CS373 Data Mining and Machine Learning Lecture 1

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(originally prepared by Tommi Jaakkola, MIT CSAIL)

Course topics

- Supervised learning
  - linear and non-linear classifiers, kernels
  - rating, ranking, collaborative filtering
  - model selection, complexity, generalization
  - conditional Random fields, structured prediction

- Unsupervised learning, modeling
  - mixture models, topic models
  - Hidden Markov Models
  - Bayesian networks
  - Markov Random Fields, factor graphs

Machine learning

- Learning from examples

+1

-1

+1

-1

training set

Machine learning

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-1

+1

-1

training set

The training set of labeled examples specifies the learning task only implicitly

Machine learning

- Learning from examples

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-1

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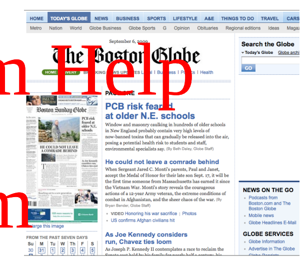
-1

training set



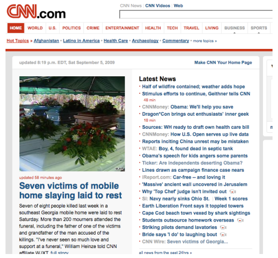
?

Our goal is to accurately label new websites that were not part of the training set

Machine learning

- Learning from examples

+1



-1

+1

-1

training set

predicted label

new website

$$y = f\Big($$  $$\Big)$$

classifier = mapping from websites to labels

“Examples”

- We will have to first represent the examples (websites) in a manner that can be easily mapped to labels

news article

White House officials consulted with the Justice Department in preparing a list of U.S. attorneys who would be removed.

(NYT 03/13/07)

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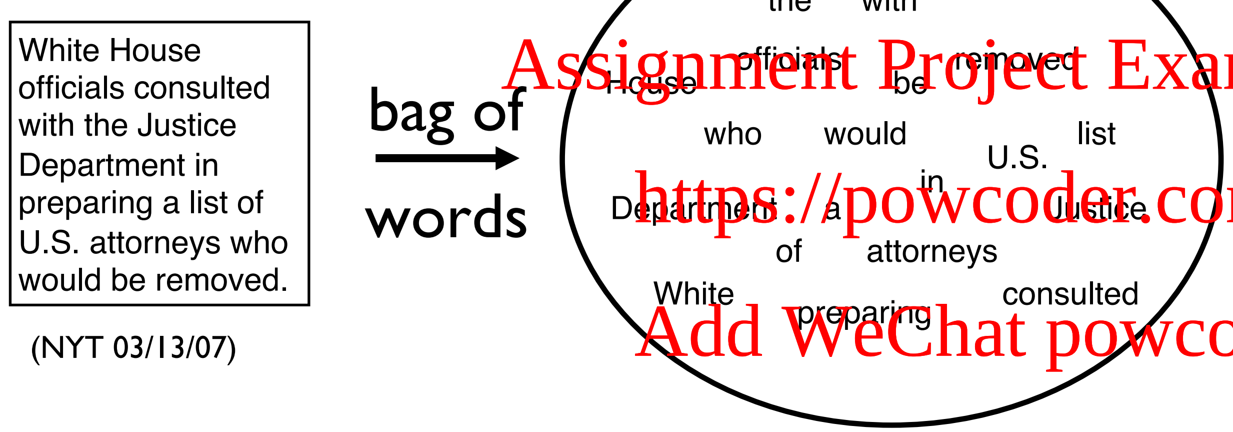

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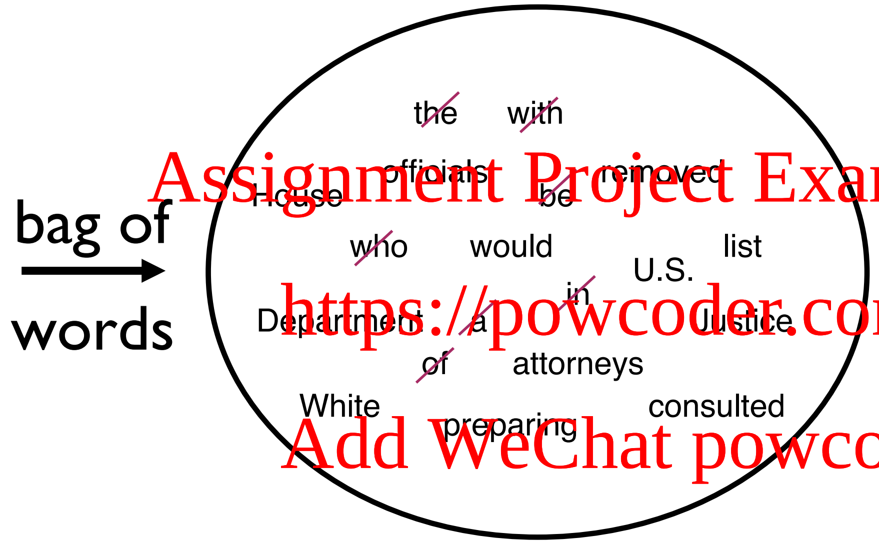

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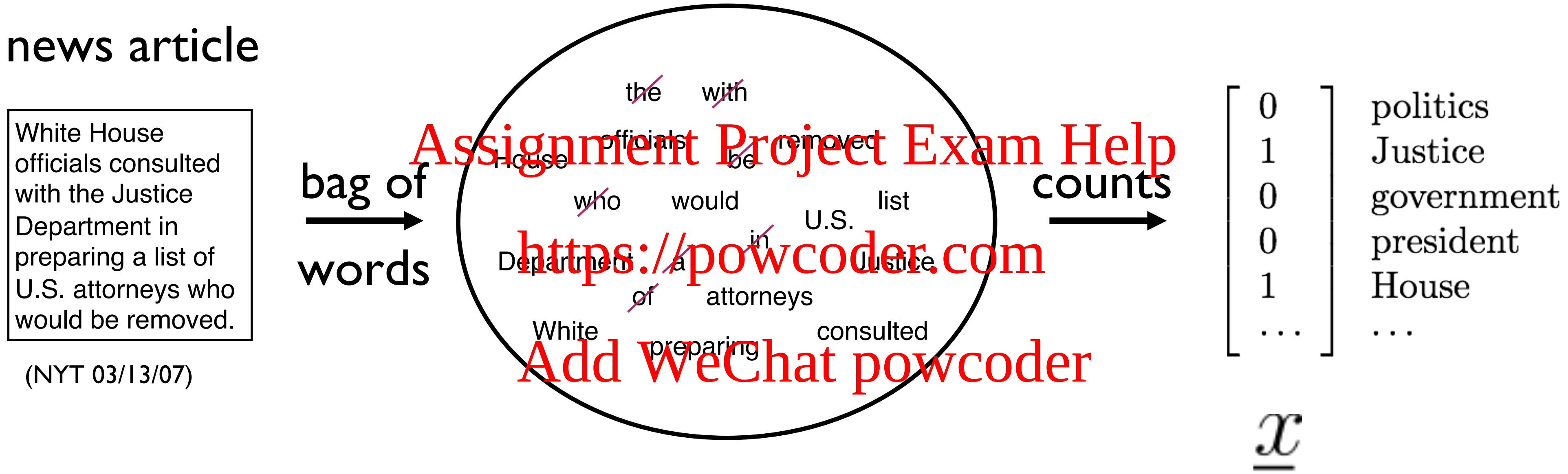
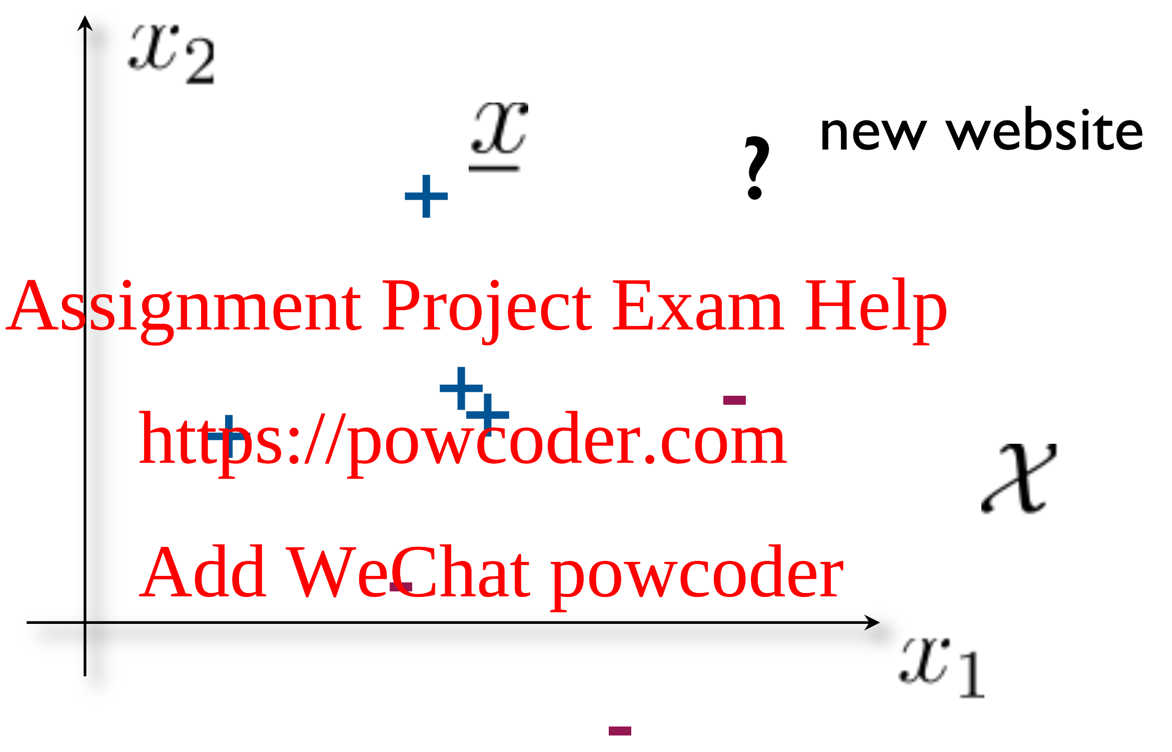


a vector whose coordinates (features) specify how many times (or whether) particular words appeared in the article

The learning task

- The training set is now a set of labeled points



- Our goal is to find a "good" classifier $f : \mathcal{X} \to \{-1, 1\}$ based on the training set $D = \{(\underline{x}_i, y_i)_{i=1,\ldots,n}\}$ so that $f(\underline{x})$ correctly labels any new websites $\underline{x}$

The learning task

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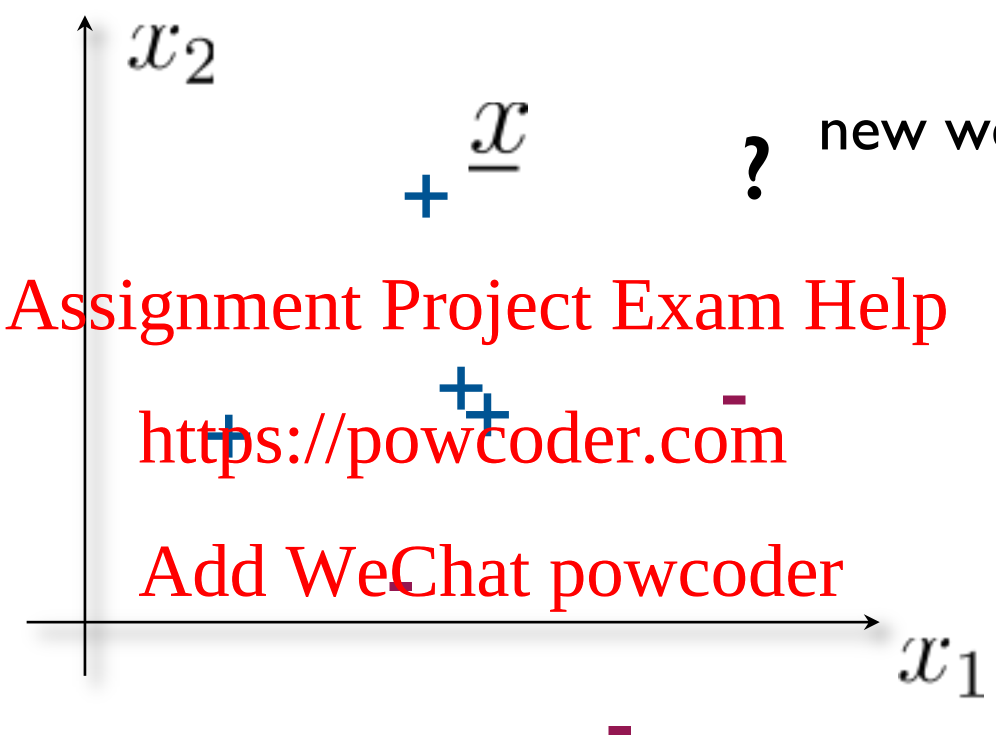
Part 1:
Model selection
what type of classifiers should we consider?



