Announcements

Reminder: self-grading forms for ps1 and ps2 due 10/5 at midnight (Boston)

Assignment Project Exam Help

- ps3 out today, due 10/8 (1 week) https://powcoder.com
- Midterm practice questions out next week Add WeChat powcoder

Today: Outline

• Neural networks cont'd: learning via gradient descent; chain rule reviews gradient representation using the backpropropagation algorithm https://powcoder.com

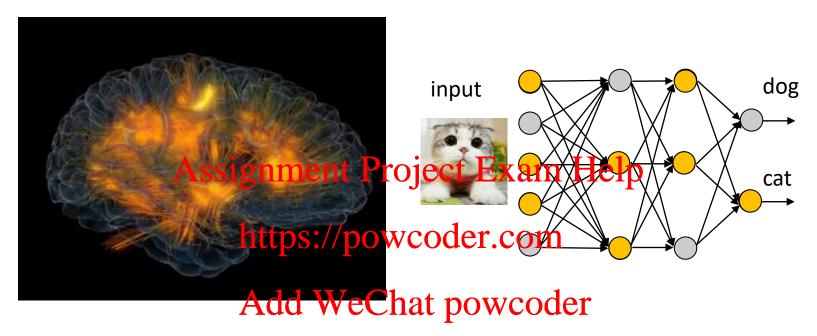
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Neural Networks II

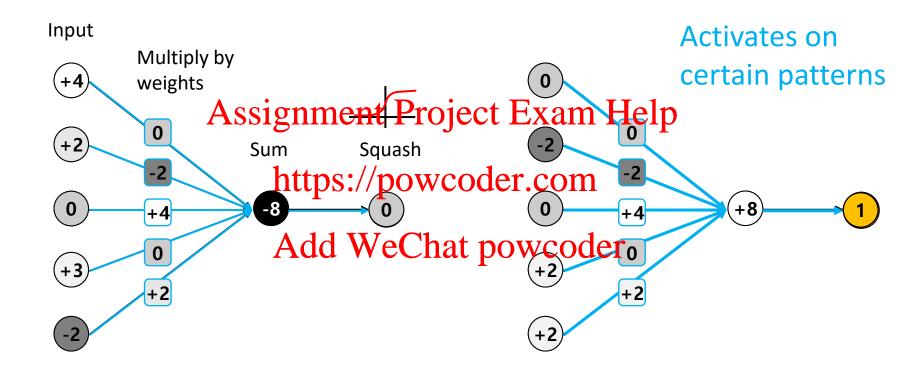
Learning

Artificial Neural Network

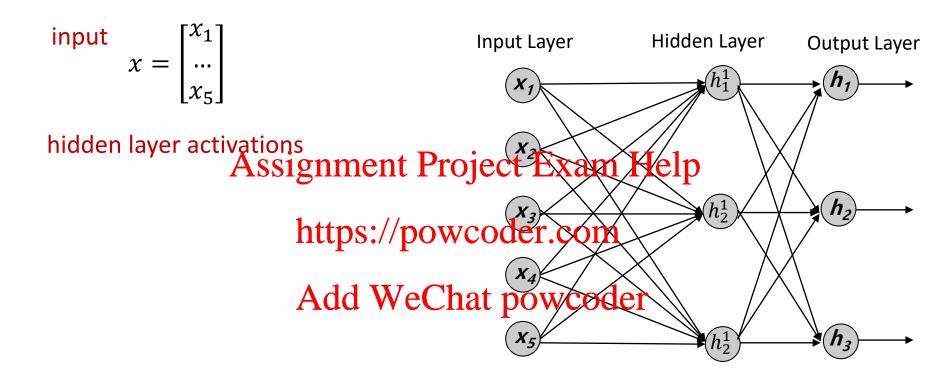


- Artificial neural networks: consist of many inter-connected neurons organized in layers
- **Neurons**: each neuron receives inputs from neurons in previous layer, passes its output to next layer
- Activation: neuron's output between 1 (excited) and 0 (not excited)

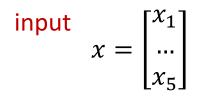
Artificial Neuron: Activation



Artificial Neural Network: notation



Artificial Neural Network: notation

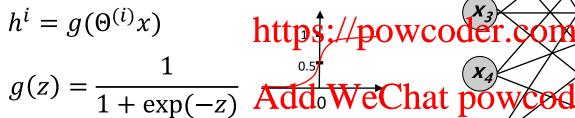


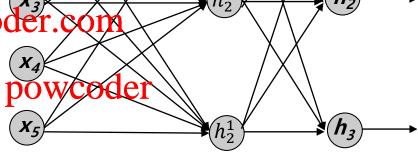
Hidden Layer Input Layer **Output Layer**

hidden layer activations Assignment Project Exam

$$h^i = g(\Theta^{(i)}x)$$

$$g(z) = \frac{1}{1 + \exp(-z)}$$





output

$$h_{\Theta}(\mathbf{x}) = g(\Theta^{(2)}h^{i}) \qquad \text{weights} \qquad \Theta^{(1)} = \begin{pmatrix} \theta_{11} & \cdots & \theta_{15} \\ \vdots & \ddots & \vdots \\ \theta_{31} & \cdots & \theta_{35} \end{pmatrix} \qquad \Theta^{(2)} = \begin{pmatrix} \theta_{11} & \cdots & \theta_{13} \\ \vdots & \ddots & \vdots \\ \theta_{31} & \cdots & \theta_{33} \end{pmatrix}$$

Cost function

Neural network: $h_{\Theta}(x) \in \mathbb{R}^K$ $(h_{\Theta}(x))_i = i^{th}$ output

Gradient computation

$$J(\Theta) = -\frac{1}{m} \left[\sum_{i=1}^{m} \sum_{k=1}^{K} y_k^{(i)} \log h_{\theta}(x^{(i)})_k + (1 - y_k^{(i)}) \log(1 - h_{\theta}(x^{(i)})_k) \right]$$

$$+ \frac{\lambda}{2m} \sum_{l=1}^{L-1} \sum_{i=1}^{s_l} \sum_{j=1}^{s_{l+1}} \text{Project Exam Help}$$

$$\text{https://powcoder.com}$$

$$\min_{\Theta} J(\Theta)$$

Add WeChat powcoder Backpropagation algorithm"

Need code to compute:

- Efficient way to compute $\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta)$

$$- J(\Theta)$$

$$- \frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta)$$

Computes gradient
 incrementally by
 "propagating" backwards
 through the network



