COSC1107 Computing Theory

(We will commence soon. We are just allowing a few minutes for people to join and set up. *Please mute your microphone unless you are speaking.* You can raise your hand or use the chat at any time.)

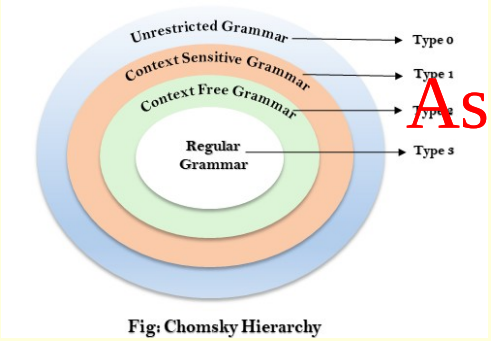
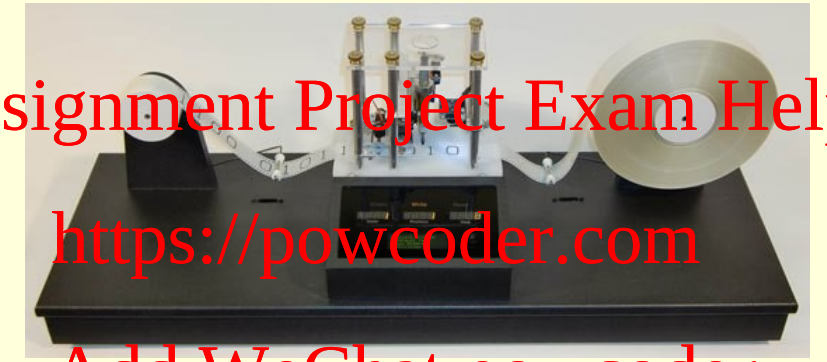


Fig: Chomsky Hierarchy



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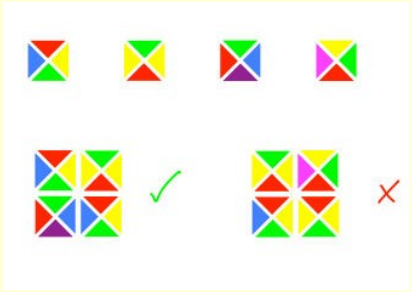

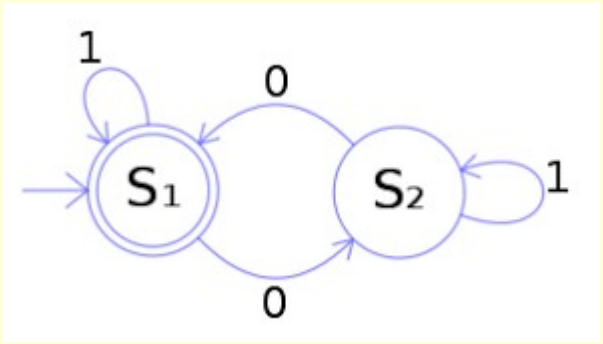

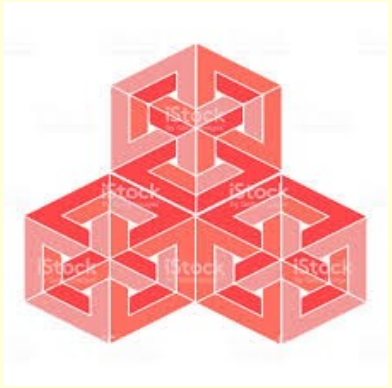

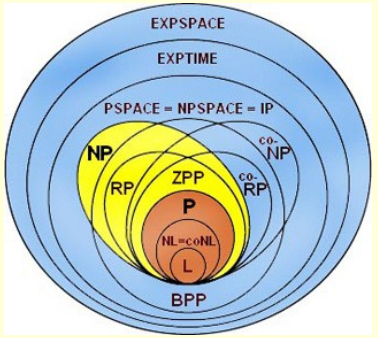

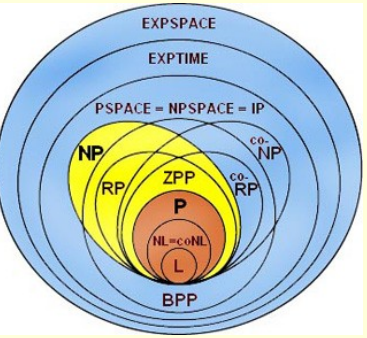

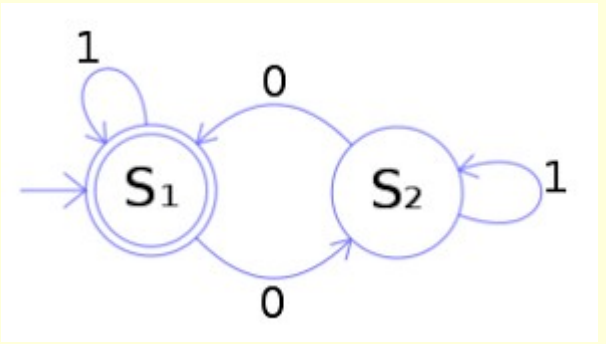

COSC1107

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NFAs and DFAs

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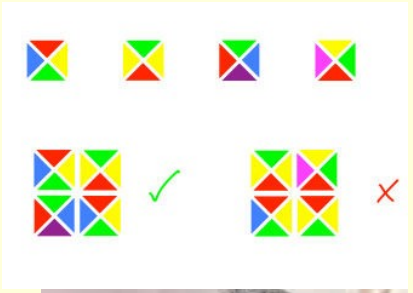
Week 6



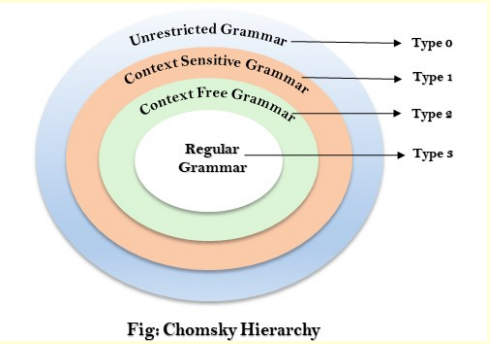


Fig: Chomsky Hierarchy

James Harland

james.harland@rmit.edu.au

* With thanks to Sebastian Sardina

Intro music 'Far Over' playing now …

Acknowledgement

RMIT University acknowledges the people of the Woi wurrung and Boon wurrung language groups of the eastern Kulin Nations on whose unceded lands we conduct the business of the University. RMIT University respectfully acknowledges their Ancestors and Elders, past and present.

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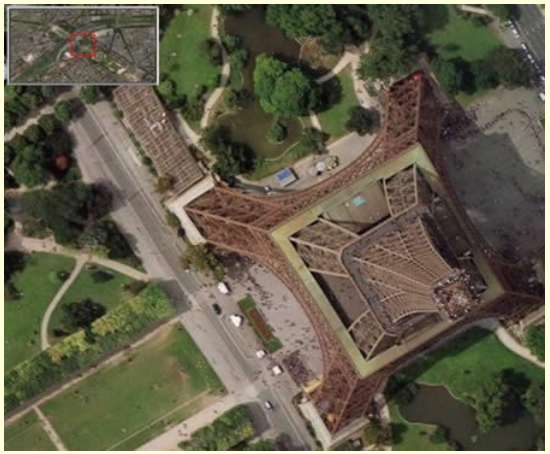
RMIT also acknowledges the Traditional Custodians and their Ancestors of the lands and waters across Australia where we conduct our business.

(add your name here to volunteer for this or email me)

Overview



- Questions?
- Converting NFAs to DFAs
- Questions?
- Pumping Lemma
- Questions?
- Platypus Game
- Questions?

What **can** be done

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What **can't** be done

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Of course!

Weekly Schedule

	Lecture/Lectorial	Tutorial	Assessment
1	Formal languages, grammars	Motivations & Mathematical preliminaries	
2	Finite State Machines	Grammars	Quiz 1
3	Pushdown Automata, nondeterminism	NFAs and DFAs	Quiz 2
4	Turing machines	Pushdown automata	Quiz 3
5	Computability, universality	Turing machines	Quiz 4
6	Pumping Lemma, NFA->DFA conversion	Computability, universality	Assignment 1, Quiz 5
7	Chomsky Hierarchy	Nondeterminism, Pumping Lemma	Quiz 6
8	Unrestricted grammars		Quiz 7
9	Complexity and intractability	Unrestricted grammars	Quiz 8
10	Zero-knowledge proofs	Complexity and intractability	Quiz 9
11	Closure properties, problem reduction	Zero-knowledge proofs	Quiz 10
12	Research and requests	Closure properties, problem reduction	Assignment 2
14-16	Week 6	--	Final exercise

Foundations

Relationships

Analysis

Assessment

Computing Theory

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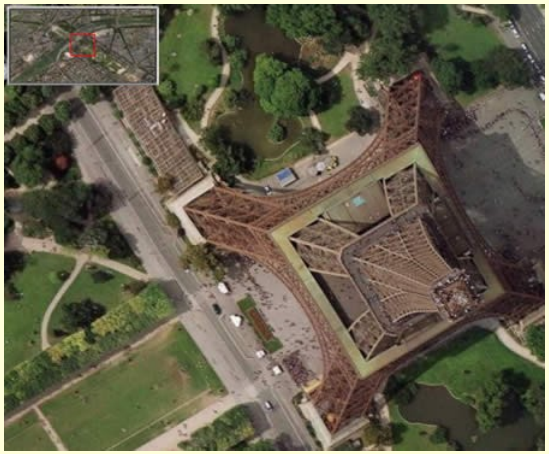
Mid-semester break



- No classes 29th August to 2nd September
- No deadlines in this time!
- RMIT encourages all to 'Take a Break' this week!
- Be kind to yourself and turn off for the week if you can

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Week 7

"We want you in NZ for one day"

"Which day?"

"You're kidding!" ☺

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Professor Sebastian Sardina

Monday 5th September only

Questions?

Questions?

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Questions?

Chomsky Hierarchy

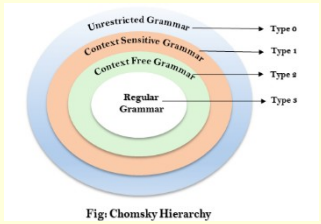

Fig: Chomsky Hierarchy

Automata	Languages	Grammars
	Undecidable languages	---
Turing Machines	Recursively enumerable languages	Unrestricted grammars
Linear Bounded Automata	Context-sensitive languages	Context-sensitive grammars
(Nondeterministic) Pushdown Automata	Context-free languages	Context-free grammars
Deterministic Pushdown Automata	?? (Deterministic CF?)	???
Nondeterministic Finite Automata & Deterministic Finite Automata	Regular languages	Regular grammars & regular expressions

Week 8

Week 7

Week 6

Week 9

Computing Theory

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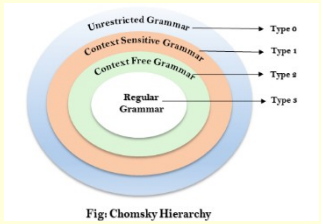

DFAs

What **can** DFAs do?

- Everything that NFAs can
- For every NFA there is an equivalent DFA
- DFA may be exponentially larger …
- There is an equivalent DFA
- For any regular grammar
- For any regular expression

Algorithm to convert an NFA to DFA

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What **can't** DFAs do?

- Recognise context-free languages
- Recognise context-sensitive languages
- Recognise recursive languages

Property of DFAs (Pumping Lemma) used to derive a contradiction

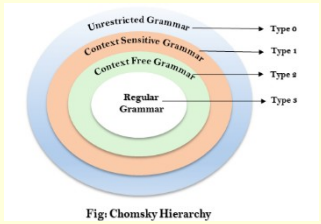

Fig: Chomsky Hierarchy

DFAs vs NFAs

DFAs

- Simpler to implement
- Harder to combine
- Simpler to show properties
- Simpler to show 'negative' results
- More 'machine-friendly'

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Define exactly the same languages (!!)

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NFAs

- Simpler to specify
- Trivial to combine
- Harder to analyse
- Simpler to show 'negative' results
- More 'human-friendly'

Quiz time!

