivation (2)

- Consider the following network where there is a packet switch at each node and the delay on each link is 1 time unit.
- Question: Assuming you want to place one controller in the network, where will you place the controller?

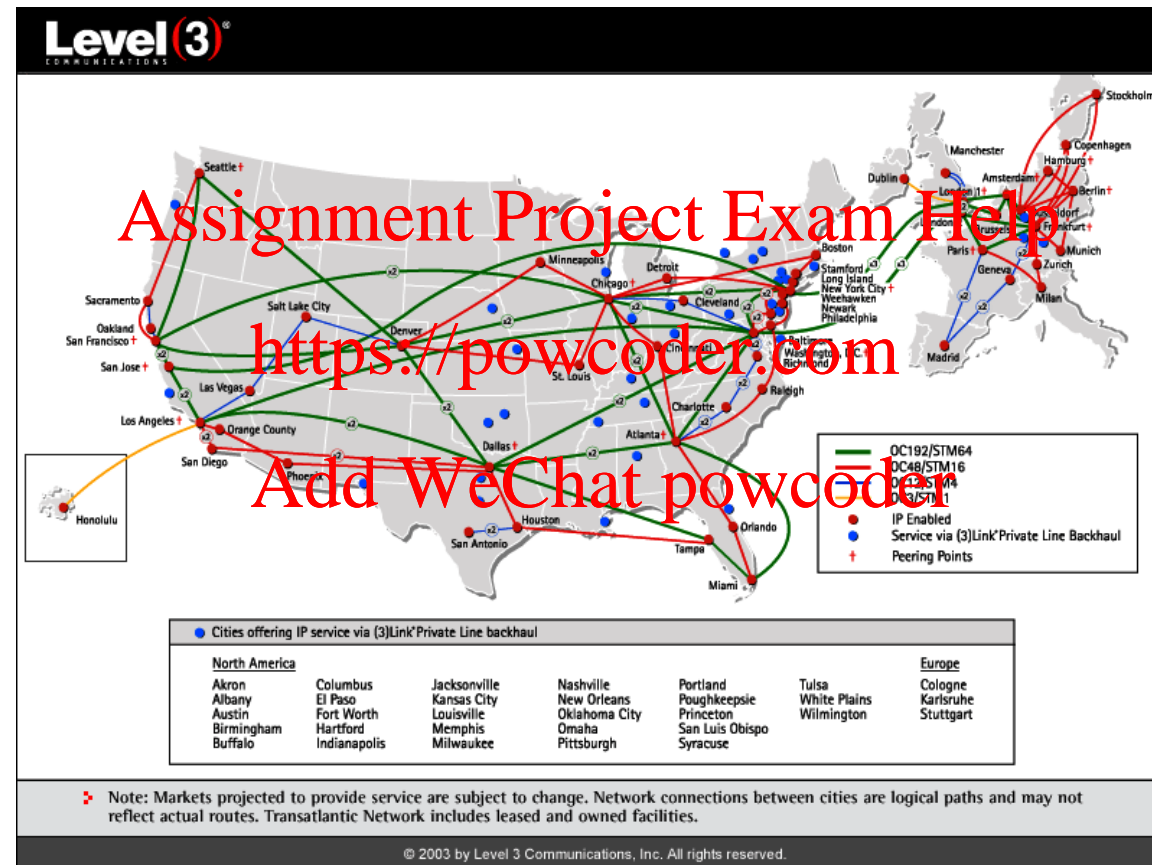
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- Question: What if you want to place two controllers?

Motivation (3)

- How about solving the same problem for a large heterogeneous network?



- Optimisation provides a systematic method to make decisions

Elements of an optimisation problem

- You want to maximise your WAM and still have a life

Maximise $WAM(x_1, x_2, x_3, \dots)$

x_1 hours/week on COMP9334

x_2 hours/week on COMPxxxx

x_3 hours/week on socialising

$x_1 \geq 10$

$x_3 \leq \text{maxSocialHours}$

$x_1 + x_2 + x_3 + \dots \leq \text{totalAwakeHours}$

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- Elements of an optimisation problem
 - Minimise or maximise an objective function
 - Decision variables: x_1, x_2, \dots etc.
 - Constraints

What is optimization?

