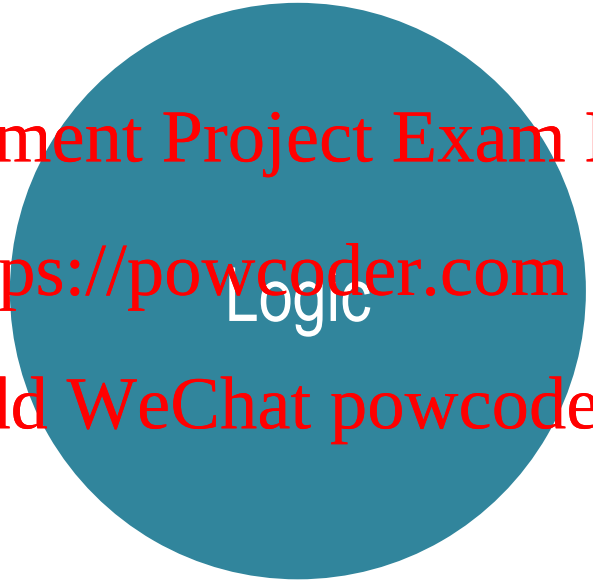


Planning

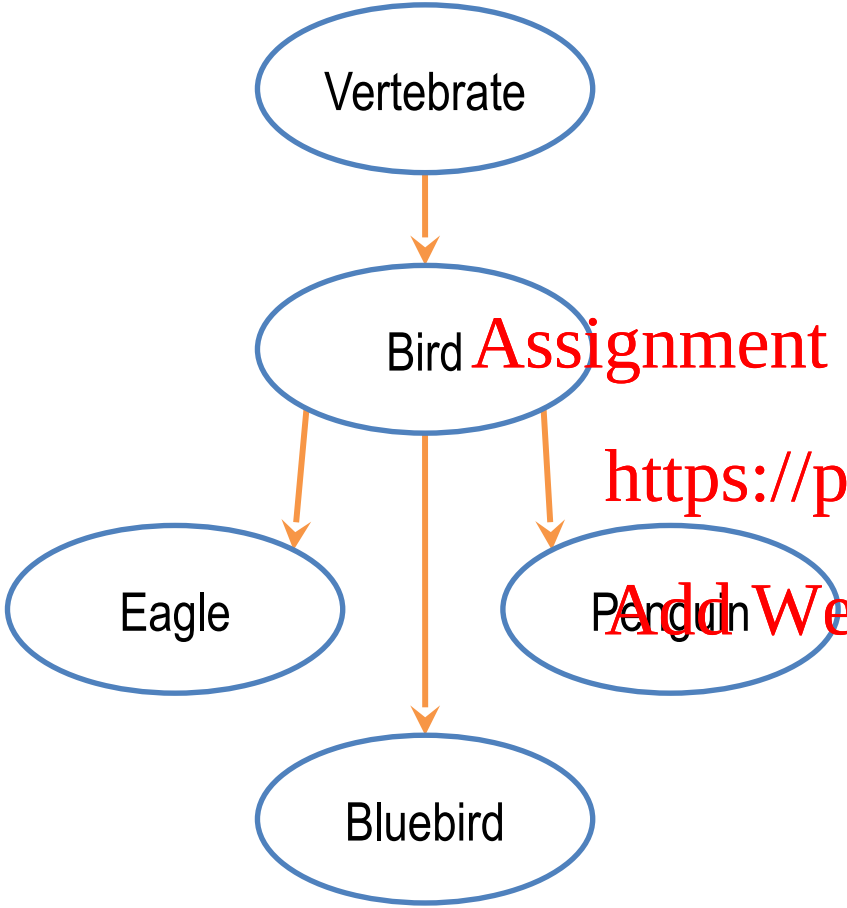
Lesson Preview

- Formal notation
- Conjunctions, disjunctions, negations, implications
- Truth tables
- Rules of inference
- Resolution theorem proving

Why do we need formal logic?