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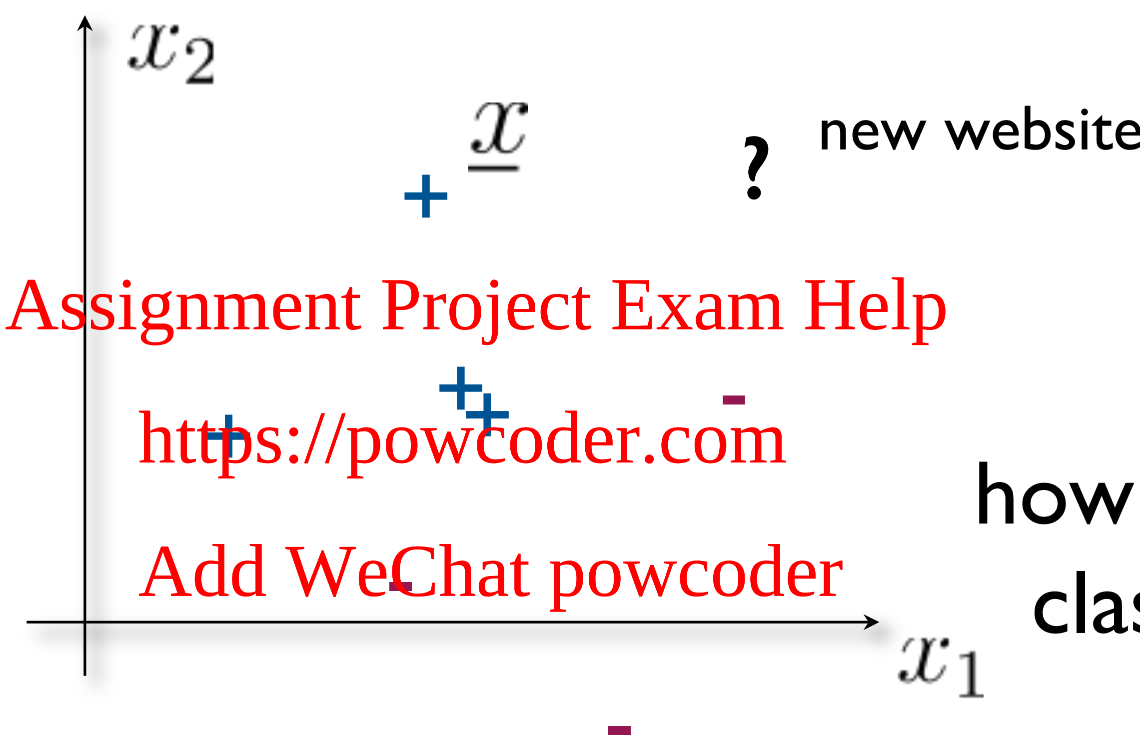

Part 2:
Estimation
how to select the best classifier in the set?

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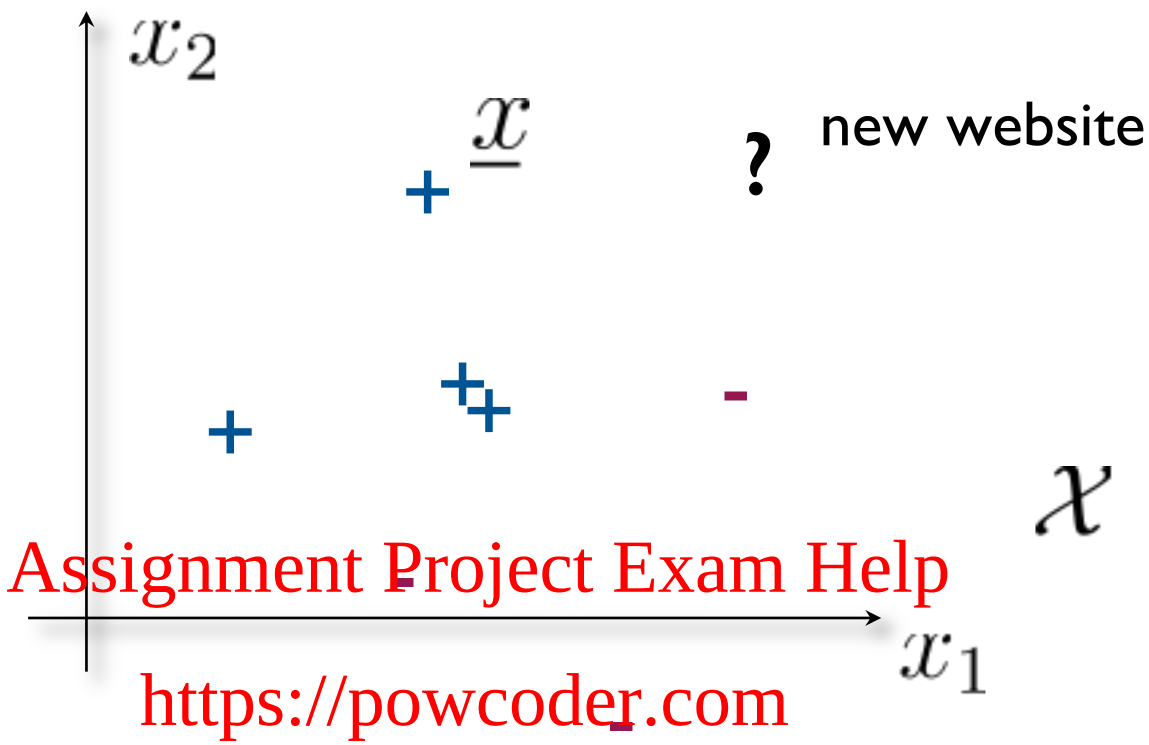
Part 1: allowing all classifiers?

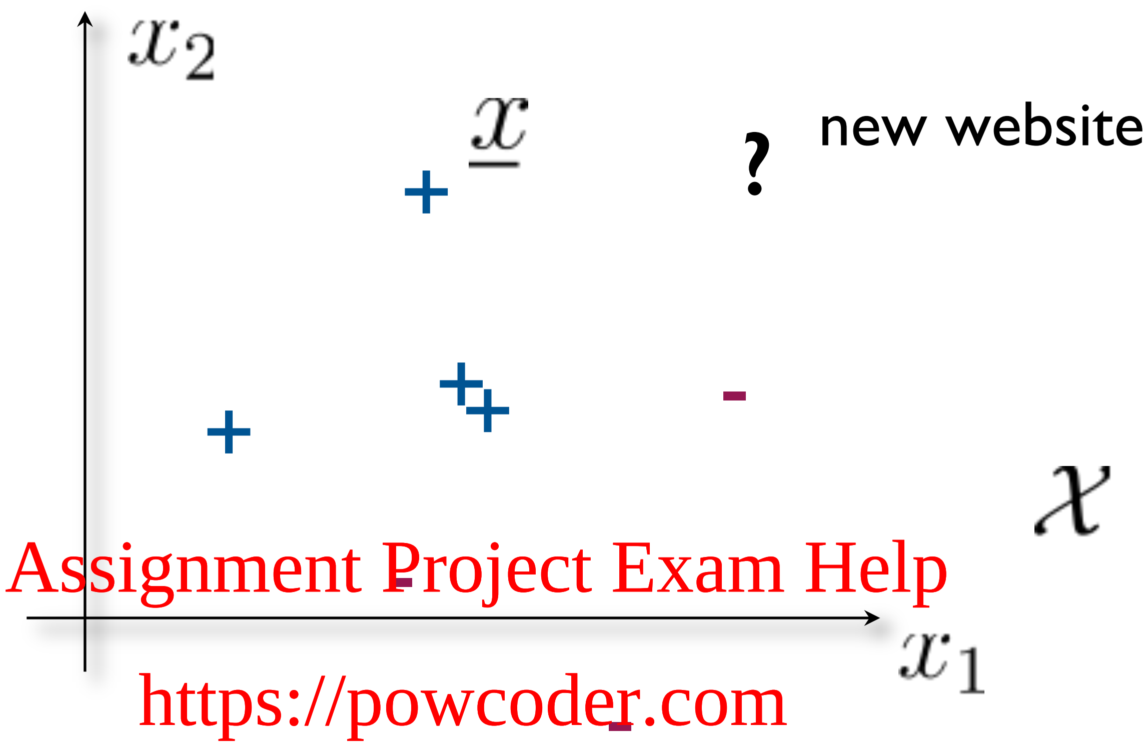


- We can easily construct a “silly classifier” that perfectly classifiers any distinct set of training points

$$f(\underline{x}) = \begin{cases} y_i, & \text{if } \underline{x} = \underline{x}_i \text{ for some } i \\ -1, & \text{otherwise} \end{cases}$$

- But it doesn’t “generalize” (it doesn’t classify new points very well)

Part 1: allowing few classifiers?



- We could instead consider very few alternatives such as

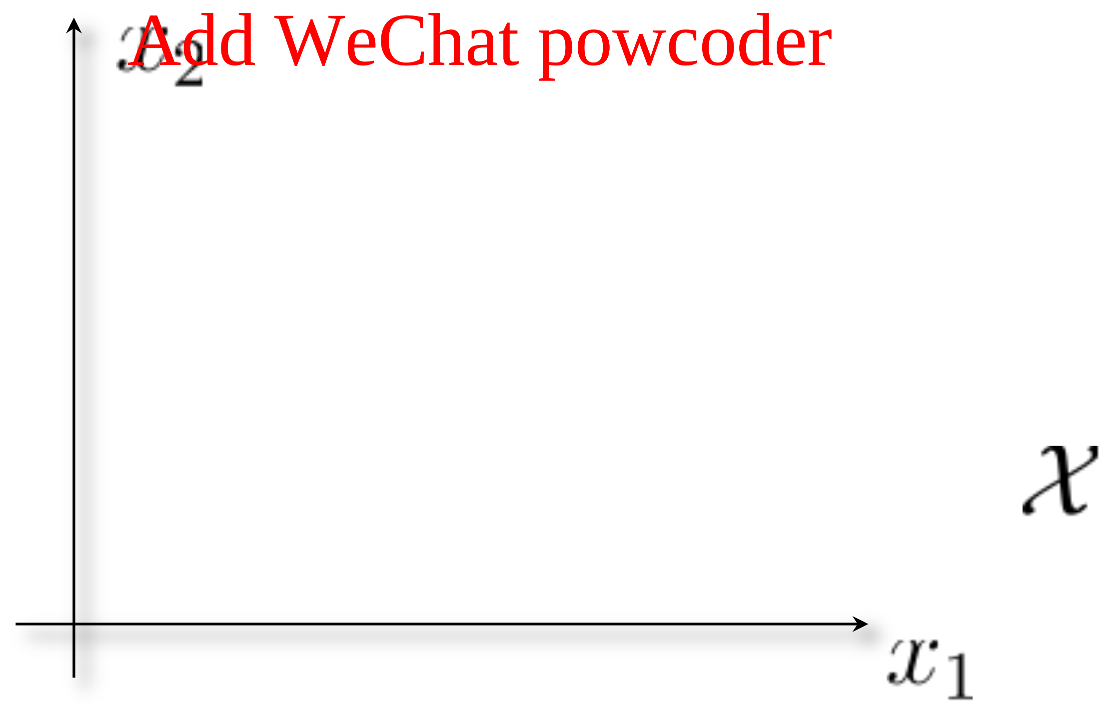
$$
\begin{aligned}
f(\underline{x}) &= 1, \quad \text{for all} \quad \underline{x} \in \mathcal{X}, \quad \text{or} \\
f(\underline{x}) &= -1, \quad \text{for all} \quad \underline{x} \in \mathcal{X},
\end{aligned}
$$

- But neither one classifies even training points very well

Part 1: linear classifiers

- A linear classifier (through origin) with parameters $\underline{\theta}$ divides the space into positive and negative halves

$$\begin{aligned} f(\underline{x};\underline{\theta}) &= \mathrm{sign}\big(\,\underline{\theta}\cdot\underline{x}\,\big) = \mathrm{sign}\big(\,\theta_1 x_1 + \ldots + \theta_d x_d\,\big) \\ &= \begin{cases} +1, & \text{if } \underline{\theta}\cdot\underline{x} > 0 \\ -1, & \text{if } \underline{\theta}\cdot\underline{x} \leq 0 \end{cases} \end{aligned}$$



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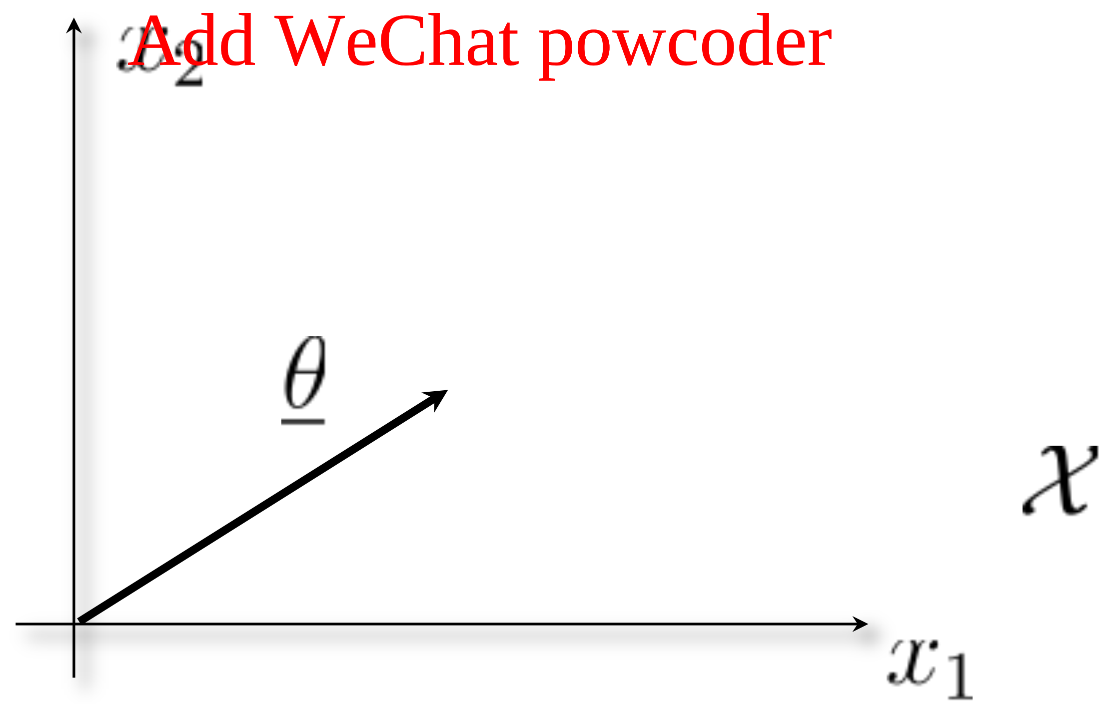

