GAC Example

(a) $Dom[X] = \{1, 2, 3, 4\}$
(b) $Dom[Y] = \{1, 2, 3, 4\}$
(c) $Dom[Z] = \{1, 2, 3, 4\}$
(d) $Dom[W] = \{1, 2, 3, 4, 5\}$

And 3 constraints:

(a) $C_1(X, Y, Z)$ which is satisfied only when $X = Y = Z$
(b) $C_2(X, W)$ which is satisfied only when $W > X$
(c) $C_3(X, Y, Z, W)$ which is satisfied only when $W = X = Z = Y$

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

GAC Example

(a) $Dom[X] = \{1, 2, 3, 4\}$
(b) $Dom[Y] = \{1, 2, 3, 4\}$
(c) $Dom[Z] = \{1, 2, 3, 4\}$
(d) $Dom[W] = \{1, 2, 3, 4, 5\}$

And 3 constraints:

(a) $C_1(X, Y, Z)$ which is satisfied only when $X = Y + Z$
(b) $C_2(X, W)$ which is satisfied only when $W > X$
(c) $C_3(X, Y, Z, W)$ which is satisfied only when $W = X + Z + Y$

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- All constraints put on GAC queue.
- Process C_3 first.
  X = 1 (X=1, Y=1, Z=1, W=3)
  X = 2 (X=2, Y=1, Z=1, W=4)
  X = 3 (X=3, Y=1, Z=1, W=5)
  X = 4 – Inconsistent.
  Dom(X) = {1, 2, 3}
  similarly
  Dom(Y) = {1, 2, 3}
  Dom(Z) = {1, 2, 3}

W = 1 – inconsistent
W = 2 – inconsistent
W = 3 – same support as X=1
W = 4 – same support as X = 2
W= 5 – same support as X = 3

Dom(W) = {3, 4, 5}

All domains pruned, but all other constraints already on GAC queue

GAC Example

(a) $Dom[X] = \{1, 2, 3, 4\}$
(b) $Dom[Y] = \{1, 2, 3, 4\}$
(c) $Dom[Z] = \{1, 2, 3, 4\}$
(d) $Dom[W] = \{1, 2, 3, 4, 5\}$

And 3 constraints:

(a) $C_1(X, Y, Z)$ which is satisfied only when $X = Y + Z$
(b) $C_2(X, W)$ which is satisfied only when $W > X$
(c) $C_3(X, Y, Z, W)$ which is satisfied only when $W = X + Z + Y$

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- Process C_2 next
  Currently
  Dom(X) = {1, 2, 3}
  Dom(W) = {3, 4, 5}

  X = 1 (X=1, W=3)
  X = 2 (X=2, W=3)
  X = 3 (X=3, W=4)

  W=3, W=4 found supports already

W=5 (X=1, W=5)

No domains pruned. Nothing added to GAC Queue

GAC Example

(a) $Dom[X] = \{1, 2, 3, 4\}$
(b) $Dom[Y] = \{1, 2, 3, 4\}$
(c) $Dom[Z] = \{1, 2, 3, 4\}$
(d) $Dom[W] = \{1, 2, 3, 4, 5\}$

And 3 constraints:

(a) $C_1(X, Y, Z)$ which is satisfied only when $X = Y + Z$
(b) $C_2(X, W)$ which is satisfied only when $W > X$
(c) $C_3(X, Y, Z, W)$ which is satisfied only when $W = X + Z + Y$

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- Process C_1 next

At this stage
Dom(X) = Dom(Y) = Dom(Z)
= {1, 2, 3}

X = 1 – inconsistent
X = 2 – (X=2, Y=1, Z=1)
X = 3 – (X=3, Y=1, Z=2)

Y = 1 – same support as X=2
Y = 2 – (X=3, Y=2, Z=1)
Y = 3 – inconsistent

Z = 1 – same support as X=2
Z = 2 – same support as X=3
Z = 3 – inconsistent

Updated domains
X = {2,3}
Y = {1,2}
Z = {1,2}

Put C_2 and C_3 back onto GAC queue

GAC Example

(a) $Dom[X] = \{1, 2, 3, 4\}$
(b) $Dom[Y] = \{1, 2, 3, 4\}$
(c) $Dom[Z] = \{1, 2, 3, 4\}$
(d) $Dom[W] = \{1, 2, 3, 4, 5\}$

And 3 constraints:

(a) $C_1(X, Y, Z)$ which is satisfied only when $X = Y + Z$
(b) $C_2(X, W)$ which is satisfied only when $W > X$
(c) $C_3(X, Y, Z, W)$ which is satisfied only when $W = X + Z + Y$