- In mathematics, also known as **mathematical programming**
 - The term **programming** refers to planning of activities to obtain an optimal result, not computer programming
 - The amount or level of each activity can be represented as a variable whose values to be determined
- **Optimization** means solving problems in which we seek to minimize or maximize the value of an **objective function** of many **decision variables**, subject to **constraints** on the decision variables

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Reference books

- Winston, “Operations Research”, 4th edition
 - Examples from this book tend to come from manufacturing, business, finance, etc
 - The abstraction power of mathematics means many optimization problems have similar mathematical formulation
 - Very often an optimization problem in networking may have a similar cousin in other application areas, and their mathematical formulation are identical
- Ahuja, Magnanti and Orlin, “Network Flows”
- Fourer, Gay and Kernighan, “AMPL: A Modeling Language for Mathematical Programming”, 2nd edition

Software

- Modeling language: AMPL and Solver CPLEX
 - High-level programming language for describing optimization problems
 - Syntax similar to mathematical formulation of optimization problems
 - Demo version of the software is available for download from:
<http://www.ampl.com>. Click *Try AMPL*, then *Download a Free Demo*
- Note: Demo version of AMPL/CPLEX is full-featured but limited to 500 variables and 500 objectives plus constraints

Motivating example 1: Cloud/Grid computing

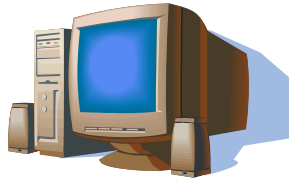
- Service providers sell computing power as an utility
 - Computing power measured in CPU cycles
- Target customers
 - Financial company, pharmaceutical company, etc.
- Quality of Service in Cloud Computing
 - Different service providers might offer the service at different levels for different costs
 - Optimization problem: How to select service providers (allocate resources) to achieve the best level of service without exceeding budget

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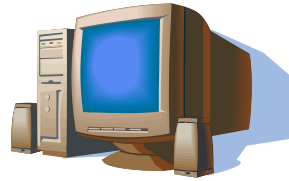
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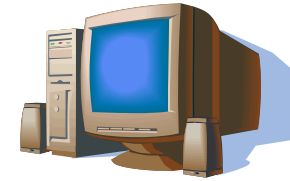
Cloud computing resource allocation



Resource 1
Speed: 1,000 million
cycles/sec
Cost: 0.1 dollars/sec



Resource 2
Speed: 2,000 million
cycles/sec
Cost: 0.25 dollars/sec



Resource 3
Speed: 3,000 million
cycles/sec
Cost: 0.6 dollars/sec

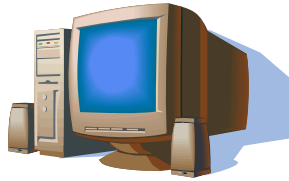
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- A computation job:
 - Requires 10^7 million cycles
 - Must be completed in at most 4,800 sec
 - Cost must not exceed 1,500 dollars
- Exercises: For the time being, let us ignore the constraint on the completion time and cost.
 - If you use Resource 1 only, what is the completion time and cost?
 - Repeat for Resources 2 and 3.

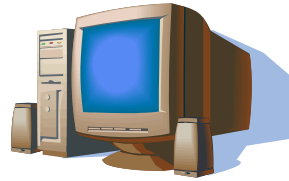
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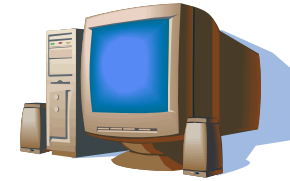
Cloud computing resource allocation (cont.)



Resource 1
Speed: 1,000 million
cycles/sec
Cost: 0.1 dollars/sec



Resource 2
Speed: 2,000 million
cycles/sec
Cost: 0.25 dollars/sec



Resource 3
Speed: 3,000 million
cycles/sec
Cost: 0.6 dollars/sec

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■ A computation job:

- Requires 10^7 million cycles
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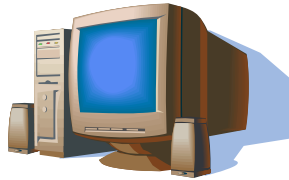
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■ Completion time and cost for each resource:

- Resource 1: Completion time = 10,000 sec, cost = 1,000 dollars
- Resource 2: Completion time = 5,000 sec, cost = 1,250 dollars
- Resource 3: Completion time = 3,333 sec, cost = 2,000 dollars

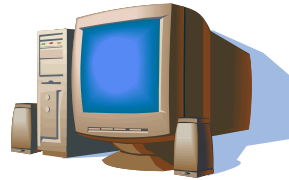
Cloud computing resource allocation (cont.)



