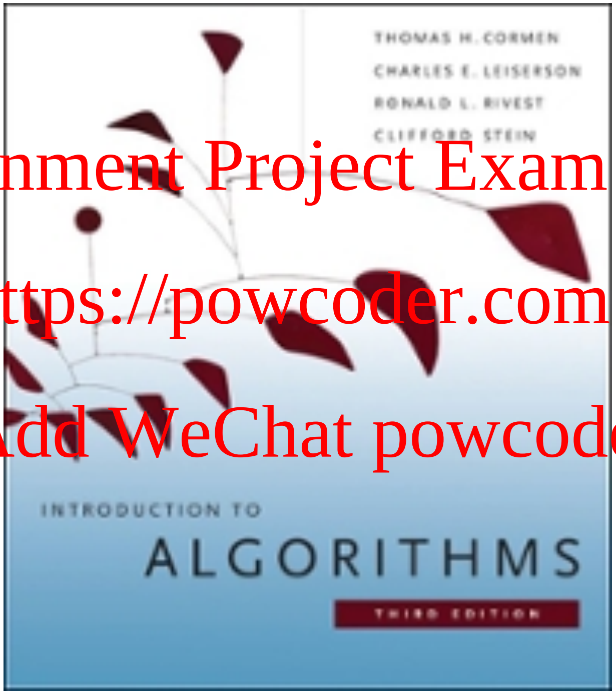
CS146 Data Structures and Algorithms



Part II: Sorting and Order Statistics

Chapter 6: Heapsort

Sorting Algorithm

- Insertion sort :
  - In place: only a constant number of elements of the input array are even sorted outside the array.

- Merge sort :
  - not in place.

- Heap sort : (chapter 6)
  - Sorts n numbers in place in O(n lgn)

Sorting Algorithm

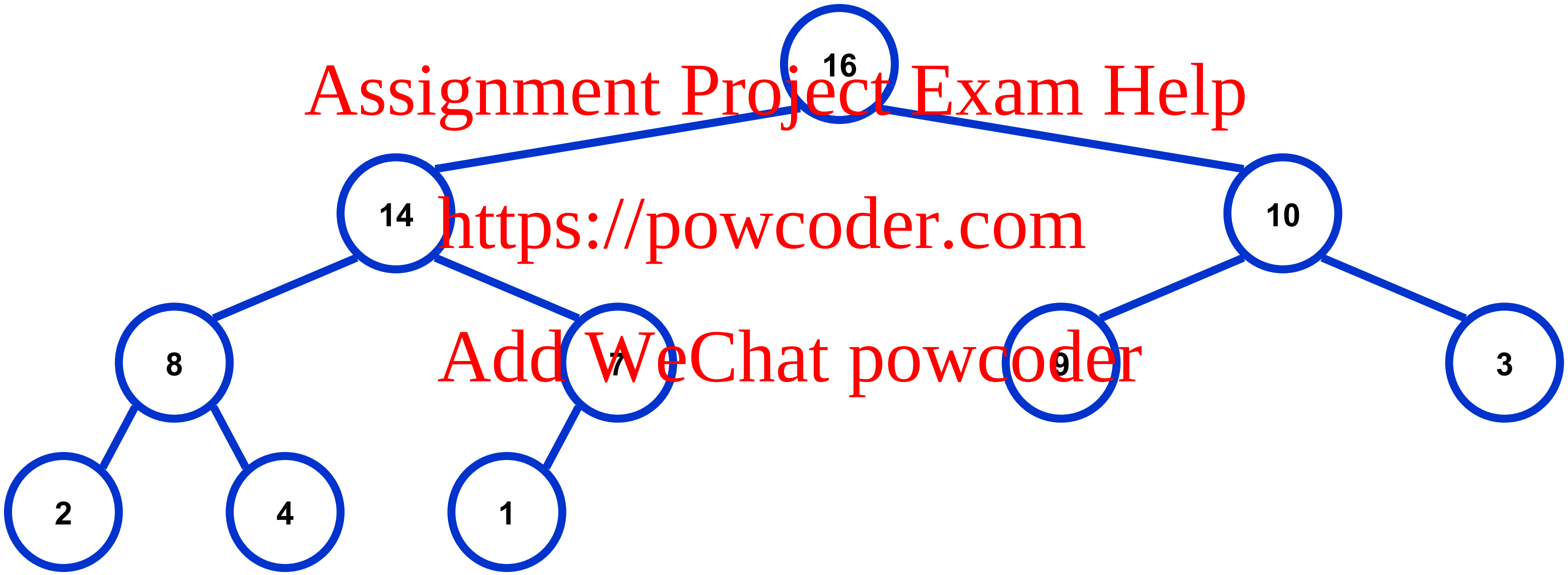
- Quick sort : (chapter 7)
  - worst time complexity O(n^2)
  - Average time complexity O(n lg n)
- Decision tree model : (chapter 8)
  - Lower bound O (n lg n)
  - Counting sort
  - Radix sort
- Order statistics

