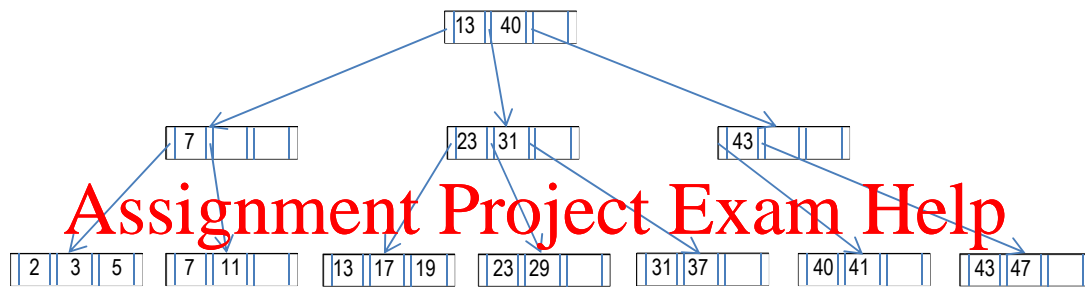1.
a)   Insert the search-key values 31, 41, 40 in order to the following B$^+$-tree.

```
                                    [13| | ]
                       ┌──────────────┴──────────────┐
                    [7| | ]                      [23|43| ]
              ┌────────┴────────┐          ┌────────┼────────┐
         [2|3|5]   [7|11]  [13|17|19]  [23|29|37]  [43|47]
```

```
                                    [13|40| ]
                  ┌──────────────────┼──────────────────┐
               [7| | ]           [23|31| ]           [43| | ]
          ┌──────┴──────┐    ┌───────┼───────┐   ┌──────┴──────┐
      [2|3|5]  [7|11]  [13|17|19]  [23|29]  [31|37]  [40|41]  [43|47]
```
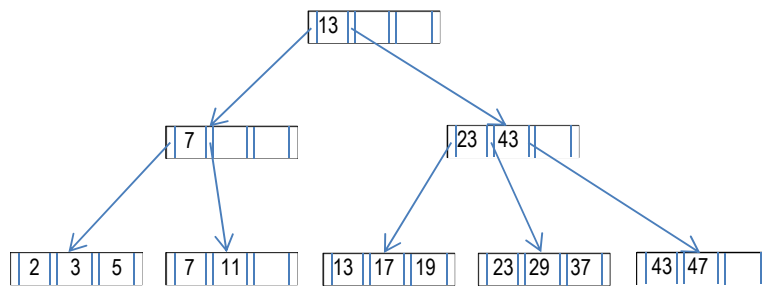
b)   For the resultant B$^+$-tree in a), show the form of the tree after deleting 7, 11, 43, 37 and 41 in order.

```
                        [13|29|31]
            ┌──────────┬──┴──┬──────────┐
        [2|3|5]  [13|17|19]  [23|29]  [31|40|47]
```
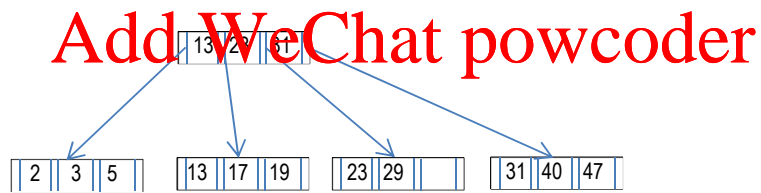
c)   Re-create the resultant B$^+$-tree in a), i.e., rebuild the tree from an empty tree, using **bottom-up B$^+$-tree construction**.

```
                                    [29| | ]
                       ┌──────────────┴──────────────┐
                    [7|17| ]                      [40|43| ]
              ┌────────┼────────┐          ┌────────┼────────┐
         [2|3|5]  [7|11|13]  [17|19|23]  [29|31|37]  [40|41]  [43|47]
```
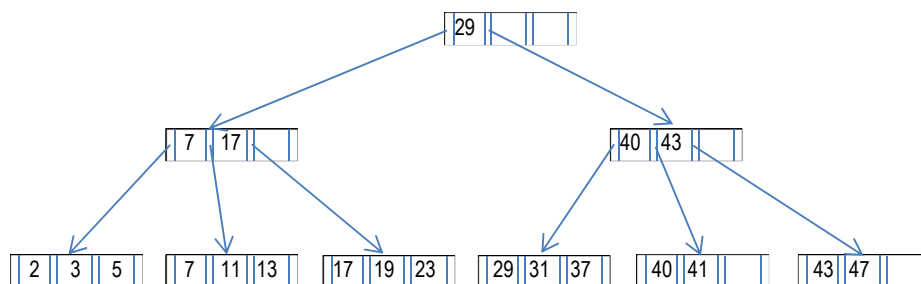
2.   Consider a hash function on integer search keys i defined by $h(i) = i^2 \bmod B$, where B is the number of buckets. What is wrong with this hash function if $B = 10$?

- Since each integer can be represented as $10a + b$ where $0 \le b < 10$ and so its square modulo 10 is the same as b^2 modulo 10.
- The squares of 0 to 9 modulo 10 are: 0, 1, 4, 9, 6, 5, 6, 9, 4, 1, and so the buckets for 2,3,7,8 will always be empty and buckets 1, 4, 6, 9 are twice as likely to be hit.
- So, this function is not uniform.

3.   A PARTS file with Part# as hash key includes records with the following Part# values: 2369, 3760, 4692, 4871, 5659, 1821, 1074, 7115, 1620, 2428, 3943, 4750, 6975, 4981, 9208. The file uses 8 buckets, numbered 0 to 7. Each bucket is one disk block and holds two records. Load these records into the file in the given order using the hash function $h(K)=K \bmod 8$. Calculate the average number of block accesses for a random retrieval on Part#.

The records will hash to the following buckets:
K $h(K)$ (bucket number)
2369   1
3760   0
4692   4
4871   7
5659   3
1821   5
1074   2
7115   3
1620   4
2428   4 overflow
3943   7
4750   6
6975   7 overflow
4981   5
9208   0

Two records out of 15 are in overflow buckets, which require an additional block access. The other records require only one block access. Hence, the average time to retrieve a random record is:
$(1 * (13/15)) + (2 * (2/15)) = 0.867 + 0.266 = 1.133$ block accesses