Resource 1

Speed: 1,000 million
cycles/sec

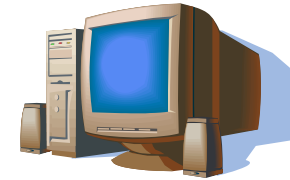
Cost: 0.1 dollars/sec



Resource 2

Speed: 2,000 million
cycles/sec

Cost: 0.25 dollars/sec



Resource 3

Speed: 3,000 million
cycles/sec

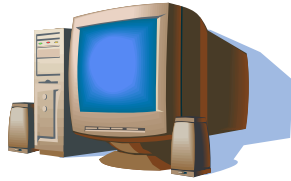
Cost: 0.6 dollars/sec

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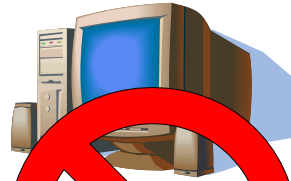
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- Assume the computation job can be arbitrarily split into up to three parallel tasks
- Question: How should the job be split, so that completion time T is minimized subject to two constraints:
 - Completion time constraint: $T \leq 4,800$ sec
 - Cost constraint: $C \leq 1,500$ dollars

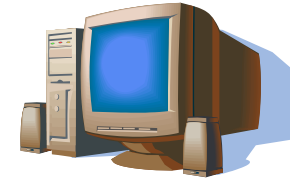
Trial and error: Solution 1



Resource 1
Speed: 1,000 million
cycles/sec
Cost: 0.1 dollars/sec



Resource 2
Speed: 2,000 million
cycles/sec
Cost: 0.25 dollars/sec



Resource 3
Speed: 3,000 million
cycles/sec
Cost: 0.6 dollars/sec

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- 48% to Resource 1, 52% to Resource 3
 - Resource 1: Completion time = 4,800 sec, cost = 480 dollars
 - Resource 3: Completion time = 1,733 sec, cost = 1,040 dollars
- Job completion time = 4,800 sec (remember jobs run in parallel)
- cost = 1,520 dollars, **Infeasible** solution

Terminology

- A solution is **feasible** if all the constraints are satisfied
- A solution is **infeasible** if not all the constraints are satisfied

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Trial and error: Solution 2

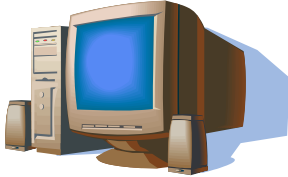


Resource 1	Resource 2	Resource 3
Speed: 1,000 million cycles/sec	Speed: 2,000 million cycles/sec	Speed: 3,000 million cycles/sec
Cost: 0.1 dollars/sec	Cost: 0.25 dollars/sec	Cost: 0.6 dollars/sec

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- 70% to Resource 2, 30% to Resource 3
- Resource 2: Completion time = 3,500 sec, cost = 875 dollars
- Resource 3: Completion time = 1,000 sec, cost = 600 dollars
- Job completion time = 3,500 sec, cost = 1,475 dollars
- Feasible solution

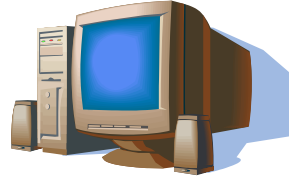
Trial and error: Solution 3



Resource 1

Speed: 1,000 million
cycles/sec

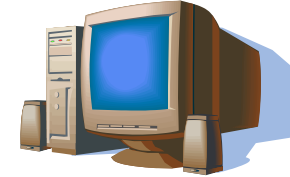
Cost: 0.1 dollars/sec



Resource 2

Speed: 2,000 million
cycles/sec

Cost: 0.25 dollars/sec



Resource 3

Speed: 3,000 million
cycles/sec

Cost: 0.6 dollars/sec

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- 30% to Resource 1, 30% to Resource 2, 40% to Resource 3
- Resource 1: Completion time = 3,000 sec, cost = 300 dollars
- Resource 2: Completion time = 1,500 sec, cost = 375 dollars
- Resource 3: Completion time = 1,333 sec, cost = 800 dollars
- Job completion time = 3,000 sec, cost = 1,475 dollars
- Feasible solution

Optimizing resource allocation

■ Given:

- Job requirement = 10^7 million cycles
- Completion time $\leq 4,800$ sec
- Budget $\leq 1,500$ dollars

■ Let:

