Machine learning lecture slides

COMS 4771 Fall 2020

Classification III: Classification objectives

- Scoring functions
- Cost-sensitive classification
- Conditional probability estimation
- Reducing multi-class to binary
- Fairness in classification

Scoring functions in general

- Statistical model: $(X, Y) \sim P$ for distribution P over $\mathcal{X} \times \{-1, +1\}$
- Binary classifiers are generally of the form

$$x \mapsto \operatorname{sign}(h(x))$$

for some *scoring function* $h \colon \mathcal{X} \to \mathbb{R}$

  - E.g. Bayes classifier uses scoring function $h(x) = \eta(x) - 1/2$ where $\eta(x) = \Pr(Y = +1 \mid X = x)$
- Use with loss functions like $\ell_{0/1}$, ℓ_{logistic}, ℓ_{sq}, ℓ_{msq}, ℓ_{hinge}

$$\mathcal{R}(h) = \mathbb{E}[\ell(Y h(X))]$$

- Issues to consider:
  - Different types of mistakes have different costs
  - How to get $\Pr(Y = +1 \mid X = x)$ from $h(x)$?
  - More than two classes

Cost-sensitive classification

- Cost matrix for different kinds of mistakes (for $c \in [0,1]$)

	$\hat{y} = -1$	$\hat{y} = +1$
$y = -1$	0	c
$y = +1$	$1 - c$	0

(Why can we restrict attention to $c \in [0,1]$?)

- *Cost-sensitive ℓ-loss*:

$$\ell^{(c)}(y, \hat{y}) = \left(\mathbf{1}_{\{y=+1\}} \cdot (1-c) + \mathbf{1}_{\{y=-1\}} \cdot c\right) \cdot \ell(y\hat{y}).$$

  - If ℓ is convex in $\hat{y}$, then so is $\ell^{(c)}(y, \cdot)$.
  - *Cost-sensitive (empirical) risk*:

$$\mathcal{R}^{(c)}(h) := \mathbb{E}[\ell^{(c)}(Y, h(X))]$$

$$\widehat{\mathcal{R}}^{(c)}(h) := \frac{1}{n}\sum_{i=1}^{n} \ell^{(c)}(y_i, h(x_i))$$

- What is the analogue of Bayes classifier for cost-sensitive (zero-one loss) risk?
- Let $\eta(x) = \Pr(Y = 1 \mid X = x)$.
- Fix x; what is conditional cost-sensitive risk of predicting $\hat{y}$?

$$\eta(x) \cdot (1 - c) \cdot \mathbf{1}_{\{\hat{y}=-1\}} + (1 - \eta(x)) \cdot c \cdot \mathbf{1}_{\{\hat{y}=+1\}}.$$

- Minimized when

$$\hat{y} = \begin{cases} +1 & \text{if } \eta(x) \cdot (1 - c) > (1 - \eta(x)) \cdot c \\ -1 & \text{otherwise} \end{cases}$$

- So use scoring function $h(x) = \eta(x) - c$
  - Equivalently, use η as scoring function, but threshold at c instead of $1/2$
- Where does c come from?

- *Balanced error rate*: $\mathrm{BER} := \frac{1}{2}\mathrm{FNR} + \frac{1}{2}\mathrm{FPR}$
- Which cost sensitive risk to try to minimize?

$$
\begin{aligned}
&2 \cdot \mathrm{BER} \\
&= \Pr(h(X) \le 0 \mid Y = +1) + \Pr(h(X) > 0 \mid Y = -1) \\
&= \frac{1}{\pi} \cdot \Pr(h(X) \le 0 \wedge Y = +1) + \frac{1}{1-\pi} \cdot \Pr(h(X) > 0 \wedge Y = -1)
\end{aligned}
$$

where $\pi = \Pr(Y = +1)$.

- Therefore, we want to use the following cost matrix:

	$\hat{y} = -1$	$\hat{y} = +1$
$y = -1$	0	$\frac{1}{1-\pi}$
$y = +1$	$\frac{1}{\pi}$	0

- This corresponds to $c = \pi$.