Go to Canvas and find the quiz Lectorial 6 Question set

- Not worth any marks
- You can consult other students if you wish
- Time limit will be 10 minutes

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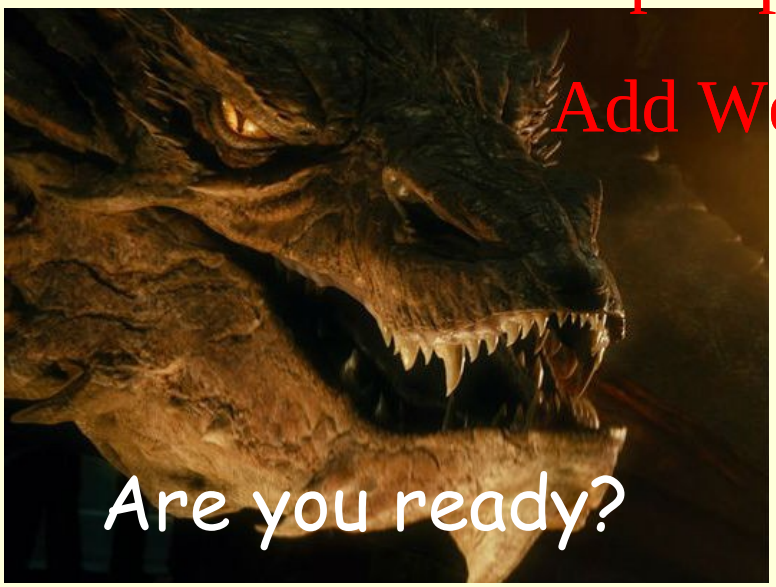

Go!

The pictures will take 10 minutes to disappear!

Thomas music means 1 minute left!

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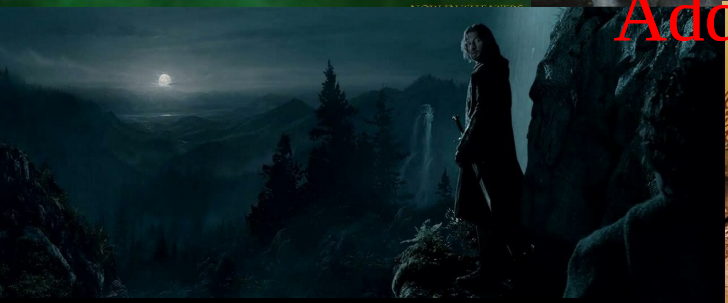

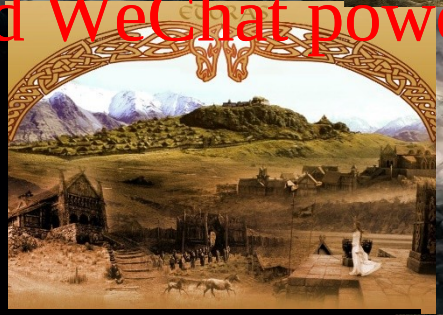

Questions?

Questions?

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Questions?

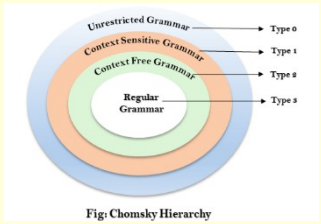

Fig: Chomsky Hierarchy

NFAs

Two key issues

- May contain transitions
- May contain more than one transition for a given state and input

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Two solutions

- Compute the -closure
  - Consider an input abc as
- Compute the possible sets of states for the NFA
  - These will be the states in the DFA (!!)

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States of the DFA are the sets of possible states in the NFA

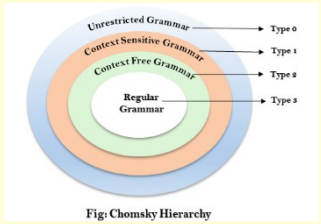

Fig: Chomsky Hierarchy

How did you go?

Question 1 Consider the NFA M below. Construct a DFA which accepts the same language as M.

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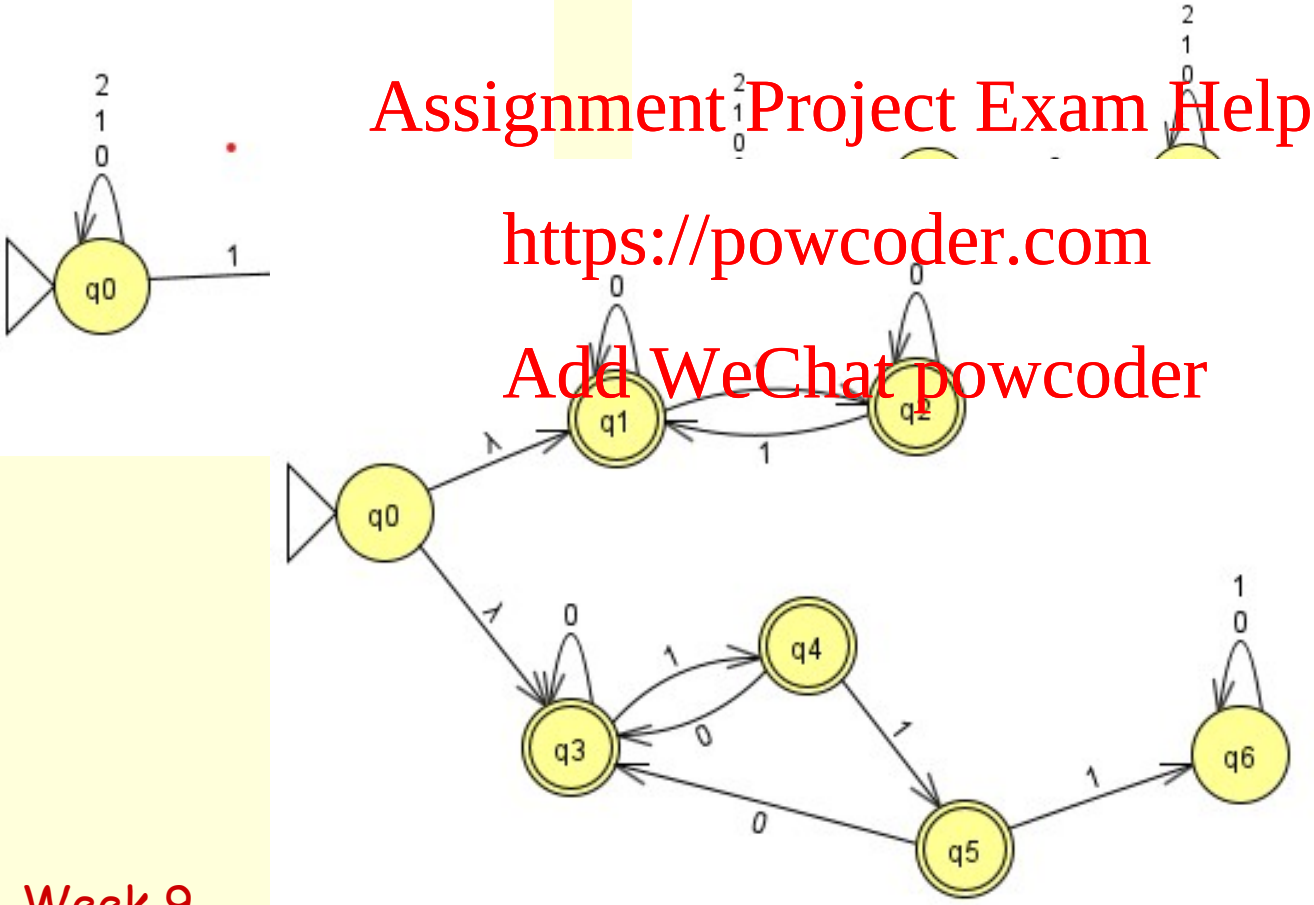
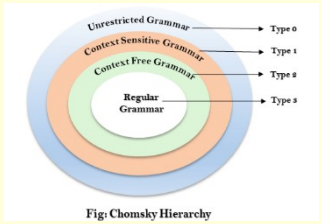
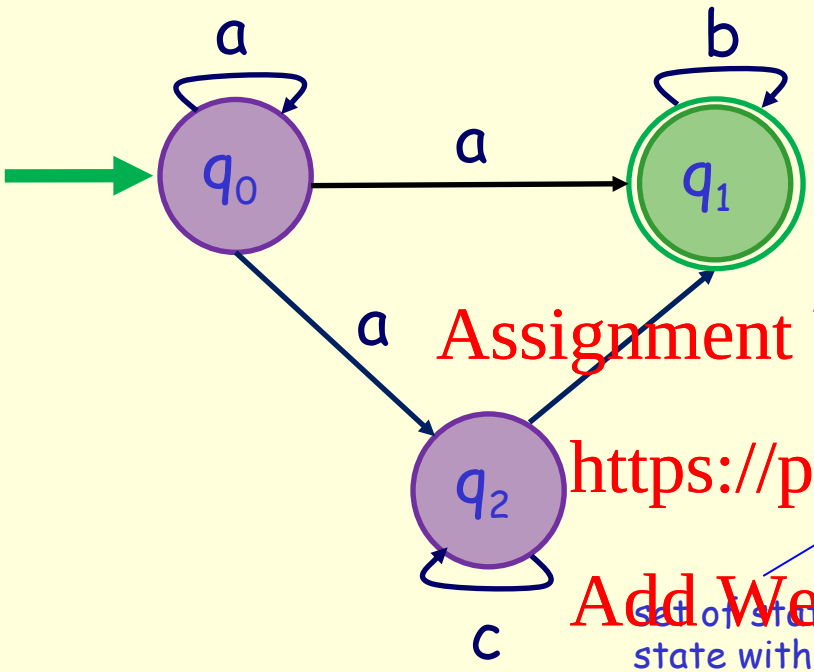




NFAs



$L(M) = a^+c^*b^*$

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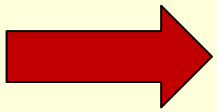
	a	b	c	
q_0	$\{q_0, q_1, q_2\}$			
q_1		$\{q_1\}$		
q_2			$\{q_2\}$	$\{q_1\}$

set of states reachable from this state with this input

"fill in" gaps in transitions

DFA construction uses this table



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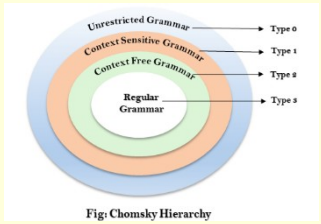
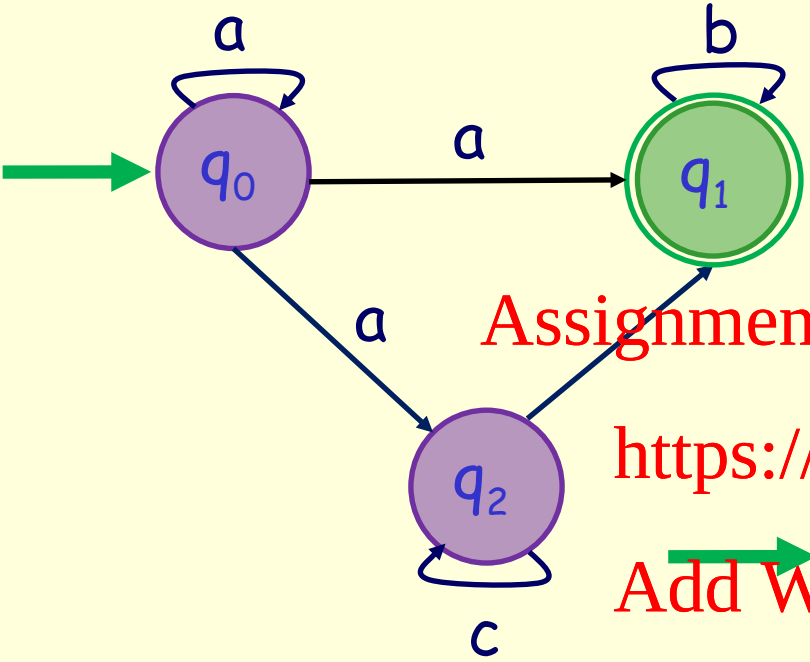
NFAs