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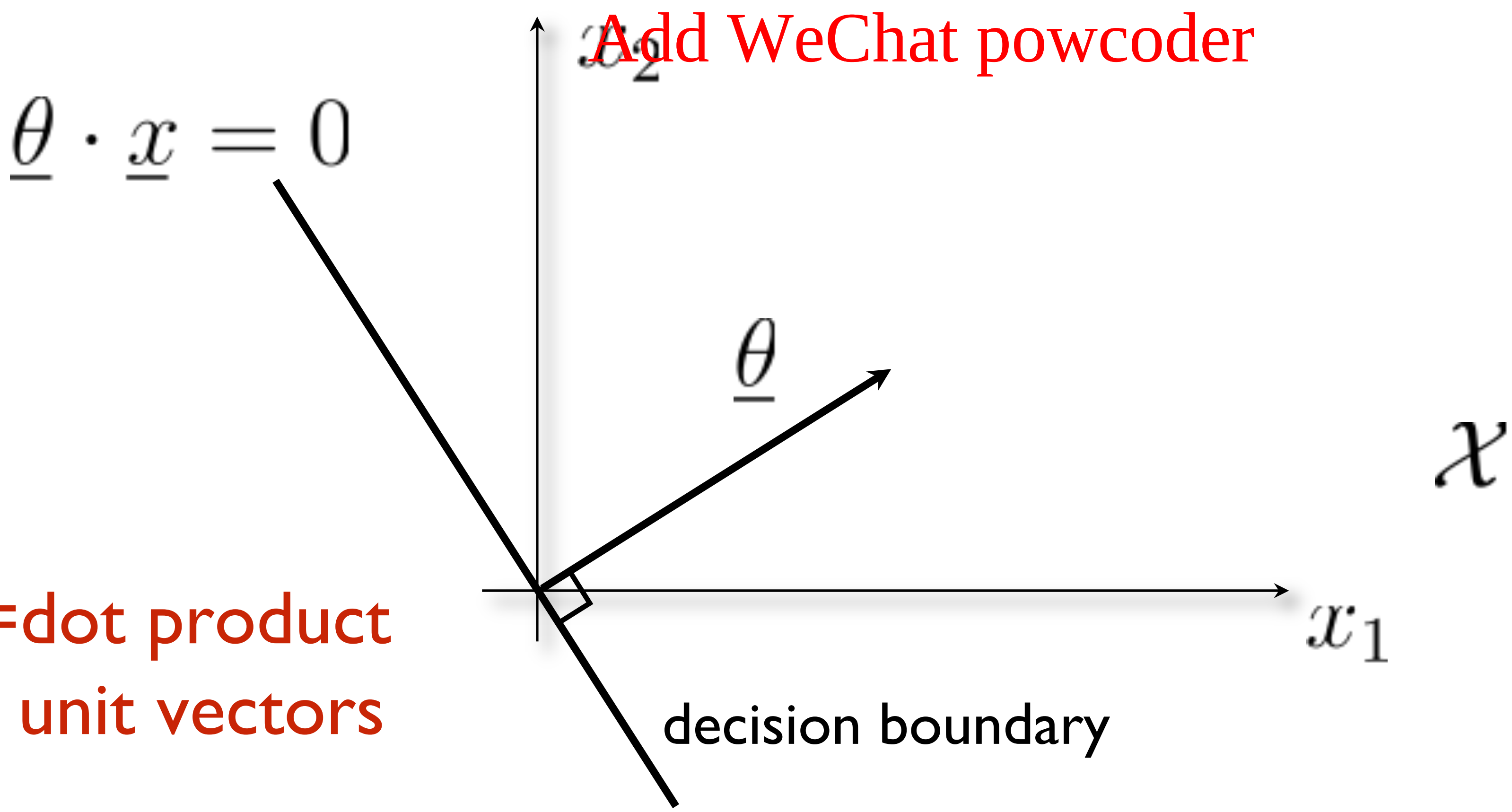

Recall:
cos(angle)=dot product
of two unit vectors
cos(90)=0

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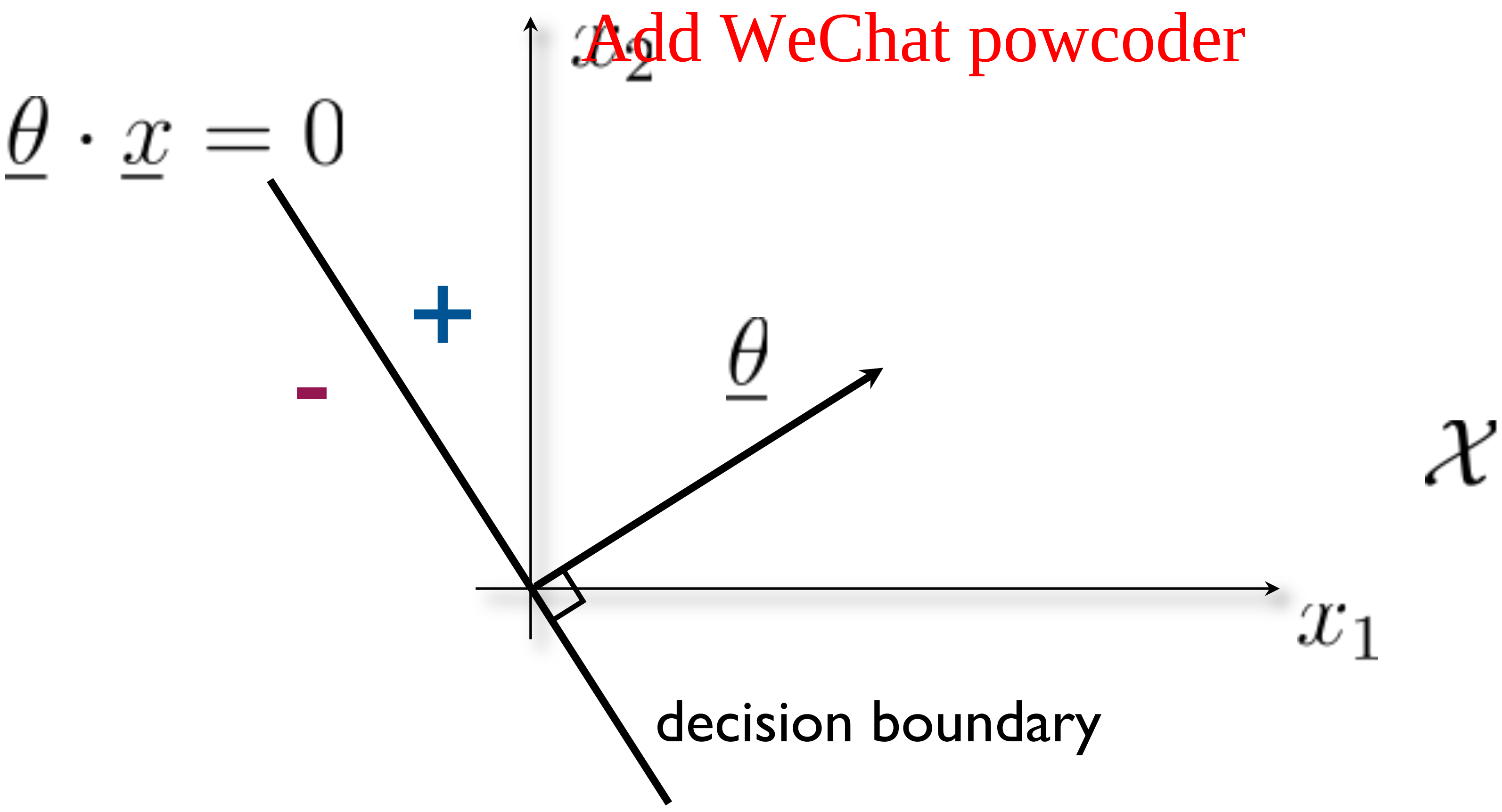

Part 2: estimation

- We can use the training error as a surrogate criterion for finding the best linear classifier through origin

$$\hat{R}_n(\underline{\theta}) = \frac{1}{n}\sum_{i=1}^{n} \text{Loss}\big(\, y_i, f(\underline{x}_i;\underline{\theta})\,\big)$$

$$\text{where} \;\; \text{Loss}\big(\, y, y' \,\big) = \begin{cases} 1, & \text{if } y \neq y' \\ 0, & \text{o.w.} \end{cases}$$

- Other choices are possible (and often preferable)

Perceptron algorithm

- The perceptron algorithm considers each training point in turn, adjusting the parameters to correct any mistakes

Initialize: $\underline{\theta} = 0$

Repeat until convergence:

for $t = 1, \ldots, n$

if $y_t(\underline{\theta} \cdot \underline{x}_t) \leq 0$ (mistake)

$\underline{\theta} \leftarrow \underline{\theta} + y_t \underline{x}_t$

- The algorithm will converge (no mistakes) if the training points are *linearly separable*, otherwise it won't converge

Perceptron algorithm: motivation

- If we make a mistake on the t^th training point, then

$$\mathrm{y}_t(\underline{\theta} \cdot \underline{x}_t) \leq 0$$

- After the update, we have

$$\underline{\theta}' = \underline{\theta} + y_t \underline{x}_t$$

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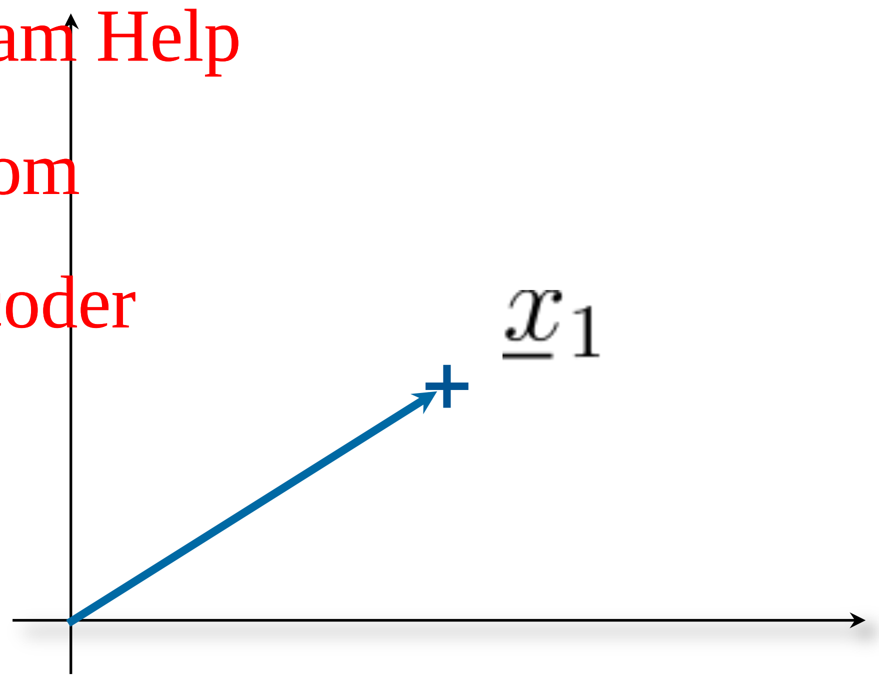
- So that $y_t(\underline{\theta}' \cdot \underline{x}_t)$ increases based on the update

Perceptron algorithm

- Iterative updates based on mistakes

$$\underline{\theta}_0 = 0$$



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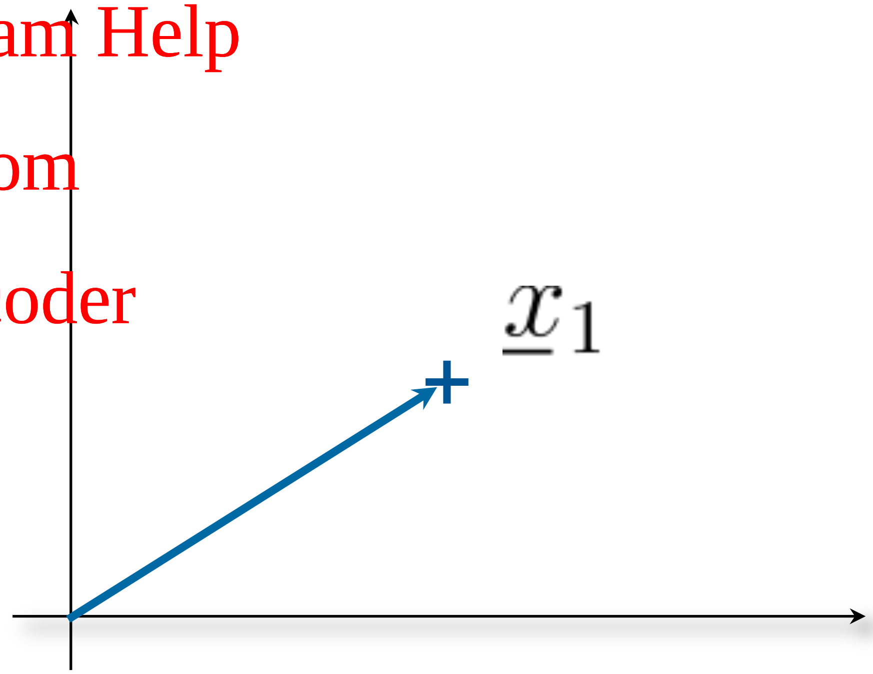

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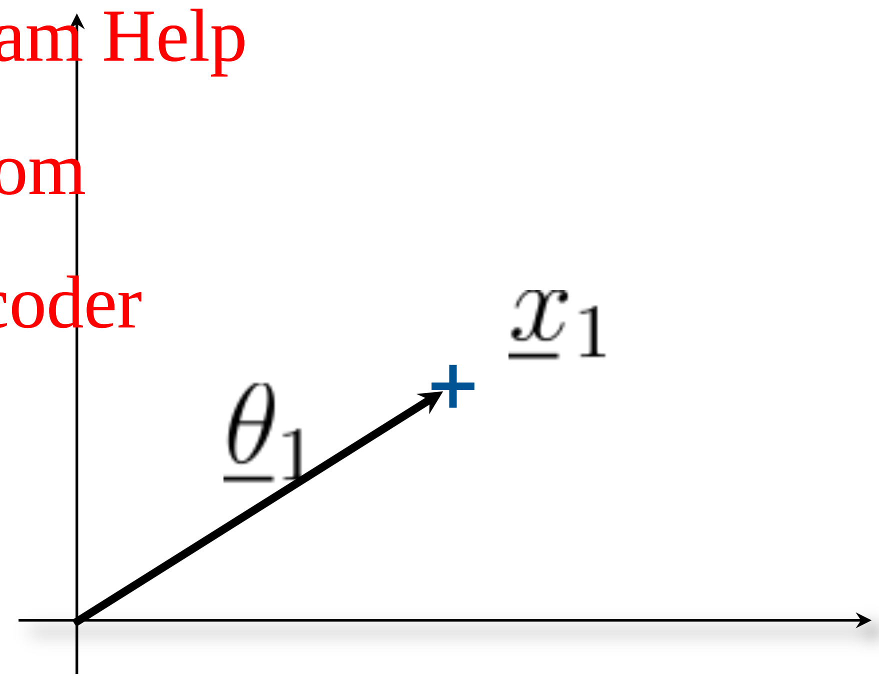