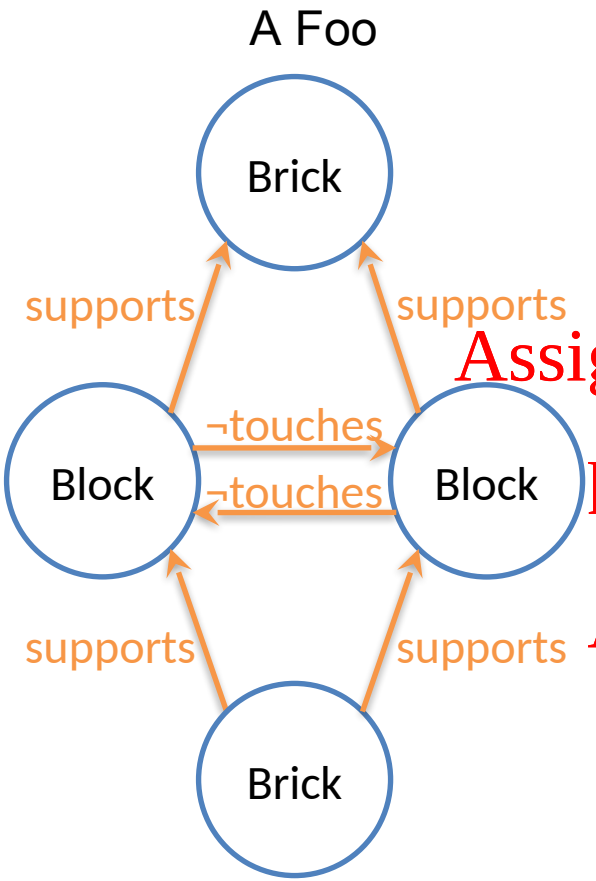
Soundness: Only valid conclusions can be proven.

Completeness: All valid conclusions can be proven.



If an animal has feathers, then it is a bird

If an animal lays eggs and it flies, then it is a bird



Mark the sufficient conditions:

If...

- ☐ A brick supports two bricks
- ■ A brick supports two blocks
- ☐ Those two blocks touch
- ■ Those two blocks do not touch
- ☐ Those two blocks support a block
- ■ Those two blocks support a brick

...then the object is a foo

Predicate:

A function that maps object arguments to true or false values

`Feathers(bluebird)`

Predicate:

A function that maps object arguments to true or false values

```
Feathers(animal)
```


If an animal has feathers, then it is a bird


```
If Feathers(animal):
    Then Bird(animal)
```

If an animal lays eggs and it flies,
then it is a bird

```
Lays-eggs(animal)
        ∧
Flies(animal)
```



```
If Lays-eggs(animal) ∧
   Flies(animal):
Then Bird(animal)
```

If an animal lays eggs or it flies,
then it is a bird

```
Lays-eggs(animal)
        v
Flies(animal)
```



```
If Lays-eggs(animal) v
    Flies(animal):
Then Bird(animal)
```

If an animal flies and is not a bird, it is a bat.

```
Flies(animal)
      ∧
¬Bird(animal)
```



```
If Flies(animal) ∧
   ¬Bird(animal):
Then Bat(Animal)
```

If Lays-eggs(animal) ∧ Flies(animal):
Then Bird(animal)

Lays-eggs(animal) ∧ Flies(animal) ⇒ Bird(animal)

Operator	Symbol	Accepted Symbol
AND	$A \wedge B$	`A & B` `A && B`
OR	$A \vee B$	`A | B` `A || B`
NOT	$\neg A$	`!A` `~A`

IMPLIES	$A \Rightarrow B$	`A = B`
		`A == B`
		`A => B`

If an animal lays eggs and does not have feathers, it is a reptile.

$$\text{Lays-eggs}(animal) \land \neg \text{Feathers}(animal) \Rightarrow \text{Reptile}(animal)$$

If an animal has feathers or has talons, it is a bird.

$$\text{Feathers}(animal) \lor \text{Talons}(animal) \Rightarrow \text{Bird}(animal)$$

If an animal lays eggs, has a beak, and flies, it is a duck.

$$\text{Lays-eggs}(animal) \land \text{Beak}(animal) \land \text{Flies}(animal) \Rightarrow \text{Duck}(animal)$$

If an animal lays eggs, has a beak, and do not fly, it is a platypus.