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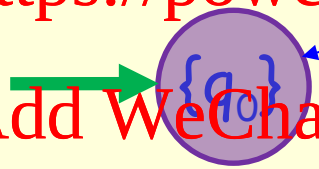

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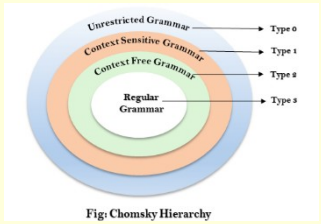

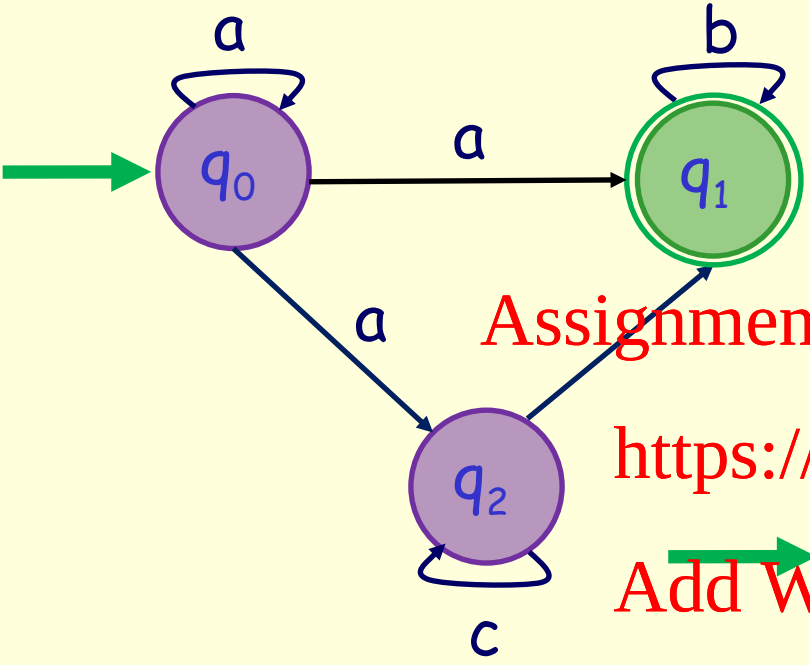

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q_1		$\{q_1\}$	
q_2			$\{q_1, q_2\}$

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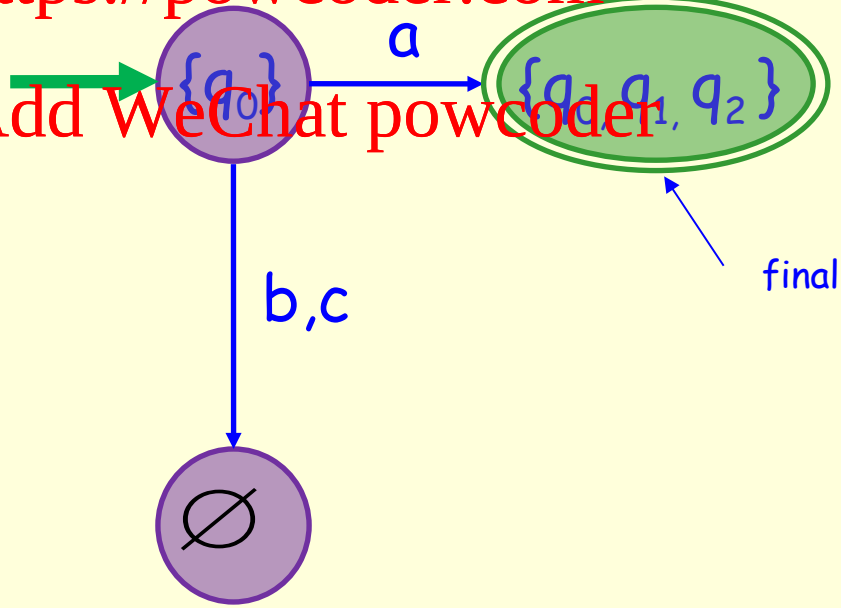
possible states after input a



final because q_1 is final

NFAs

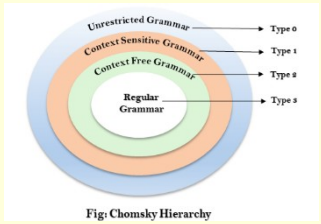
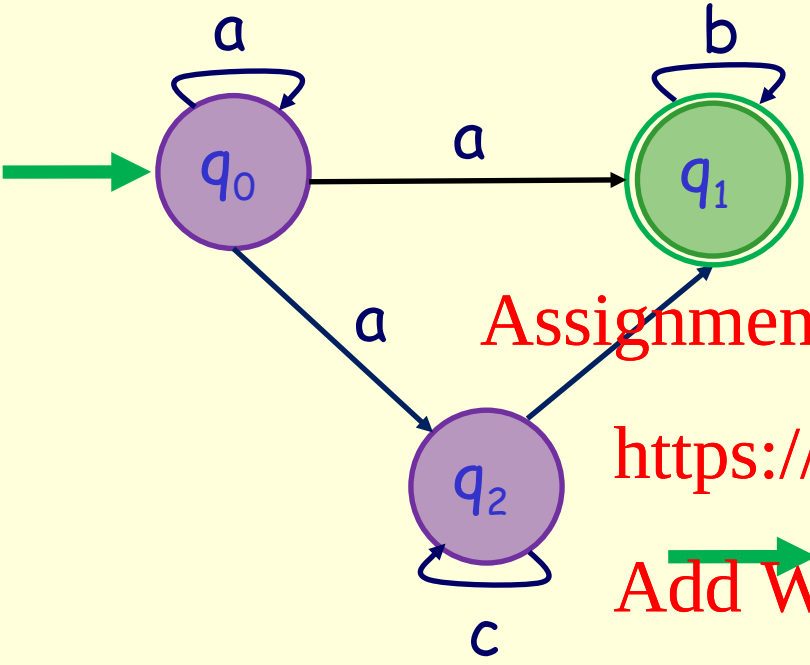


Fig: Chomsky Hierarchy



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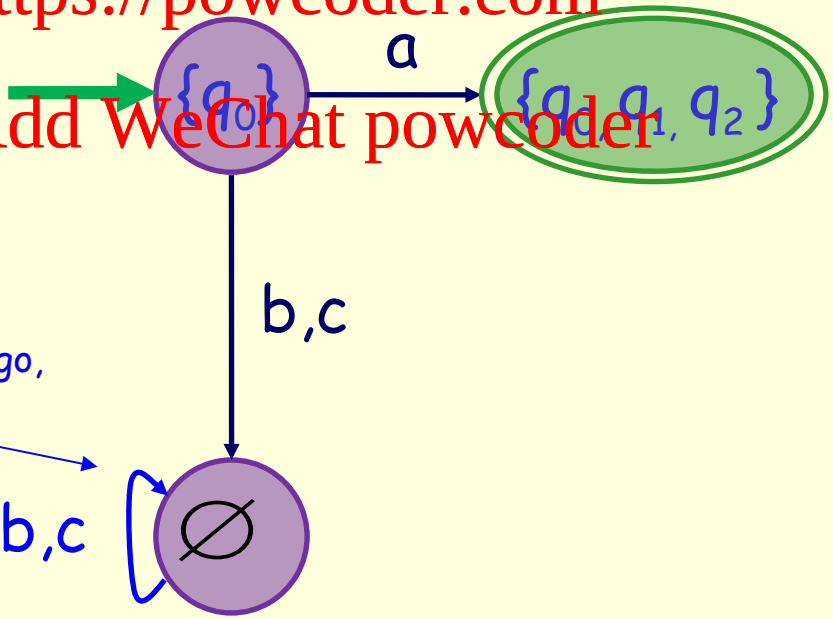

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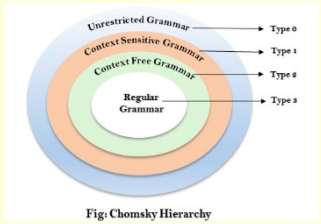

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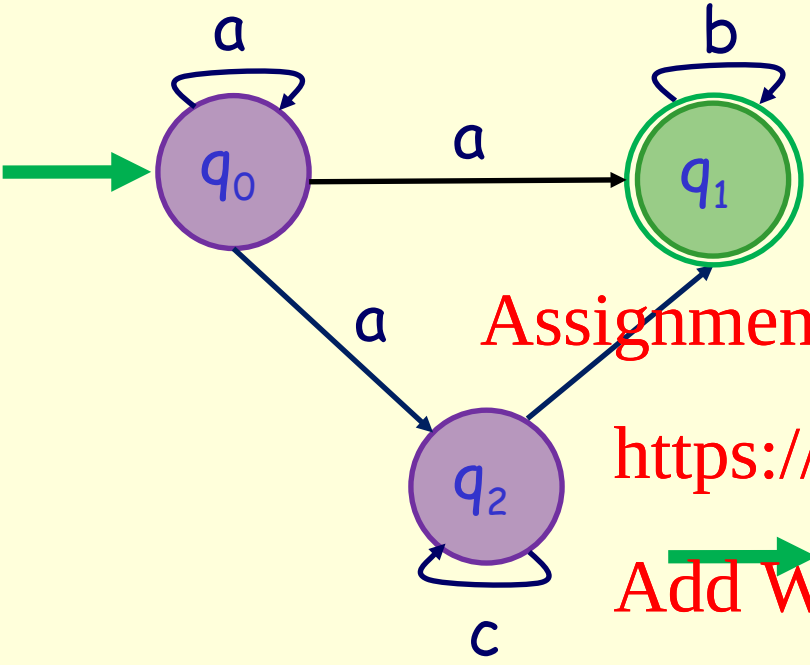

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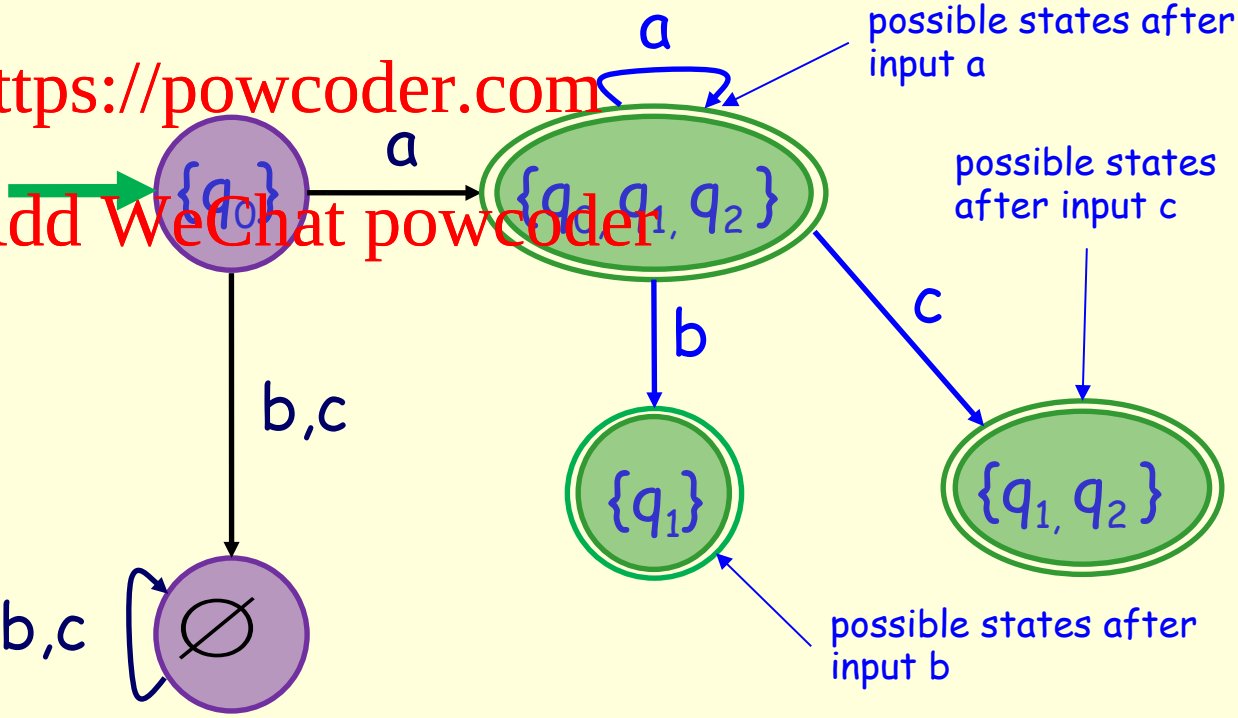