Neural Networks II

backpropagation

Chain Rule

Need to compute gradient of

$$\log(h_{\Theta}(\mathbf{x})) = \log(g(\Theta^{(2)}g(\Theta^{(1)}x))) \quad \text{w.r.t}\Theta$$

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 How can we compute the gradient of several chained functions?

$$f(\theta) = f_1(\frac{\text{https://powecoder.goin}_2(\theta)}{*}) * f_2'(\theta)$$

$$f(\theta) = f_1(f_2) + f_3(\theta) + f_4(\theta) + f$$

What about functions of multiple variables?

$$f(\theta_1, \theta_2) = f_1(f_2(\theta_1, \theta_2))$$
 $\frac{\partial f}{\partial \theta_1} = \frac{\partial f}{\partial \theta_2} = \frac{\partial f}{\partial \theta_2}$

Backpropagation: Efficient Chain Rule

Partial gradient computation via chain rule:

$$\frac{\partial f}{\partial \theta_1} = \frac{\partial f_1}{\partial s} \left(f_2(f_3(\theta)) \right) * \frac{\partial f_2}{\partial s} \left(f_3(\theta) \right) * \frac{\partial f_3}{\partial \theta_1} (\theta)$$
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$$\frac{\partial f}{\partial \theta_2} = \frac{\partial f_1}{\partial f_2} \left(\frac{\text{https://powcooder.com}}{f_2(f_3(\theta))} \right) * \frac{\partial g}{\partial f_3} \left(\frac{\partial g}{\partial g} \right) * \frac{\partial f_3}{\partial \theta_2} (\theta)$$
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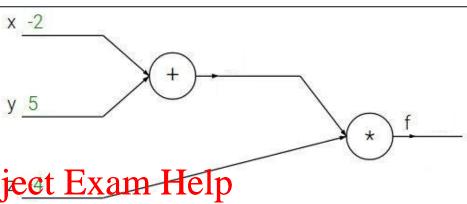
$$\frac{\partial f}{\partial \theta_3} = \frac{\partial f_1}{\partial f_2} \left(f_2 \left(f_3(\theta) \right) \right) * \frac{\partial f_2}{\partial f_3} \left(f_3(\theta) \right) * \frac{\partial f_3}{\partial \theta_3} (\theta)$$

- need to re-evaluate functions many times
- Very inefficient! E.g. 100,000-dim parameters

Chain Rule with a Computational Graph

$$f(x,y,z) = (x+y)z$$

e.g.
$$x = -2$$
, $y = 5$, $z = -4$



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Chain Rule with a Computational Graph

$$f(x,y,z) = (x+y)z$$

e.g.
$$x = -2$$
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$$q = x + y$$
 $\frac{\partial q}{\partial x} = 1$, $\frac{\partial q}{\partial y} = 1$ Project Exam Help

$$f=qz$$
 $\frac{\partial f}{\partial q}=z_{A}\frac{\partial f}{\partial t}=q_{C}$ hat powcoder

Want:
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$

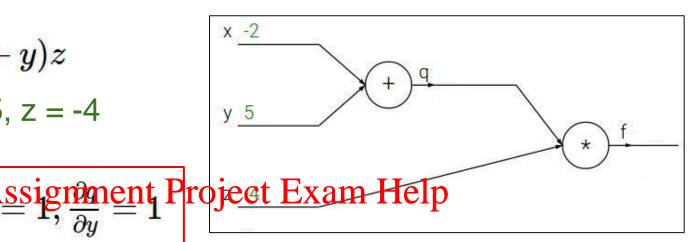
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Computation Graph: Forward

$$f(x,y,z)=(x+y)z$$

e.g.
$$x = -2$$
, $y = 5$, $z = -4$

$$q = x + y$$
 $\frac{\partial q}{\partial x} = i$, $\frac{\partial m}{\partial y} = 1$



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m Chat\ powcoder}$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

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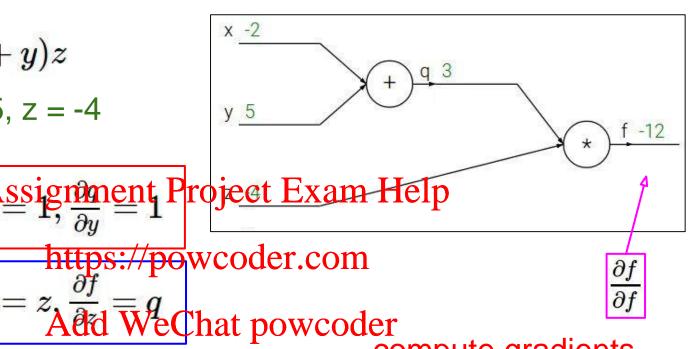
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Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

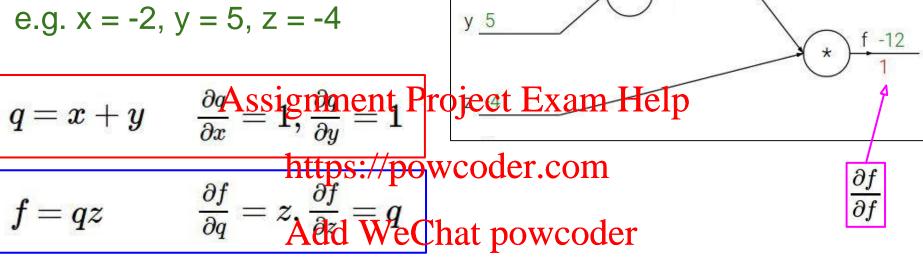


compute gradients

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$$f(x,y,z) = (x+y)z$$

e.g. x = -2, y = 5, z = -4

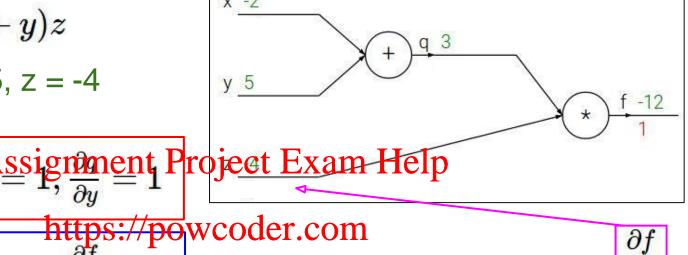


Want:
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$

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$$f(x, y, z) = (x + y)z$$

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$$q = x + y$$
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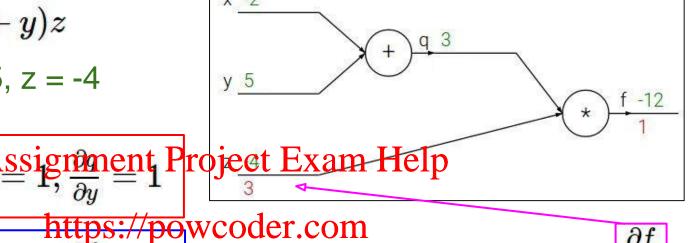
 $= z \frac{\partial f}{\partial t} = q$ Add WeChat powcoder

