```
Rational implementation using functions:
                                                               List comprehensions:
                                                                                                                                                           List & dictionary mutation:
                                                                  [<map exp> for <name> in <iter exp> if <filter exp>]
                                                                                                                                                           >>> a = [10]
                                                                                                                                                                                       >>> a = [10]
 def rational(n, d):
                                                                                                                                                                                       >>> b = [10]
        def select(name):
                                                                   Short version: [<map exp> for <name> in <iter exp>]
                                                  This
                                                                                                                                                           >>> a == b
                                                                                                                                                                                       >>> a == b
               if name == 'n':
                                               function
                                                                                                                                                           True
                                                                                                                                                                                       True
                                                               A combined expression that evaluates to a list using this
                      return n
                                              represents
                                                                                                                                                           >>> a.append(20)
                                                                                                                                                                                       >>> b.append(20)
                                                               evaluation procedure:
               elif name == 'd':
                                             a rational
                                                                                                                                                           >>> a == b
                                                                                                                                                                                       >>> a
                                                               1. Add a new frame with the current frame as its parent
                                                number
                                                                                                                                                           True
                                                                                                                                                                                       [10]
                      return d
                                                               2. Create an empty result list that is the value of the
                                                                                                                                                           >>> a
                                                                                                                                                                                       >>> b
        return select
                                                                  expression
                                                                                                                                                           [10, 20]
                                                                                                                                                                                       [10, 20]
                                                               3. For each element in the iterable value of <iter exp>:
def numer(x):
                                                                                                                                                           >>> b
                                                                                                                                                                                       >>> a == b
                                                                  A. Bind <name> to that element in the new frame from step 1
                                  Constructor is a
                                                                                                                                                                                       False
       return x('n')
                                                                                                                                                           [10, 20]
                                                                  B. If <filter exp> evaluates to a true value, then add
                               higher-order function
def denom(x):
                                                                                                                                                           >>> nums = {'I': 1.0, 'V': 5, 'X': 10}
>>> nums['X']
                                                                      the value of <map exp> to the result list
       return x('d')
                                  Selector calls x
                                                              The result of calling repr on a value is
                                                                                                                            >>> 12e12
                                                                                                                                                           10
                                                              Lists:
                                                                                                                                                           >>> nums['I'] = 1
                                                                                                                            >>> print(repr(12e12))
1200000000000000.0
                                                              The result of calling str on a value is
                                                                                                                                                           >>> nums['L'] = 50
>>> digits = [1, 8, 2, 8]
                                                                                                                                                           >>> nums
                                                              what Python prints using the print function
>>> len(digits)
                                                                                                                                                           {'X': 10, 'L': 50, 'V': 5, 'I': 1}
                     digits __
                                                                                                                            >>> print(today)
                                                                >>> today = datetime.date(2019, 10, 13)
                                                                                                                                                           >>> sum(nums.values())
>>> digits[3]
                                                                                                                            2019-10-13
                                            8
                                                 2 8
                                                                                                                                                          66
                                                              str and repr are both polymorphic; they apply to any object
                                                                                                                                                           >>> dict([(3, 9), (4, 16), (5, 25)])
 >>> [2, 7] + digits * 2
                                                              repr invokes a zero-argument method __repr__ on its argument
                                                                                                                                                           {3: 9. 4: 16.
 [2, 7, 1, 8, 2, 8, 1, 8, 2, 8]
                                                                                                                                                           >>> nums.get('A', 0)
                                                                >>> today.__repr__()
                                                                                                            >>> today.__str__()
                                                                                                            2019-10-13
                                                                'datetime.date(2019, 10, 13)'
>>> pairs = [[10, 20], [30, 40]]
                                                                                                                                                           >>> nums.get('V', 0)
 >>> pairs[1]
                     pairs 1
                                                              Type dispatching: Look up a cross-type implementation of an
 [30, 40]
                                                10 20
                                                              operation based on the types of its arguments
 >>> pairs[1][0]
                                                                                                                                                           >>> {x: x*x for x in range(3,6)}
                                                              Type coercion: Look up a function for converting one type to
                                                                                                                                                           {3: 9, 4: 16, 5: 25}
30
                                                              another, then apply a type-specific implementation.
