Import statement

```
1  from math import pi
2  tau = 2 * pi
```

Assignment statement

Global frame

Name | pi 3.1416 | Value

Binding

Code (left):

Statements and expressions
Red arrow points to next line.
Gray arrow points to the line just executed

Frames (right):

A name is bound to a value

In a frame, there is at most one binding per name

208

mul(add(2, mul(4, 6)), add(3, 5))

mul

26
add(2, mul(4, 6))

add | 2 | 24
mul(4, 6)

mul | 4 | 6

8
add(3, 5)

add | 3 | 5

Pure Functions

–2 ▶ *abs*(number): ▶ 2

2, 10 ▶ *pow*(x, y): ▶ 1024

Non–Pure Functions

–2 ▶ *print*(...): ▶ None

display "–2"

```
1  from operator import mul
2  def square(x):
3      return mul(x, x)
4  square(-2)
```

Built–in function

func mul(...) [parent=Global]

Intrinsic name of function called

func square(x) [parent=Global]

mul
square

User–defined function

Local frame

f1: square [parent=Global]

x -2

Formal parameter bound to argument

Return value 4

Return value is not a binding!

Defining:

>>> *def square*(x):

Formal parameter | Return expression

Def statement

return mul(x, x)

Body (*return statement*)

Call expression: square(2+2)

operand: 2+2
argument: 4

operator: square
function: func square(x)

Calling/Applying: 4 ▶ *square*(x):

Argument

Intrinsic name

return mul(x, x) ▶16

Return value

Compound statement | Clause

```
<header>:
    <statement>
    <statement>
```

Suite

```
<separating header>:
    <statement>
    <statement>
    ...
...
```

1 statement,
3 clauses,
3 headers,
3 suites,
2 boolean contexts

```
def abs_value(x):
    if x > 0:
        return x
    elif x == 0:
        return 0
    else:
        return -x
```

```
1  from operator import mul
2  def square(x):
3      return mul(x, x)
4  square(square(3))
```

A name evaluates to the value bound to that name in the earliest frame of the current environment in which that name is found.

Global frame

mul
square

f1: square [parent=Global]

x 3

f2: square [parent=Global]

x 9

Return value 81

Evaluation rule for call expressions:

1. Evaluate the operator and operand subexpressions.
2. Apply the function that is the value of the operator subexpression to the arguments that are the values of the operand subexpressions.

Applying user–defined functions:

1. Create a new local frame with the same parent as the function that was applied.
2. Bind the arguments to the function's formal parameter names in that frame.
3. Execute the body of the function in the environment beginning at that frame.

Execution rule for def statements:

1. Create a new function value with the specified name, formal parameters, and function body.
2. Its parent is the first frame of the current environment.
3. Bind the name of the function to the function value in the first frame of the current environment.

Execution rule for assignment statements:

1. Evaluate the expression(s) on the right of the equal sign.
2. Simultaneously bind the names on the left to those values, in the first frame of the current environment.

Execution rule for conditional statements:

Each clause is considered in order.
1. Evaluate the header's expression.
2. If it is a true value, execute the suite, then skip the remaining clauses in the statement.

Evaluation rule for or expressions:

1. Evaluate the subexpression <left>.
2. If the result is a true value v, then the expression evaluates to v.
3. Otherwise, the expression evaluates to the value of the subexpression <right>.

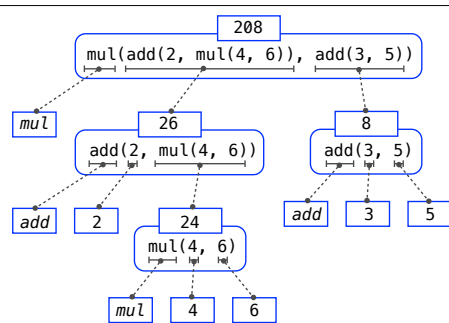**Evaluation rule for and expressions:**

1. Evaluate the subexpression <left>.
2. If the result is a false value v, then the expression evaluates to v.
3. Otherwise, the expression evaluates to the value of the subexpression <right>.