Importance-weighted risk

- Perhaps the world tells you how important each example is
- Statistical model: $(X, Y, W) \sim P$
  - W is (non-negative) *importance weight* of example (X, Y)
- *Importance-weighted ℓ-risk* of h:

$$\mathbb{E}[W \cdot \ell(Y h(X))]$$

- Estimate from data $(x_1, y_1, w_1), \ldots, (x_n, y_n, w_n)$:

$$\frac{1}{n} \sum_{i=1}^{n} w_i \cdot \ell(y_i h(x_i))$$

Conditional probability estimation (1)

- How to get estimate of $\eta(x) = \Pr(Y = +1 \mid X = x)$?
- Useful if want to know expected cost of a prediction

$$\mathbb{E}[\ell_{0/1}^{(c)}(Yh(X)) \mid X = x] = \begin{cases} (1-c)\cdot\eta(x) & \text{if } h(x) \leq 0 \\ c\cdot(1-\eta(x)) & \text{if } h(x) > 0 \end{cases}$$

- Squared loss risk minimized by scoring function

$$h(x) = 2\eta(x) - 1.$$

  Therefore, given h, can estimate η using $\hat{\eta}(x) = \frac{1+h(x)}{2}$
- Recipe:
  - Find scoring function h that (approximately) minimizes (empirical) squared loss risk
  - Construct conditional probability estimate $\hat{\eta}$ using above formula

Conditional probability estimation (2)

- Similar strategy available for logistic loss
- But not for hinge loss!
  - Hinge loss risk is minimized by $h(x) = \text{sign}(2\eta(x) - 1)$
  - Cannot recover η from h
- Caveat: If using insufficiently expressive functions for h (e.g., linear functions), may be far from minimizing squared loss risk
  - Fix: use more flexible models (e.g., feature expansion)

Application: Reducing multi-class to binary

- Multi-class: Conditional probability function is vector-valued function

$$\eta(x) = \begin{bmatrix} \Pr(Y = 1 \mid X = x) \\ \vdots \\ \Pr(Y = K \mid X = x) \end{bmatrix}$$

- Reduction: learn K scalar-valued functions, the k-th function is supposed to approximate

$$\eta_k(x) = \Pr(Y = k \mid X = x).$$

  - This can be done by create K binary classification problems, where in problem k, label is $\mathbf{1}_{\{y=k\}}$.
- Given the K learned conditional probability functions $\hat{\eta}_1, \ldots, \hat{\eta}_K$, we form a final predictor $\hat{f}$

$$\hat{f}(x) = \operatorname*{arg\,max}_{k=1,\ldots,K} \hat{\eta}_k(x).$$

- If learned conditional probability functions $\hat{\eta}_k$ are accurate, then behavior of one-against-all classifier $\hat{f}$ is similar to optimal classifier

$$f^\star(x) = \arg\max_{k=1,\dots,K} \Pr(Y = k \mid X = x).$$

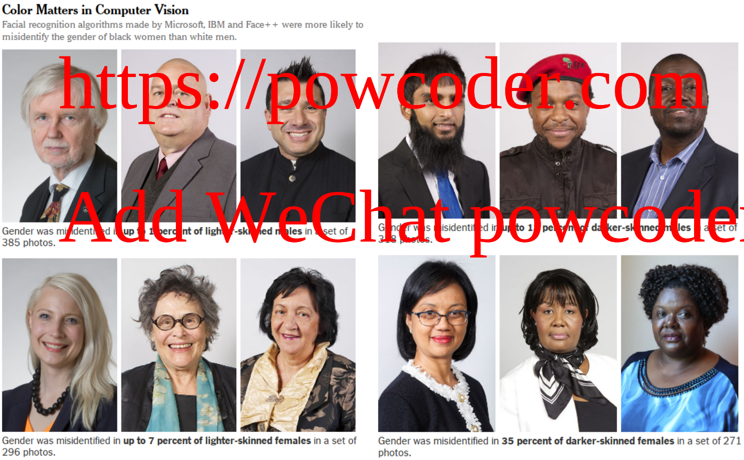
- **Claim**:

$$\operatorname{err}(\hat{f}) \leq \operatorname{err}(f^\star) + 2 \cdot \mathbb{E}[\max_k |\hat{\eta}_k(X) - \eta_k(X)|].$$

Fairness

- Use of predictive models (e.g., in admissions, hiring, criminal justice) has raised concerns about whether they offer "fair treatment" to individuals and/or groups
  - We will focus on *group-based fairness*
  - *Individual-based fairness* also important, but not as well-studied

- Often predictive models work better for some groups than for others
  - Example: face recognition (Buolamwini and Gebru, 2018; Lohr, 2018)



Possible causes of unfairness

- People deliberately being unfair
- Disparity in number of available training data for different groups
- Disparity in usefulness of available features for different groups
- Disparity in relevance of prediction problem for different groups
- ...

