CS 314 Principles of Programming Languages

Lecture 14: Functional Programming

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Class Information

- Homework 5 will be posted this weekend.
- Project 1 due 10/23, in less than one week.

Review: LISP

- Functional language developed by John McCarthy in the mid 50’s
- Semantics based on *Lambda Calculus*
- All functions operate on lists or symbols called: “S-expression”
- Only five basic functions:
  list functions ***con***, ***car***, ***cdr***, ***equal***, ***atom***,
  & one conditional construct: ***cond***
- Useful for LISt-Processing (LISP) applications
- Program and data have the same syntactic form “S-expression”
- Originally used in Artificial Intelligence

Review: SCHEME

- Developed in 1975 by Gerald J. Sussman and Guy L. Steele
- A dialect of LISP
- Simple syntax, small language
- Closer to initial semantics of LISP as compared to COMMON LISP
- Provide basic list processing tools
- Allows functions to be first class objects

SCHEME

- Expressions are written in prefix, parenthesized form

  (*function* arg_1 arg_2 … arg_n)

  (+ 4 5)

  (+ (* 3 4) (- 5 3))

- Operational semantics:

  In order to evaluate an expression

  1. Evaluate function to a function value
  2. Evaluate each arg_i in order to obtain its value
  3. Apply function value to these values

S-expression

S-expression ::= Atom | (S-expression) | S-expression S-expression
Atom ::= Name | Number | #t | #f | ε

#t
()
(a b c)
(a (b c) d)
((a b c) (d e (f)))
(1 (b) 2)