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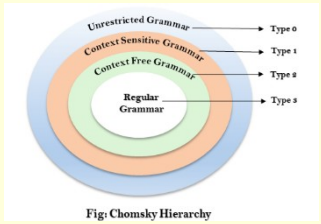

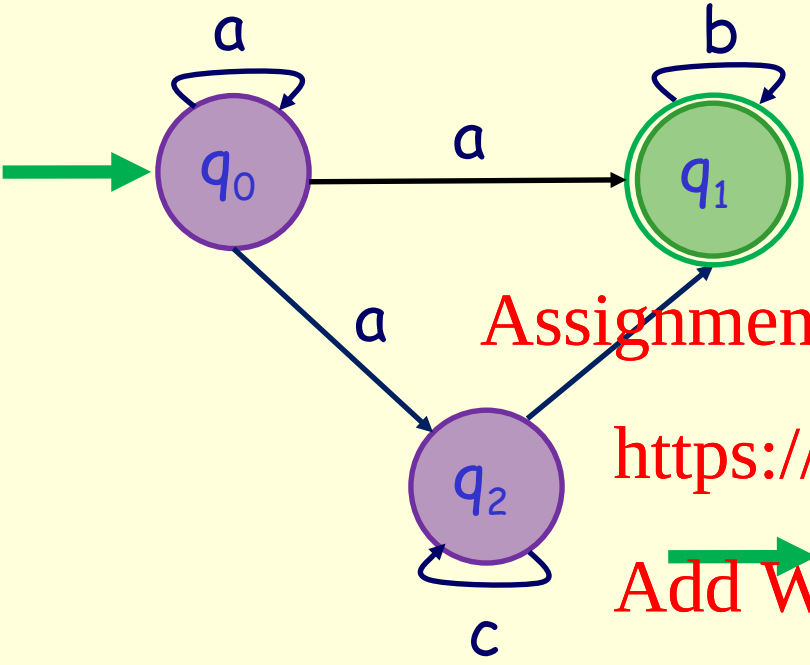

	a	b	c
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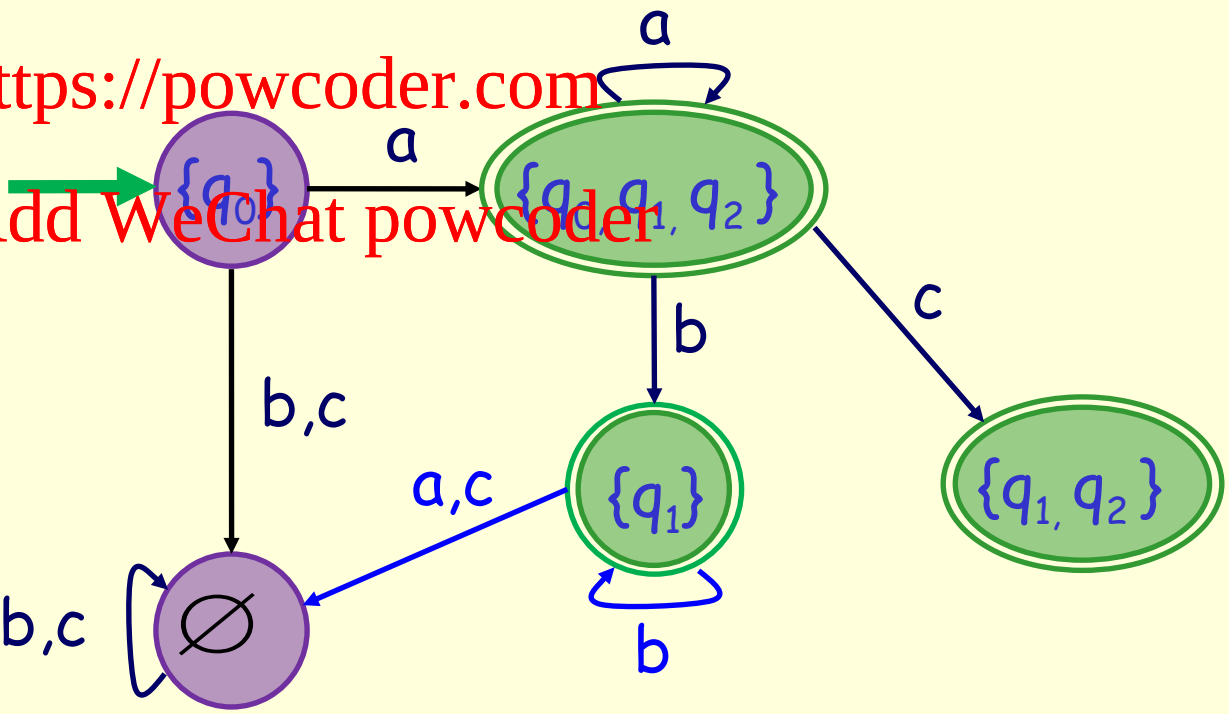
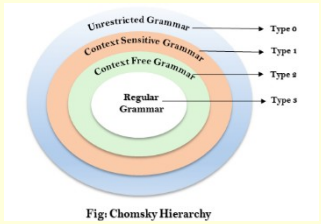
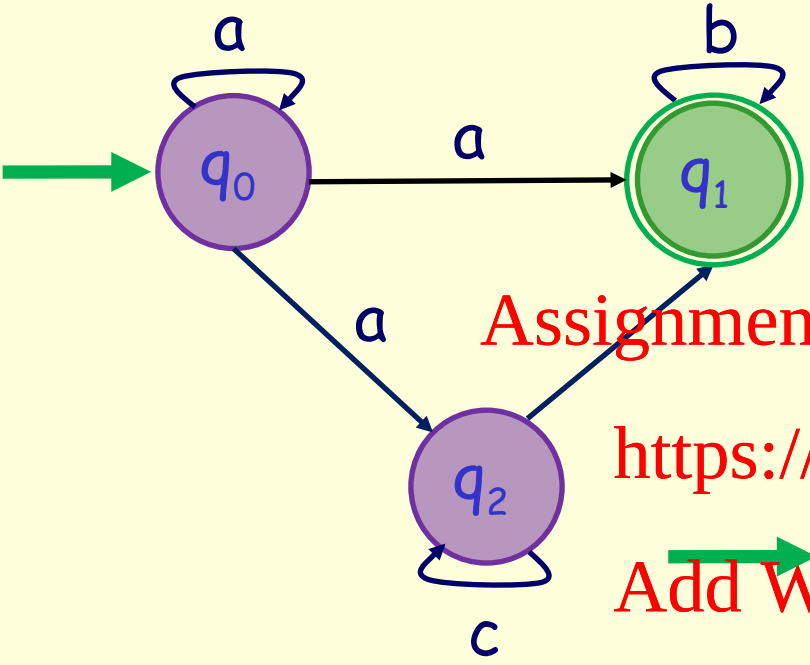


NFAs





	a	b	c
q_0	$\{q_0, q_1, q_2\}$		
q_1		$\{q_1\}$	
q_2			$\{q_1, q_2\}$

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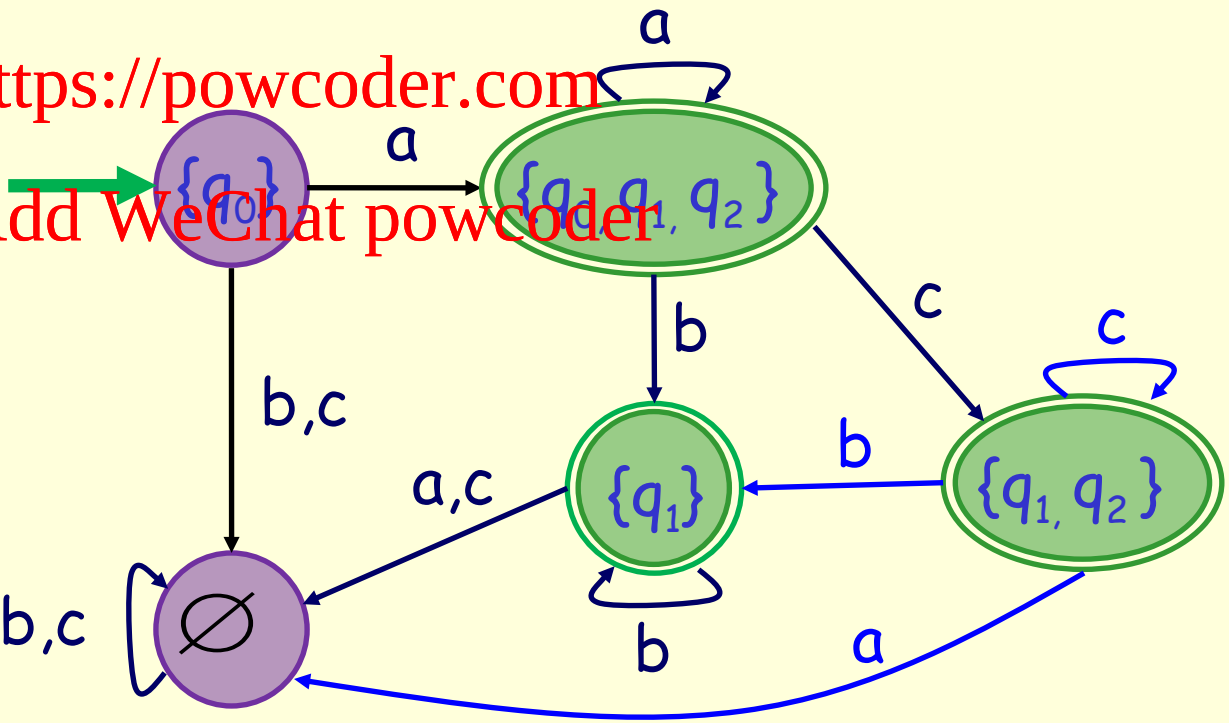
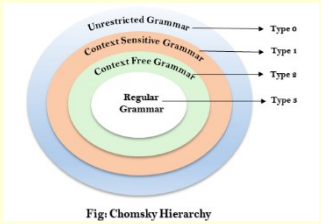
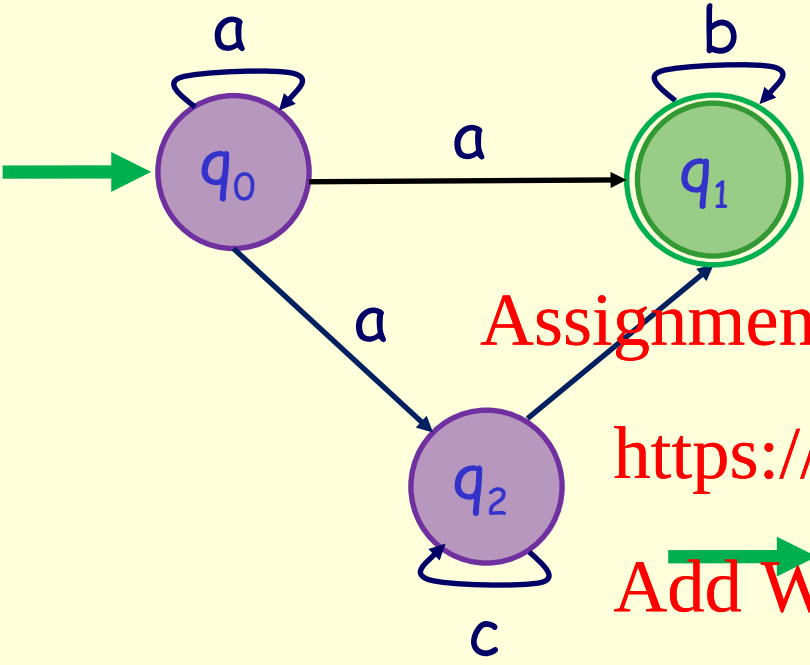


