

# NUMERICAL OPTIMISATION

## TUTORIAL 9: NONSMOOTH OPTIMISATION

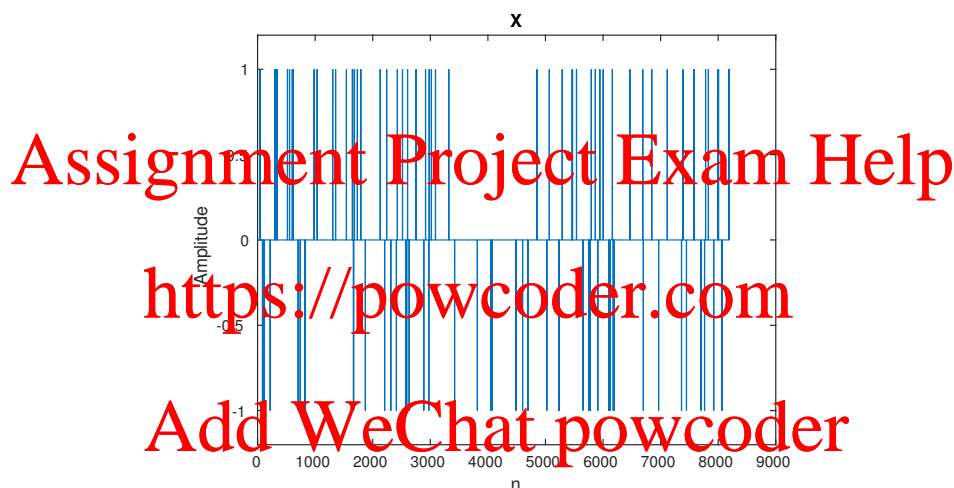
MARTA BETCKE  
KIKO RUL-LAN

### EXERCISE 1

Consider a *sparse* signal  $x = \{x_n\}_{n=1\dots N}$  of length  $N = 2^{13}$  consisting of:

- $T = 100$  randomly distributed spikes with values  $\{\pm 1\}$ .
- the remaining,  $N - T$ , values equal to 0.

A possible realisation of the signal  $x$  is:



Following the compressed sensing paradigm we take  $K = 2^{10}$  *compressed measurements* of the form

$$y_i = a_i^T x,$$

where  $a_i \in \mathcal{R}^N$  is an appropriately chosen sensing vector. Assuming that the measurement is corrupted with zero-mean Gaussian noise with standard deviation  $\sigma = 0.005$ , the measurements can be compactly written as

$$\tilde{y} = Ax + e,$$

where  $a_i^T$  is the  $i$ th row of the measurement matrix  $A \in \mathcal{R}^{K \times N}$  and  $e \in \mathcal{N}(0, \sigma)$  is the normally distributed noise vector.

We consider two different measurement types:

- $A$  is a orthogonal random matrix (use `randn()` and `orth()` to construct it)
- $A$  is a subsampled Welsh-Hadamard transform (the forward and inverse WH transform is available in Matlab: `fwht()`, `ifwht()`)

While the problem to recover  $x$  from  $\tilde{y}$  is underdetermined, under assumption of *sparsity* it is still possible to recover sparse signals from such incomplete measurements (under certain assumptions on  $A$  and sparsity of the signal vs. number of measurements). While forcing sparsity would lead to a

combinatorial problem, the  $L_1$  norm has been proven to provide a good relaxation resulting in the following compressed sensing recovery problem

$$x_{CS} = \arg \min_x \frac{1}{2} \|Ax - y\|_2^2 + \lambda \|x\|_1, \quad (\text{CS})$$

where  $\lambda$  is the regularisation parameter chosen depending on the properties of  $A$  and the standard deviation of the noise.

Implement and solve the compressed sensing recovery problem (CS) with

- ISTA
- FISTA
- ADMM

## EXERCISE 2

Repeat the same experiment in 2D. This is to demonstrate how to apply the same methods in image settings.

# Assignment Project Exam Help

## <https://powcoder.com>

## Add WeChat powcoder