Evaluation rule for not expressions:

1. Evaluate <exp>; The value is True if the result is a false value, and False otherwise.

Execution rule for while statements:

1. Evaluate the header's expression.
2. If it is a true value, execute the (*whole*) suite, then return to step 1.

```
1  def f(x, y):
2      return g(x)
3
4  def g(a):
5      return a + y
6
7  result = f(1, 2)
```

"y" is not found
Error

Global frame

f
g

func f(x, y) [parent=Global]
func g(a) [parent=Global]

f1: f [parent=Global]

x 1
y 2

f2: g [parent=Global]

a 1

"y" is not found

• An environment is a sequence of frames

• An environment for a non–nested function (no def within def) consists of one local frame, followed by the global frame

```
1  from operator import mul
2  def square(square):
3      return mul(square, square)
4  square(4)
```

A | B

Global frame

mul
square

f1: square [parent=Global]

square 4

Return value 16

A call expression and the body of the function being called are evaluated in different environments

```
def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1   # Zeroth and first Fibonacci numbers
    k = 1               # curr is the kth Fibonacci number
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

```
def cube(k):
    return pow(k, 3)
```

Function of a single argument (not called term)

```
def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

A formal parameter that will be bound to a function

The cube function is passed as an argument value

$0 + 1^3 + 2^3 + 3^3 + 4^3 + 5^5$

The function bound to term gets called here

Higher–order function: A function that takes a function as an argument value or returns a function as a return value

Nested def statements: Functions defined within other function bodies are bound to names in the local frame

```
square = lambda x,y: x * y
```

Evaluates to a function. No "return" keyword!

A function
with formal parameters x and y
that returns the value of "x * y"

Must be a single expression

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

A function that returns a function

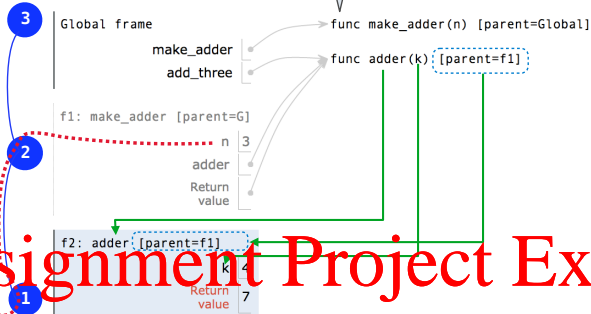The name add_three is bound to a function

A local def statement

Can refer to names in the enclosing function

- Every user-defined **function** has a *parent frame* (often global)
- The parent of a **function** is the frame in which it was *defined*
- Every local **frame** has a *parent frame* (often global)
- The parent of a **frame** is the parent of the function *called*

A function's signature has all the information to create a local frame

```
1  def make_adder(n):
2      def adder(k):
           return k + n
       return adder

6  add_three = make_adder(3)
7  add_three(4)
```

Nested def

③

Global frame → func make_adder(n) [parent=Global]
make_adder → func adder(k) [parent=f1]
add_three

f1: make_adder [parent=G]
② n 3
adder
Return value

①
f2: adder [parent=f1]
k 4
Return value 7

```
1  def square(x):
2      return x * x
3
4  def make_adder(n):
5      def adder(k):
6          return k + n
7      return adder
8
9  def compose1(f, g):
10     def h(x):
11         return f(g(x))
12     return h
13
14 compose1(square, make_adder(2))(3)
```

Return value of make_adder is an argument to compose1

Global frame
square
make_adder
compose1

func square(x) [parent=Global]
func make_adder(n) [parent=Global]
func compose1(f, g) [parent=Global]
func adder(k) [parent=f1]
func h(x) [parent=f2]

f1: make_adder [parent=Global]
n 2
adder
Return value

f2: compose1 [parent=Global]
f
g
h
Return value

f3: h [parent=f2]
x 3

f4: adder [parent=f1]
k 3

```
1  def print_sums(n):
2      print(n)
3      def next_sum(k):
4          return print_sums(n+k)
5      return next_sum
6
7  print_sums(1)(3)(5)
```

