

Homework 7

Due date: 3/3/2021 right before class

Problem 1. Q8 page 418

Statistically, the arrival of spring typically results in increased accidents and increased need for emergency medical treatment, which often requires blood transfusions. Consider the problem faced by a hospital that is trying to evaluate whether its blood supply is sufficient. The basic rule for blood donation is the following. A person's own blood supply has certain antigens present (we can think of antigens as a kind of molecular signature); and a person cannot receive blood with a particular antigen if their own blood does not have this antigen present. Concretely, this principle underpins the division of blood into four types: A , B , AB , and O . Blood of type A has the A antigen, blood of type B has the B antigen, blood of type AB has both, and blood of type O has neither. Thus, patients with type A can receive only blood types A or O in a transfusion, patients with type B can receive only B or O , patients with type O can receive only O , and patients with type AB can receive any of the four types.

1. Let s_O, s_A, s_B , and s_{AB} denote the supply in whole units of the different blood types on hand. Assume that the hospital knows the projected demand for each blood type d_O, d_A, d_B , and d_{AB} for the coming week. Give a polynomial-time algorithm to evaluate if the blood on hand would suffice for the projected need.
2. Consider the following example. Over the next week, they expect to need at most 100 units of blood. The typical distribution of blood types in U.S. patients is roughly 45 percent type O , 42 percent type A , 10 percent type B , and 3 percent type AB . The hospital wants to know if the blood supply it has on hand would be enough if 100 patients arrive with the expected type distribution. There is a total of 105 units of blood on hand. The table below gives these demands, and the supply on hand.

blood type	supply	demand
O	50	45
A	36	42
B	11	8
AB	8	3

Is the 105 units of blood on hand enough to satisfy the 100 units of demand? Find an allocation that satisfies the maximum possible number

of patients. Use an argument based on a minimum-capacity cut to show why not all patients can receive blood. Also, provide an explanation for this fact that would be understandable to the clinic administrators, who have not taken a course on algorithms. (So, for example, this explanation should not involve the words flow, cut, or graph in the sense we use them in this book.)

Problem 2. Q10 page 419

Suppose you are given a directed graph $G = (V, E)$, with a positive integer capacity c_e on each edge e , a source $s \in V$, and a sink $t \in V$. You are also given a maximum $s - t$ flow in G , defined by a flow value f_e on each edge e . The flow f is acyclic: There is no cycle in G on which all edges carry positive flow. The flow f is also integer-valued. Now suppose we pick a specific edge $e^* \in E$ and reduce its capacity by 1 unit. Show how to find a maximum flow in the resulting capacitated graph in time $O(m + n)$, where m is the number of edges in G and n is the number of nodes.

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Problem 3. Q12 page 419

Consider the following problem. You are given a flow network with unit capacity edges: It consists of a directed graph $G = (V, E)$, a source $s \in V$, and a sink $t \in V$; and $c_e = 1$ for every $e \in E$. You are also given a parameter k .

The goal is to delete k edges so as to reduce the maximum $s - t$ flow in G by as much as possible. In other words, you should find a set of edges $F \subseteq E$ so that $|F| = k$ and the maximum $s - t$ flow in $G = (V, E - F)$ is as small as possible subject to this. Give a polynomial-time algorithm to solve this problem.

Problem 4. Q14 page 421

We define the Escape Problem as follows. We are given a directed graph $G = (V, E)$ (picture a network of roads). A certain collection of nodes $X \subset V$ are designated as populated nodes, and a certain other collection $S \subset V$ are designated as safe nodes. (Assume that X and S are disjoint.) In case of an emergency, we want evacuation routes from the populated nodes to the safe nodes. A set of evacuation routes is defined as a set of paths in G so that (i) each node in X is the tail of one path, (ii) the last node on each path lies in S , and (iii) the paths do not share any edges. Such a set of paths gives a way for the occupants of the populated nodes to “escape” to S , without overly congesting any edge in G .

1. Given G, X , and S , show how to decide in polynomial time whether such a set of evacuation routes exists.

2. Suppose we have exactly the same problem as in (a), but we want to enforce an even stronger version of the “no congestion” condition (iii). Thus we change (iii) to say “the paths do not share any nodes.” With this new condition, show how to decide in polynomial time whether such a set of evacuation routes exists. Also, provide an example with the same G , X , and S , in which the answer is yes to the question in (a) but no to the question in (b).

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