Want:

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$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



$$q = x + y$$
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 $\frac{\partial f}{\partial f} = \frac{\partial f}{\partial f}$

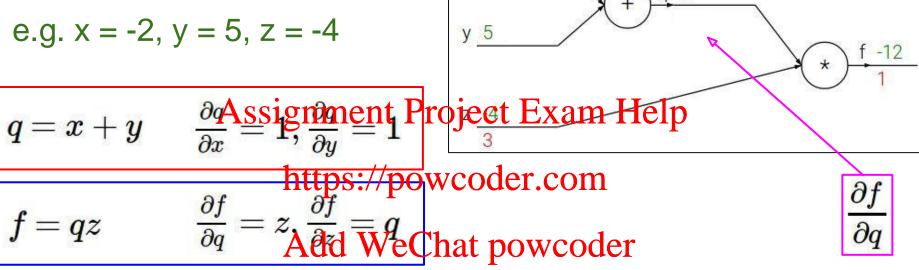
$$\frac{\partial f}{\partial q} = z_{\text{Add}} = q$$
That powcoder

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

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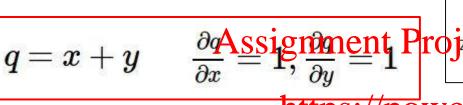


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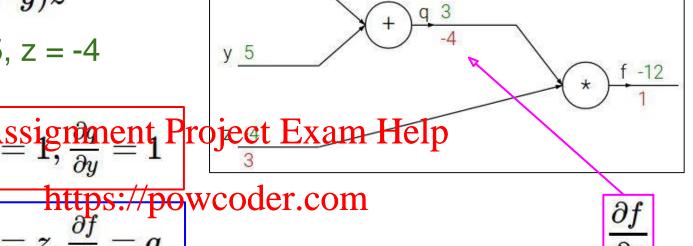
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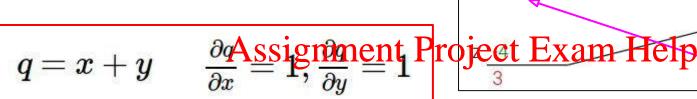


Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

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m A} rac{\partial f}{\partial d} = q \ {
m AeChat\ powcoder}$

 $\frac{\partial f}{\partial y}$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

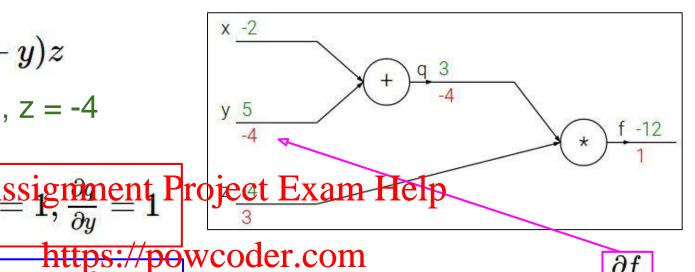
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Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



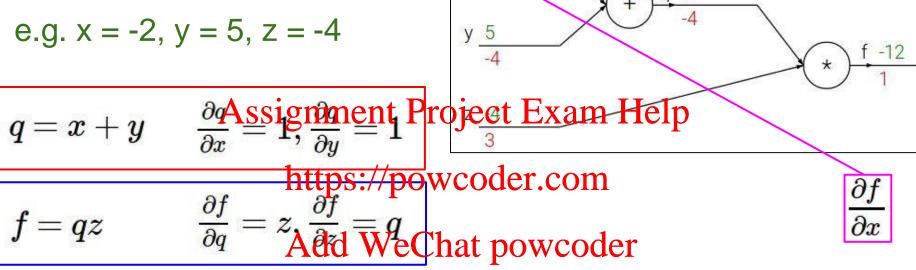
=z $\stackrel{\dot{=}}{\to}$ $\frac{g}{WeC}$ hat powcoder $\frac{\partial f}{\partial f}$ $\frac{\partial g}{\partial f}$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

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$$f(x,y,z) = (x+y)z$$

e.g. x = -2, y = 5, z = -4



Want:
$$\frac{\partial f}{\partial x}$$
, $\frac{\partial f}{\partial y}$, $\frac{\partial f}{\partial z}$

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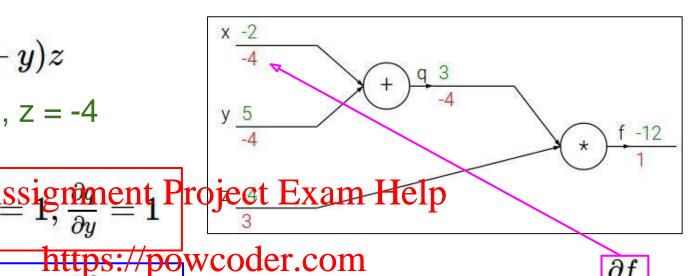
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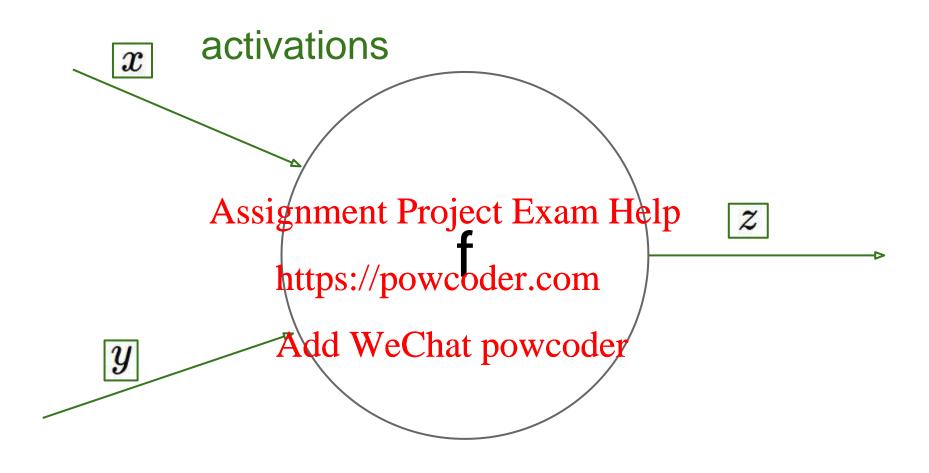
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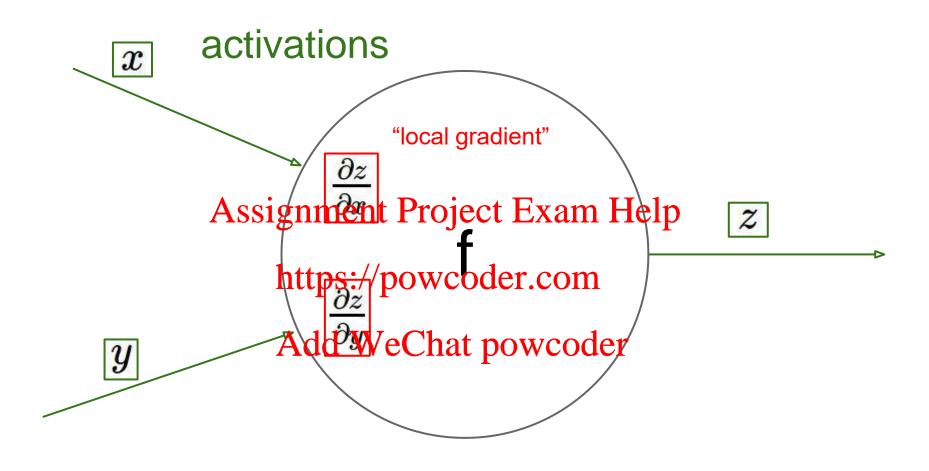