Sorting Revisited

- So far we've talked about two algorithms to sort an array of numbers
  - What is the advantage of merge sort?
    - Answer: $O(n \lg n)$ worst-case running time
  - What is the advantage of insertion sort?
    - Answer: sorts in place
    - Also: When array "nearly sorted", runs fast in practice
- Next on the agenda: *Heapsort*
  - Combines advantages of both previous algorithms

6.1 Heaps

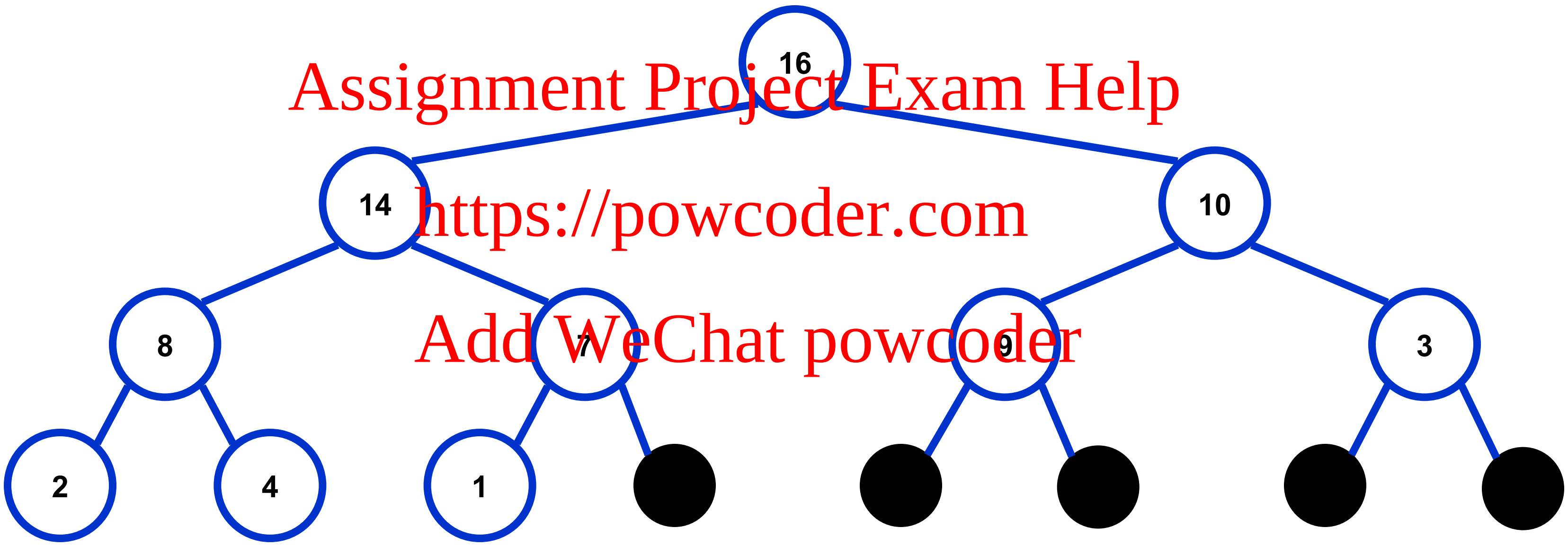
- A *heap* can be seen as a complete binary tree:



  - *What makes a binary tree complete?*
  - *Is the example above complete?*

Heaps

- A *heap* can be seen as a complete binary tree:



  - The book calls them "nearly complete" binary trees; can think of unfilled slots as null pointers

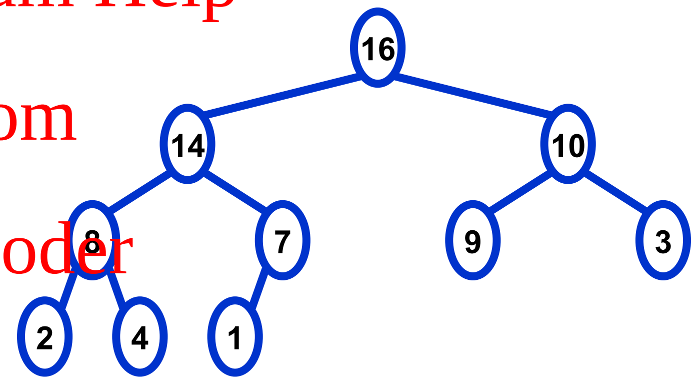
Heaps

- In practice, heaps are usually implemented as arrays:

