BINARY NUMBERS

Binary Digits (bits)

- data within a computer system are stored in one of 2 physical states (hence the use of binary digits)
  - 0V and 5V
  - charge / NO charge on a transistor gate
  - ferrite core magnetised clockwise or counter clockwise
  - …
- binary digits (bits) and are represented by the values 0 and 1
- binary digits are normally grouped together so they are easier to work with
  - 4 bits = nibble or nybble
  - 8 bits = byte (or 2 nibbles)
  - 16 bits = halfword (or 2 bytes)
  - 32 bits = word (or 4 bytes)

intel
16 bits = WORD
32 bits = DWORD (double word)

1110

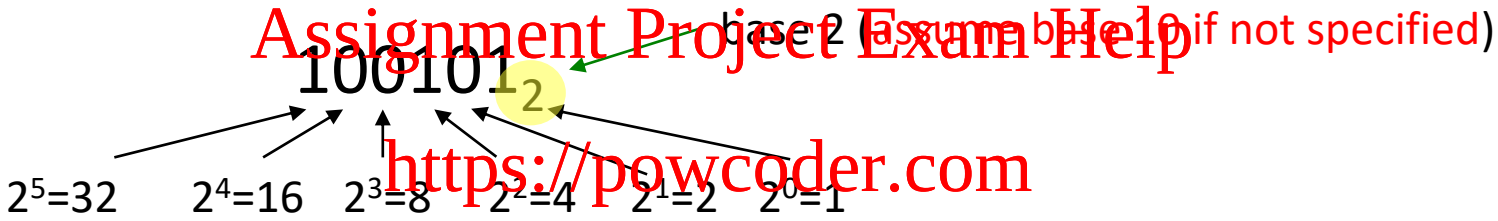
01101001

01111000 01001011

11101101 01111101 01100111 01100101

Unsigned Binary Integers

- unsigned == positive integers ONLY
- converting binary to decimal



$100101_2 = 1\times2^5 + 0\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$100101_2 = 32 + 0 + 0 + 4 + 0 + 1$

$100101_2 = 37$

- similar decimal calculation (base 10)

$37 = 3\times10^1 + 7\times10^0$

$403 = 4\times10^2 + 0\times10^1 + 3\times10^0$

Converting a positive decimal integer to binary

- keep dividing by 2 until 0 and remember remainders
- convert 37 to binary

2	37	
2	18	1
2	9	0
2	4	1
2	2	0
2	1	0
	0	1

37 ÷ 2 = 18 remainder 1

read from bottom ⇒ $37 = 0010\ 0101_2$

- what is 42 in binary? $0010\ 1010_2$
- what is 16 in binary? $0001\ 0000_2$

BINARY NUMBERS

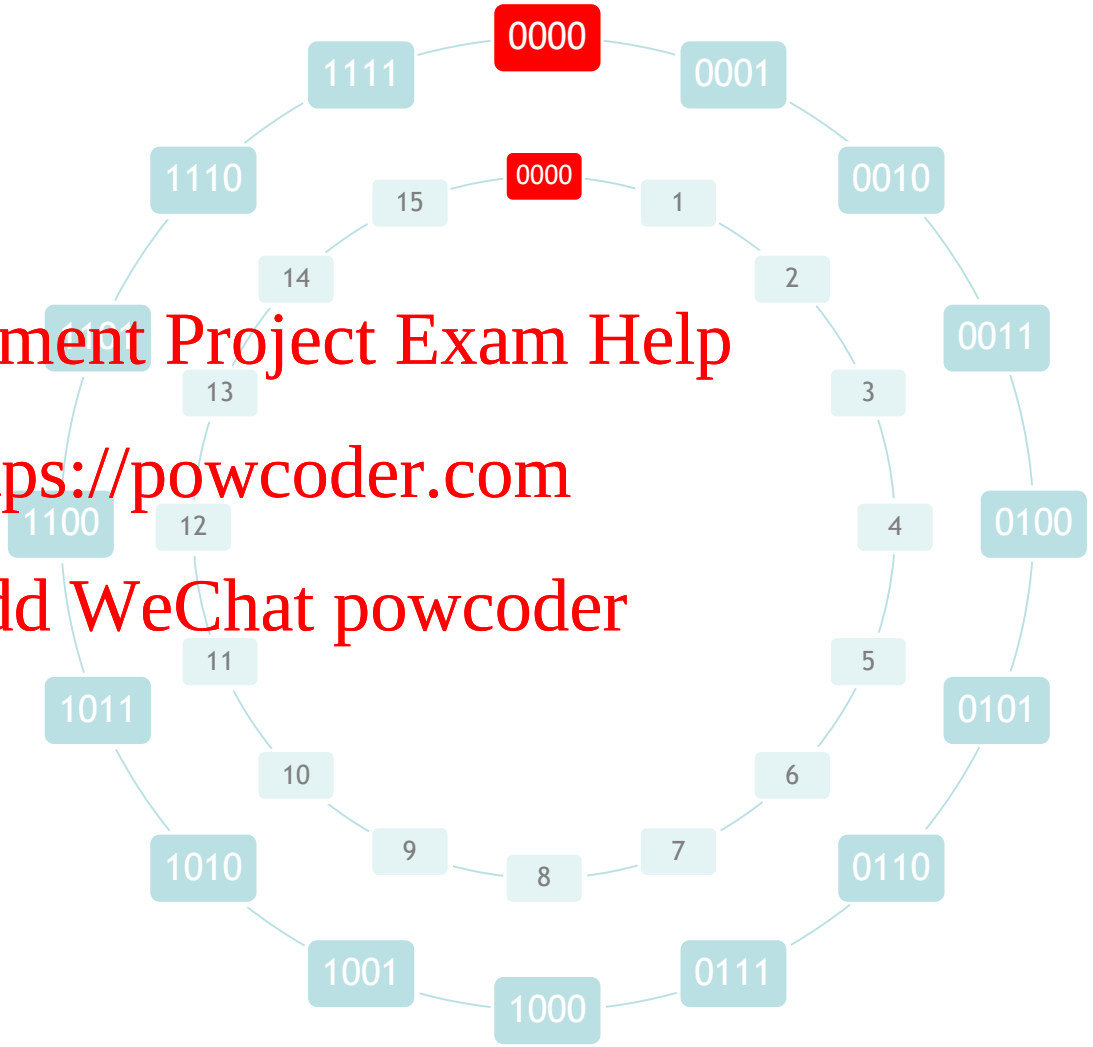
- pictorial view of a 4 bit unsigned binary integer