NFAs





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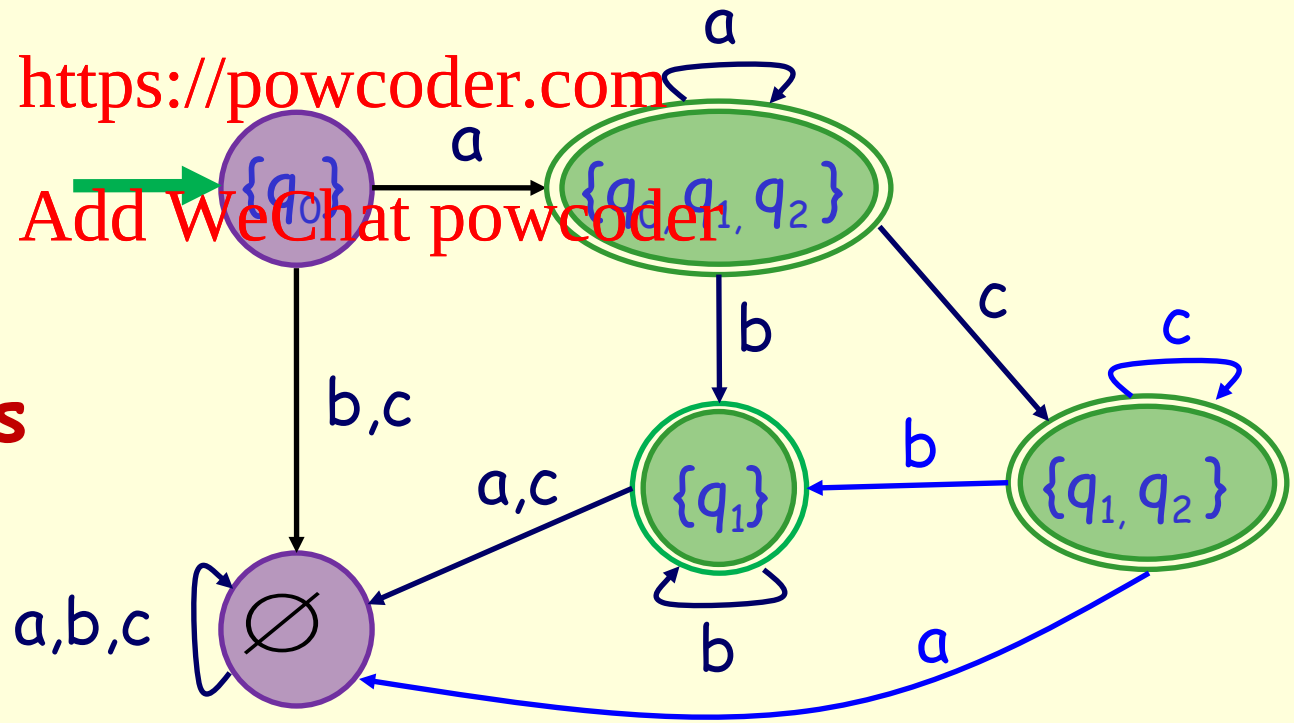

DONE! This is the DFA

Questions?

Questions?

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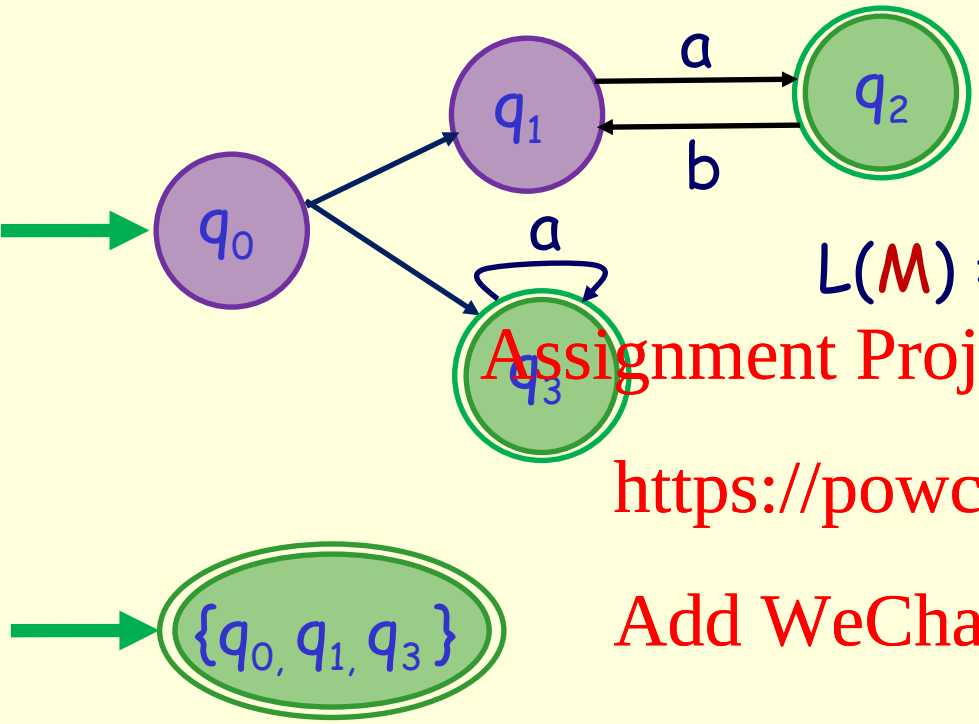
Questions?

Another NFA



$L(M) = a(ba)^* \mid a^*$

	a	b
q_0	$\{q_2, q_3\}$	
q_1	$\{q_2\}$	
q_2		
q_3	$\{q_3\}$	

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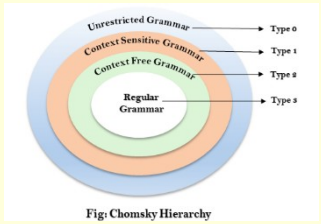

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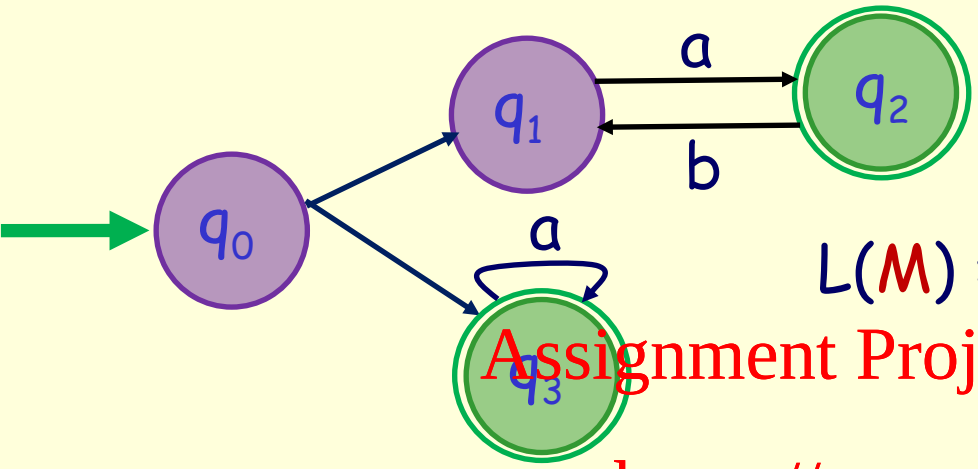

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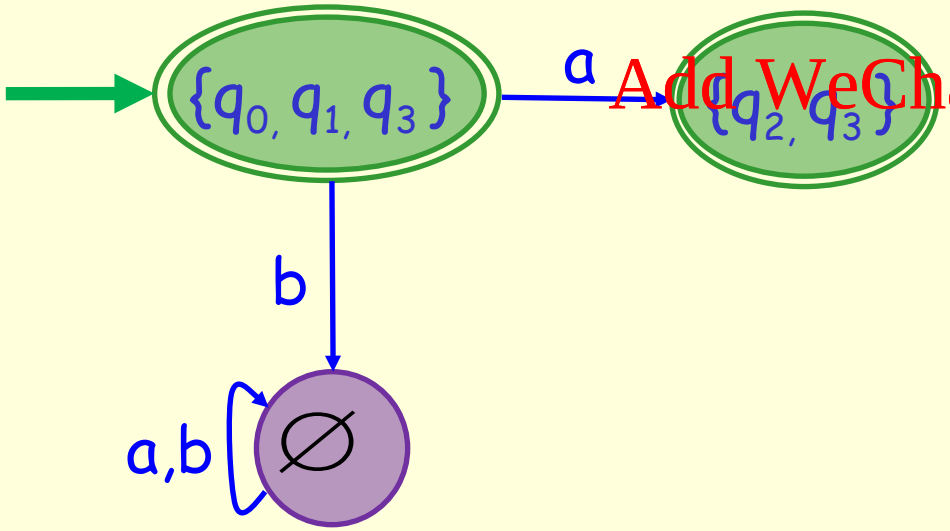

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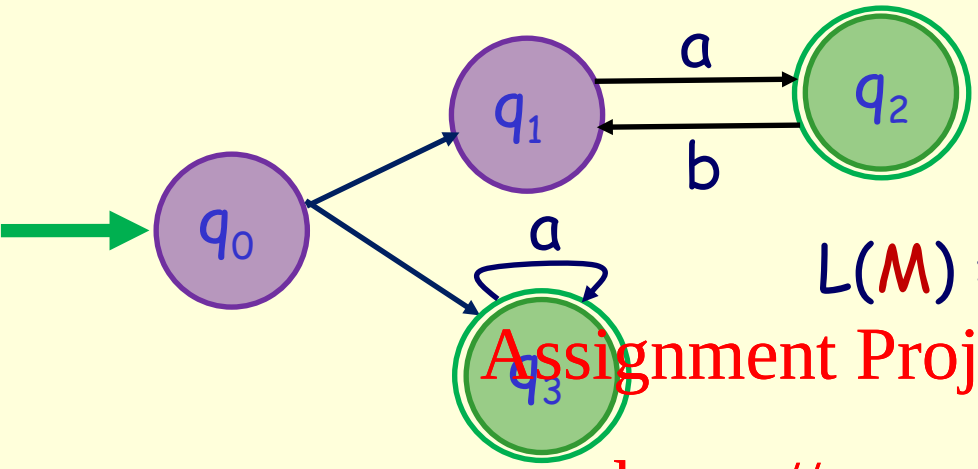

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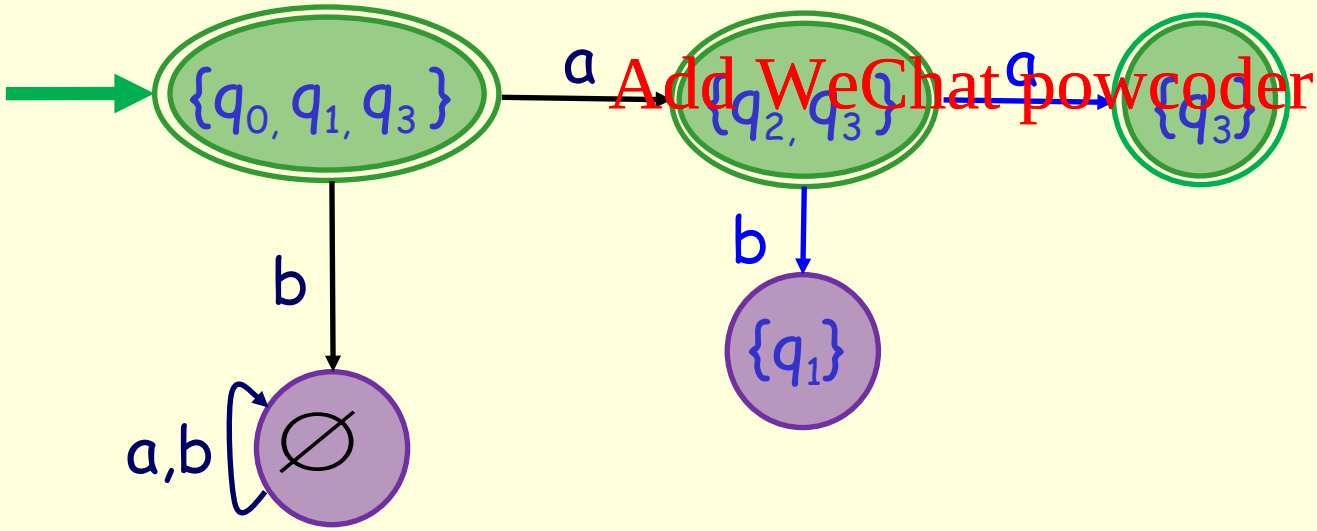