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- Process C_3 next current domains:
  Dom(X) = {2, 3}
  Dom(Y) = {1, 2}
  Dom(Z) = {1, 2}
  Dom(W) = {3,4,5}

X = 2 – {X=2, W=4, Y=1, Z=1}
X = 3 – {X=3, W=5, Y=1, Z=1}

Y = 1 – found support
Y = 2 – {X=2, W=5, Y=2, Z=1}

Z = 1 – found support
Z = 2 – {X=2, W=5, Y=1, Z=2}

W = 3 inconsistent
W = 4 – found support
W = 5 – found support

Pruned domains
W = {4, 5}

C_2 already on GAC queue

GAC Example

(a) $Dom[X] = \{1, 2, 3, 4\}$
(b) $Dom[Y] = \{1, 2, 3, 4\}$
(c) $Dom[Z] = \{1, 2, 3, 4\}$
(d) $Dom[W] = \{1, 2, 3, 4, 5\}$

And 3 constraints:

(a) $C_1(X, Y, Z)$ which is satisfied only when $X = Y + Z$
(b) $C_2(X, W)$ which is satisfied only when $W > X$
(c) $C_3(X, Y, Z, W)$ which is satisfied only when $W = X + Z + Y$

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- Process C_2 next current domains: No Domains pruned.
  Dom(X) = {2, 3}
  Dom(W) = {4,5}
  Nothing added to queue
  Queue Empty

  X = 2 – {X=2, W=4}
  X = 3 – {X=3, W=4}

  W = 4 – found support
  W = 5 – {X=3, W=5}

GAC finished.

GAC domains:

X = {2,3}
Z = {1, 2}
Y = {1, 2}
W = {4,5}

GAC Example

(a) $Dom[X] = \{1, 2, 3, 4\}$
(b) $Dom[Y] = \{1, 2, 3, 4\}$
(c) $Dom[Z] = \{1, 2, 3, 4\}$
(d) $Dom[W] = \{1, 2, 3, 4, 5\}$

And 3 constraints:

(a) $C_1(X, Y, Z)$ which is satisfied only when $X = Y + Z$
(b) $C_2(X, W)$ which is satisfied only when $W > X$
(c) $C_3(X, Y, Z, W)$ which is satisfied only when $W = X + Z + Y$

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

- Note GAC enforce does not find a solution
  To find a solution we must use do search while enforcing GAC.

- Branch on X.
  X = 2
  GAC(C_1) ➔ Y = 1, Z=1
  GAC(C_2) ➔ no changes
  GAC(C_3) ➔ W = 4
  This is a solution.

- Branch On X = 3
  GAC(C_1) ➔ no changes
  GAC(C_2) ➔ no changes
  GAC(C_3) ➔ Prune W=4
  Prune Y = 2
  Prune Z = 2
  Current Domains
  X={3}, Y={1}, Z={1}, W={5}
  GAC(C_1) ➔ Prune Y={1} DWO

NOTE No solution with X=3 but X=3 not pruned by GAC enforce.

Example

- C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

- C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

- C3(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

- Dom[V1]…Dom[V5] = {a, b, c}

Example

- C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

- C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

- C3(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

- V1=C: no support
- V2=C: no support
- V3=A: no support

- V1={a,b}
- V2={a,b}
- V3={b,c}

Example

- C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

- C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

- C3(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

- V1=C: no support
- V2=C: no support
- V3=A: no support

- V1={a,b}
- V2={a,b}
- V3={b,c}

Example

- C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

- C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

- C3(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

- V1=C: no support
- V2=C: no support
- V3=A: no support

- V1={a,b}
- V2={a,b}
- V3={b,c}

- V4=A: no support
- V5=A: no support
- V5=C: no support

- V4={C,B}
- V5={B}

Example

- C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

- V1=C: no support
- V2=C: no support
- V3=A: no support

- V1={a,b}
- V2={a,b}
- V3={b,c}

- C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

- V4=A: no support
- V5=A: no support
- V5=C: no support

- V4={C,B}
- V5={B}

- C2(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

Example

- C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

- V1=C: no support
- V2=C: no support
- V3=A: no support

- V1={a,b}
- V2={a,b}
- V3={b,c}

- C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

- V4=A: no support
- V5=A: no support
- V5=C: no support

- V4={C,B}
- V5={B}

- C2(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

- V2=A: no support
- V3=B: no support

- V2={B}
- V3={C}

Example

- C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

- C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

- C2(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

- V1=B has no support
- V1={A}
- V4={C,B}
- V5={B}
- V2={B}
- V3={C}

Example

- C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

- V1=B has no support
- V1={A}

- C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

- V4=B has no support
- V4={B}
- V5={B}
- V3=C has no support

- V3={} DWO

- C2(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

- V2={B}
- V3={C}