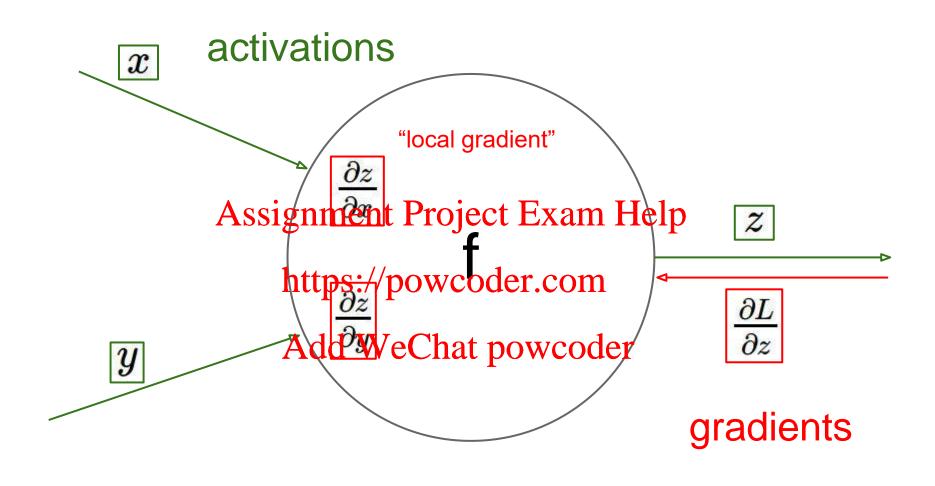
 $=z_{A}\frac{\partial f}{\partial t} = q_{Chat powcoder}$ Chain rule:

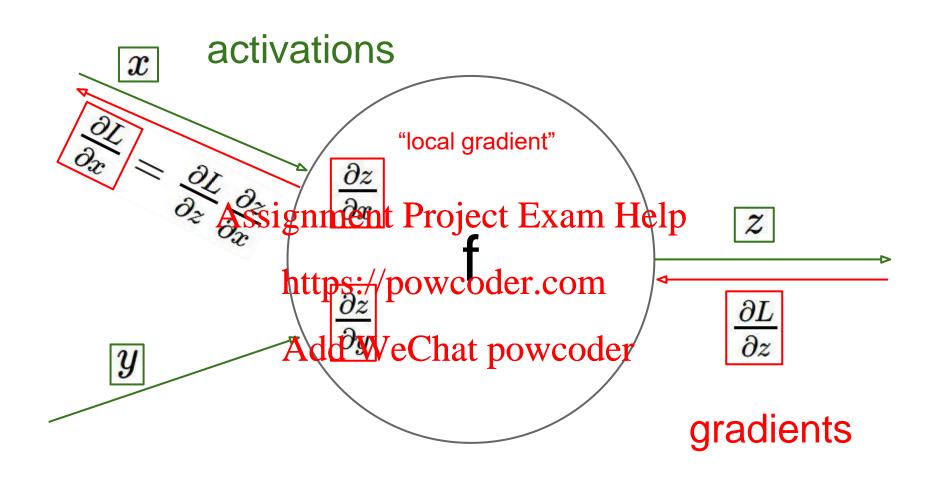
$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$

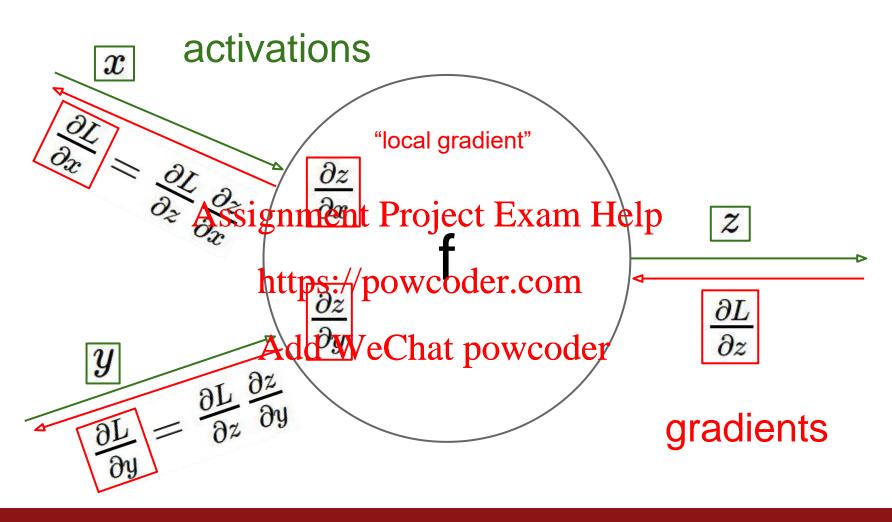
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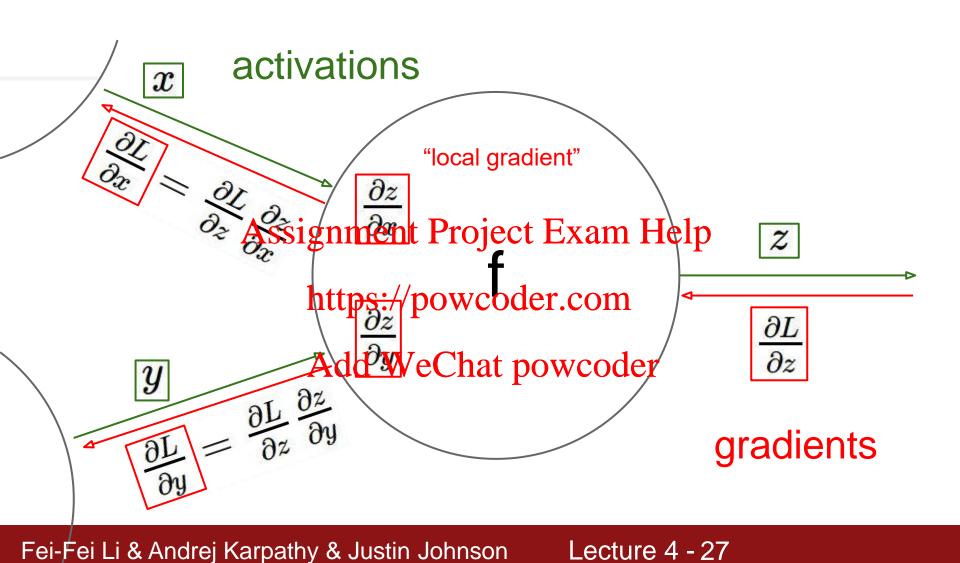