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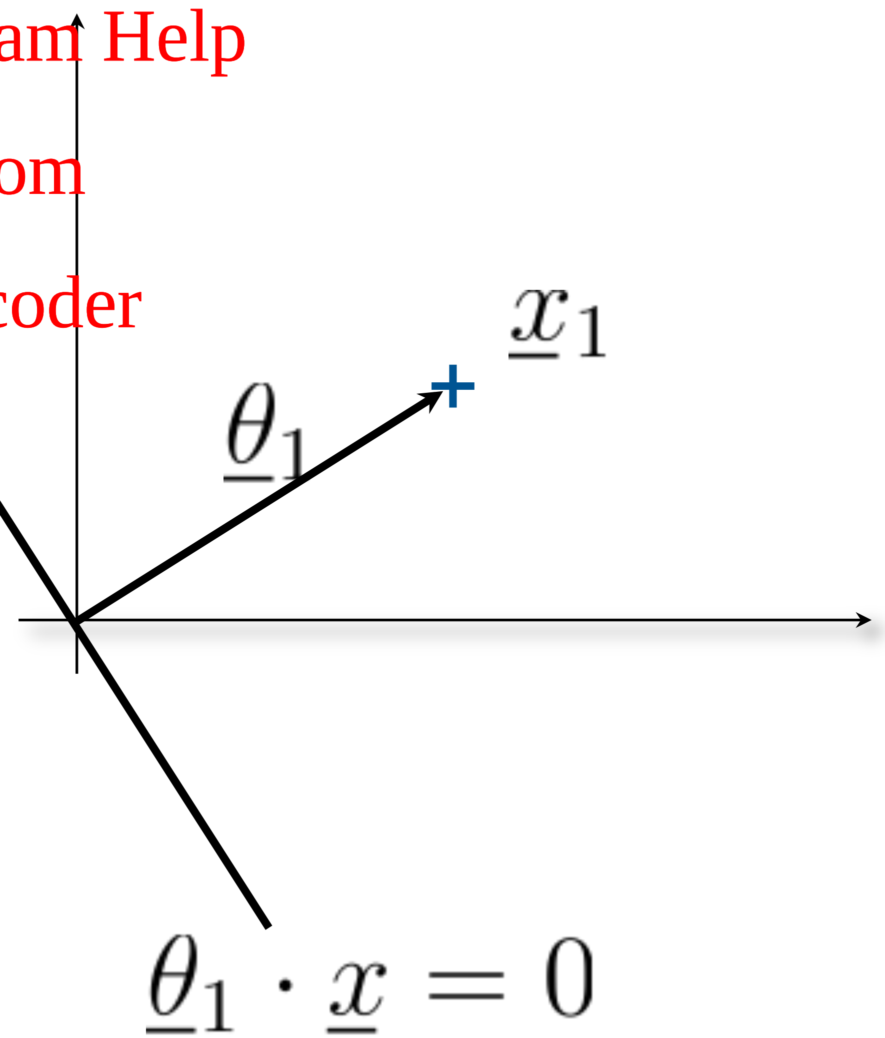

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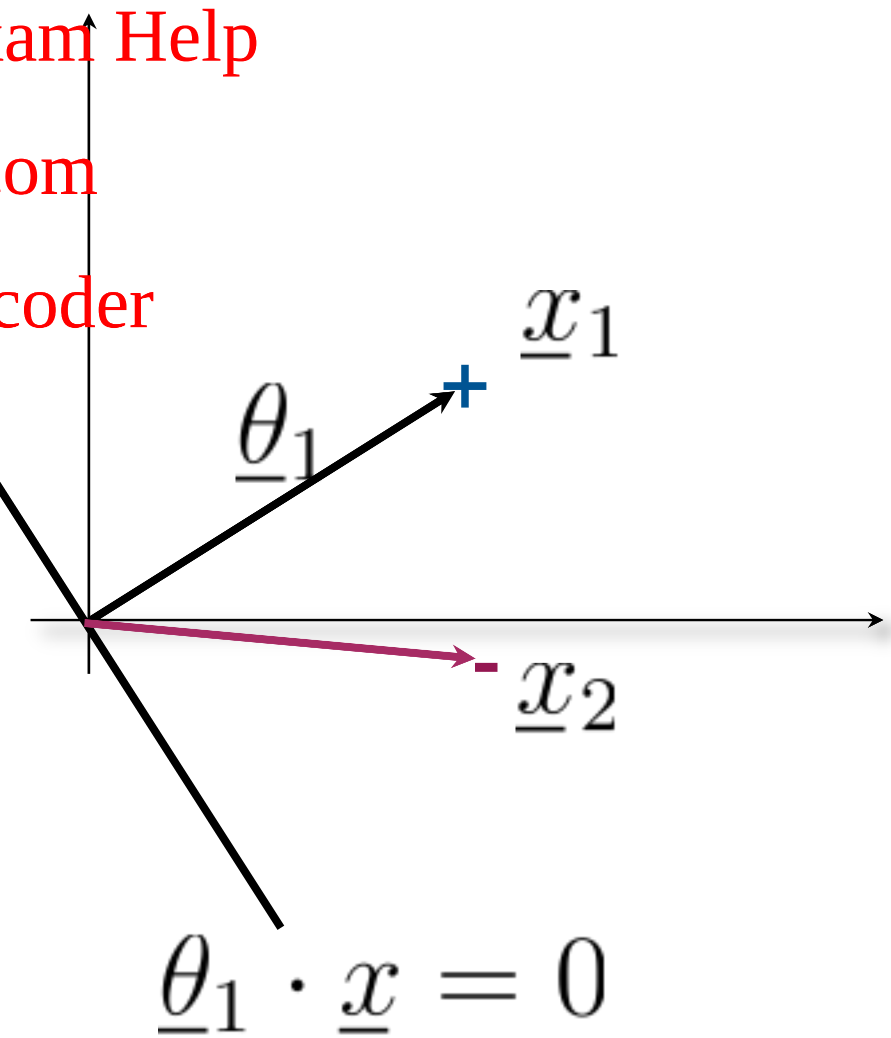

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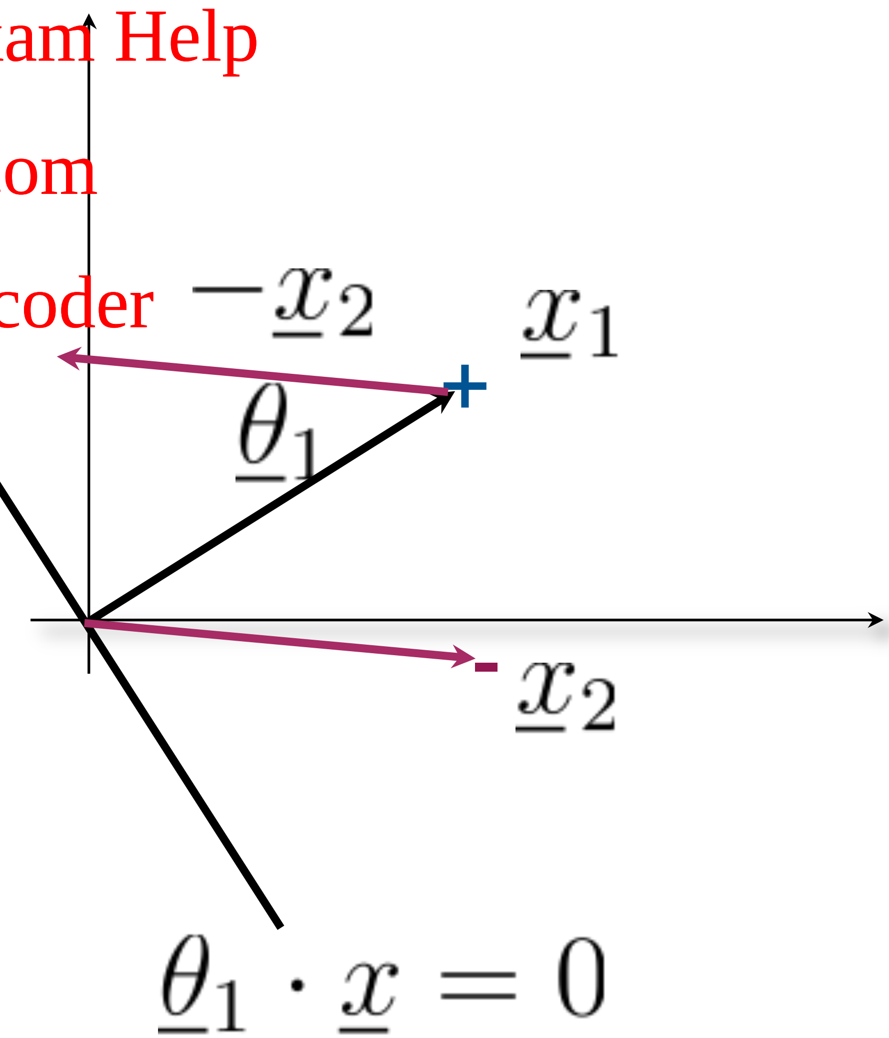

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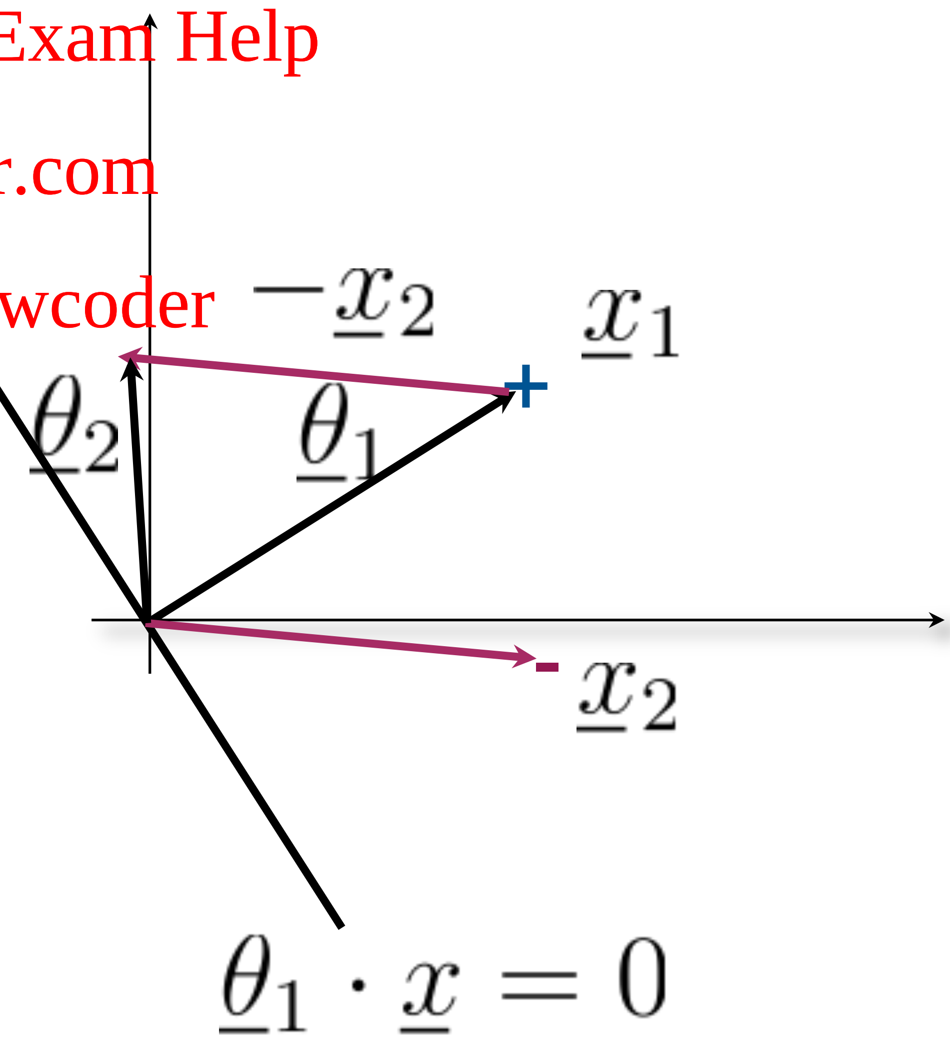