- 4 bit unsigned binary integer range

  0000_2 *to* 1111_2
  0 *to* 15

- n bit unsigned binary integer range

  0 *to* 2^n-1



BINARY NUMBERS

There are 10 types of people in the world: those who understand binary and those who don't.

Signed Binary Integers

- SIGNED = positive and negative integers
- 2's complement notation
- convert +5 to -5 by taking the 2's complement (invert bits and add 1)

5	0101_2
invert bits	1010_2
add 1	0001_2
-5	1011_2
invert bits	0100_2
add 1	0001_2
5	0101_2

Signed Binary Integers

- same effect achieved by subtracting from 0 (modulo 16 in this case)

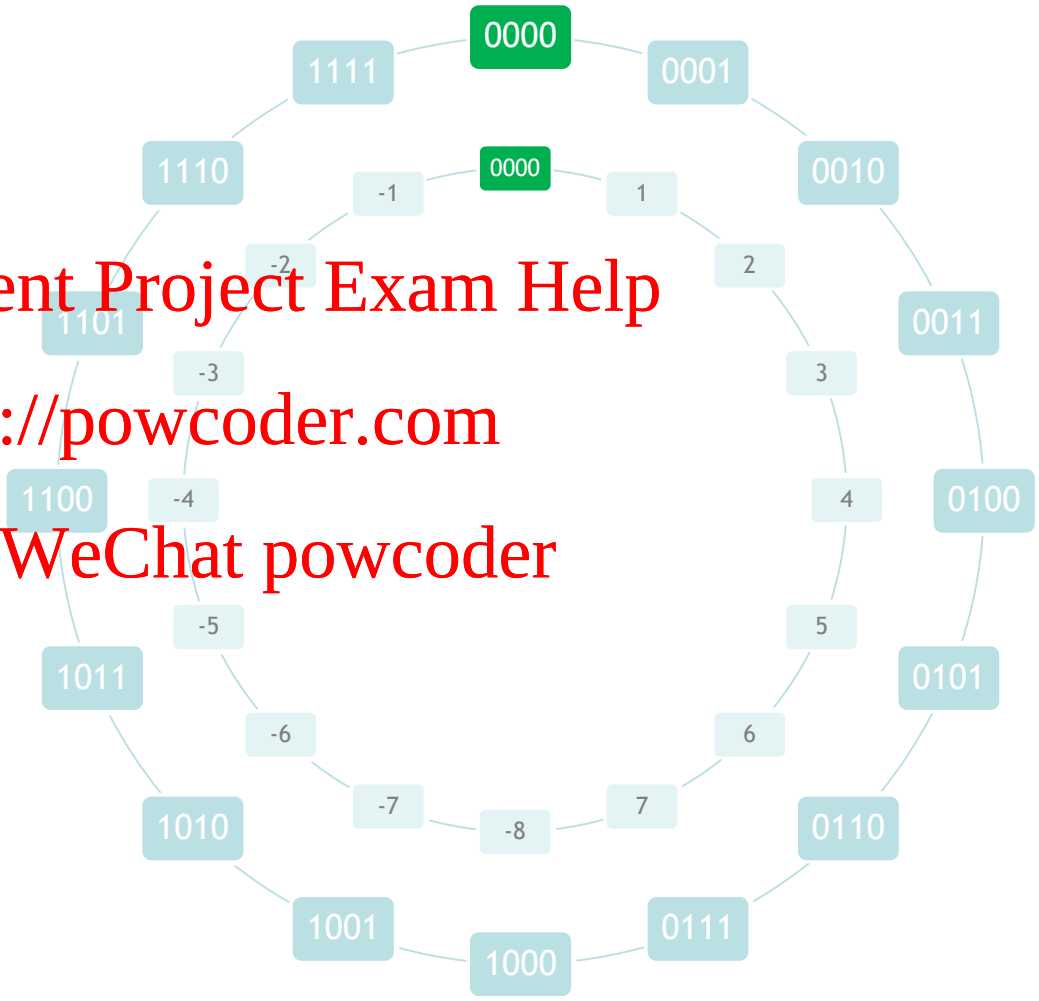
zero	0000_2
subtract 5	0101_2
-5	1011_2

ignore bits beyond the first 4

- 4 bit signed binary integer
  - positive range 0000_2 *to* 0111_2 0 *to* 7
  - negative range 1111_2 *to* 1000_2 -1 *to* -8
- **n** bit signed binary integer range: -2^{n-1} *to* $2^{n-1} - 1$
- most significant bit (MSB) indicates sign (0 - positive, 1 - negative)
- note asymmetrical range – only one zero (do have a +0 and a -0)

BINARY NUMBERS

- pictorial view of a 4 bit signed binary integer
- if unsigned, inner ring would have values 0 to 15
- value depends whether the binary numbers are interpreted as unsigned or signed (the programmer should know!)
- 2's complement notation used because the same CPU hardware can perform unsigned and signed binary arithmetic simultaneously



Binary Numbers

Try these