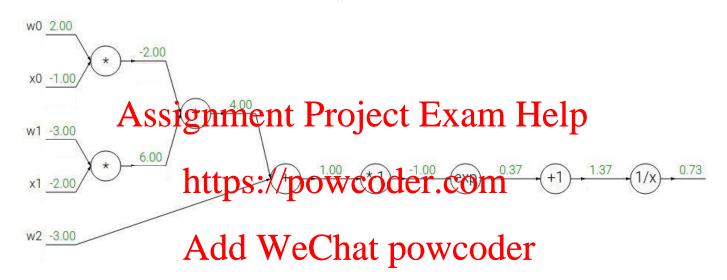






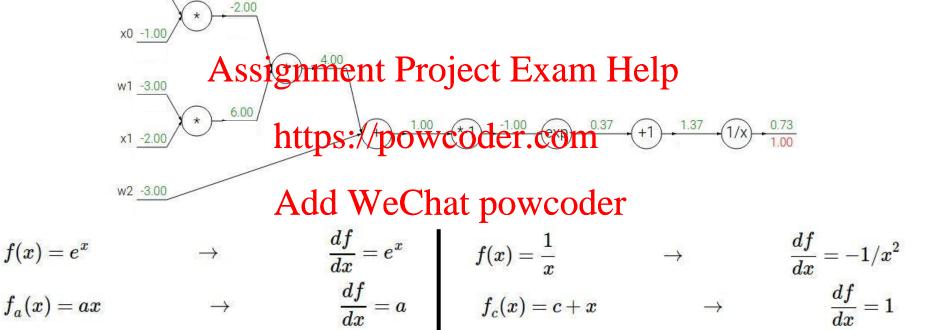


$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$



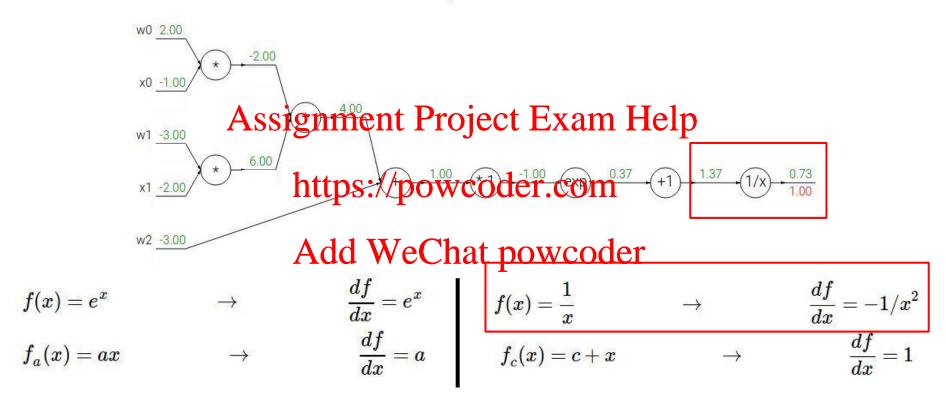
w0 2.00

$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

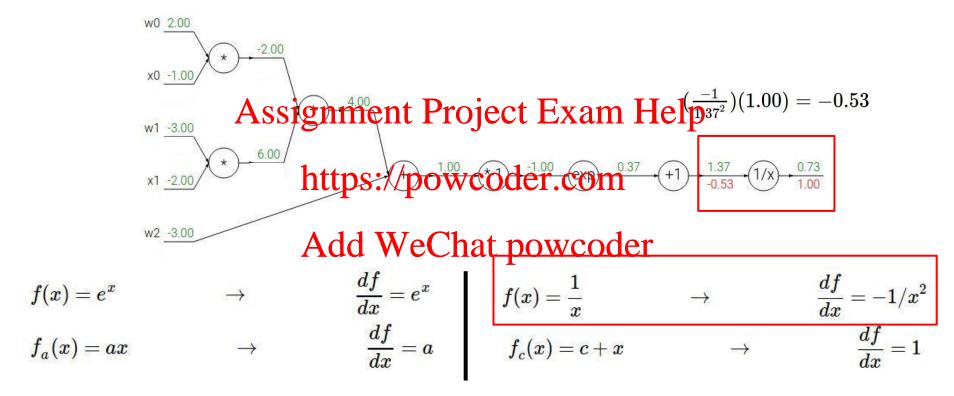


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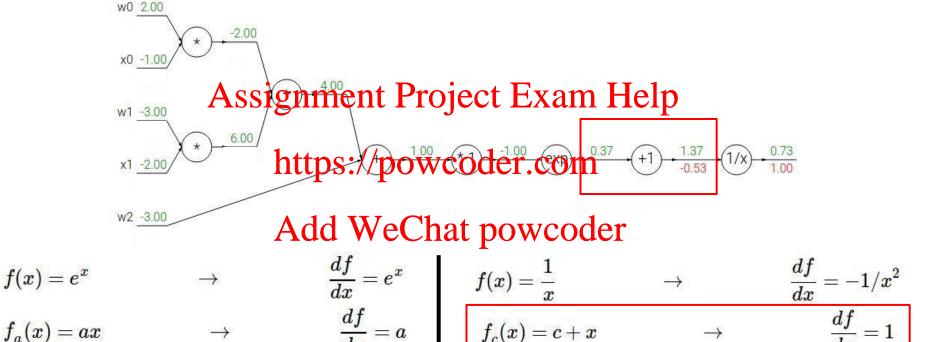
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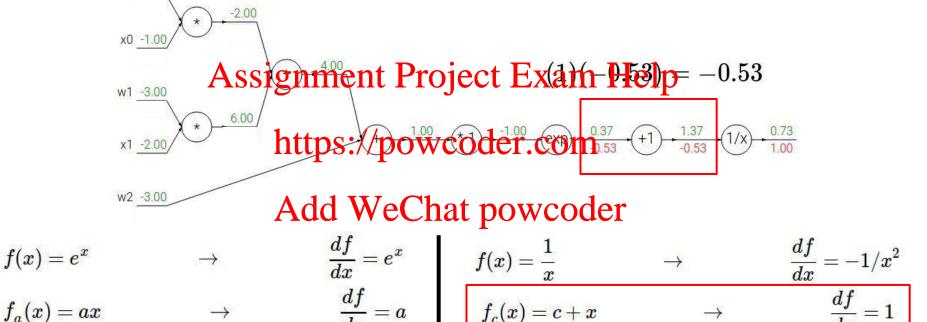


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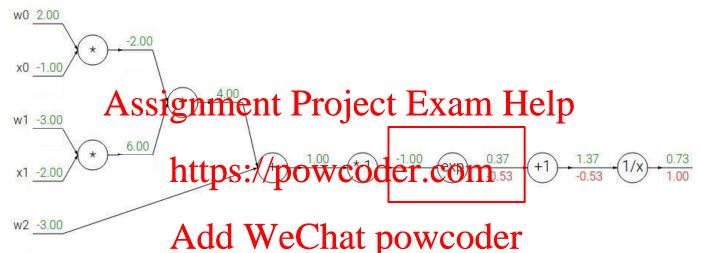
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$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$



$$egin{aligned} f(x) = e^x &
ightarrow & rac{df}{dx} = e^x \ & & \ f_a(x) = ax &