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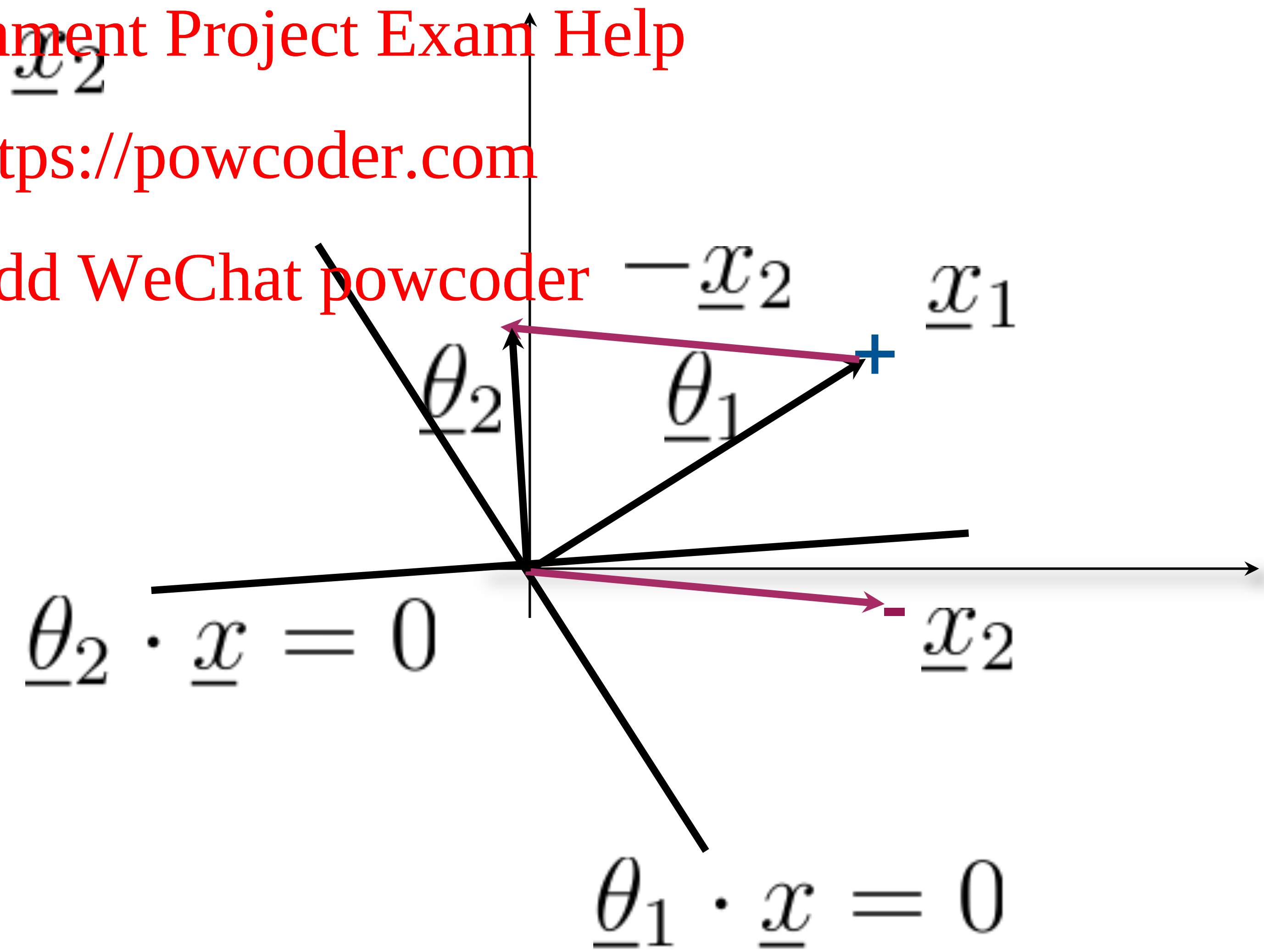

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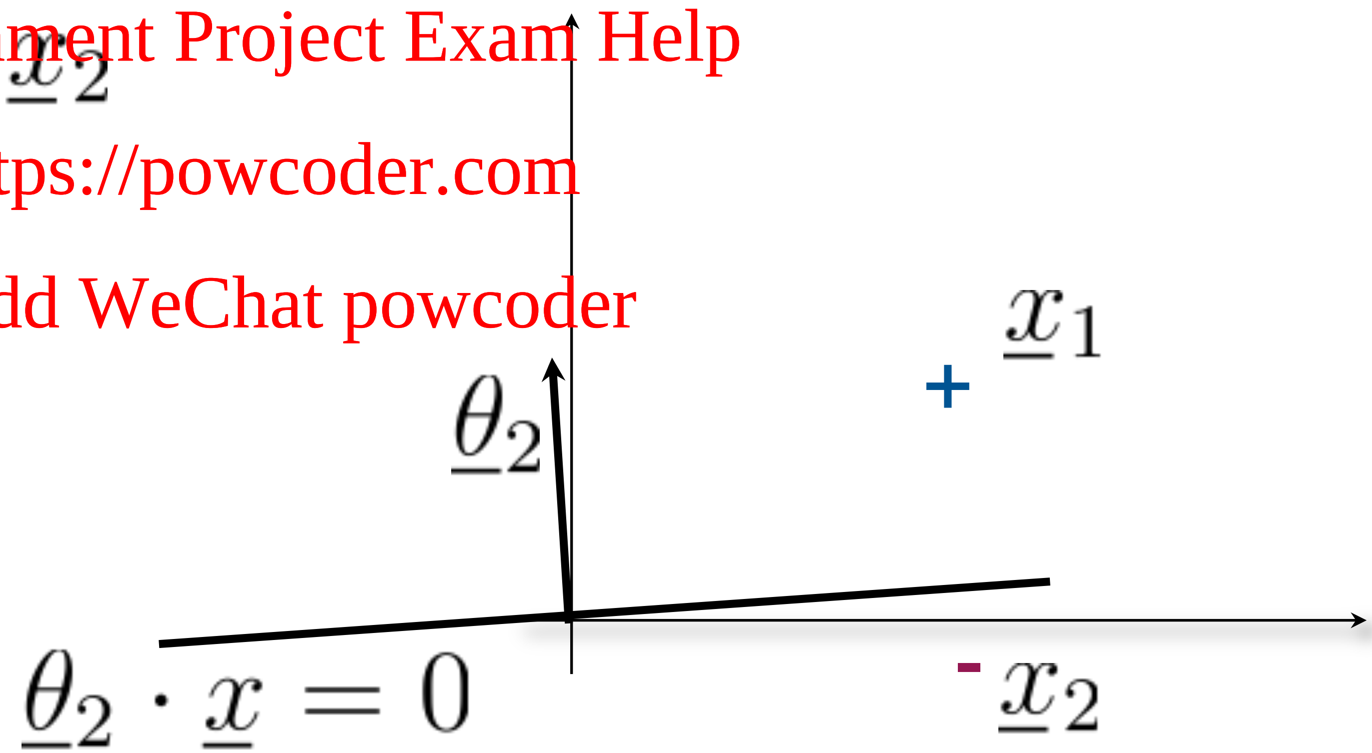

Perceptron algorithm (take 2)

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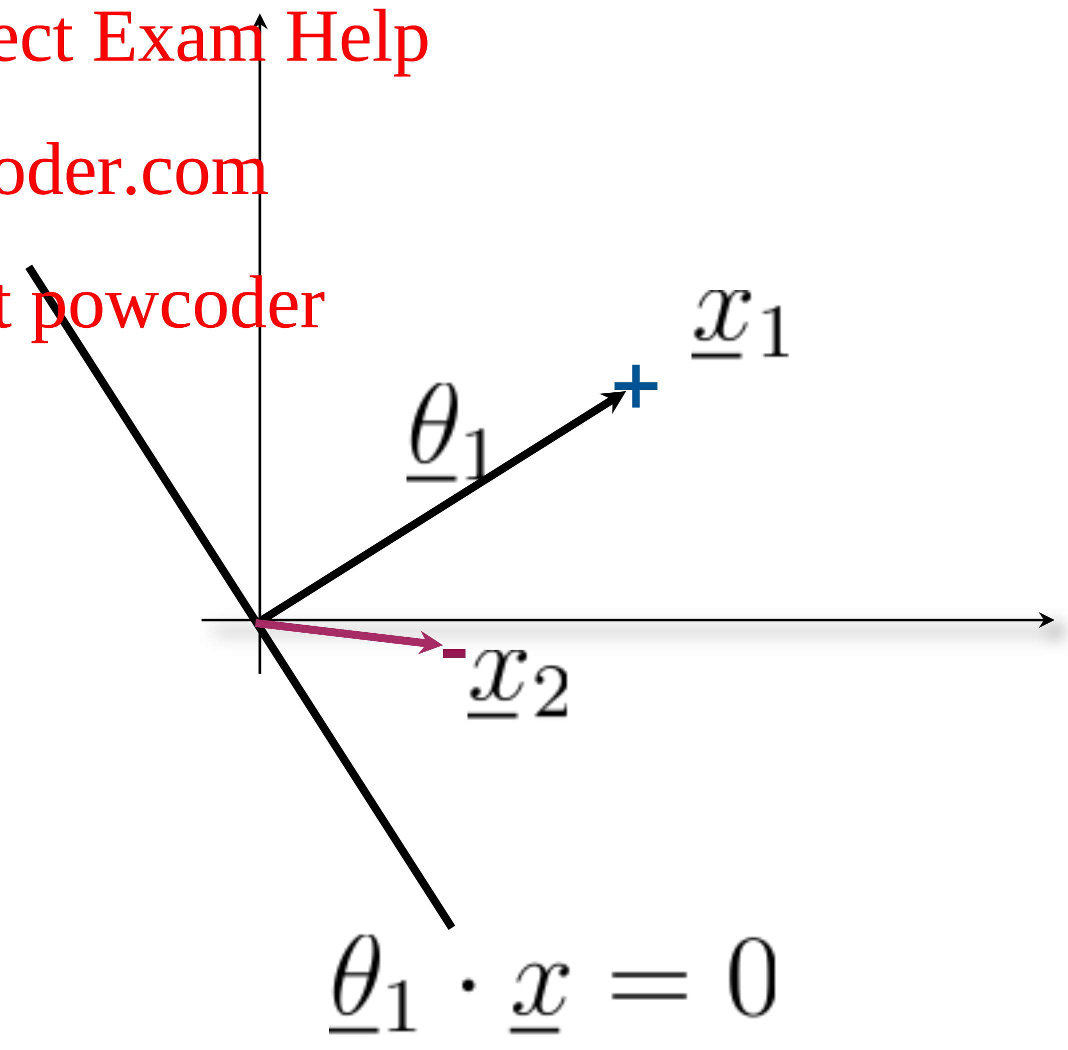

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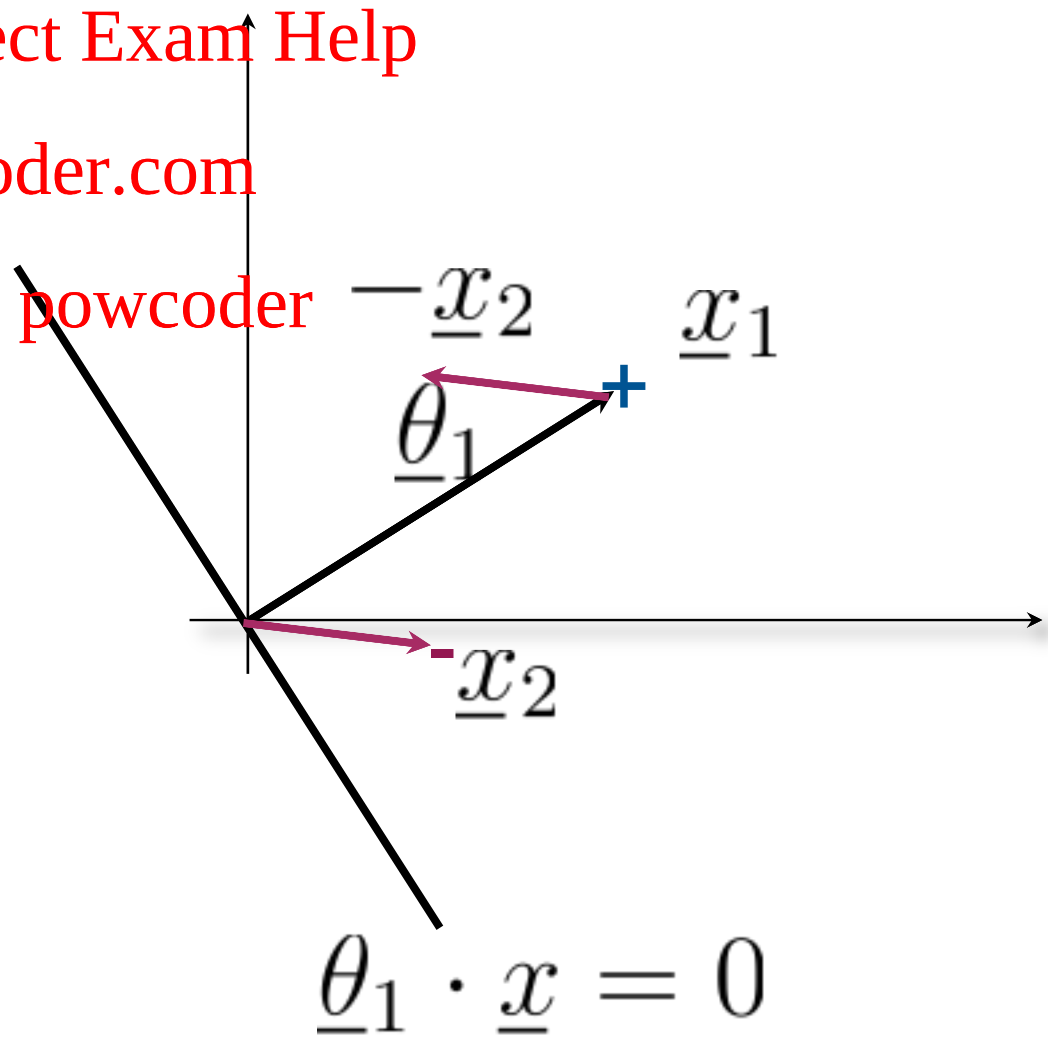