- what is -42 in binary?

  +42 $0010\ 1010_2$

  invert bits and add 1 $1101\ 0110_2$

- what is -16 in binary?

  +16 $0001\ 0000_2$

  invert bits and add 1 $1111\ 0000_2$

Hexadecimal Notation

- base 16
- easier to handle large binary numbers by grouping 4 binary bits into a hexadecimal digit (starting at the least significant bit)
- consider the following 16 bit unsigned binary integer

$1111\ 1010\ 1100\ 1110_2 = FACE_{16}$

$F\times16^3 + A\times16^2 + C\times16^1 + E\times16^0$
$15\times16^3 + 10\times16^2 + 12\times16^1 + 14\times16^0$
$15\times4096 + 10\times256 + 12\times16 + 14\times1$
$64{,}206$

- what about?

$0000\ 1011\ 1010\ 1101_2 = 0BAD_{16}$

$0\times16^3 + 11\times16^2 + 10\times16^1 + 13\times16^0$
$2{,}989$

BINARY	DEC	HEX
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F

Binary Numbers

Try this

- what decimal value is $FACE_{16}$ if interpreted as a 16 bit signed integer?
- MSB is 1, hence negative so take 2's complement by inverting bits and adding 1

	$FACE_{16}$
invert bits	0531_{16}
add 1	0532_{16}

- convert 0532_{16} to decimal

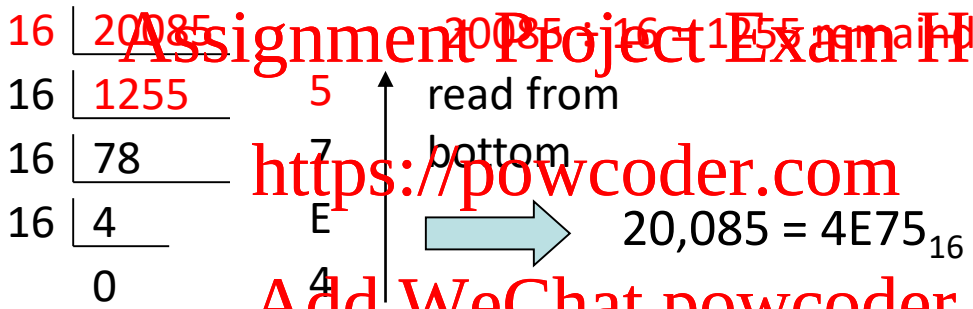
$0\times16^3 + 5\times16^2 + 3\times16^1 + 2\times16^0$
1,330

- $FACE_{16}$ when interpreted as a 16 bit signed integer = -1,330

Binary	Dec	Hex
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F

Decimal to hexadecimal conversion

- convert 20,085 to hexadecimal
- keep dividing by 16 until 0 and remember remainders