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$$f(x)=rac{1}{x} \qquad \qquad
ightarrow \qquad rac{df}{dx}=-1/x^2 \ f_c(x)=c+x \qquad \qquad
ightarrow \qquad rac{df}{dx}=1$$

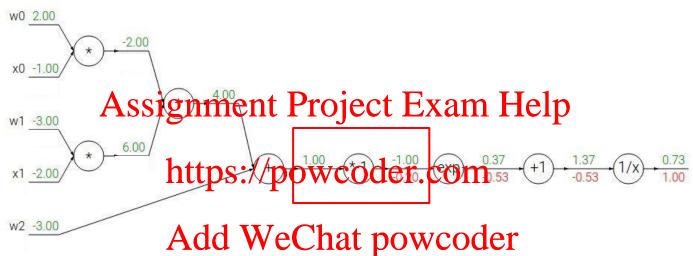
$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$



$$f(x)=e^x \qquad \qquad o \qquad \qquad rac{df}{dx}=e^x \ f_a(x)=ax \qquad \qquad o \qquad \qquad rac{df}{dx}=a$$

$$f(x) = rac{1}{x}
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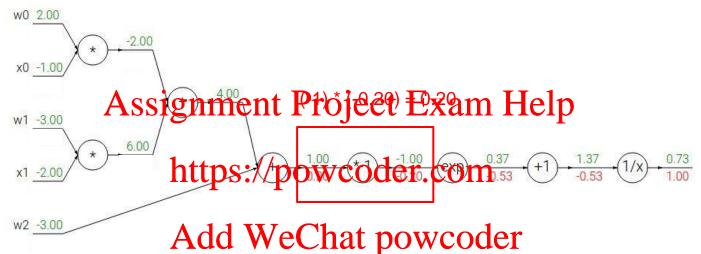
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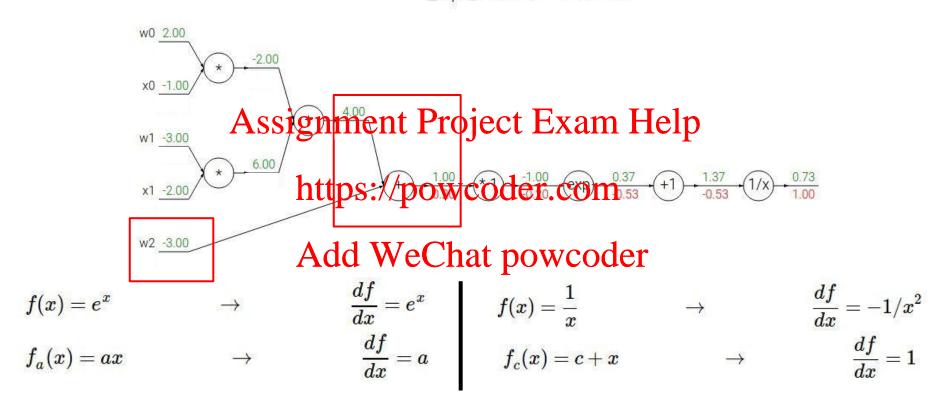
$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$



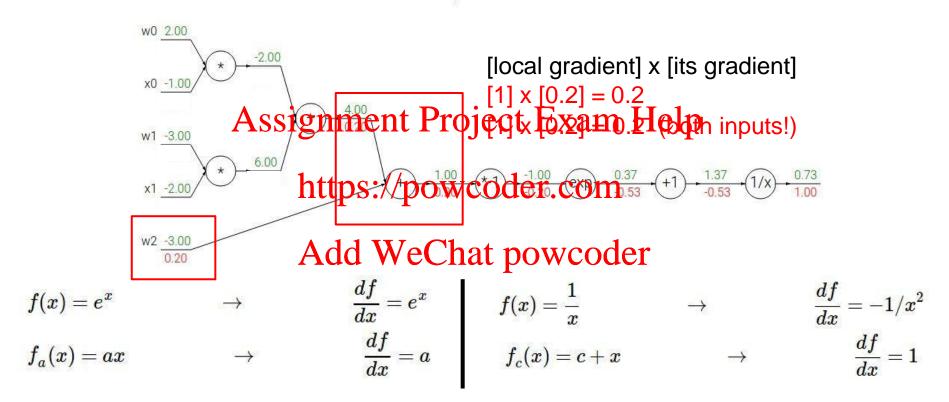
$$f(x) = e^x \hspace{1cm} o \hspace{1cm} rac{df}{dx} = e^x \ f_a(x) = ax \hspace{1cm} o \hspace{1cm} rac{df}{dx} = a$$