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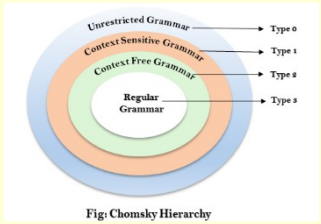

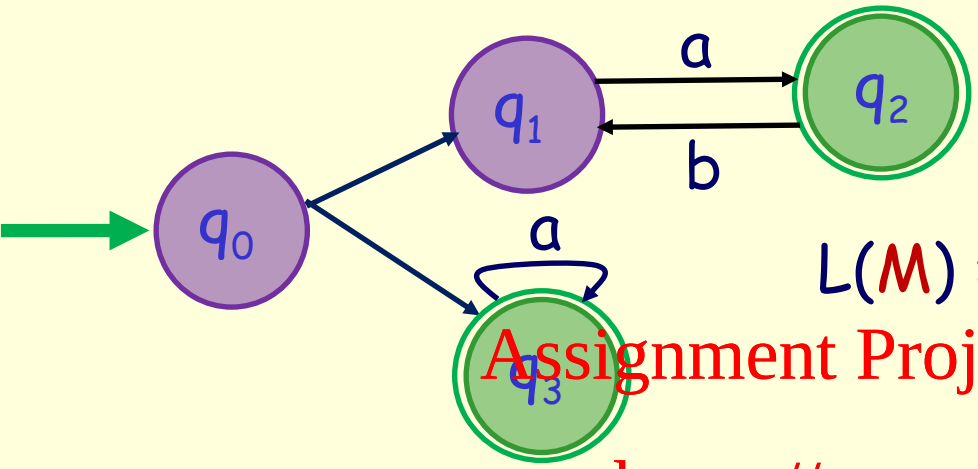

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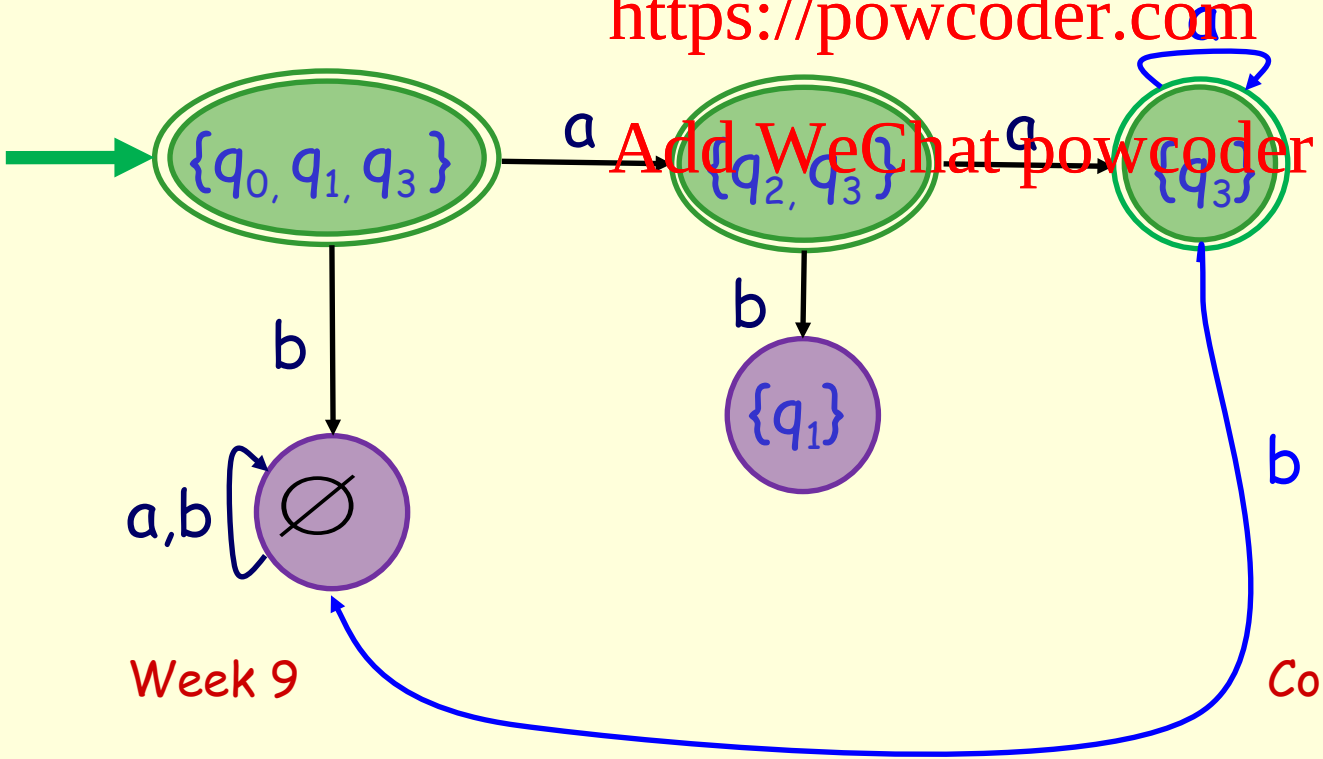

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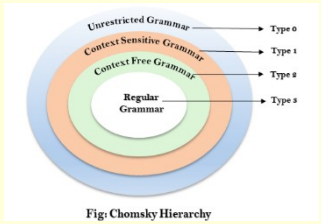

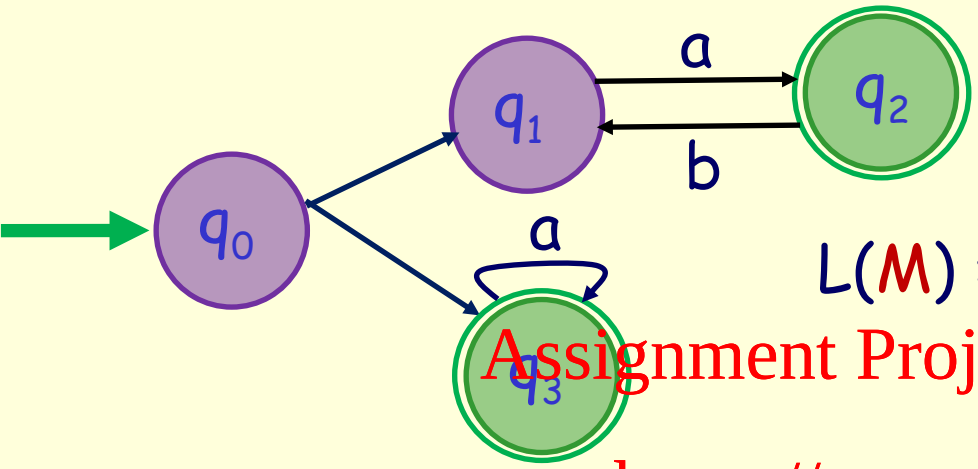

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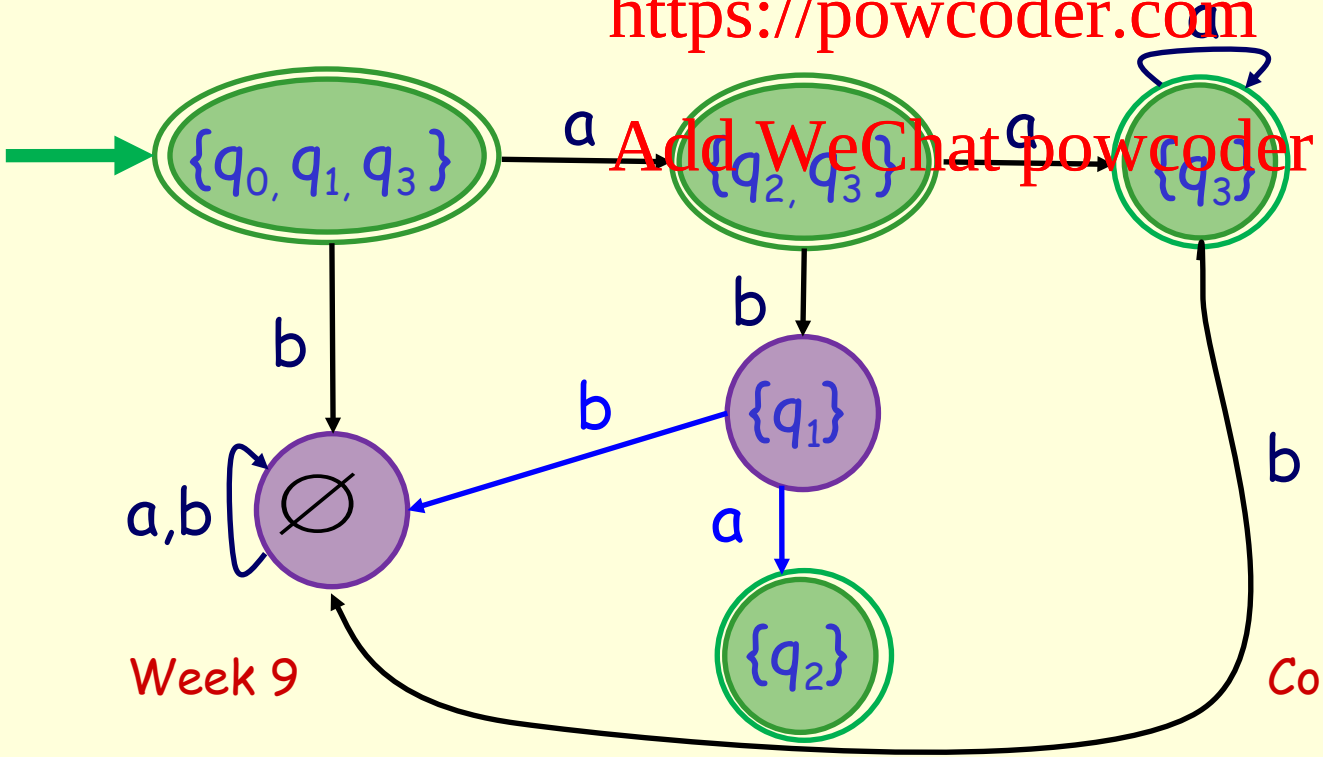

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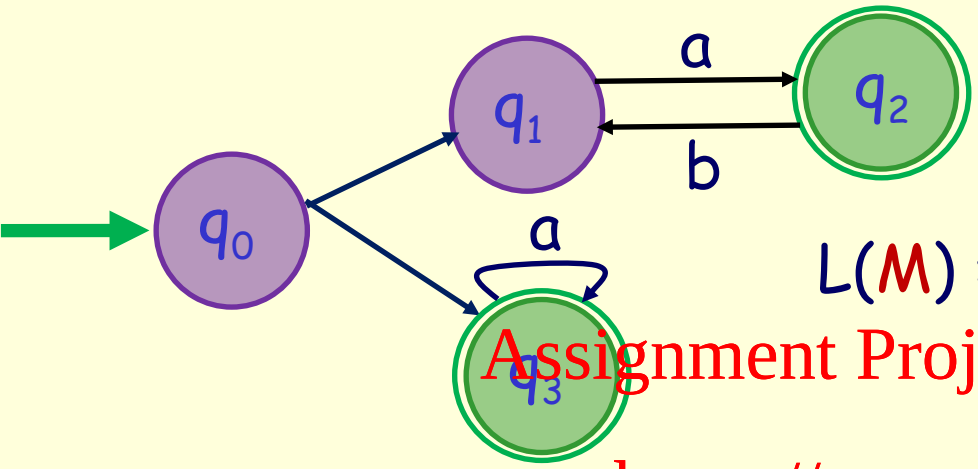