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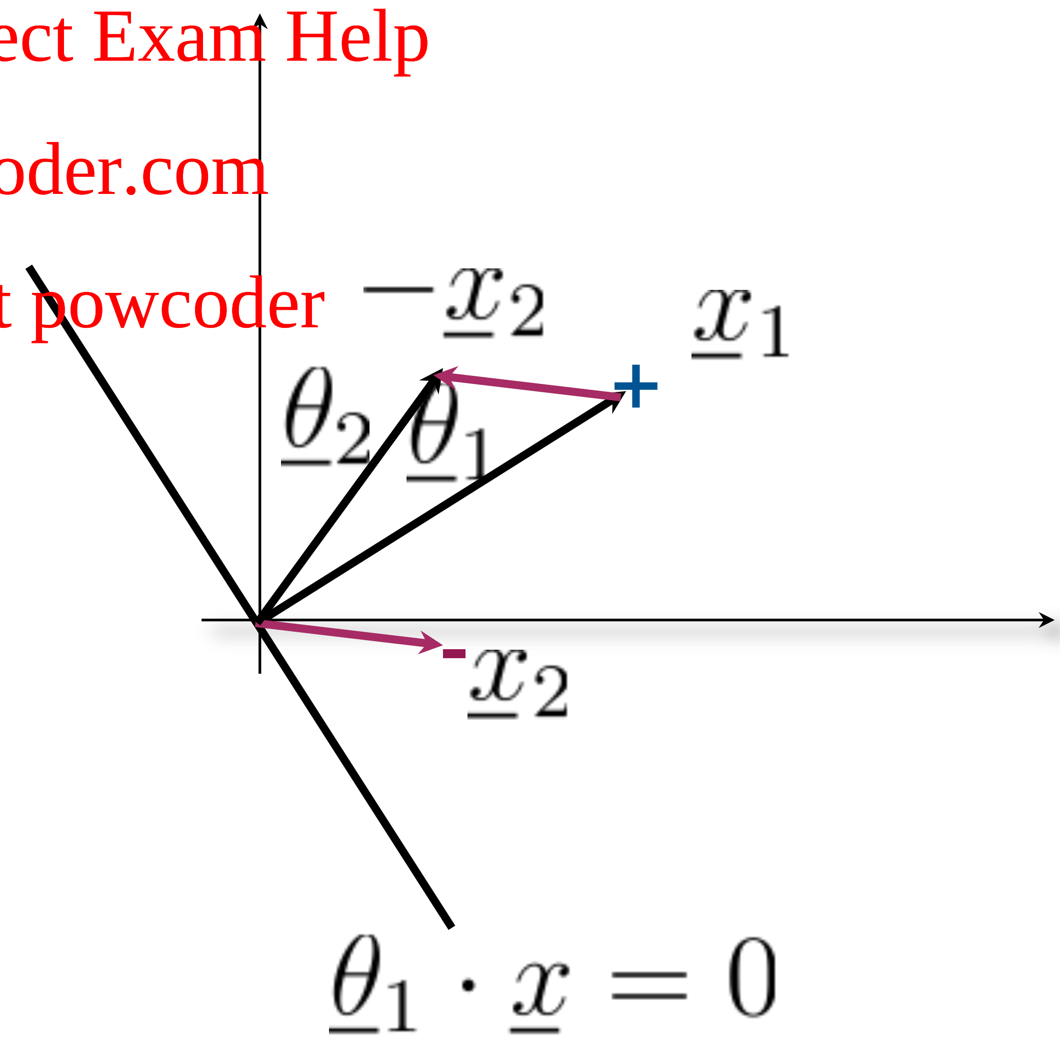

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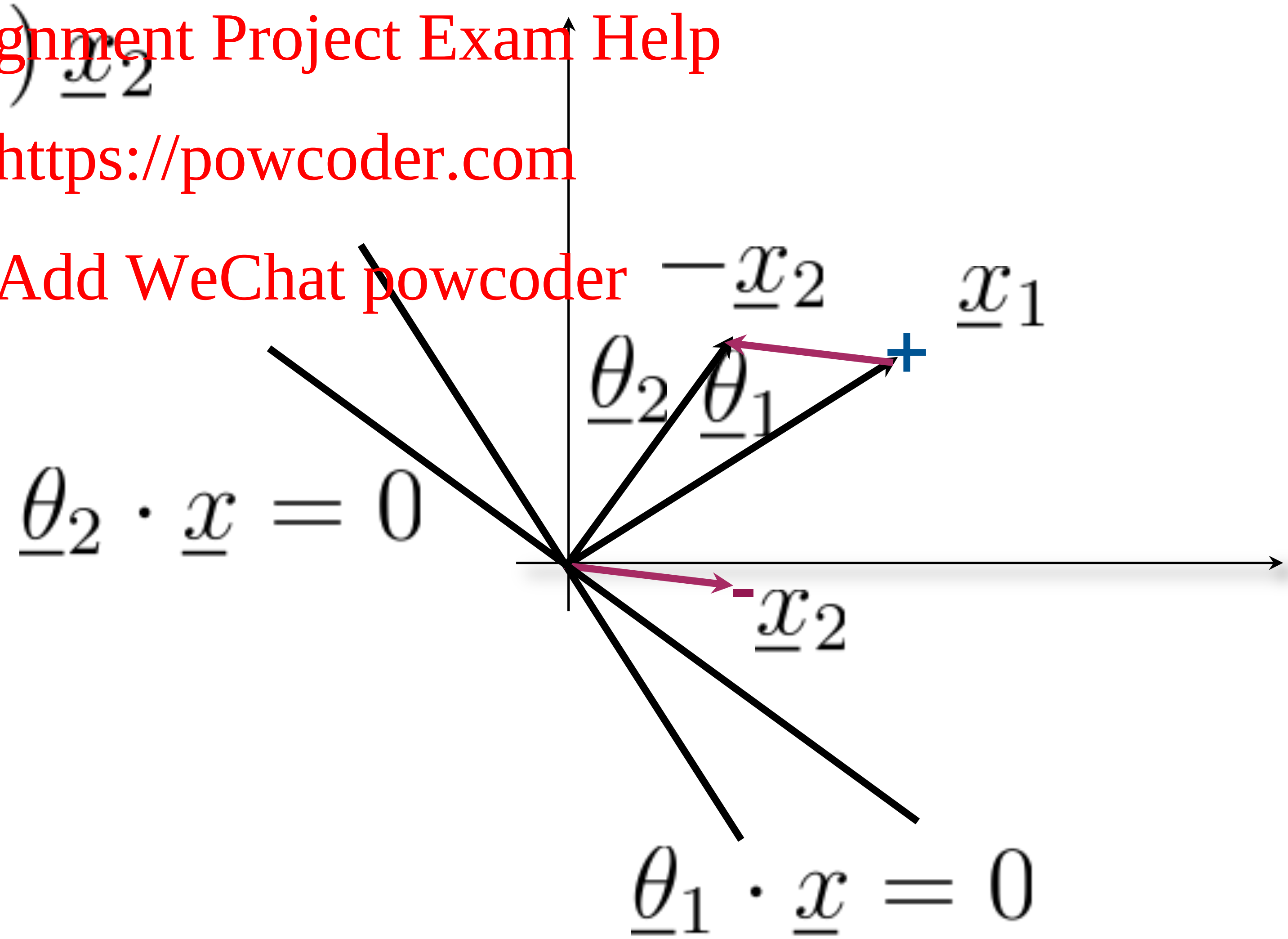

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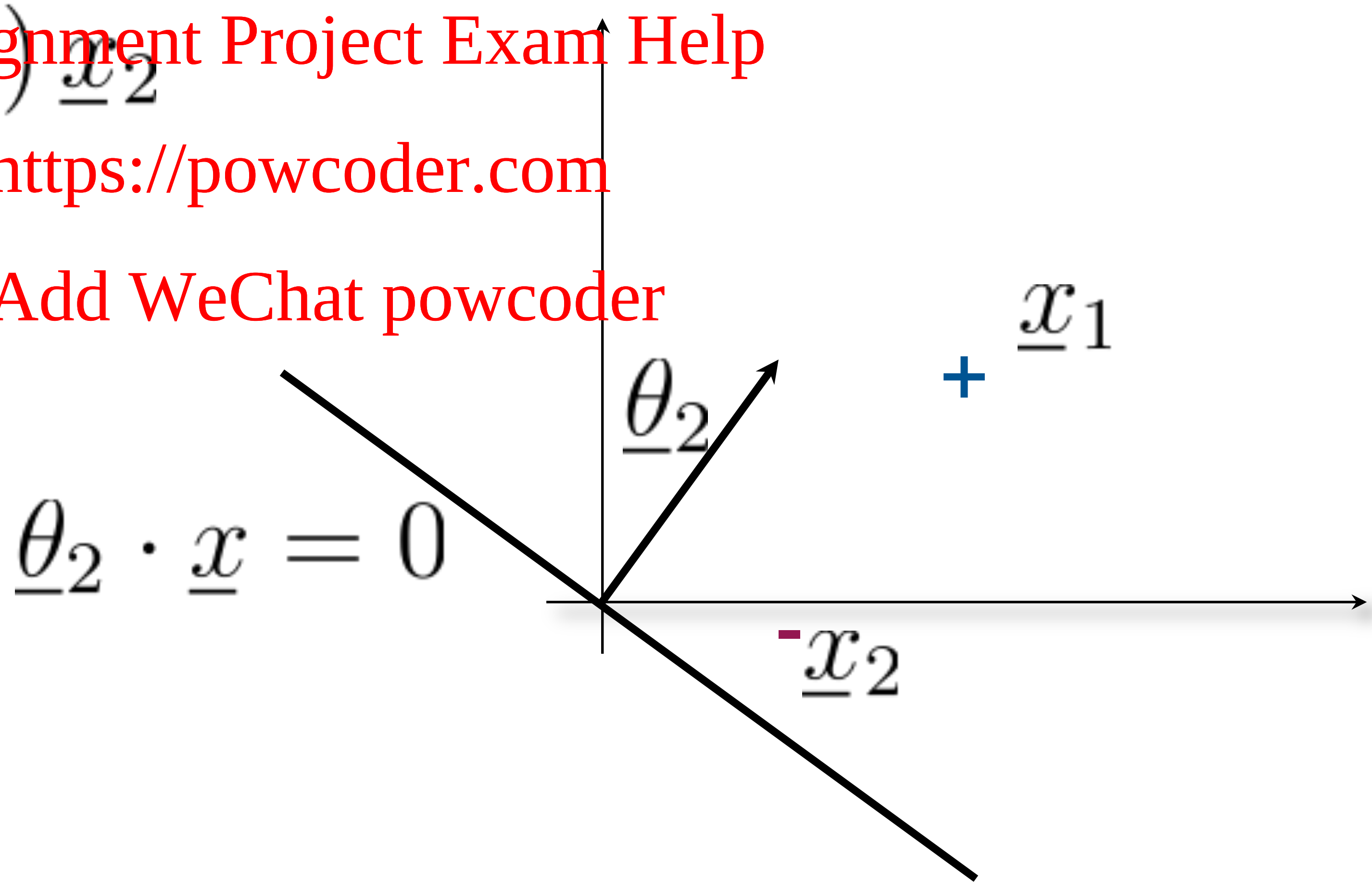

Perceptron algorithm (take 3)

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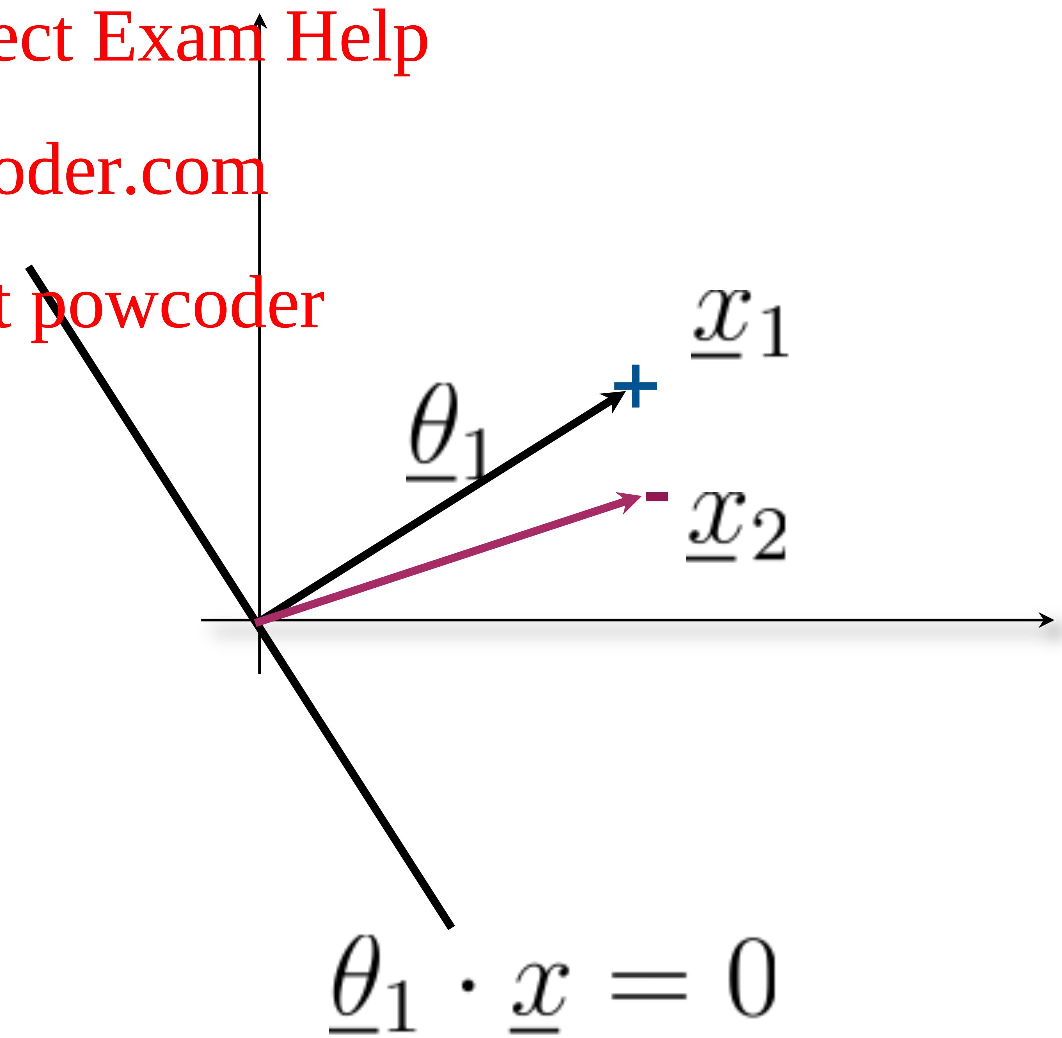