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- x_1 = fraction of the job to Resource 1
- x_2 = fraction of the job to Resource 2
- x_3 = fraction of the job to Resource 3

■ Find x_1 , x_2 and x_3 such that

- All requirements are met
- Completion time is minimized

Formulating optimization problem

■ Completion time:

- Resource 1 = $\frac{10^7 \times x_1}{1000} = 10000 \times x_1$

- Resource 2 = $\frac{10^7 \times x_2}{2000} = 5000 \times x_2$

- Resource 3 = $\frac{10^7 \times x_3}{3000} = \frac{10000}{3} \times x_3$

- Job completion time $T = \max(10000 \times x_1, 5000 \times x_2, \frac{10000}{3} \times x_3)$

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■ Cost:

- Resource 1 = $0.1 \times 10000 \times x_1 = 1000 \times x_1$

- Resource 2 = $0.25 \times 5000 \times x_2 = 1250 \times x_2$

- Resource 3 = $0.6 \times \frac{10000}{3} \times x_3 = 2000 \times x_3$

- Cost $C = 1000 \times x_1 + 1250 \times x_2 + 2000 \times x_3$

Formulating optimization problem (cont.)

- Mathematically, the optimization problem can be formulated as

$$\min T$$

subject to

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$$T \geq 10000 \times x_1$$

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$$T \geq 5000 \times x_2$$

$$T \geq \frac{10000}{3} \times x_3$$

$$T \leq 4800$$

$$1000 \times x_1 + 1250 \times x_2 + 2000 \times x_3 \leq 1500$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

Components of an optimization problem

- Given parameters

- Decision variables

- In this example, they are x_1 , x_2 , x_3 and T
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- Objective function

- Can be minimization or maximization
- Can be single objective or multi-objective

- Constraints

Exercise

- Consider the following optimization problem where x is the decision variable:

$$\min_x 2x - 1$$

subject to

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- What are the feasible solutions?
- What is the optimal solution?

LP solvers

- Many commercial and free software are available for solving LP problems
- Commercial software
 - Capable of solving large LP problems, e.g. millions of variables
 - A 50,000-variable LP problem takes about 5 seconds on a standard linux PC
 - You can try out many commercial solvers at the NEOS web site
 - <https://neos-server.org/neos/>
- Free software / demo version
 - <http://www.ampl.com>
 - <http://ampl.com/try-ampl/download-a-demo-version/>

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LP solvers (cont.)

- LP solvers require the user to write the problem in fixed format
- Can be embedded in C, C++ or Java, e.g.

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```
model.add(IloMinimize(env, -9*x[0] + x[1] + 4*x[2]));  
model.add(-x[0] + x[2] == -3);  
model.add(x[0] - x[1] <= 1);
```

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- Can be used with some modeling languages
 - AMPL
 - MPS
 - GAMS

AMPL/CPLEX for solving example 1

- In AMPL, the grid computing problem formulated earlier becomes

```
var T;  
var x_1 >= 0;  
var x_2 >= 0;  
var x_3 >= 0;  
minimize time: T;  
subject to T_1: T >= 10000*x_1;  
subject to T_2: T >= 5000*x_2;  
subject to T_3: T >= 10000/3*x_3;  
subject to T_max: T <= 4800;  
subject to C_max: 1000*x_1+1250*x_2+2000*x_3 <= 1500;  
subject to x_sum: x_1+x_2+x_3 = 1;
```

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- This is saved in the file `grid_lp.mod`

AMPL/CPLEX for solving example 1 (cont.)

- The problem can be solved by CPLEX with the batch file `grid_lp_batch`