Executing a for statement:
                                                               Functions that aggregate iterable arguments
 for <name> in <expression>:
                                                                                                                                                       >>> sum([1, 2])
                                                                                                                                                                                      >>> any([False, True])
                                                30
                                                      40
      <suite>
                                                               •sum(iterable[, start]) -> value
                                                                                                                       sum of all values
                                                                                                                                                                                      True
                                                               •max(iterable[, key=func]) -> value
                                                                                                                       largest value
                                                                                                                                                       >>> sum([1, 2], 3)
                                                                                                                                                                                      >>> any([])
 1. Evaluate the header <expression>
                                                                                                                                                                                      False
    which must yield an iterable value
                                                                max(a, b, c, ...[, key=func]) -> value
                                                                                                                                                                                      >>> max(1, 2)
                                                                                                                                                       >>> sum([])
     (a list, tuple, iterator, etc.)
                                                                min(iterable[, key=func]) -> value
                                                                                                                       smallest value
                                                                min(a, b, c, ...[, key=func]) \rightarrow value
 2. For each element in that sequence,
                                                                                                                                                       >>> all([False, True])
                                                                                                                                                                                      >>> max([1, 2])
                                                                                                                       whether all are true
Thethet XX 2 119
                                                            eal(iterable) -> boot Proje

Pany built-in map(func, iterable)
    in order:
   A. Bind <name> to that element in C
                                                                                                                                                                                      >>> max([1, -2], key=abs)
       the current frame
                                                                                       map(func, iteraple):
   B. Execute the <suite>
                                                              Python sequence
                                                                                          Iterate over func(x) for x in iterable
 Unpacking in a
                                A sequence of
                                                              operations
                                                                                       filter(func, iterable):
 for statement:
                                                                                                                                                           You can copy a list by calling the list
                         fixed-length sequences
                                                              return
                                                                                          Iterate over x in iterable if func(x)
                                                                                                                                                           constructor or slicing the list from the
                                                              iterators that
                                                                                       powcoder, com

Persed (sequence):
                                                                                                                                                           beginning to the end.
 >>> pairs=[[1, 2], [2, 2], [3, 2], [4, 4]]
                                                             John It Desutts
 >>> same_count = 0
                                                                                                                                                           >>> suits = ['coin', 'string', 'myriad']
        A name for each element in a fixed-length sequence
                                                                                                                                                           >>> suits.pop()—
                                                                                          Iterate over x in a sequence in reverse order
                                                                                                                                                                                                Remove and return
                                                                                                                                                           'mvriad
                                                                                                                                                                                               the last element
                                                               To view the
                                                                                                                                                           >>> suits.remove('string')
                                                                                       list(iterable):
>>> for (x, y) in pairs:
    if x == y:
                                                                                                                                                                                               Remove a value
                                                               contents of
                                                                                         Created list containing all x in iterable clitering to possible to partial all x in iterable create a tuple containing all x in iterate a tuple containing all x in iterate a tuple containing all x in iterate a tuple containing a tuple con
                                                                                                                                                           >>> suits.append('cup')
                                                               n iterator,
page the
                                                                                                                                                            >> suits.extend(['sword', 'club'])
                 same_count = same_count + 1
                                                                                                              Con DIO W COCK
                                                                                                                                                           >>> suits[2] = 'spade'
 >>> same count
                                                               resulting
                                                                                                                                                           >>> suits
['coin', 'cup', 'spade', 'club']
>>> suits[0:2] = ['diamond']
                                                                                       sorted(iterable):
                                                                                                                                                                                                            values
                                                               elements into
                                                                                          Create a sorted list containing x in iterable
                                                                                                                                                                                                         Replace a
                                                               a container
     ..., -3, -2, -1, 0, 1, 2, 3, 4, ...