ProPublica study

- ProPublica (investigative journalism group) studied a particular predictive model being used to determine "pre-trial detention"
  - Angwin et al., 2016
  - Judge needs to decide whether or not an arrested defendant should be released while awaiting trial
  - Predictive model ("COMPAS") provides an estimate of $\Pr(Y = 1 \mid X = x)$ where $Y = \mathbb{1}_{\{\text{will commit (violent) crime if released}\}}$ and X is "features" of defendant.
- Study argued that COMPAS treated black defendants unfairly in a certain sense
  - What sense? How do they make this argument?

Fairness criteria

- Setup:
  - X: features for individual
  - A: group membership attribute (e.g., race, sex, age, religion)
  - Y: outcome variable to predict (e.g., "will repay loan", "will re-offend")
  - $\hat{Y}$: prediction of outcome variable (as function of (X, A))
  - For simplicity, assume A, Y, and $\hat{Y}$ are $\{0, 1\}$-valued
- Many fairness criteria are based on joint distribution of

$$(A, Y, \hat{Y})$$

- Caveat: Often, we don't have access to Y in training data

- Fairness criterion: *Classification parity*

$$\Pr(\hat{Y} = 1 \mid A = 0) \approx \Pr(\hat{Y} = 1 \mid A = 1)$$

- Sounds reasonable, but easy to satisfy with perverse methods
- Example: trying to predict $Y = \mathbf{1}_{\{\text{will repay loan if given one}\}}$
- Suppose conditional distributions of $(Y, \hat{Y})$ given A are as follows:

$(A = 0)$	$\hat{Y} = 0$	$\hat{Y} = 1$
$Y = 0$	$1/2$	0
$Y = 1$	0	$1/2$

$(A = 1)$	$\hat{Y} = 0$	$\hat{Y} = 1$
$Y = 0$	$1/4$	$1/4$
$Y = 1$	$1/4$	$1/4$

- For $A = 0$ people, correctly give loans to people who will repay
- For $A = 1$ people, give loans randomly $(\mathrm{Bernoulli}(1/2))$
- Satisfies criterion, but bad for $A = 1$ people

Equalized odds (1)

- Fairness criterion: *Equalized odds*

$$\Pr(\hat{Y} = 1 \mid Y = y, A = 0) \approx \Pr(\hat{Y} = 1 \mid Y = y, A = 1)$$

for both $y \in \{0, 1\}$.

- In particular, FPR and FNR must be (approximately) same across groups.
  - Could also just ask for *Equalized FPR*, or *Equalized FNR*

- Previous example fails to satisfy equalized odds:

$(A = 0)$	$\hat{Y} = 0$	$\hat{Y} = 1$
$Y = 0$	$1/2$	0
$Y = 1$	0	$1/2$

$(A = 1)$	$\hat{Y} = 0$	$\hat{Y} = 1$
$Y = 0$	$1/4$	$1/4$
$Y = 1$	$1/4$	$1/4$

E.g., $A = 0$ group has 0% FPR, while $A = 1$ has 50% FPR.

- Criteria imply constraints on the classifier / scoring function
  - Can try to enforce constraint during training

Equalized odds (2)

- ProPublica study:
  - Found that FPR for $A = 0$ group (black defendants; 45%) was higher than FPR for $A = 0$ group (white defendants; 23%)

$(A = 0)$	$\hat{Y} = 0$	$\hat{Y} = 1$
$Y = 0$	0.27	0.22
$Y = 1$	0.14	0.37

$(A = 1)$	$\hat{Y} = 0$	$\hat{Y} = 1$
$Y = 0$	0.46	0.14
$Y = 1$	0.19	0.21