- convert -20,085 to hexadecimal (assume 16 bit signed integer)

20,085	$4E75_{16}$
invert bits	$B18A_{16}$
add 1	0001_{16}
-20,085	$B18B_{16}$

BINARY	DEC	HEX
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F

Alternative notation

- when writing ARM Assembly Language, can use the following notion for decimal, hexadecimal and binary integers

1000	no prefix usually means decimal
0x1000	hexadecimal (also used by C/C++ and Java)
&1000	alternative hexadecimal notation
2_1000	binary
n_1000	base n eg. 8_777 is octal (base 8)

Adding Hexadecimal Numbers

- compute 0xA89F + 0x09A1

		unsigned	signed
	$A89F_{16}$	43,167	-22,369
+	$09A1_{16}$	2,465	2,465
	$B240_{16}$	45,632	-19,904

- remember hexadecimal/binary numbers can be interpreted as being unsigned or signed

BINARY	DEC	HEX
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F

Subtracting Hexadecimal Numbers

- compute 0xA89F - 0x09A1

		unsigned	signed
	$A89F_{16}$	43,167	-22,369
-	$09A1_{16}$	2,465	2,465
	$9EFE_{16}$	40,702	-24,834

- remember hexadecimal/binary numbers can be interpreted as being unsigned or signed

BINARY	DEC	HEX
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F

Real Binary Numbers

- binary point (rather than a decimal point)
- what is the value of the following binary number?

11.010101_2

$2^1=2$ $2^0=1$ $2^{-1}=0.5$ $2^{-2}=0.25$ $2^{-3}=0.125$ $2^{-4}=0.0625$ …

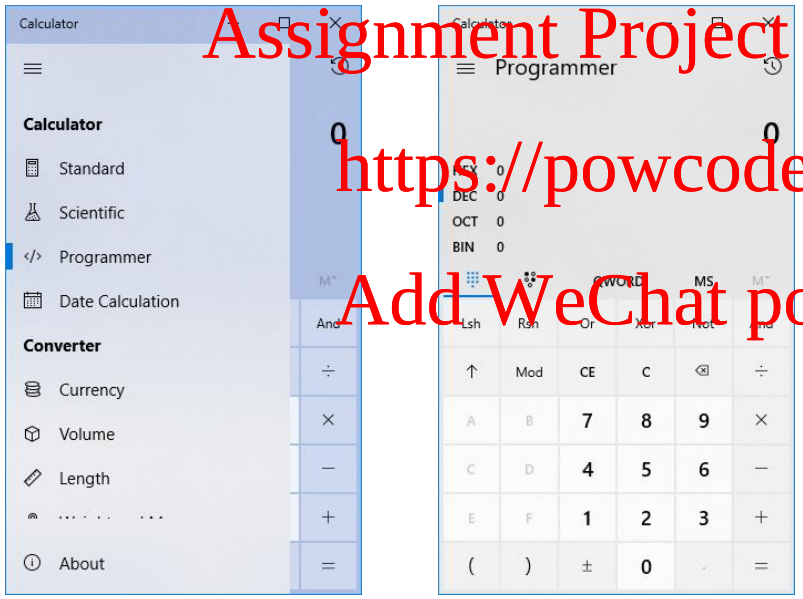
- 2 + 1 + 0.25 + 0.0625 + 0.015625 = 3.328125
- shows how real numbers can be represented as *floating point binary numbers* inside a computer, but further detail is beyond the scope of CS1021

Larger units

- larger units of information
  - 1 kilobyte (KB) = 2^{10} bytes = 1,024 bytes
  - 1 megabyte (MB) = 1024 x 1024 bytes = 1,024 KB = 2^{20} bytes = 1,048,576 bytes
  - 1 gigabyte (GB) = 1,024 MB = 2^{30} bytes = 1,073,741,824 bytes
  - 1 terabyte (TB) = 1024 GB = 2^{40} bytes = 1,099,511,627,776 bytes
- the following units are used when expressing data rates (eg. Mb/s – note the lowercase b)
  - 1 kilobit (Kb/s) = 1,000 bits per second
  - 1 megabit (Mb/s) = 1,000 kilobits = 1,000,000 bits per second
- IEC prefixes KiB, MiB, GiB, … used to differentiate between 1000 and 1024
  - technically 1KB = 1000 bytes and 1KiB = 1024bytes (although KB is often used to mean 1024)

Programmer Calculator

- many calculators have a programmer mode (eg. Windows 10 calculator) for performing binary and hexadecimal arithmetic



- don't use one until you know how to do the calculations "by hand"
- calculators NOT allowed in the CS1021 mid-term test or exams