                                                                                                                                                           >>> suits
                                                                                                                                                                                                           values
                                                             def cascade(n):
                                                                                          >>> cascade(123)
                                                                                                                                                           ['diamond', 'spade', 'club']
>>> suits.insert(0, 'heart')
Add an element
                                                                                                                       fib(n): 0, 1, 1, 2, 3, 5, 8, 13, 21,
                                                                                         123
                                                                  if n < 10:
                                                                                                             def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
                                                                                         12
                                                                      print(n)
                                                                                                                                                                                                    at an index
                                                                                                                                                           >>> suits
                  range(-2, 2)
                                                                  else:
                                                                                                                                                           ['heart', 'diamond', 'spade', 'club']
                                                                      print(n)
 Length: ending value - starting value
                                                                       cascade(n//10) 123
                                                                                                                  else:
 Element selection: starting value + index
                                                                       print(n)
                                                                                                                                                                                       >>> bool(0)
                                                                                                                        return fib(n-2) + fib(n-1)
                                                                                                                                                           False values:
  >>> list(range(-2, 2)) \ List constructor
                                                                                                                                                            Zero
                                                                                                                                                                                       False
                                                               Global frame

→func make withdraw(balance) [parent=Global]
                                                                                                                                                                                       >>> bool(1)
  [-2, -1, 0, 1]
                                                                                                                                                           False
                                                                                 make_withdraw
                                                                                                                                                           None
                                                                                                                                                                                       True
                                                                                                           func withdraw(amount) [parent=f1]
                               Range with a 0
                                                                                                                                                                                       >>> bool('')
 >>> list(range(4)) {

    An empty string,

                                                                                      withdraw
                                                                                                          >>> withdraw = make_withdraw(100)
                              starting value
                                                                                                                                                                                      False
  [0, 1, 2, 3]
                                                                                                                                                            list, dict, tuple
                                                                                                          >>> withdraw(25)
                                                                                                                                                                                       >>> bool('0')
Membership:
                                    Slicing:
                                                               f1: make withdraw [parent=Global]
                                                                                                          75
                                                                                                                                                                                       True
                                                                                                                                                           All other values
                                    >>> digits[0:2]
>>> digits = [1, 8, 2, 8]
                                                                                      balance 50
                                                                                                                                                                                       >>> bool([1)
                                                                                                          >>> withdraw(25)
                                                                                                                                                           are true values.
                                                                   The parent
                                    [1, 8]
>>> 2 in digits
                                                                                                                                                                                       False
                                                                                      withdraw
                                                                                                         50
                                                                frame contains
                                    >>> digits[1:]
                                                                                                                                                                                       >>> bool([[]])
True
                                                                                                         def make_withdraw(balance):
                                                                                       Return
                                                                the balance of
                                    [8, 2, 8]
>>> 1828 not in digits
                                                                                        value
                                                                                                                                                                                      True
                                                                                                             def withdraw(amount):
                                                                    withdraw
                                                                                                                                                                                      >>> bool({})
                Slicing creates a new object
True
                                                                                                                    nonlocal balance
                                                                                                                                                                                      False
                                                               f2: withdraw [parent=f1]
                                                                                                                    if amount > balance:
    return 'No funds
Identity:
                                                                                                                                                                                       >>> bool(())
                                                                                       amount 25
<exp0> is <exp1>
                                                                   Every call
