143 In natural variables s and n prove

$$P \;\Leftarrow\; \textbf{if } n{=}0 \textbf{ then } ok \textbf{ else } n{:=}\ n{-}1.\ \ s{:=}\ s{+}2^n{-}n.\ \ t{:=}\ t{+}1.\ \ P \textbf{ fi}$$

where $P \;=\; s' = s + 2^n - n{\times}(n{-}1)/2 - 1 \;\wedge\; n'{=}0 \;\wedge\; t' = t{+}n$.

§ Proof by parts (3 of them) and by cases (2 of them), so 6 things to prove. First part, first case, starting with the right side:

$n{=}0 \wedge ok$ — expand ok

$=\ \ n{=}0 \wedge n'{=}n \wedge s'{=}s \wedge t'{=}t$ — arithmetic and specialization

$\Rightarrow\ \ s' = s + 2^n - n{\times}(n{-}1)/2 - 1$

First part, last case:

$n{>}0 \wedge (n{:=}\ n{-}1.\ \ s{:=}\ s{+}2^n{-}n.\ \ t{:=}\ t{+}1.\ \ s' = s + 2^n - n{\times}(n{-}1)/2 - 1)$ — Substitution Law 3 times

$=\ \ n{>}0 \;\wedge\; s' = s + 2^{n-1} - (n{-}1) + 2^{n-1} - (n{-}1){\times}(n{-}1{-}1)/2 - 1$ — arithmetic

$=\ \ n{>}0 \;\wedge\; s' = s + 2^n - n{\times}(n{-}1)/2 - 1$ — specialization

$\Rightarrow\ \ s' = s + 2^n - n{\times}(n{-}1)/2 - 1$

Middle part, first case:

$n{=}0 \wedge ok$ — expand ok

$=\ \ n{=}0 \wedge n'{=}n \wedge s'{=}s \wedge t'{=}t$ — transitivity and specialization

$\Rightarrow\ \ n'{=}0$

Middle part, last case:

$n{>}0 \wedge (n{:=}\ n{-}1.\ \ s{:=}\ s{+}2^n{-}n.\ \ t{:=}\ t{+}1.\ \ n'{=}0)$ — Substitution Law 3 times

$=\ \ n{>}0 \;\wedge\; n'{=}0$ — specialization

$\Rightarrow\ \ n'{=}0$

Last part, first case:

$n{=}0 \wedge ok$ — expand ok

$=\ \ n{=}0 \wedge n'{=}n \wedge s'{=}s \wedge t'{=}t$ — arithmetic and specialization

$\Rightarrow\ \ t' = t{+}n$

Last part, last case:

$n{>}0 \wedge (n{:=}\ n{-}1.\ \ s{:=}\ s{+}2^n{-}n.\ \ t{:=}\ t{+}1.\ \ t' = t{+}n)$ — Substitution Law 3 times

$=\ \ n{>}0 \;\wedge\; t' = t{+}1{+}n{-}1$ — arithmetic and specialization

$\Rightarrow\ \ t' = t{+}n$