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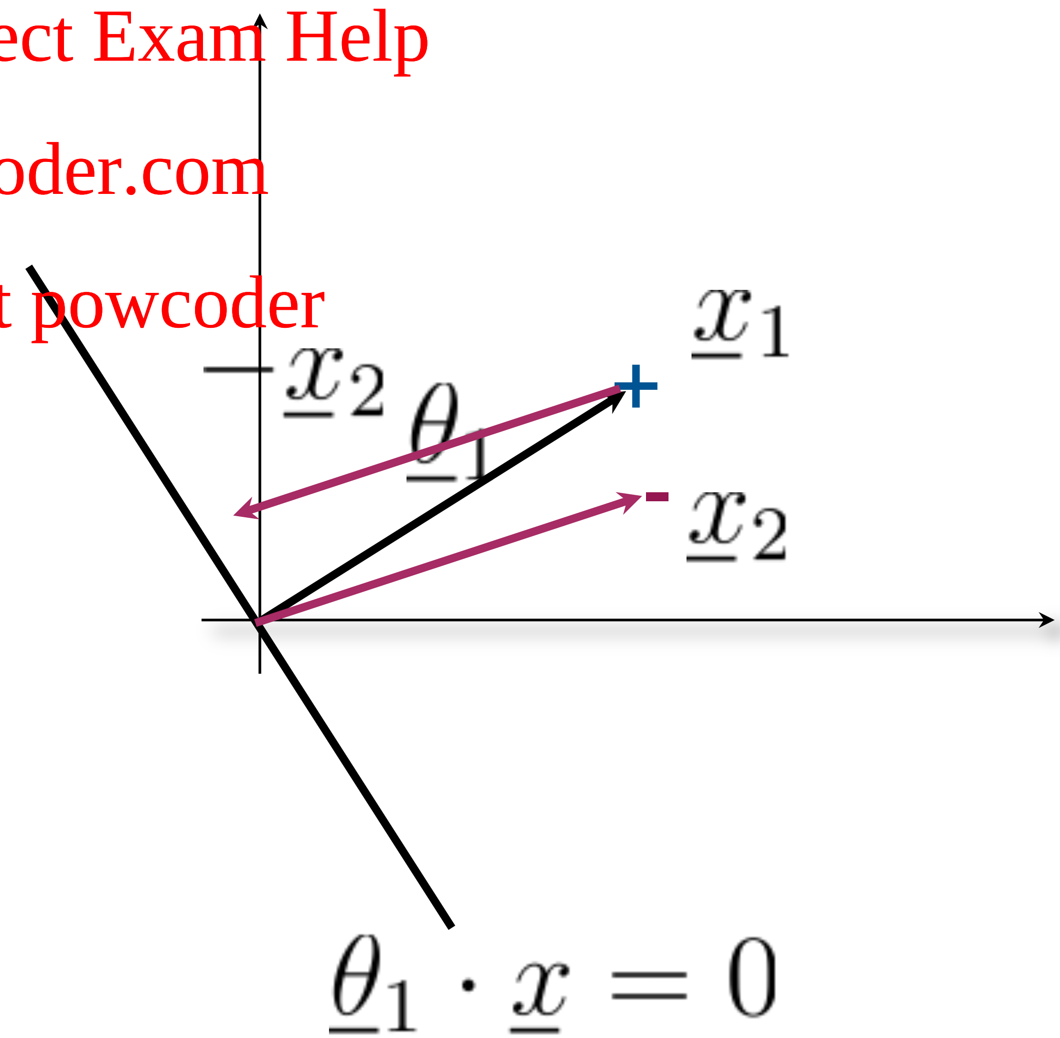

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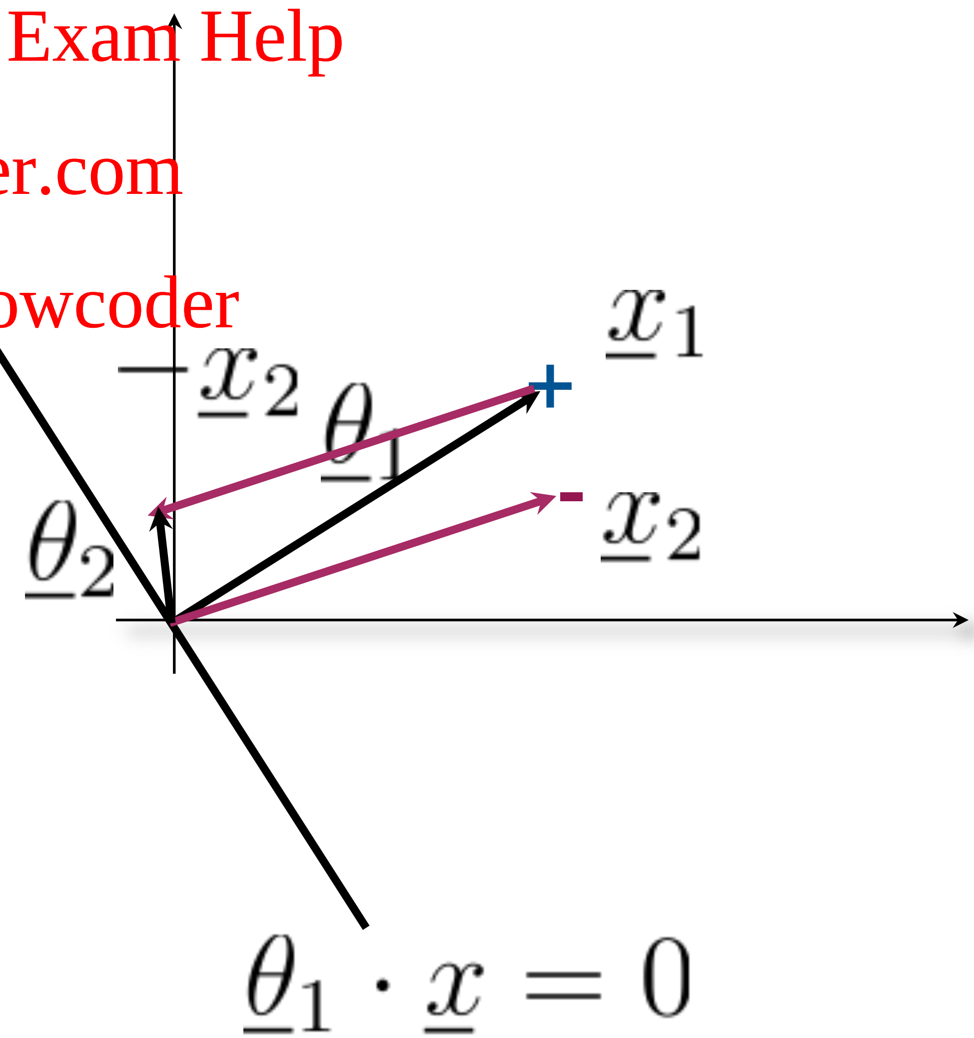

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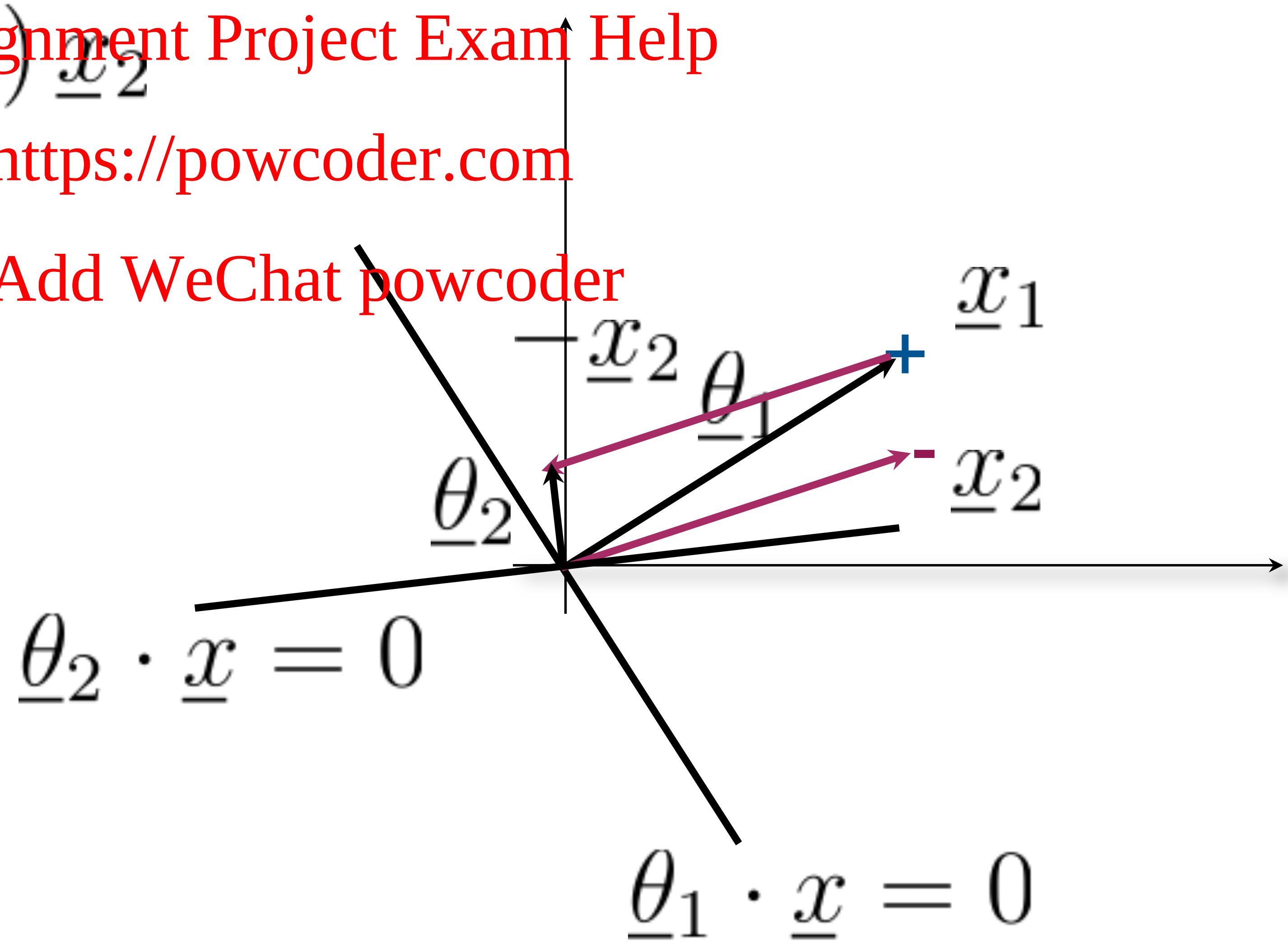

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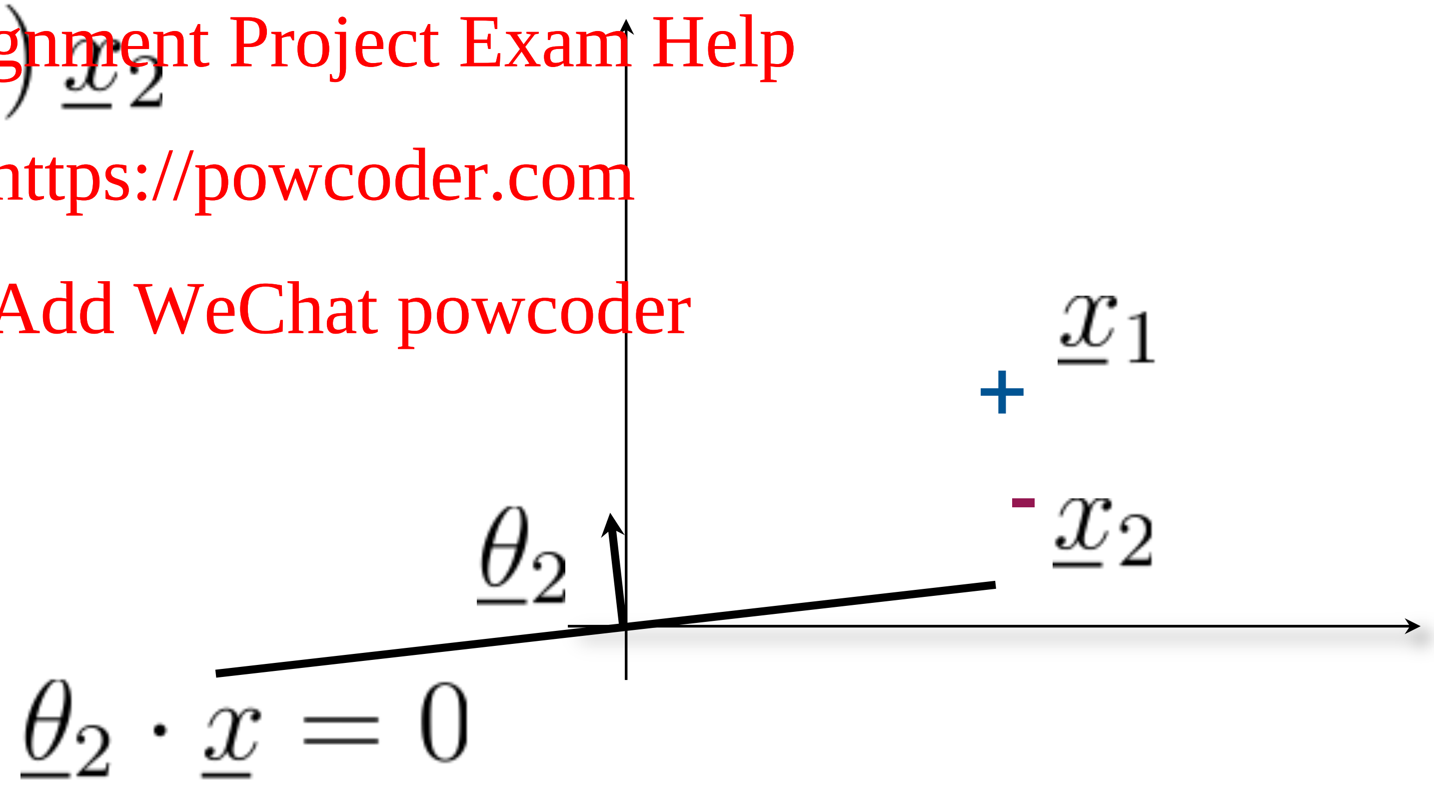
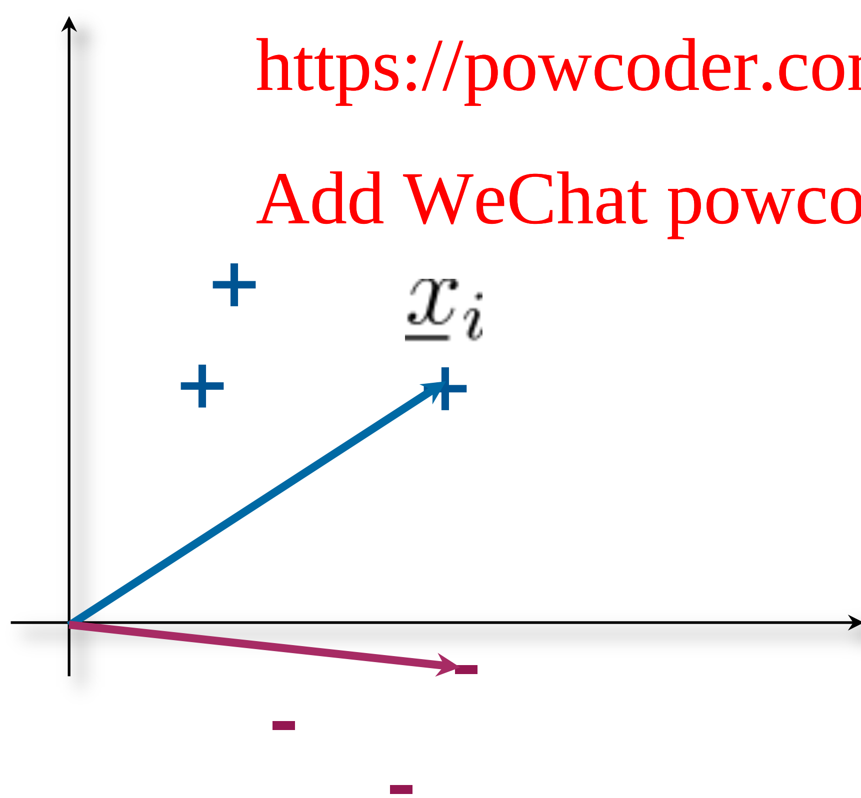