                                                                                                                                                                                       False
                                                                                                                    balance = balance - amount
                                                                                       Return
value 75
evaluates to True if both <exp0> and
                                                                decreases the
                                                                                                                                                                                       >>> bool(lambda x: 0)
                                                                                                                    return balance
<exp1> evaluate to the same object
                                                                 same balance
                                                                                                                                                                                      True
                                                                                                               return withdraw
Equality:
<exp0> == <exp1>
                                                               f3: withdraw [parent=f1]
                                                                                                           •No nonlocal statement
                                                                                                              Status
                                                                                                                                                    Effect
                                                                                      amount 25
                                                                                                                                                 Create a new binding from name "x" to number 2
evaluates to True if both <exp0> and
                                                                                                           •"x" is not bound locally
                                                                                                                                                 in the first frame of the current environment
<exp1> evaluate to equal values
                                                                                       Return
value 50
Identical objects are always equal values
                                                                                                                                                 Re-bind name "x" to object 2 in the first frame

    No nonlocal statement

                                                         >>> d = {'one': 1, 'two': 2, 'three': 3}
iter(iterable):
Return an iterator
over the elements of
                                >>> s = [3, 4, 5]
>>> t = iter(s)
                                                                                                           •"x" is bound locally
                                                                                                                                                 of the current environment
                                                         >>> k = iter(d) >>> v = iter(d.values())
                                                                                                           •nonlocal x
                                 >>> next(t)
                                                         >>> next(k)
                                                                           >>> next(v)
                                                                                                                                                 Re-hind "x" to 2 in the first non-local frame of
 an iterable value
                                                                                                           •"x" is bound in a
                                                          one
                                                                                                                                                 the current environment in which "x" is bound
next(iterator):
                                                         >>> next(k)
                                                                                                            non-local frame
                                 >>> next(t)
                                                                           >>> next(v)
 Return the next element
                                                          'two
                                                                                                           •nonlocal x
                                                                                                                                                 SyntaxError: no binding for nonlocal 'x' found
A generator function is a function that yields values instead of returning them.
                                                                                                           •"x" is not bound in
>>> def plus_minus(x):
                              >>> t = plus_minus(3)
                                                            def a_then_b(a, b):
                                                                                                            a non-local frame
          yield x
                              >>> next(t)
                                                                yield from a
                                                                                                           •nonlocal x
                                                                vield from b
          yield -x
                                                                                                           •"x" is bound in a
                                                                                                                                                 SyntaxError: name 'x' is parameter and nonlocal
                              >>> next(t)
                                                            >>> list(a_then_b([3, 4], [5, 6]))
                                                                                                            non-local frame
                                                            [3, 4, 5, 6]
                                                                                                           •"x" also bound locally
```

```
Root or Root Node
                                                                                   Python object system:
                                                                     - Nodes
  Recursive description:
                                                        Path
                                                                                    Idea: All bank accounts have a balance and an account holder;
  •A tree has a root label
                                      Root label
                                                    34)
                                                                                    the Account class should add those attributes to each of its instances
                                                                           ahels
   and a list of branches
                                   Branch-
  •Each branch is a tree
                                                                                                               >>> a = Account('Jim')
                                                                                      A new instance is
  •A tree with zero branches
                                                                                                               >>> a.holder
                                                                                     created by calling a
                                           1
                                                                                                               'Jim'
   is called a leaf
                                                                                             class
                                                                                                               >>> a.balance
  Relative description:
                                                                                                                                         An account instance
                                       0
                                                 1
                                                       1
  •Each location is a node
                                                                                   When a class is called:
                                                                                                                                                  holder: 'Jim'
                                                                                                                                   balance: 0
  •Each node has a label
                                                                                   1.A new instance of that class is created:
  •One node can be the
                                      Leaf 
                                                            6
                                                                                   2. The __init__ method of the class is called with the new object as its first
   parent/child of another
                                                                                     argument (named self), along with any additional arguments provided in the
  def tree(label, branches=[]):
                                                                                     call expression.