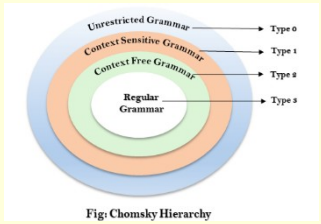

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q_2		
q_3	$\{q_3\}$	

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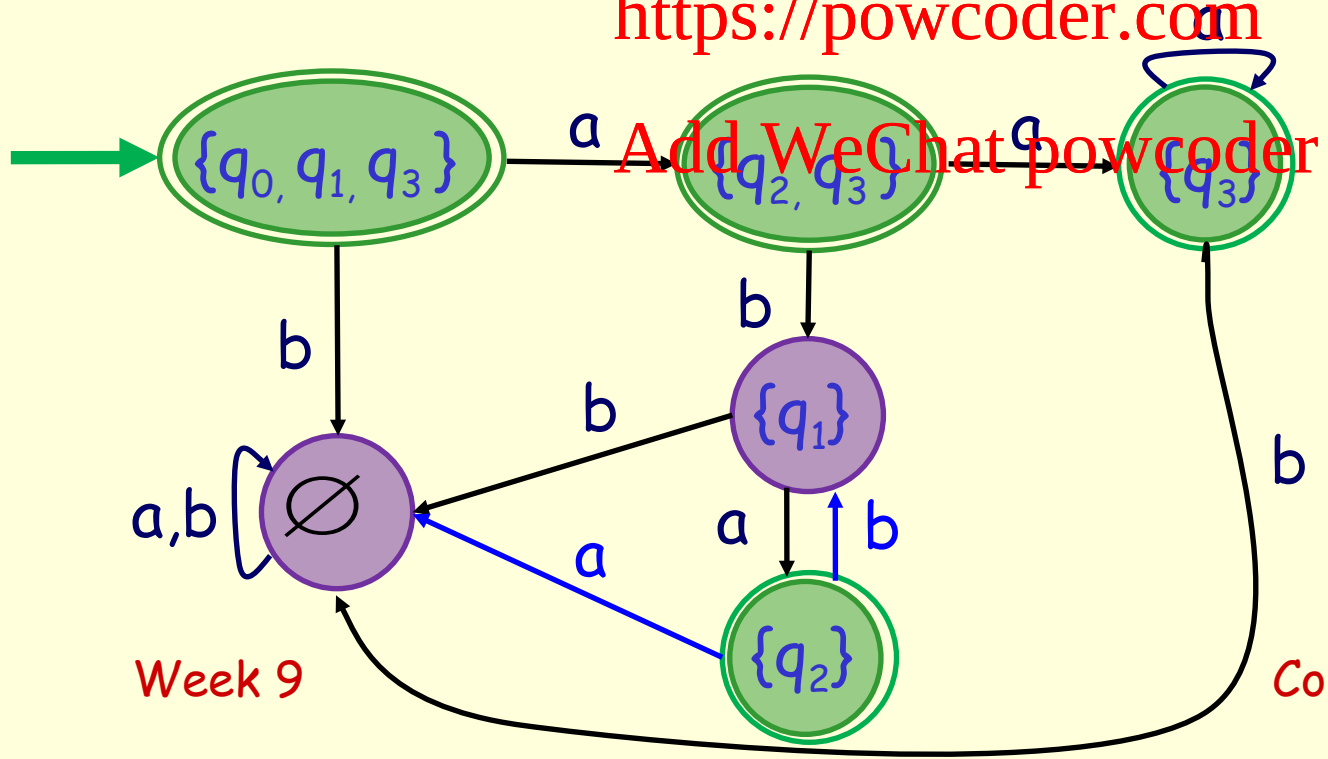
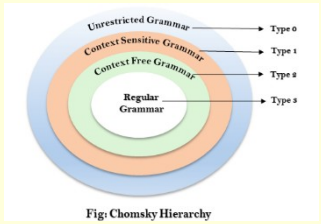


DONE! This is the DFA



NFA to DFA

1. Start with -closure for each state
2. Generate t table
3. Initial state of DFA is -closure of initial state in NFA
4. Construct DFA
   - Keep adding states until all transitions are to existing states

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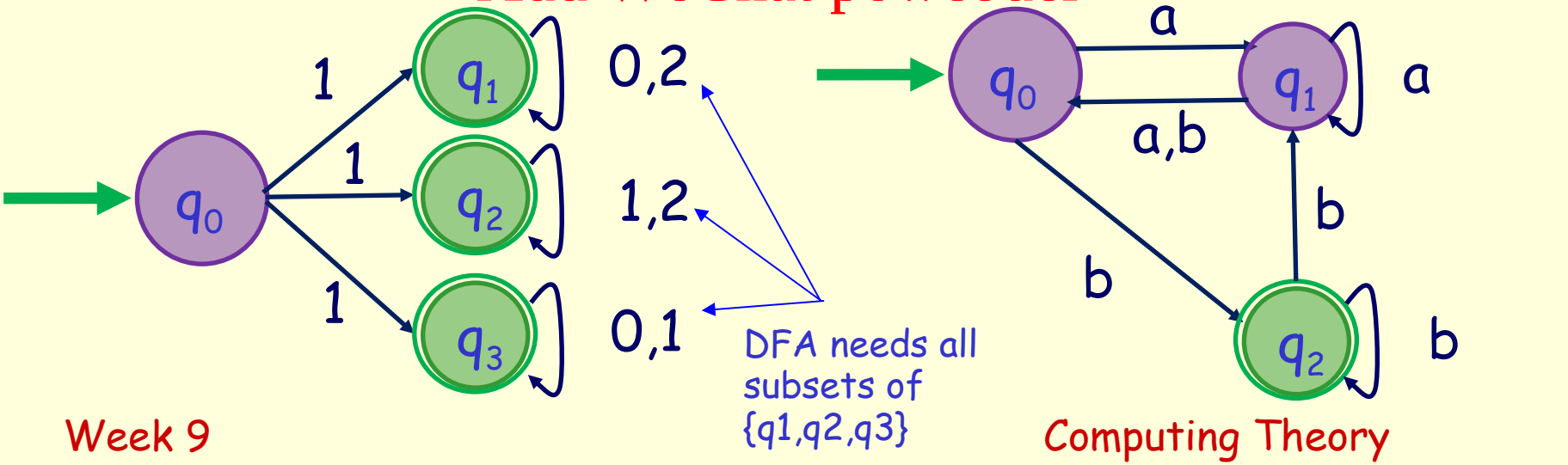
DFA may be exponentially larger (!!)
n states in NFA up to 2^n states in DFA



Questions?

Questions?

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Questions?

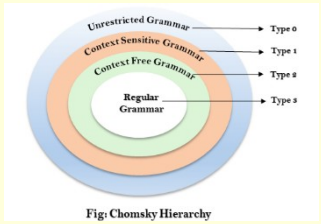

Fig: Chomsky Hierarchy

DFAs

What **can** DFAs do?

- Everything that NFAs can
- For every NFA there is an equivalent DFA
- DFA may be exponentially larger …
- There is an equivalent DFA
- For any regular grammar
- For any regular expression

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What **can't** DFAs do?

- Recognise context-free languages
- Recognise context-sensitive languages
- Recognise recursive languages

Limitation of DFAs

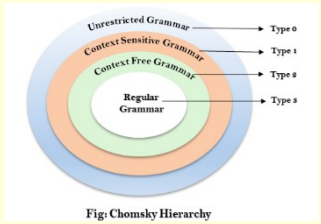

Fig: Chomsky Hierarchy

Type	Memory Size	Memory access
DFAs	Bounded*	Defined by machine
PDAs	Unbounded	Top of stack only
(Linear Bounded Automata)	*?? (see Week 8)*	*?? (see Week 8)*
Turing Machines	Infinite	Random (ie unlimited)

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* bounded means the number is fixed and known in advance

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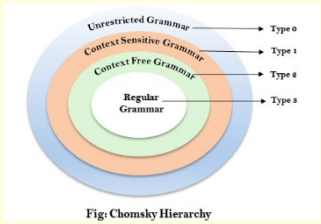
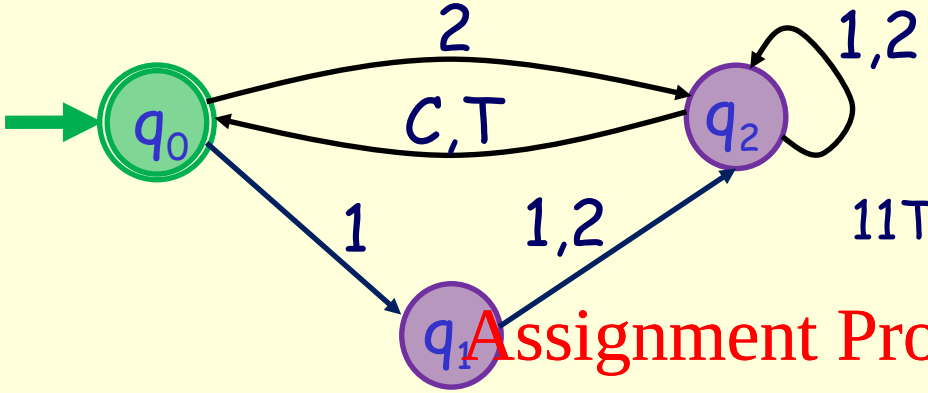
What happens when a DFA processes a string longer than the number of states?

"Now you are getting some

Some states must be repeated!

Limitation of DFAs





2C11T: $q_0\ q_2\ q_0\ q_1\ q_2\ q_0$

11T11C2C2T: $q_0\ q_1\ q_2\ q_0\ q_1\ q_2\ q_0\ q_2\ q_0\ q_2\ q_0$

cycle cycle cycle cycle

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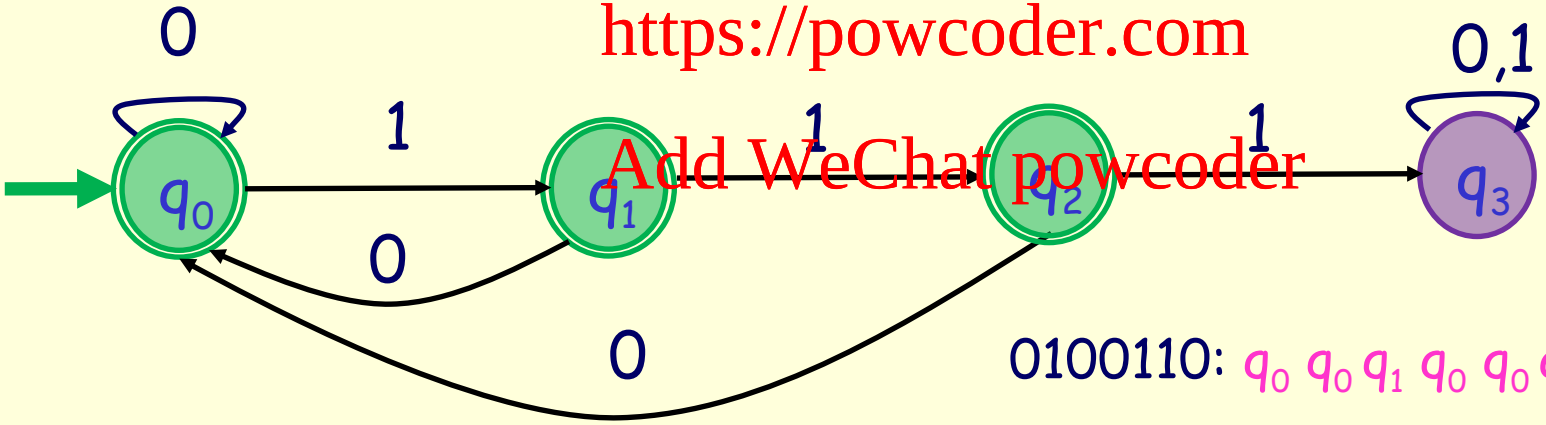
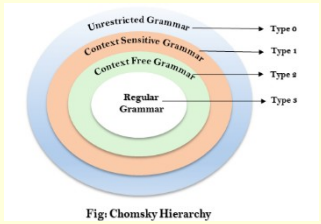