$$\text{Lays-eggs}(animal) \land \text{Beak}(animal) \land \neg \text{Flies}(animal) \Rightarrow \text{Platypus}(animal)$$

A	B	A ∨ B
True	True	True
True	False	True
False	True	True
False	False	False

A	B	A ∨ ¬B
True	True	True
True	False	True
False	True	False
False	False	True

A	B	¬A ∧ ¬B
True	True	False
True	False	False
False	True	False
False	False	True

A	B	C	A ∨ (B ∧ ¬C)
True	True	True	True
True	True	False	True
True	False	True	True
True	False	False	True
False	True	True	False
False	True	False	True
False	False	True	False
False	False	False	False

Commutative Property

A	B	A ∧ B	B ∧ A
True	True	True	True
True	False	False	False
False	True	False	False
False	False	False	False

Distributive Property

A	B	C	A ∧ (B ∨ C)	(A ∧ B) ∨ (A ∧ C)
True	True	True	True	True
True	True	False	True	True
True	False	True	True	True
True	False	False	False	False
False	True	True	False	False
False	True	False	False	False
False	False	True	False	False
False	False	False	False	False

Associative Property

A	B	C	A ∨ (B ∨ C)	(A ∨ B) ∨ C
True	True	True	True	True
True	True	False	True	True
True	False	True	True	True
True	False	False	True	True
False	True	True	True	True
False	True	False	True	True
False	False	True	True	True
False	False	False	False	False

de Morgan's Law

A	B	¬(A ∧ B)	¬A ∨ ¬B
True	True	False	False
True	False	True	True
False	True	True	True
False	False	True	True

Truth of Implications

A	B	A ⇒ B
True	True	True
True	False	False
False	True	True
False	False	True

Implication Elimination

Given:

$a \Rightarrow b$

Rewrite as:

$\neg a \vee b$

Given:

$Feathers \Rightarrow Bird$

Rewrite as:

$\neg Feathers \vee Bird$

Rules of Inference: Instantiate general rules to prove specific claims.

Modus Ponens

Sentence 1: $p \Rightarrow q$
Sentence 2: p
$\therefore$ Sentence 3: q

$\text{Feathers} \Rightarrow \text{Bird}$
Feathers
$\therefore$ Bird

Modus Tollens

Sentence 1: $p \Rightarrow q$
Sentence 2: $\neg q$
$\therefore$ Sentence 3: $\neg p$

$\text{Feathers} \Rightarrow \text{Bird}$
$\neg$Bird
$\therefore \neg$Feathers

Prove: Harry is a bird

S1: Feathers(animal) ⇒ Bird(animal)

S2: Feathers(Harry)

By Modus Ponens

S3: Bird(Harry)

Prove: Harry is a bird

S1: Feathers(animal) ⇒ Bird(animal)

S2: Feathers(Harry)

S3: Bird(Harry)

Prove: Buzz does not have feathers

S1: Feathers(animal) ⇒ Bird(animal)

S2: ¬Bird(Buzz)

By Modus Tollens

S3: ¬Feathers(Buzz)

Prove: Buzz does not have feathers

S1: Feathers(animal) ⇒ Bird(animal)

S2: ¬Bird(Buzz)

S3: ¬Feathers(Buzz)

For one animal:

$$\text{Lays-eggs}(\text{animal}) \land \text{Flies}(\text{animal}) \Rightarrow \text{Bird}(\text{animal})$$

For all animals:

$$\forall x[\text{Lays-eggs}(x) \land \text{Flies}(x) \Rightarrow \text{Bird}(x)]$$

“Universal Quantifier”

For one animal:

$$\text{Lays-eggs}(\text{animal}) \wedge \text{Flies}(\text{animal}) \Rightarrow \text{Bird}(\text{animal})$$

For at least one animal:

$$\exists y[\text{Lays-eggs}(y) \wedge \text{Flies}(y) \Rightarrow \text{Bird}(y)]$$

“Existential Quantifier”

We know:

S1: ¬can-move ⇒ ¬liftable

We find:

S2: ¬can-move

How do we prove the box is not liftable?