                                         Verifies the
      for branch in branches:
                                                                                                          class Account:
          assert is tree(branch) tree definition
                                                                                                                   __init__(self, account_holder):
                                                                                                              > def
                                                                                       init is called a
                                                                                                                    self.balance = 0
       return [label] + list(branches)
                                                                                          constructor
                                                                                                                    self.holder = account_holder
  def label(tree):
                                                                                                               def deposit(self, amount):
                           Creates a list from a
       return tree[0]
                                                                                                                    self.balance = self.balance + amount
                           sequence of branches
                                                                                                                   return self.balance
  def branches(tree):
                                                                3
                                                                                     self should always be
                                                                                                                   withdraw(self, amount):
  if amount > self.balance:
    return 'Insufficient funds'
                                                                                                               def
                          Verifies that tree is
                                                                                    bound to an instance of
       return tree[1:]
                             bound to a list
                                                                                     the Account class or a
  def is_tree(tree):
                                                                                      subclass of Account
                                                                                                                    self.balance = self.balance - amount
      if(type(tree) != list)or len(tree) < 1:</pre>
                                                                                                                    return self.balance
           return False
       for branch in branches(tree):
                                                                                                            >>> type(Account.deposit)
                                                                                     Function call: all
                                                                                                           <class 'function'
                                           >>> tree(3, [tree(1),
           if not is_tree(branch):
                                                                                                           >>> type(a.deposit)
                                                          tree(2, [tree(1)
                                                                                      arguments within
                                            . . .
               return False
                                                                    tree(1)))))
                                                                                        parentheses
                                                                                                            <class 'method'>
       return True
                                            [3, [1], [2, [1], [1]]]
  def is_leaf(tree):
                                                                                                            >>> Account.deposit(a, 5)
                                                                                     Method invocation:
       return not branches(tree) |def fib_tree(n):
                                                                                      One object before
  def leaves(t):
    """The leaf values in t.
                                        if n == 0 or n == 1:
                                                                                      the dot and other
                                                                                                            >>> a.deposit(2)
                                                                                                                                          Call expression
                                             return tree(n)
                                                                                                           12
                                                                                      arguments within
       >>> leaves(fib_tree(5))
                                                                                         parentheses
       [1, 0, 1, 0, 1, 1, 0, 1]
                                             left = fib\_tree(n-2),
                                                                                                                  Dot expression
                                             right = fib_tree(n-1)
fib_n = label(left) + label(right)
       if is_leaf(t):
           return [label(t)]

return sum([leaves(b) for b in brances(t)], [])
                                                                                        Cession danvocan Talid Vice of Esion.
       else:
                                                                                    Evaluates to the value of the attribute looked up by <name> in the object
        Tree:
  class
                                                                                    that is the value of the <expression>.
      def __init__(self, label, branches=[]):
                                                       Built-in isinstance
                                                                                    To evaluate a dot expression:
           self.label = label
                                                    function: returns True if
                                                                                       Evaluate the <expression> to the left of the dot, which yields
           for branch in branches:
                                                     branch has a dlass that
               assert isinstance(branch, Tree
                                                                                        the days ssion he instance attributes of that object;
           self.branches = list(branches)
                                                                                         if an attribute with that name exists, its value is returned
                                        def fib_tree(n):
   if n == 0 or n ==
      def is leaf(self):
                                                                                        If not, <name> is looked up in the class, which yields a class
          return not self.branches
                                                                                         attribute value
                                                return Tree(n)
                                                                                         That value is returned unless it is a function, in which case a
                                            else:
                                                                                     bound method is returned i stead

Addingent stateMents with a corpression on their left-hand side affect attributes for the object of that dot expression
  def leaves(tree):
                                                len = fb Fee(n-2)
rzgm
fib_n = teft.label+right.labet
      The leaf values in a tree."