$$egin{aligned} f(x) = rac{1}{x} &
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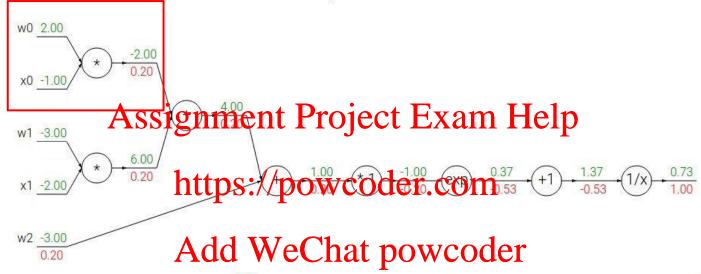
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$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$f(x) = e^x$$

$$f_a(x)=ax$$

$$\frac{df}{dt} = e^x$$

$$egin{aligned} rac{df}{dx} &= e^x & f(x) &= rac{1}{x} &
ightarrow & \ rac{df}{dx} &= a & f_c(x) &= c + x &
ightarrow &
ightarro$$

$$f(x) = \frac{1}{x}$$

$$f_c(x) = c + x$$

$$rac{df}{dx} = -1/x^2$$

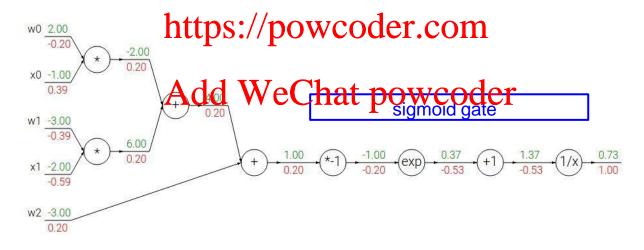
$$rac{df}{dx}=1$$

Another example:
$$f(w,x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$
 [local gradient] x [its gradient] x [its

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\sigma(x) = \frac{1}{1+e^{-x}}$$
 sigmoid function
$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{\left(1+e^{-x}\right)^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right) \left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\sigma(x)$$

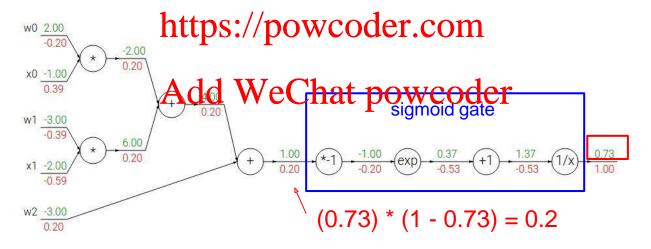
$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{\left(1+e^{-x}\right)^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right) \left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\sigma(x)$$

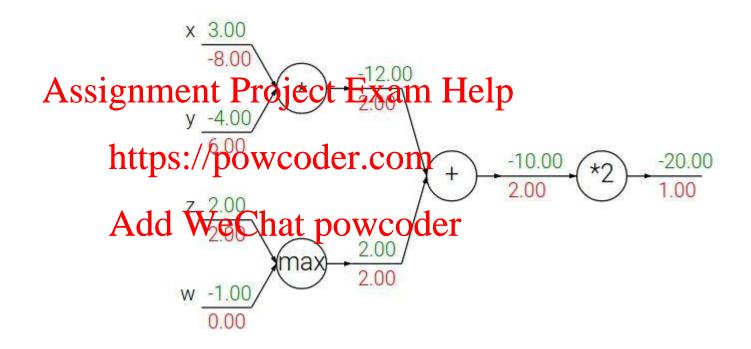


$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

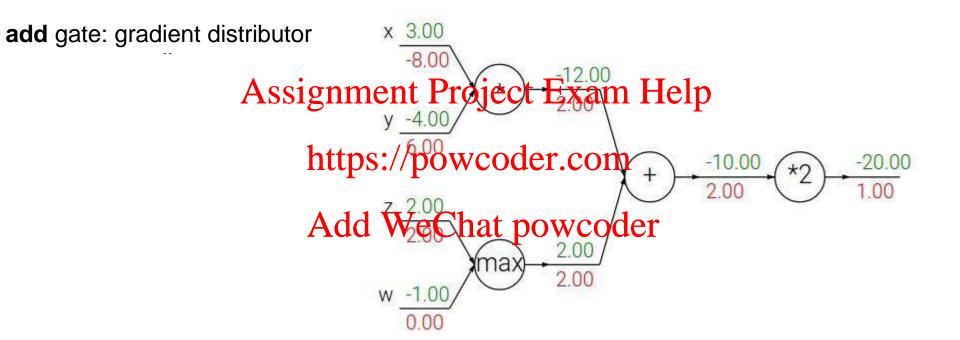
$$\sigma(x) = \frac{1}{1+e^{-x}}$$
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$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right) \left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\sigma(x)$$

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{(1+e^{-x})^2} = \frac{1}{(1+e^{-x})^2}$$
 sigmoid function
$$\frac{d\sigma(x)}{dx} = \frac{1}{(1+e^{-x})^2} = \frac{1}{(1+e^{-x})^2$$