A =

16	*14*	*10*	*8*	*7*	*9*	*3*	*2*	*4*	*1*

=



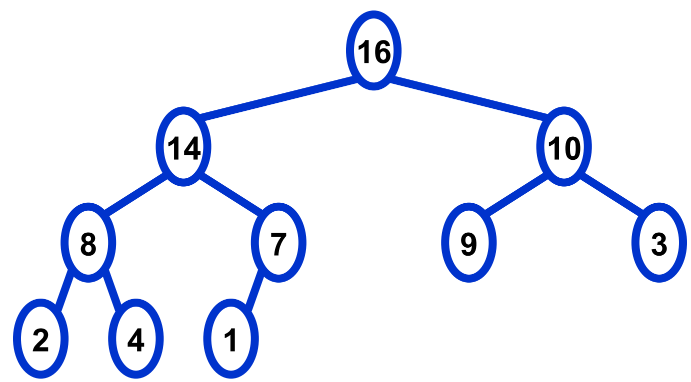
Heaps

- To represent a complete binary tree as an array:
  - The root node is A[1]
  - Node *i* is A[*i*]
  - The parent of node *i* is
    - A[*i*/2] (note: integer divide)
  - The left child of node *i* is
    - A[2*i*]
  - The right child of node *i* is
    - A[2*i* + 1]

A =

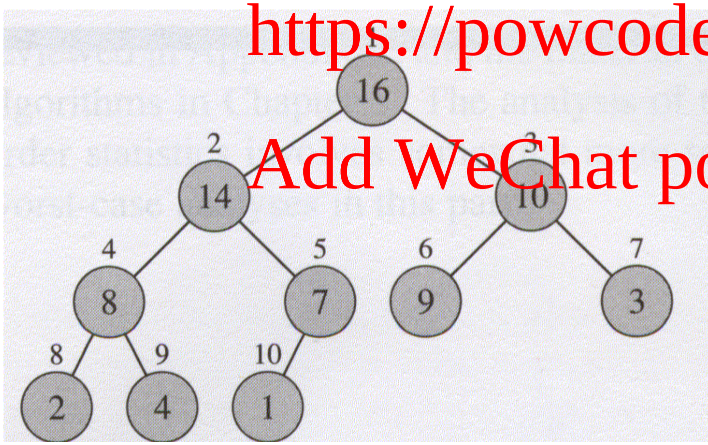
16	14	10	8	7	9	3	2	4	1

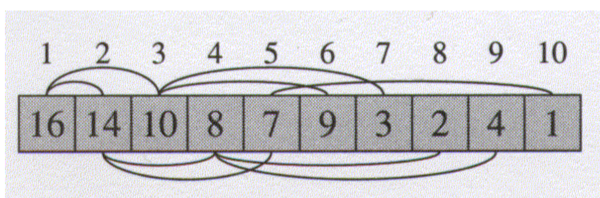
=



Heaps (Binary heap)

- The *binary heap* data structure is an array object that can be viewed as a complete tree.





Parent(i)

  return $\lfloor i/2 \rfloor$

Left(i)

  return 2i

Right(i)

  return 2i+1

Referencing Heap Elements

- So…

```
Parent(i) { return ⌊i/2⌋; }
Left(i) { return 2*i; }
right(i) { return 2*i + 1; }
```


- An aside: *How would you implement this most efficiently?*
  - Trick question, I was looking for "**`i << 1`**", etc.
  - But, any modern compiler is smart enough to do this for you (and it makes the code hard to follow)

The Heap Property

- Heaps also satisfy the *heap property*:

  A[*Parent*(i)] $\geq$ A[i]   for all nodes $i > 1$

  - In other words, the value of a node is at most the value of its parent

  - *Where is the largest element in a heap stored?*

Heap Height

- Definitions:
  - The *height* of a node in the tree = the number of edges on the longest downward path to a leaf
  - The height of a tree = the height of its root
- *What is the height of an n-element heap? Why?*
- This is nice: basic heap operations take at most time proportional to the height of the heap

The Heap Property

- Max-heap : $A[Parent(i)] \geq A[i]$

- Min-heap : $A[Parent(i)] \leq A[i]$

- The height of a node in a tree: the number of edges on the longest simple downward path from the node to a leaf.

- The height of a tree: the height of the root
- The height of a heap: $O(\lg n)$.

Pop Quiz

1. What are the minimum and maximum numbers of elements in a heap of height *h* ?

- There is a most $2^{h+1} - 1$ vertices in a complete binary tree of height *h*. Since the lower level need not be filled we may only have 2^h vertices.

Pop Quiz

2. Show that an n-element heap has height (lg n)

- Since the height of an n-element heap must satisfy that $2^h \le n \le 2^{h+1}-1 < 2^{h+1}$.

- We have $h \le \lg n < h+1$.
- h is an integer so $h = \lg n$.