     if tree.is_leaf():
          return [tree.label]
                                                return Tree(fib_n,[left, right])
                                                                                     • If the object is an instance, then assignment sets an instance attribute
      else:
           return sum([leaves(b) for b in tree.branches], [])
                                                                                     • If the object is a class, then assignment sets a class attribute
                      Some zero
 class Link:
                                                                                               Account class
                                                                                                                   interest: 0.02 0.04 0.05
     empty = () < length sequence</pre>
                                                                                                attributes
                                                                                                                   (withdraw, deposit, _
                                                                                                                                           init
          _init__(self, first, rest=empty):
         assert rest is Link.empty or isinstance(rest, Link)
                                                                                                                                                 balance:
                                                                                         Instance
                                                                                                         balance:
                                                                                                                    0
                                                                                                                                 Instance
         self.first = first
                                                                                                                     'Jim'
                                                                                                                                                             'Tom'
                                                                                                         holder:
                                                                                                                                                 holder:
                                                     Link instance
                                                                    Link instance
                                                                                      attributes of
                                                                                                                              attributes of
         self.rest = rest
                                                                                                         interest: 0.08
                                                                                       jim_account
                                                                                                                               tom account
                                                      first:
                                                                     first:
           repr (self):
     def
                                                                                                                                 >>> jim_account.interest = 0.08
                                                                                     >>> jim_account = Account('Jim')
         if self.rest:
                                                                                         tom_account = Account('Tom')
                                                                                                                                >>> jim_account.interest
                                                      rest:
                                                                      rest
                        ' + repr(self.rest)
             rest = ',
                                                                                                                                0.08
                                                                                    >>> tom_account.interest
         else:
                                                     >>> s = Link(4, Link(5))
                                                                                    0.02
                                                                                                                                >>> tom account.interest
            rest = ''
                                                     >>> s
                                                                                                                                 0.04
                                                                                    >>> jim_account.interest
         return 'Link('+repr(self.first)+rest+')'
                                                    Link(4, Link(5))
                                                                                                                                 >>> Account.interest = 0.05
                                                                                    0.02
                                                     >>> s.first
                                                                                                                                 >>> tom_account.interest
                                                                                    >>> Account.interest = 0.04
           _str__(self):
                                                                                                                                0.05
                                                                                    >>> tom_account.interest
                                                     >>> s.rest
         string = '<
                                                                                                                                 >>> jim_account.interest
                                                                                    0.04
         while self.rest is not Link.empty:
                                                     Link(5
                                                                                                                                0.08
                                                                                    >>> jim_account.interest
             string += str(self.first) +
                                                     >>> print(s)
                                                                                    0.04
             self = self.rest
                                                     >>> print(s.rest)
         return string + str(self.first) + '>'
                                                                                    class CheckingAccount(Account):
                                                     <5>
                                                                                           "A bank account that charges for withdrawals."""
                                                     >>> s.rest.rest is Link.empty
                                                     True
                                                                                        withdraw fee = 1
                                             def sum_digits(n):
    -- +he digits of positive integer n."
Anatomy of a recursive function:
                                                                                         interest = 0.01
The def statement header is like any function Conditional statements check for base cases Base cases are evaluated without recursive calls Recursive cases are evaluated with recursive calls
                                                                                        if n
                                                  all_but_last, last = n // 10, n % 10
                                                                                             return super() withdraw(
                                                                                                                              amount + self.withdraw_fee)
                                                  return sum digits(all but last) + last
Recursive decomposition: finding
                                   def count_partitions(n, m):
                                                                                     To look up a name in a class:
simpler instances of a problem.
E.g., count_partitions(6, 4)
                                       if n == 0:
                                                                                     1. If it names an attribute in the class, return the attribute value.
                                           return 1
                                                                                     2. Otherwise, look up the name in the base class, if there is one.
Explore two possibilities:
•Use at least one 4
                                        elif n < 0:
                                                                                     >>> ch = CheckingAccount('Tom') # Calls Account.__init_
                                           return 0
                                                                                                           # Found in CheckingAccount
                                                                                     >>> ch.interest
                                        elif m == 0:
•Don't use any 4
Solve two simpler problems:
                                                                                     0.01
                                            return 0
                                                                                     >>> ch.deposit(20) # Found in Account
count_partitions(2, 4)
count_partitions(6, 3)
                                        else:
                                                                                     20
                                         with m = count partitions(n-m, m)
Tree recursion often involves
                                           without_m = count_partitions(n, m-1)
                                                                                     >>> ch.withdraw(5) # Found in CheckingAccount
exploring different choices.
                                            return with_m + without_m
```