```
model grid_lp.mod;  
option solver cplex;  
solve;  
display x_1;  
display x_2;  
display x_3;  
display T;
```

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AMPL/CPLEX for solving example 1 (Command line version)

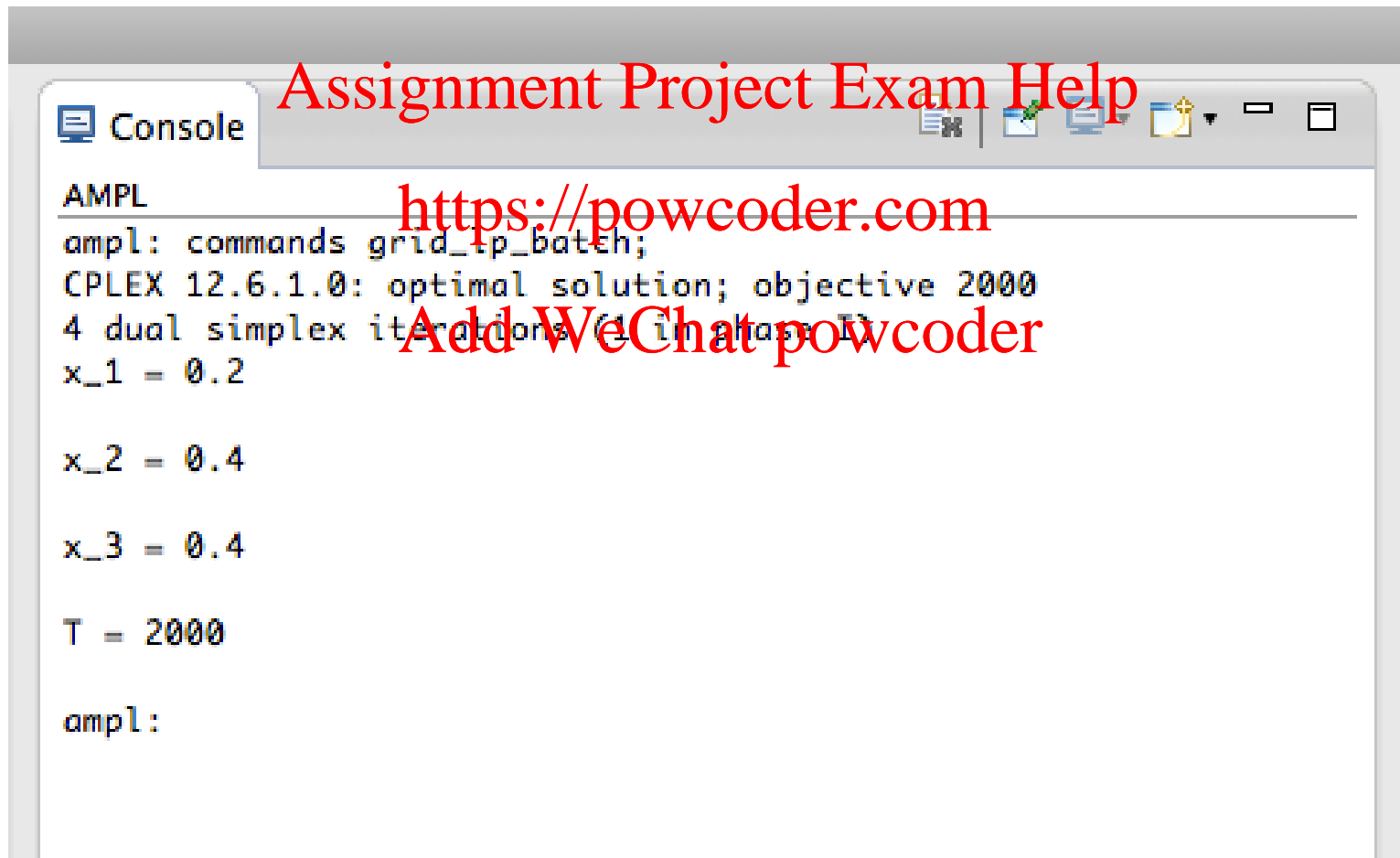
- At the ampl command prompt, type `commands grid_lp_batch;`, it returns

```
commands grid_lp_batch;  
CPLEX 12.6.0.0: optimal solution; objective 2000  
4 dual simplex iterations (1 in phase I)  
x_1 = 0.2  
  
x_2 = 0.4  
  
x_3 = 0.4  
  
T = 2000  
  
1000*x_1 + 1250*x_2 + 2000*x_3 = 1500
```

- Note: All these files can be downloaded from the course web site

AMPL/CPLEX for solving example 1 (IDE version)

- At the AMPL prompt, type `commands grid_lp_batch;`
- Need to `reset;` before working on a new problem



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```
Console
AMPL
ampl: commands grid_lp_batch;
CPLEX 12.6.1.0: optimal solution; objective 2000
4 dual simplex iterations (1 in phase II)
x_1 = 0.2

x_2 = 0.4

x_3 = 0.4

T = 2000

ampl:
```

Acknowledgment

- Grid computing example based on Menascé and Casalicchio, “QoS in computing”, IEEE Internet Computing, pp. 85–87, Jul./Aug. 2004.

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