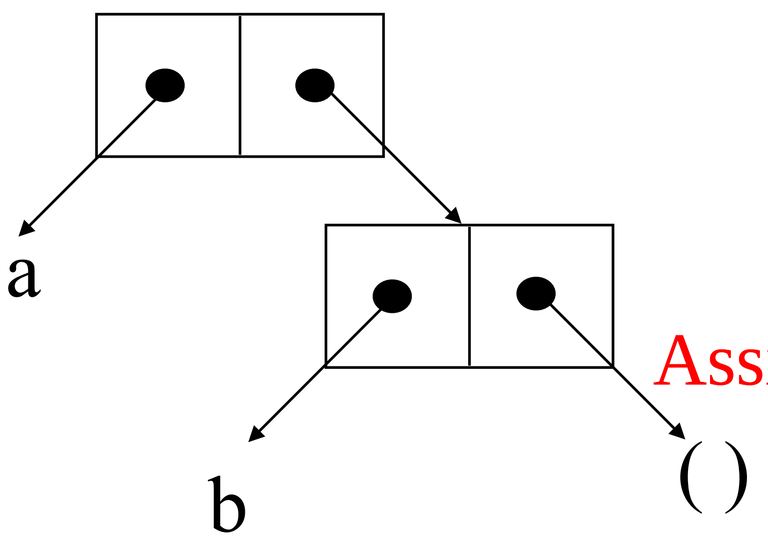
Lists have nested structure

Lists in Scheme

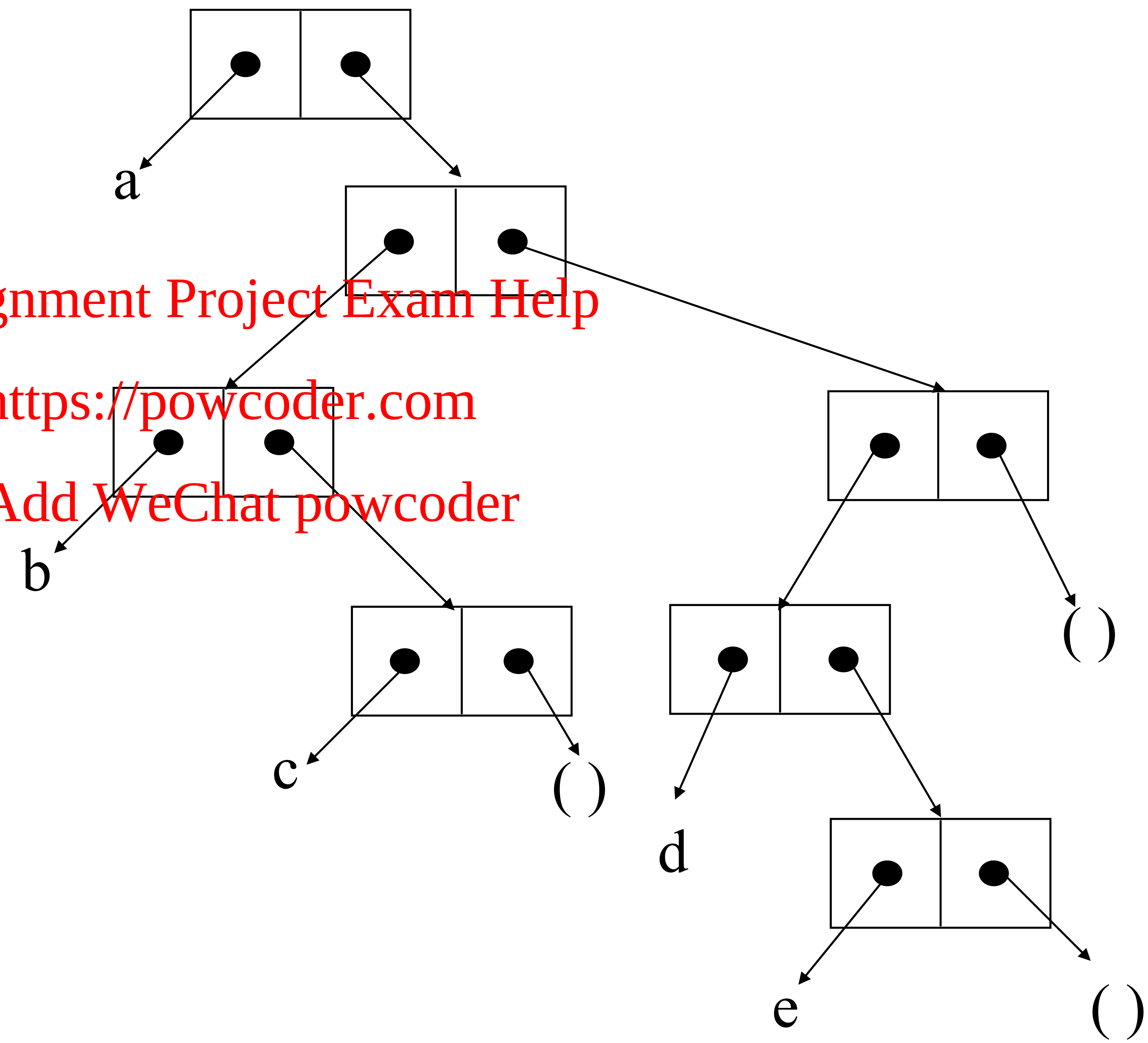
The building blocks for lists are pairs or cons-cells.

Proper lists use the empty list "( )" as an "end-of-list" marker.

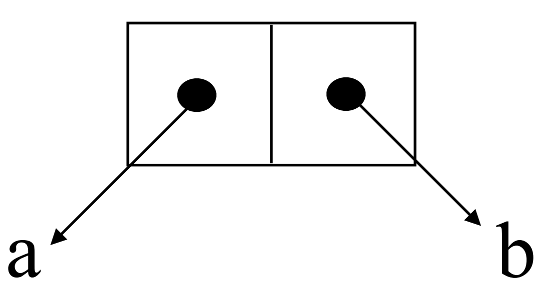
(a b)



(a (b c) (d e))



(a . b)



Special (Primitive) Functions

- **eq?**: identity on names (atoms)
- **null?**: is list empty?
- **car**: select first element of the list
  (contents of address part of register)
- **cdr**: select rest of the list
  (contents of decrement part of register)

- **(cons element list)**: constructs lists by adding **element** to the front of **list**

- **quote or ':** produces constants

Do not evaluate the ' the content after '. Treat them as list of literals.

Quotes Inhibit Evaluation

```
> ( cons 'a (cons 'b  '(c d)) )
(a b c d)

;; Now if we quote the second argument
>  ( cons 'a '(cons 'b  '(c d)) )
(a cons 'b '(c d))
```



```
;; If we unquote the first argument
> ( cons a (cons 'b  '(c d)) )
a: undefined;
 cannot reference undefined identifier
   context …
```

Special (Primitive) Functions

- '( ) is an empty list
- (car '(a b c)) = a
- (car '((a) b (c d))) = (a)
- (cdr '(a b c)) = (b c)
- (cdr '((a) b (c d))) = (b (c d))

Special (Primitive) Functions

- **car** and **cdr** can break up any list:

  (car (cdr (cdr '((a) b (c d))))) = (c d)

  (cdr '((a) b (c d))) = (b (c d))

- **cons** can construct any list:

  (cons 'a '( )) = (a)

  (cons 'd '(e)) = (d e)

