NUMERICAL OPTIMISATION
ASSIGNMENT 7

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EXERCISE 1

Implement the Gauss-Newton method for solution of nonlinear least square problems. As Gauss-Newton is a line search method, it can be easiest implemented inside the function `descentLineSearch.m`. More help is provided in `Cody Coursework`.
Submit your implementation via `Cody Coursework`. **[20pt]**

EXERCISE 2 (optional)

You are given an implementation of the Levenberg-Marquardt method. This implementation plugs in the Levenberg-Marquardt solver `solverCMlevenberg.m` into the trust region function `trustRegion.m`. Briefly answer the following questions about the solver in `solverCMlevenberg.m`

(i) What is the effect of the `for` loop in line 36 and what is the reason for sequential execution?

(ii) Explain the formula for calculating the L-M direction p.

(iii) Explain what is q and how it is calculated.

(iv) Explain what is λ and the formula used for its update.

Submit your solution via `TurnitIn`. **[0pt]**

EXERCISE 3

Consider a model

$$\varphi(x_1, x_2, x_3; t) = (x_1 + x_2 t^2) \exp(-x_3 t)$$

with parameters (x_1, x_2, x_3). Such models are relevant e.g. in optics (photon counting).

Simulate the measurements sampling this model for a fixed choice of parameters $(x_1, x_2, x_3) = (3, 150, 2)$ at 200 equi-spaced points in $t_i \in (0, 4]$ and adding Gaussian noise $n(t_i) \sim \mathcal{N}(0, \sigma^2)$ drawn from a normal distribution with 0 mean and standard deviation 5% of the maximal amplitude of the sampled model signal $\sigma = 0.05 \max_{t_i} |\varphi(t_i)|$,

$$\tilde{\varphi}(x_1, x_2, x_3; t_j) = \varphi(x_1, x_2, x_3; t_j) + n(t_j).$$

(a) Formulate the least-squares problem for fitting the model φ and derive its Jacobian.
Submit your solution via `TurnitIn`. **[30pt]**

(b) Estimate the parameters (x_1, x_2, x_3) from your simulated measurements using

  (i) Gauss-Newton (implemented in Ex 1)

  (ii) Levenberg-Marquardt (provided)

  Specify all the relevant parameters and explain the results. Visualise the fit by plotting the estimated signal versus the measurements.
  Submit your solution via `TurnitIn`. **[50pt]**

Remark. The submission to `TurnitIn` should not exceed 4 pages. Avoid submitting code unless explicitly asked for and focus on explaining your results.