0100110: $q_0\ q_0\ q_1\ q_0\ q_0\ q_1\ q_2\ q_0$

0000010: $q_0\ q_0\ q_0\ q_0\ q_0\ q_0\ q_1\ q_0$

For any string w L(M) with |w| ≥ #states in M, there is a cycle



Limitation of DFAs

Let L be any regular language
Then there is a DFA M such that L = L(M)

Let the number of states in M be n

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Consider w L(M) with $|w| \geq n$, so $w = a_1 a_2 a_3 \ldots a_n a_{n+1} \ldots a_{n+k}$

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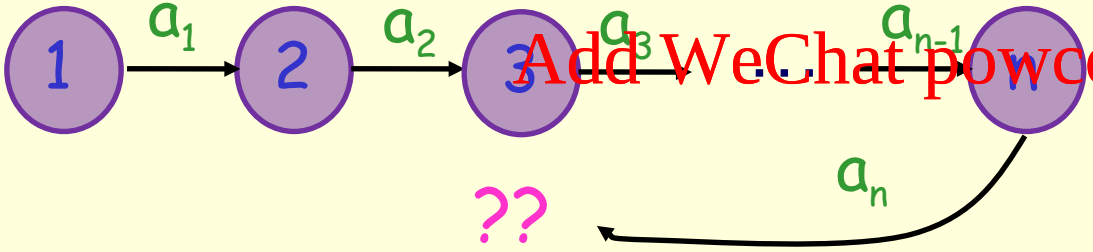

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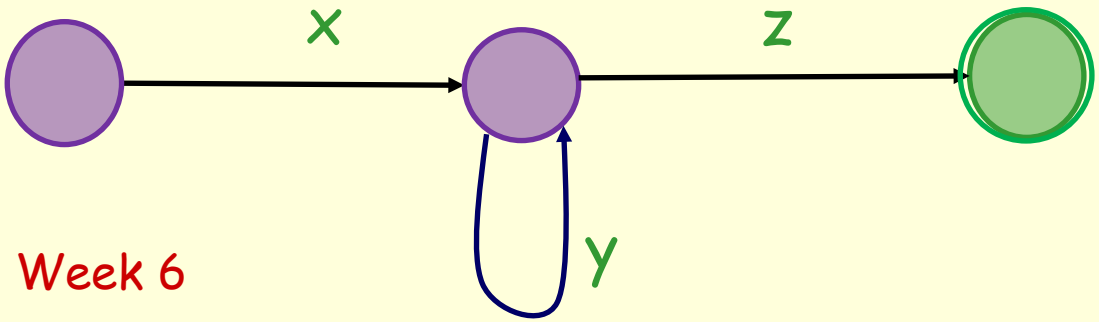
Must repeat a state somewhere …



$w = xyz, y$

$|xy| \leq n$

Questions?

Questions?

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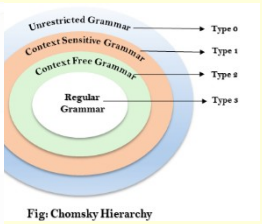
Questions?

Pumping Lemma



For any regular language L, there is $n \geq 1$ such that for any $w \in L$ with $|w| \geq n$, there exist x, y, z such that $w = xyz$ and

1. $|xy| \leq n$
2. $y \neq \varepsilon$
3. $x y^i z \in L$ for all $i \geq 0$

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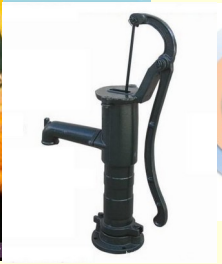
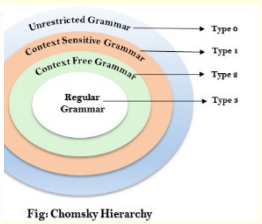
So for any L, for some n find $w = xyz \in L$ with $|w| \geq n$

- If L is regular, then $xz, xyz, xyyz, xyyyz, xyyyyz, \ldots$ are all in L
- If $xy^jz \notin L$ for some $j \geq 0$, then L is not regular

Pumping Lemma





Usual use is to show languages not regular by contradiction

1. Assume L is regular
2. Apply Pumping Lemma
3. Choose string w ∈ L
4. Use |xy| ≤ n to get information about y
5. Choose i such that xyⁱz ∉ L (i = 2 usually works)
6. Contradiction!

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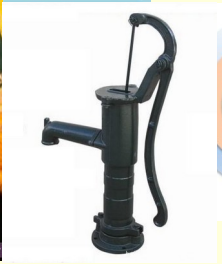
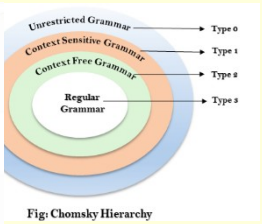
Conclude that L is not regular

All such proofs the same except steps 3 & 5

Pumping Lemma





The language $L = \{a^i b^i \mid i \geq 0\}$ is not regular

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Proof: Assume L is regular. Then the Pumping Lemma applies and there is an $n \geq 1$ such that for all $w \in L$ such that $|w| \geq n$, $w = xyz$

1. $|xy| \leq n$
2. y
3. $x y^i z \in L$ for all $i \geq 0$

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Choose $w = a^n b^n$ and so $w \in L$ and $|w| \geq n$. So by the Pumping Lemma, $w = xyz = a^n b^n$ and $|xy| \leq n$. So $y = a^j$ for some $1 \leq j \leq n$.

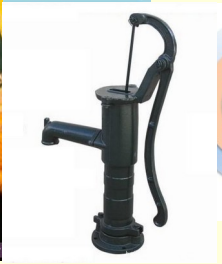
First n characters of xy are a's

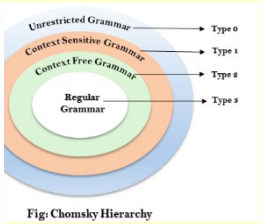
Choose $i = 2$ and consider $xyyz = a^{n+j} b^n$

By the Pumping Lemma, $a^{n+j} b^n \in L$ (!!)

Hence L is not regular.

Pumping Lemma





The language $L = \{ xx^R \mid x \in \{a,b\}^* \}$ is not regular

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Proof: Assume L is regular. Then the Pumping Lemma applies and there is an $n \geq 1$ such that for all $w \in L$ such that $|w| \geq n$, $w = xyz$

1. $|xy| \leq n$
2. y
3. $x y^i z \in L$ for all $i \geq 0$

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Choose $w = a^n bba^n$ and so $w \in L$ and $|w| \geq n$. So by the Pumping Lemma, $w = xyz = a^n bba^n$ and $|xy| \leq n$. So $y = a^j$ for some $1 \leq j \leq n$.

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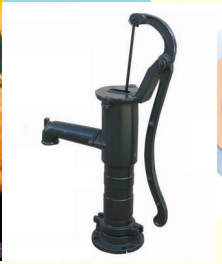
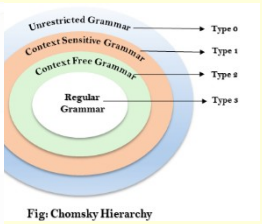
Hence L is not regular.

Pumping Lemma

The language $L = \{a^i \mid i \text{ is prime}\}$ is not regular

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Proof: Assume L is regular. Then the Pumping Lemma applies and there is an $n \geq 1$ such that for all $w \in L$ such that $|w| \geq n$, $w = xyz$

1. $|xy| \leq n$
2. y
3. $x y^i z \in L$ for all $i \geq 0$

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Choose $w = a^m$ where m is prime and $m > n$. So $w \in L$ and $|w| \geq n$. So by the Pumping Lemma, $w = xyz = a^m$ and $|xy| \leq n$.

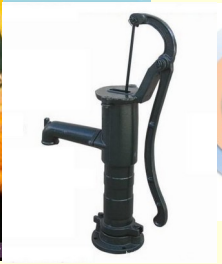
This means $|xyz| = m$

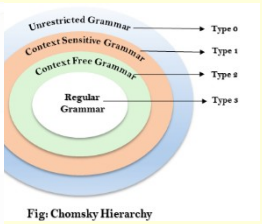
Choose $i = m+1$ and consider $|xy^{m+1}z| = |xyz| + m|y| = m + m|y| = m(|y|+1)$

By the Pumping Lemma, $a^{m(|y|+1)} \in L$ but $m(|y|+1)$ is not prime (!!)

Hence L is not regular.

Pumping Lemma





The language L = { ??? } is not regular

Proof: Assume L is regular. Then the Pumping Lemma applies and there is an n ≥ 1 such that for all w ∈ L such that |w| ≥ n, w = xyz

1. |xy| ≤ n
2. y
3. $x y^i z$ ∈ L for all i ≥ 0

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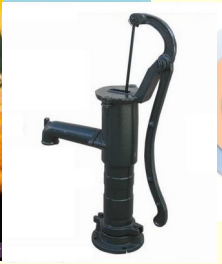
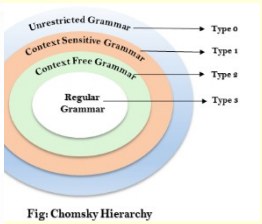
Choose w = ??? and so w ∈ L and |w| ≥ n. So by the Pumping Lemma, w = xyz = ???? and |xy| ≤ n. So y = ????

Choose i = ??? (try 2 first) and consider xy^iz = ????
By the Pumping Lemma, ??????? ∉ L (!!)

Hence L is not regular.

Regular languages





Regular languages

- Generated by regular grammars
- Specified by regular expressions
- Accepted by NFAs
- Accepted by DFAs
- Pumping Lemma holds
- Pumping Lemma used by contradiction to show languages not regular
- Any NFA can be converted into an equivalent DFA
- DFA can be exponentially larger than the NFA

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(there is also a Pumping Lemma for context-free languages)

Questions?

Questions?

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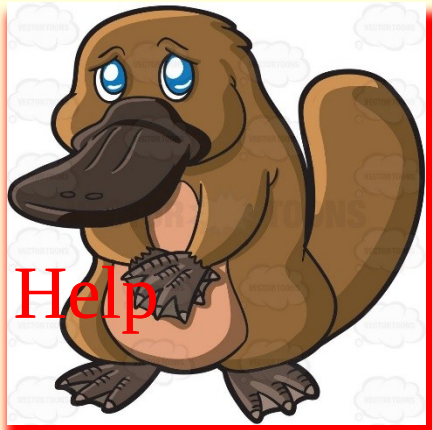
Questions?

The Platypus Game



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https://www.youtube.com/watch?reload=9&v=0gM5TjSOQ48
https://zoneringtones.com

The Platypus Game

Assignment 2

- Round-robin + knockout tournament
- 2,500 machines each
- 3,126,250 matches
- 0.59 seconds per 1,000 matches
- About 30 minutes
- I will take your top 10 for the knock-out phase
- Will post more about this on Ed soon

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That's it!

I am out of here!

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Break time! (We resume when all the pictures are gone! This will take 3 minutes!)

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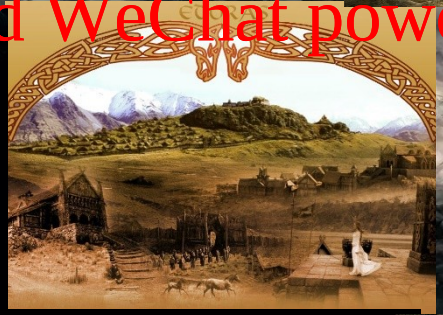

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I AM BACK!

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Alternative Scheme?

Troll	Dreadful	Poor	Acceptable	Exceeds Expectations	Outstanding

Outstanding – CONGRATULATIONS! Your exemplary powers of deduction and a formidable knowledge of the inner workings of the magical world reveal you to be a witch or wizard of genuine skill and learning.

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Exceeds Expectations – Well done – a most creditable performance!

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Acceptable – demonstrates real magical potential.

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Poor – Alas – we regret to inform you that you have narrowly failed. This may have been due to factors outside your control (eg: poltergeist intervention, examination nerves or a malfunctioning quill.) Please do not disconsolate.

Dreadful – We are sorry to inform you that you have failed.

Troll – You would appear either to have abandoned the test due to factors outside your control (eg, earthquake, poltergeist attack), or else you are a troll, in which case you are to be congratulated on being able to use a computer and have achieved the grade of O.F.T. (Outstanding for Trolls).

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