  (cons '(a b) '(c d)) = ((a b) c d)

  (cons '(a b c) '((a) b)) = ((a b c) (a) b)

Review: Defining Scheme Functions

(define <fcn-name> **(lambda (** *<fcn-params>* **)** *<expression>* **))**

<u>Example:</u> Given function **pair?** (true for non-empty lists, false o/w) and function **not** (boolean negation):

<fcn-params>

```
(define  atom?
         ( lambda ( object )
                 ( not ( pair? object ) )
         )
 )
```

<expression>

Conditional Execution: if

(if *<condition> <result1> <result2>***)**

1. Evaluate <condition>
2. If the result is a “true value” (i.e., anything but #f), then evaluate and return <result1>
3. Otherwise, evaluate and return <result2>

```
(define abs-val
        ( lambda (x)
            ( if ( >= x 0) x (- x) )
        )
)
```



```
(define rest-if-first
        ( lambda (e l)
                  ( if ( eq? e (car l) ) ( cdr l ) '() )
        )
)
```

Conditional Execution: cond

(**cond** (*<condition1> <result1>*)
(*<condition2> <result2>*)
...
(*<conditionN> <resultN>*)
(**else** *<else-result>*)) *; optional else clause*

1. Evaluate conditions in order until obtaining one that returns a #t value
2. Evaluate and return the corresponding result
3. If none of the conditions returns a true value, evaluate and return <else-result>

Conditional Execution: cond

```
(define rest-if-first
  (lambda (e l)
          (cond ( (null? l)         '( )  )
                ( (eq? e (car l) ) ( cdr l) )
                ( else              '( ) )
          )
  )
)
```

If first item matches, return the rest of the list.


```
(define abs-val
  (lambda ( x )
          ( cond ( (>= x 0) x )
                 (else (- x)  )
          )
  )
)
```

If “x” is non-negative, return x. Otherwise, return -x.

Recursive Scheme Functions: abs-List

```
(define abs-list
  (lambda (l)
          (if (null? l) '()
                    (cons (abs (car l)) (abs-list (cdr l)))
          )
   )
)
```


- (abs-list '(1 -2 -3 4 0)) ⇒ (1 2 3 4 0)
- (abs-list '()) ⇒ ()

Recursive Scheme Functions: Append

- (append '(1 2) '(3 4 5)) ⇒ (1 2 3 4 5)
- (append '(1 2) '(3 (4) 5)) ⇒ (1 2 3 (4) 5)
- (append '() '(1 4 5)) ⇒ (1 4 5)
- (append '(1 4 5) '()) ⇒ (1 4 5)
- (append '() '()) ⇒ ()


```
(define append
  (lambda (x y)
          (cond ((null? x) y)
                ((null? y) x)
                (else (cons (car x) (append (cdr x) y)))
          )
  )
)
```

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  )
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```

Equality Checking

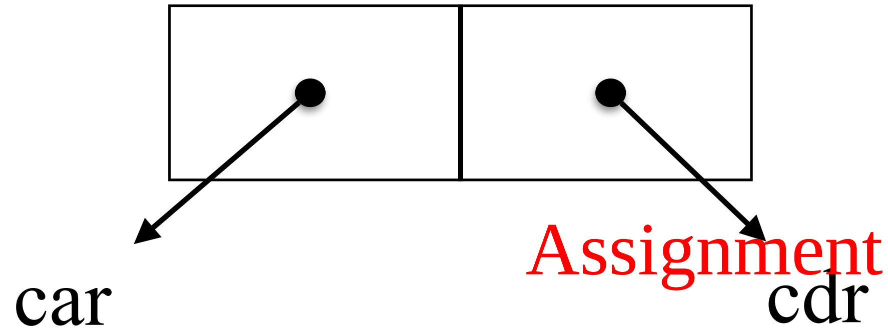
The eq? predicate does not work for lists.

Why not?

- (**cons** 'a '()) produces a new list
- (**cons** 'a '()) produces another new list
- **eq?** checks whether two arguments are the *same*
- (**eq?** (**cons** 'a '()) (**cons** 'a '())) evaluates to #f

Equality Checking

Lists are stored as pointers to the first element (car) and the rest of the list (cdr). This "elementary" data structure, the building block of a list, is called a *pair*



Symbols are stored uniquely, so **eq?** works on them.

