

# NUMERICAL OPTIMISATION

## ASSIGNMENT 6

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### Minimal Surface Cost Function

[Adapted from Exercise 7.7 from Nocedal-Wright]

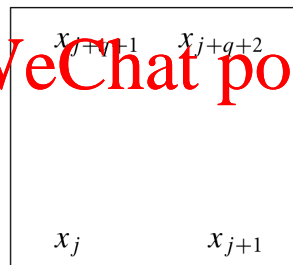
The minimum surface problem is a classical application of the calculus of variations and can be found in many textbooks. We wish to find the surface of minimum area, defined on the unit square, that interpolates a prescribed continuous function on the boundary of the square. In the standard discretization of this problem, the unknowns are the values of the sought-after function  $z(x, y)$  on a  $q \times q$  rectangular mesh of points over the unit square.

More specifically, we divide each edge of the square into  $q$  intervals of equal length, yielding  $(q + 1)^2$  grid points. We label the grid points as:

$$x_{(i-1)(q+1)+1}, \dots, x_{i(q+1)} \quad \text{for } i = 1, 2, \dots, q + 1,$$

so that each value of  $i$  generates a line. With each point, we associate a variable  $z_i$  that represents the height of the surface at this point. The values of the function are fixed by the boundary conditions at the  $4q$  grid points on the boundary. The optimization problem is to determine the other  $(q + 1)^2 - 4q$  variables  $z_i$  so that the total surface area is minimized.

A typical subsquare in this partition looks as follows:



We denote this square by  $A_j$  and note that its area is  $1/q^2$ . The desired function is  $z(x, y)$ , and we wish to compute its surface over  $A_j$ . Calculus books show that the area of the surface is given by:

$$f_j(\mathbf{x}) = \int \int_{(x,y) \in A_j} \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dx dy$$

Approximating the derivatives by finite differences,  $f_j$  has the form

$$f_j(\mathbf{x}) = \frac{1}{q^2} \left[ 1 + \frac{q^2}{2} [(z(x_j) - z(x_{j+q+2}))^2 + (z(x_{j+1}) - z(x_{j+q+1}))^2] \right]^{\frac{1}{2}}. \quad (1)$$

The function  $F$  in `surfaceFunc_vector.m` implements the sum of all  $f_j$  over the unit square. The gradient and the Hessian of the cost function  $F$  are provided in `surfaceGrad.m` and `surfaceHess.m`, respectively.

## EXERCISE 1

Implement the Line Search Newton - Conjugate Gradient (LS-Newton-CG) algorithm (Algorithm 7.1 from Nocedal-Wright). More help is provided in *Cody Coursework*.

*Submit your implementation via Cody Coursework.*

[30pt]

## EXERCISE 2

You are given an adaptation of the trust region SR-1 function in `trustRegionLS.m` and the 2D subspace in `solverCM2dSubspaceExtLS.m`. Point out and explain the relevant modifications in the solver `solverCM2dSubspaceExtLS.m`.

*Submit your solution via Turnitin.*

[20pt]

## EXERCISE 3

Compute and plot a minimal area surface function with given the boundary conditions:

- ★ Bottom edge:  $B_B(t) = \sin(\pi t), t \in [0, 1]$ .
- ★ Left edge:  $B_L(t) = \sin(\pi t + \pi), t \in [0, 1]$ .
- ★ Top edge:  $B_T(t) = \sin(3\pi t), t \in [0, 1]$ .
- ★ Right edge:  $B_R(t) = \sin(3\pi t + \pi), t \in [0, 1]$ .

(a) using the LS-Newton-CG algorithm,

(b) using the BFGS algorithm (provided),

(c) using the Trust Region-SR1 algorithm (provided in **Ex2**).

Discretization with  $q = 14$  yields a problem of reasonable size. Provide any parameters and explanations that you consider relevant in the minimisation.

*Submit your solution via Turnitin.*

[50pt]

**Remark.** The submission to Turnitin should not exceed 6 pages. Avoid submitting code unless explicitly asked for and focus on explaining your results.