“Margin”

- We can get a handle on convergence by assuming that there exists a target classifier with good properties
- One such property is margin, i.e., how close the separating boundary is to the points



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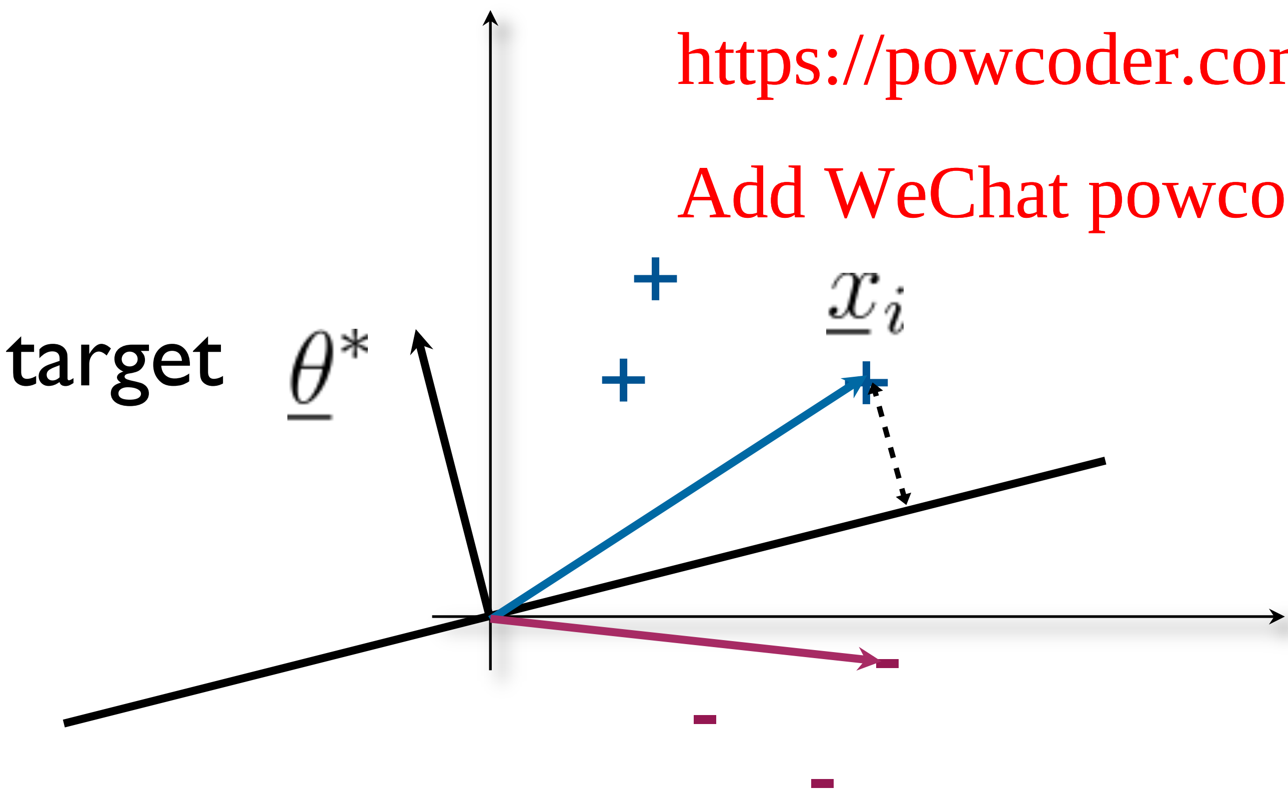

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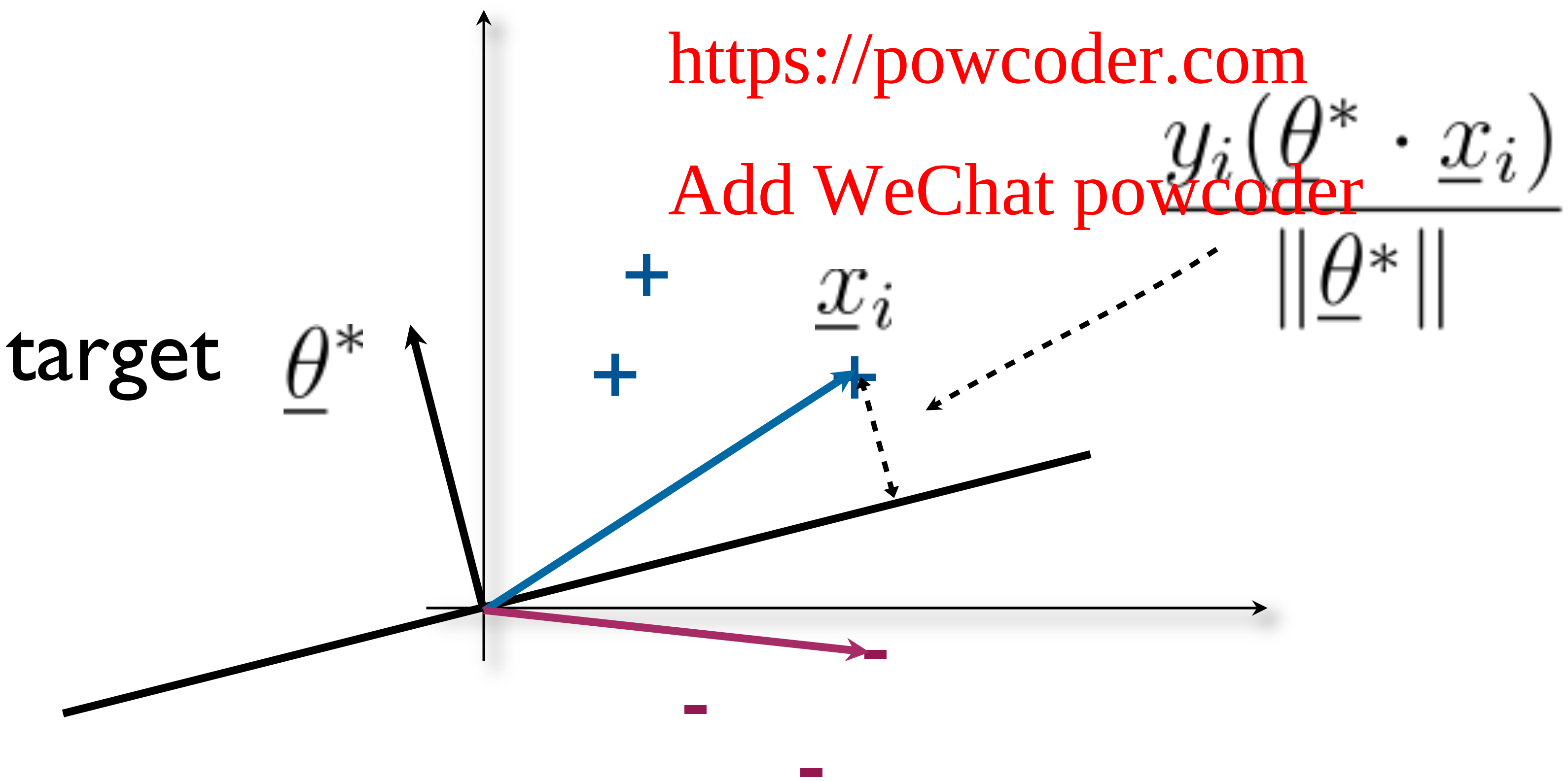

Recall:
cos(angle)=dot product of two unit vectors
dotted distance = radius * cos(angle)

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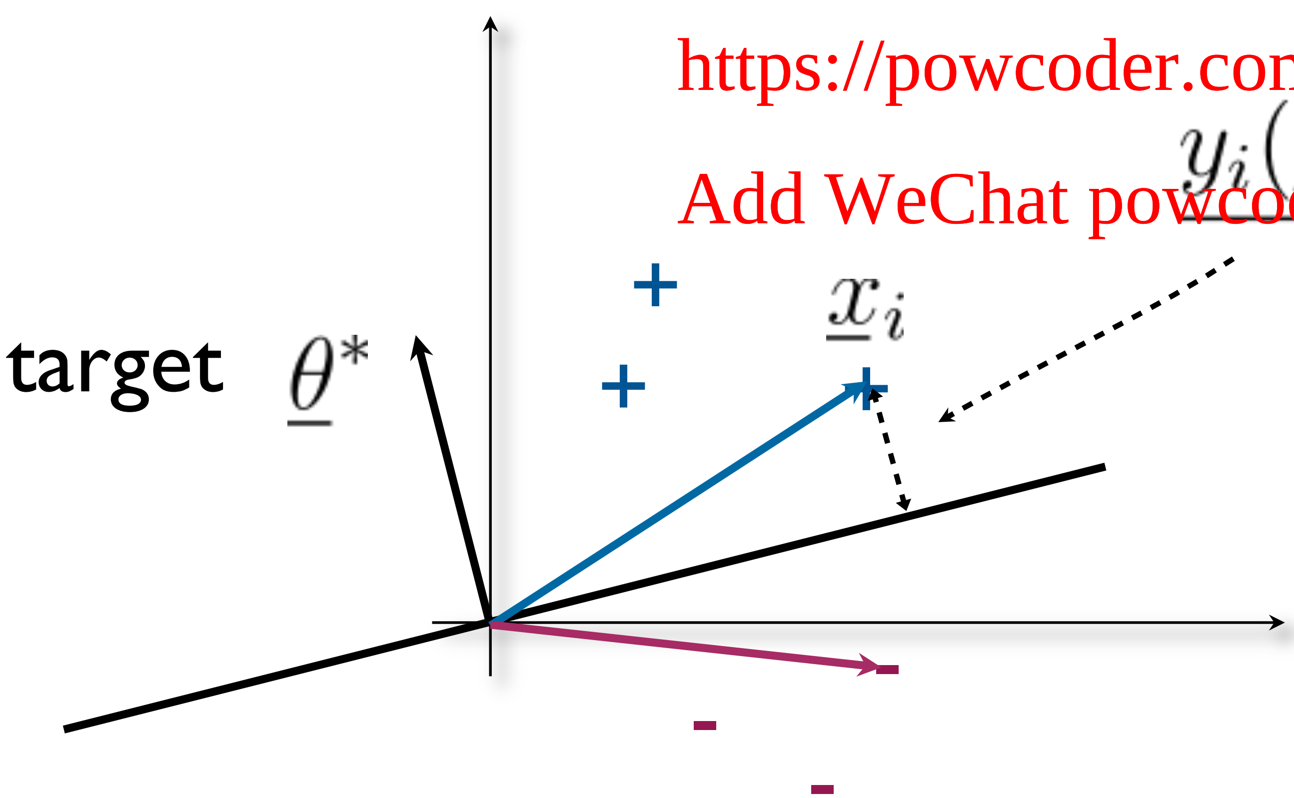

$$\frac{y_i(\underline{\theta}^* \cdot \underline{x}_i)}{\|\underline{\theta}^*\|} \geq \frac{\gamma}{\|\underline{\theta}^*\|}$$

Perceptron convergence theorem

- If there exists $\underline{\theta}^*$ such that

$$y_i(\underline{\theta}^* \cdot \underline{x}_i) \geq \gamma, \quad i = 1, \ldots, n$$

and $\|\underline{x}_i\| \leq R$ then the perceptron algorithm makes at most

$$\frac{\|\underline{\theta}^*\|^2 R^2}{\gamma^2}$$

mistakes (on the training set).

- Note that the result does NOT depend on $dim(\underline{\theta}), \quad n$