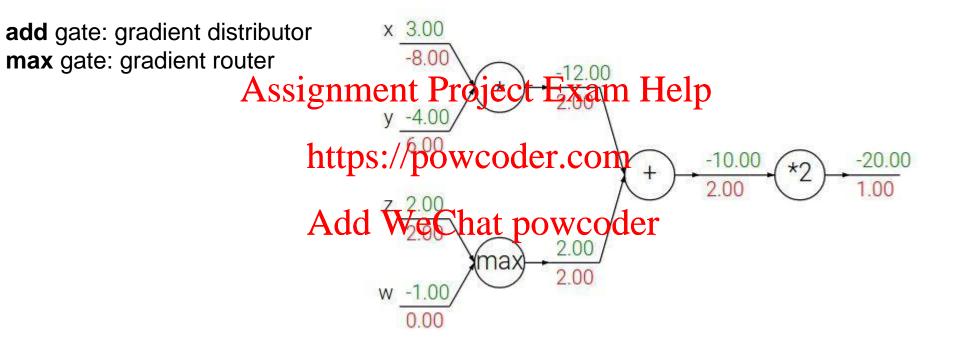




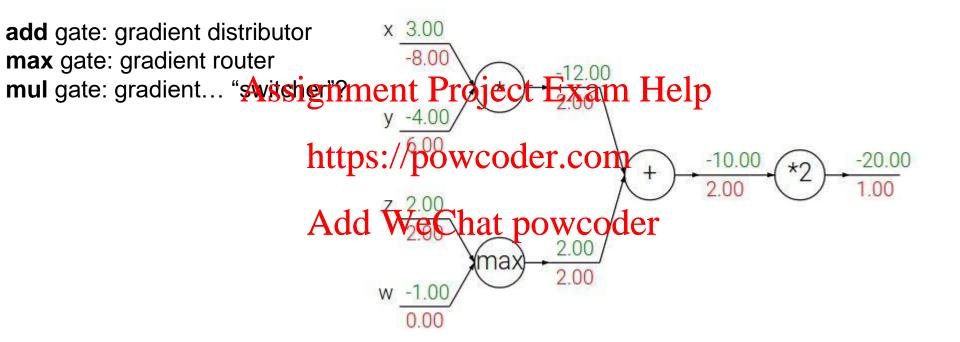
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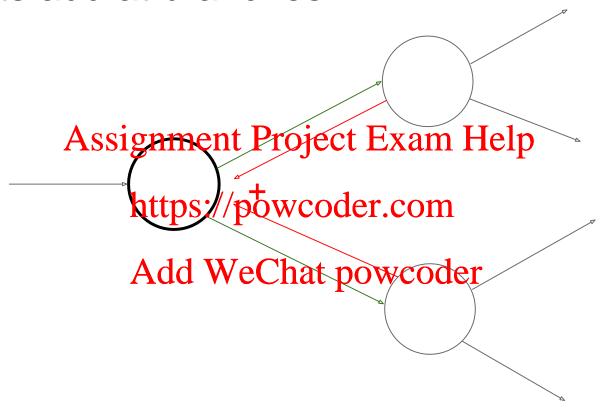


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Gradients add at branches



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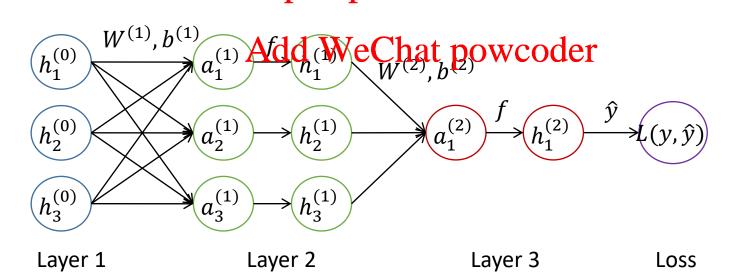


Neural Networks II

Vectorized Backpropagation

Forward Pass

```
Require: Network depth, l
Require: \mathbf{W}^{(i)}, i \in \{1, \dots, l\}, the weight matrices of the model
Require: \mathbf{b}^{(i)}, i \in \{1, \dots, l\}, the bias parameters of the model
Require: \mathbf{x}, the input to process
Require: \mathbf{y}, the target output
\mathbf{h}^{(0)} = \mathbf{x}
for k = 1, \dots, l do
\mathbf{a}^{(k)} = \mathbf{b}^{(k)} + \mathbf{W}^{(k)} \mathbf{h}^{(k-1)}
\mathbf{h}^{(k)} = f(\mathbf{a}^{(k)})
end for
\mathbf{Assignment\ Project\ Exam\ Help}
\hat{\mathbf{y}} = \mathbf{h}^{(l)}
J = L(\hat{\mathbf{y}}, \mathbf{y}) + \lambda \Omega(\theta)
https://powcoder.com
```



Backward Pass

After the forward computation, compute the gradient on the output layer:

$$\boldsymbol{g} \leftarrow \nabla_{\hat{\boldsymbol{y}}} J = \nabla_{\hat{\boldsymbol{y}}} L(\hat{\boldsymbol{y}}, \boldsymbol{y})$$

for
$$k = l, l - 1, ..., 1$$
 do

Convert the gradient on the layer's output into a gradient into the prenonlinearity activation (element-wise multiplication if f is element-wise):

$$g \leftarrow \nabla_{\boldsymbol{a}^{(k)}} J = g \odot f'(\boldsymbol{a}^{(k)})$$

Compute gradients on weights and biases (including the regularization term, where needed):

$$\nabla_{\boldsymbol{b}^{(k)}} J = \boldsymbol{g} + \lambda \nabla_{\boldsymbol{b}^{(k)}} \mathbf{A}(\mathbf{s}) \mathbf{signment} \text{ Project Exam Help} \\ \nabla_{\boldsymbol{W}^{(k)}} J = \boldsymbol{g} \ \boldsymbol{h}^{(k-1)\top} + \lambda \nabla_{\boldsymbol{W}^{(k)}} \Omega(\boldsymbol{\theta})$$

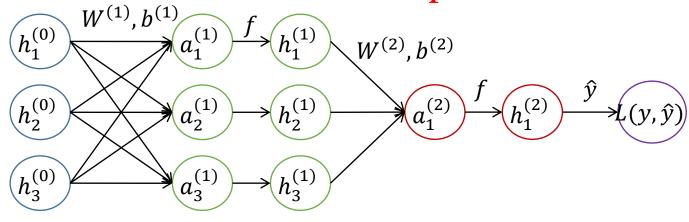
$$\nabla_{\boldsymbol{W}^{(k)}} J = \boldsymbol{g} \; \boldsymbol{h}^{(k-1)\top} + \lambda \nabla_{\boldsymbol{W}^{(k)}}^{\boldsymbol{\cup}} \Omega(\boldsymbol{\theta})$$

Propagate the gradients w.r.t. the next lower-level hidden layer's activations: $g \leftarrow \nabla_{h^{(k-1)}} J = W^{(k)\top} g$ https://powcoder.com

$$\boldsymbol{g} \leftarrow \nabla_{\boldsymbol{h}^{(k-1)}} J = \boldsymbol{W}^{(k)\top} \boldsymbol{g}$$

end for

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Layer 1

Layer 2

Layer 3

Loss

Backpropagation example with effet Pred predients am Help

https://web.stanford.edin/ttps/c//p/n/wedin/e/gredinn-notes.pdf

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Next Class

Neural Networks III: Convolutional Nets:

Convolutional networks.

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Reading: Bishop Ch 5.5