Printed output:
1
4
9

Global frame
print_sums

func print_sums(n) [parent=Global]
func next_sum(k) [parent=f1]
func next_sum(k) [parent=f3]
func next_sum(k) [parent=f5]

f1: print_sums [parent=Global]
n 1
next_sum
Return value

f2: next_sum [parent=f1]
k 3
Return value

f3: print_sums [parent=Global]
n 4
next_sum
Return value

f4: next_sum [parent=f3]
k 5

f5: print_sums [parent=Global]
n 9
next_sum
Return value

```
square = lambda x: x * x    VS    def square(x):
                                      return x * x
```

- Both create a function with the same domain, range, and behavior.
- Both functions have as their parent the environment in which they were defined.
- Both bind that function to the name square.
- Only the def statement gives the function an intrinsic name.

When a function is defined:
1. Create a **function value**: func *<name>*(*<formal parameters>*)
2. Its parent is the current frame.

   f1: make_adder   func adder(k) [parent=f1]

3. Bind *<name>* to the **function value** in the current frame (which is the first frame of the current environment).

When a function is called:
1. Add a **local frame**, titled with the *<name>* of the function being called.
2. Copy the parent of the function to the **local frame**: [parent=*<label>*]
3. Bind the *<formal parameters>* to the arguments in the **local frame**.
4. Execute the body of the function in the environment that starts with the **local frame**.

```
>>> min(2, 1, 4, 3)      >>> 2 + 3
1                        5
>>> max(2, 1, 4, 3)      >>> 2 * 3
4                        6
>>> abs(-2)              >>> 2 ** 3
2                        8
>>> pow(2, 3)            >>> 5 / 3
8                        1.6666666666666667
>>> len('word')          >>> 5 // 3
4                        1
>>> round(1.75)          >>> 5 % 3
2                        2
>>> print(1, 2)          >>> print(print(1))
1 2                      1
                         None
```

```
def search(f):
    """Return the smallest non-negative
    integer x for which f(x) is a true value.
    """
    x = 0
    while True:
        if f(x):
            return x
        x += 1

def is_three(x):
    """Return whether x is three.

    >>> search(is_three)
    3
    """
    return x == 3

def inverse(f):
    """Return a function g(y) that returns
    x such that f(x) == y.

    >>> sqrt = inverse(lambda x: x * x)
    >>> sqrt(16)
    4
    """
    return lambda y: search(lambda x: f(x)==y)
```

```
from operator import add, mul

def curry2(f):
    """Curry a two-argument function.

    >>> m = curry2(add)
    >>> add_three = m(3)
    >>> add_three(4)
    7
    >>> m(2)(1)
    3
    """
    def g(x):
        def h(y):
            return f(x, y)
        return h
    return g
```

```
from operator import floordiv, mod
def divide_exact(n, d):
    """Return the quotient and remainder of dividing N by D.

    >>> q, r = divide_exact(2012, 10)
    >>> q
    201
    >>> r
    2
    """
    return floordiv(n, d), mod(n, d)
```
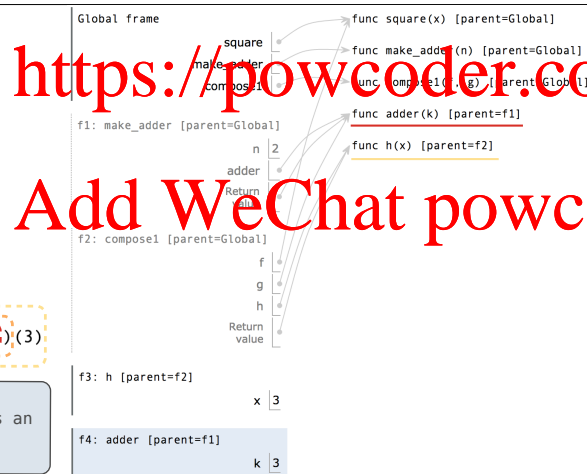
Multiple assignment to two names

Two return values, separated by commas