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In each of the following, replace  $p$  by

$$\langle x: \text{int} \rightarrow \langle y: \text{int} \rightarrow \langle z: \text{int} \rightarrow x \geq 0 \wedge x^2 \leq y \wedge \forall z: \text{int}. z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle$$

and simplify, assuming  $x, y, z, u, w: \text{int}$ .

(a)

§

$$p(x+y)(2xu+w)z$$

$$\langle x: \text{int} \rightarrow \langle y: \text{int} \rightarrow \langle z: \text{int} \rightarrow x \geq 0 \wedge x^2 \leq y \wedge \forall z: \text{int}. z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle (x+y)(2xu+w)z$$

Variables  $x$ ,  $y$ , and  $z$  appear both locally and nonlocally.Variable  $z$  is introduced twice locally.To avoid confusion, I will rename the local variables to  $a$ ,  $b$ ,  $c$ , and  $d$ .(Since the first local  $z$  is unused, I don't need to rename it to  $c$ , but I will anyway.)

$$= \langle a: \text{int} \rightarrow \langle b: \text{int} \rightarrow \langle c: \text{int} \rightarrow a \geq 0 \wedge a^2 \leq b \wedge \forall d: \text{int}. d^2 \leq b \Rightarrow d \leq a \rangle \rangle \rangle (x+y)(2xu+w)z$$

apply 3 times

$$= x+y \geq 0 \wedge (x+y)^2 \leq 2xu+w \wedge \forall d: \text{int}. d^2 \leq 2xu+w \Rightarrow d \leq x+y$$

(b)

§

$$p(x+y)(2xu+w)$$

$$\langle x: \text{int} \rightarrow \langle y: \text{int} \rightarrow \langle z: \text{int} \rightarrow x \geq 0 \wedge x^2 \leq y \wedge \forall z: \text{int}. z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle (x+y)(2xu+w)$$

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apply 2 times

$$= \langle c: \text{int} \rightarrow x+y \geq 0 \wedge (x+y)^2 \leq 2xu+w \wedge \forall d: \text{int}. d^2 \leq 2xu+w \Rightarrow d \leq x+y \rangle$$

note that  $c$  is unused.

$$= \text{int} \rightarrow x+y \geq 0 \wedge (x+y)^2 \leq 2xu+w \wedge \forall d: \text{int}. d^2 \leq 2xu+w \Rightarrow d \leq x+y$$

(c)

§

$$p(x+z)(y+y)(2+z)$$

$$\langle x: \text{int} \rightarrow \langle y: \text{int} \rightarrow \langle z: \text{int} \rightarrow x \geq 0 \wedge x^2 \leq y \wedge \forall z: \text{int}. z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle (x+z)(y+y)(2+z)$$

Variables  $x$ ,  $y$ , and  $z$  appear both locally and nonlocally.Variable  $z$  is introduced twice locally.To avoid confusion, I will rename the local variables to  $a$ ,  $b$ ,  $c$ , and  $d$ .(Since the first local  $z$  is unused, I don't need to rename it to  $c$ , but I will anyway.)

$$= \langle a: \text{int} \rightarrow \langle b: \text{int} \rightarrow \langle c: \text{int} \rightarrow a \geq 0 \wedge a^2 \leq b \wedge \forall d: \text{int}. d^2 \leq b \Rightarrow d \leq a \rangle \rangle \rangle (x+z)(y+y)(2+z)$$

apply 3 times

$$= x+z \geq 0 \wedge (x+z)^2 \leq y+y \wedge \forall d: \text{int}. d^2 \leq y+y \Rightarrow d \leq x+z$$

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