We know:

~~S1: ¬can-move → ¬liftable~~

By implication elimination:

S1: can-move ∨ ¬liftable

We find:

S2: ¬can-move

We assume:

S3: liftable

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable

S2: ¬can-move

S3: liftable

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable

S2: ¬can-move

S3: liftable

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable

S2: ¬can-move

S3: liftable

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable

S2: ¬can-move

S3: liftable

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable

S2: ¬can-move

S3: liftable

How do we prove the box is not liftable?

We know:

S1: ¬can-move ∧ battery-full ⇒ ¬liftable

We find:

S2: ¬can-move

S3: battery-full

How do we prove the box is not liftable?

We know:

~~S1: ¬can-move ∧ battery-full → ¬liftable~~

By implication elimination:

S1: ¬(¬can-move ∧ battery-full) ∨ ¬liftable

We find:

S2: ¬can-move

S3: battery-full

How do we prove the box is not liftable?

We know:

~~S1: ¬can-move ∧ battery-full → ¬liftable~~

By implication elimination:

~~S1: ¬(¬can-move ∧ battery-full) ∨ ¬liftable~~

By deMorgan's Law:

S1: can-move ∨ ¬battery-full ∨ ¬liftable

We find:

S2: ¬can-move

S3: battery-full

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable ∨ ¬battery-full

S2: ¬can-move

S3: battery-full

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable ∨ ¬battery-full

S2: ¬can-move

S3: battery-full

S4: liftable

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable ∨ ¬battery-full

S2: ¬can-move

S3: battery-full

S4: liftable

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable ∨ ¬battery-full

S2: ¬can-move

S3: battery-full

S4: liftable

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable ∨ ¬battery-full

S2: ¬can-move

S3: battery-full

S4: liftable

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable ∨ ¬battery-full

S2: ¬can-move

S3: battery-full

S4: liftable

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable ∨ ¬battery-full

S2: ¬can-move

S3: battery-full

S4: liftable

How do we prove the box is not liftable?

S1: can-move ∨ ¬liftable ∨ ¬battery-full

S2: ¬can-move

S3: battery-full

S4: liftable

How do we prove the box is not liftable?

If an animal has wings and does not have fur, it is a bird.

Write in formal logic:

$\texttt{has-wings} \wedge \neg\texttt{has-fur} \Rightarrow \texttt{bird}$

(Use the predicates `has-wings`, `has-fur`, and `bird`)

How do we prove this is a bird?

has-wings ∧ ¬has-fur ⇒ bird

Use implication elimination to rewrite as a conditional:

¬(has-wings ∧ ¬has-fur) ∨ bird

(Use the predicates has-wings, has-fur, and bird)

How do we prove this is a bird?

$\neg(\text{has-wings} \wedge \neg\text{has-fur}) \vee \text{bird}$

Use de Morgan's Law to rewrite in conjunctive normal form:

$\neg\text{has-wings} \vee \text{has-fur} \vee \text{bird}$

(Use the predicates has-wings, has-fur, and bird)

How do we prove this is a bird?

S1:
¬has-wings ∨ has-fur ∨ bird
S2: has-wings
S3: ¬has-fur

What sentence would be assumed to facilitate the proof?

S4: ¬bird

How do we prove this is a bird?

S1:
¬has-wings ∨ has-fur ∨ bird
S2: has-wings
S3: ¬has-fur
S4: ¬bird

What part of S1 would we eliminate first?

- ¬has-wings
- has-fur
- bird

How do we prove this is a bird?

S1:

¬has-wings ∨ has-fur ∨ bird

S2: has-wings

S3: ¬has-fur

S4: ¬bird

What part of S1 would we eliminate first?

- ¬has-wings
- has-fur
- bird

How do we prove this is a bird?

S1:

¬has-wings ∨ has-fur ∨ bird

S2: has-wings

S3: ¬has-fur

S4: ¬bird

What do we do next?

- Resolve on S2 and ¬has-wings from S1
  - o Resolve on S2 and has-fur from S1
- o Resolve on S3 and ¬has-wings from S1
  - o Resolve on S3 and has-fur from S1

How do we prove this is a bird?

Assignment

How would you represent Raven's progressive matrices using formal logic?

To recap…

- Formal notation
- Properties of truth values
- Rules of inference
- Proof by refutation