Equality Checking for Lists

For lists, need a comparison function to check for the **same structure** in two lists.

```
(define equal?
  (lambda (x y)
            (or ( and (atom? x) (atom? y) (eq? x y) )
                ( and (not (atom? x)) (not (atom? y))
                      (equal? (car x) (car y))
                      (equal? (cdr x) (cdr y))
                )
            )
  )
)
```

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                    (equal? (cdr x) (cdr y))
              )
          )
  )
)
```


- (equal? 'a 'a) evaluates to #t
- (equal? 'a 'b) evaluates to #f
- (equal? '(a) '(a)) evaluates to #t
- (equal? '((a)) '(a)) evaluates to #f

Scheme: Functions as First Class Values (Higher-Order)

Functions as arguments:

(define f (**lambda** (g x) (g x)))

- (f number? 0) ⇒ (number? 0) ⇒ #t
- (f length '(1 2)) ⇒ (length '(1 2)) ⇒ 2
- (f (lambda (x) (* 2 3)) 3) ⇒ ((lambda (x) (* 2 3)) 3) ⇒ (* 2 3) ⇒ 6

Scheme: Functions as First Class Values (Higher-Order)

Computation, i.e., **function application** is performed by reducing the initial S-expression (program) to an S-expression that represents a value. **Reduction** is performed by **substitution**, i.e., replacing formal by actual parameters in the function body.

Examples for S-expressions that directly represent values, i.e., cannot be further reduced:

- function values (e.g.: (**lambda** (x) e))
- constants (e.g.: 3, #t)

Computation completes when S-expression cannot be further reduced

Higher-Order Functions (Cont.)

Functions as returned value:

```
(define plusn
        ( lambda (n)
                (lambda (x) (+ n x))
        )
)
```

Return a function

- **(plusn 5)** evaluates to a function that adds 5 to its argument:

  Question: How would you write down the value of (plusn 5)?

  (lambda (x) (+ 5 x))

- **((plusn 5) 6) = ?**

Higher-Order Functions (Cont.)

In general, any n-ary function

(**lambda** (x_1 x_2 … x_n) e)

can be rewritten as a nest of *n* unary functions:

(**lambda** (x_1)

(**lambda** (x_2)

(… (**lambda**(x_n) e) …)))

Higher-Order Functions (Cont.)

In general, any n-ary function

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can be rewritten as a nest of *n* unary functions:

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(**lambda** (x_2)

(… (**lambda**(x_n) e) …)))

This translation process is called currying. It means that having functions with multiple parameters do not add anything to the expressiveness of the language:

((**lambda** (x_1 x_2 ... x_n) e) v_1 v_2 ... v_n)

$\downarrow$

(… (((**lambda** (x_1)
(**lambda** (x_2)
...
(**lambda** (x_n) e)...)) v_1) v_2) ... v_n)

Higher-order Functions: map

function

```
(define map
  (lambda (f l)
    (if (null? l)
       '( )
       ( cons (f (car l))  ((map f (cdr l)) )  )
    )
  )
)
```

list

Apply *f* to the first element of *l*

Apply *map and f* to the rest of *l*

- **map** takes two arguments: a function **f** and a list **l**
- **map** builds a new list by applying the function to every element of the (old) list

Higher-Order Functions: map

- Example:
  (map **abs** '(-1 2 -3 4)) ⇒ (1 2 3 4)
  (map (**lambda** (x) (+ 1 x)) '(-1 2 3) ⇒ (0 3 4)

- Actually, the built-in **map** can have more than two arguments:
  (map + '(1 2 3) '(4 5 6)) ⇒ (5 7 9)

More on Higher-Order Functions

reduce Higher order function that takes a binary, associative operation and uses it to “roll up” a list

```
(define reduce
  (lambda ( op l id)
            ( if (null? l)
                id
                (op (car l) (reduce op (cdr l) id)) ) ) )
```


Example:

```
(reduce + '(10 20 30) 0) ⇒
(+  10 (reduce + '(20 30) 0)) ⇒
(+  10 (+ 20 (reduce + '(30) 0))) ⇒
(+  10 (+ 20 (+ 30 (reduce + '() 0)))) ⇒
(+  10 (+ 20 (+ 30 0))) ⇒
60
```

Higher-Order Functions

Compose higher order functions to form compact powerful functions

```
(define sum
        (lambda (f  l)
                (reduce + ( map f  l ) 0) ) )
```



```
(sum (lambda (x) (* 2 x)) '(1 2 3) ) ⇒
```



```
(reduce (lambda (x y) (+ 1 y)) '(a b c) 0) ⇒
```

Next Lecture

Things to do:

- Read Scott, Chapter 11.1 - 11.3