22 (tennis) An advertisement for a tennis magazine says "If I'm not playing tennis, I'm watching tennis. And if I'm not watching tennis, I'm reading about tennis.". Assuming the speaker cannot do more than one of these activities at a time,

(a) prove that the speaker is not reading about tennis.

§ Abbreviations: p = (I'm playing tennis) ; w = (I'm watching tennis) ; r = (I'm reading about tennis) . The given information is on the first line of the proof.

$(\neg p \Rightarrow w) \wedge \underline{(\neg w \Rightarrow r)} \wedge \neg(p \wedge w) \wedge \neg(p \wedge r) \wedge \neg(r \wedge w)$ contrapositive

$= \underline{(\neg p \Rightarrow w) \wedge (\neg r \Rightarrow w)} \wedge \neg(p \wedge w) \wedge \neg(p \wedge r) \wedge \neg(r \wedge w)$ antidistributive

$= \underline{(\neg p \vee \neg r} \Rightarrow w) \wedge \neg(p \wedge w) \wedge \neg(p \wedge r) \wedge \neg(r \wedge w)$ duality

$= \underline{(\neg(p \wedge r) \Rightarrow w)} \wedge \neg(p \wedge w) \wedge \underline{\neg(p \wedge r)} \wedge \neg(r \wedge w)$ discharge

$= \underline{w} \wedge \neg(p \wedge \underline{w}) \wedge \neg(p \wedge r) \wedge \neg(r \wedge \underline{w})$ use the first w as context

$= w \wedge \neg p \wedge \neg(p \wedge r) \wedge \neg r$ duality

$= w \wedge \neg p \wedge (\underline{\neg p} \vee \underline{\neg r}) \wedge \neg r$ use $\neg p$ and then $\neg r$ to simplify

$= w \ \wedge \ \neg p \ \wedge \ \neg r$ specialize

$\Longrightarrow \ \neg r$

Perhaps the second given sentence meant "And if I'm also not watching tennis, I'm reading about tennis.". If so, it should be formalized as $\neg p \wedge \neg w \Rightarrow r$.

$(\neg p \Rightarrow w) \wedge \underline{(\neg p \wedge \neg w \Rightarrow r)} \wedge \neg(p \wedge w) \wedge \neg(p \wedge r) \wedge \neg(r \wedge w)$ portation

$= \underline{(\neg p \Rightarrow w) \wedge (\neg p \Rightarrow (\neg w \Rightarrow r))} \wedge \neg(p \wedge w) \wedge \neg(p \wedge r) \wedge \neg(r \wedge w)$ distribution

$= (\neg p \Rightarrow w) \wedge (\neg p \Rightarrow (\neg w \Rightarrow r)) \wedge \neg(p \wedge w) \wedge \neg(p \wedge r) \wedge \neg(r \wedge w)$

use first w as context for second w

$= (\neg p \Rightarrow w) \wedge \underline{\neg(p \wedge w)} \wedge \neg(p \wedge r) \wedge \neg(r \wedge w)$ duality

$= (\neg p \Rightarrow w) \wedge (\underline{\neg p \vee \neg w}) \wedge \neg(p \wedge r) \wedge \neg(r \wedge w)$ material implication

$= \underline{(\neg p \Rightarrow w) \wedge (w \Rightarrow \neg p)} \wedge \neg(p \wedge r) \wedge \neg(r \wedge w)$ double implication

$= \underline{\neg p = w} \wedge \neg(p \wedge r) \wedge \neg(r \wedge \underline{w})$ use context $\neg p = w$ to replace w with $\neg p$

$= \neg p = w \wedge \underline{\neg(p \wedge r)} \wedge \underline{\neg(r \wedge \neg p)}$ duality twice

$= \underline{\neg p = w} \wedge \underline{(\neg p \vee \neg r) \wedge (\neg r \vee p)}$ exclusion and factor out $\neg r$

$= p \neq w \wedge (\neg r \vee (\neg p \wedge p))$ noncontradiction and identity

$= p \neq w \wedge \neg r$ specialize

$\Longrightarrow \ \neg r$

In either interpretation, the speaker is not reading about tennis, so it is not a good advertisement for the magazine.

(b) what is the speaker doing?

§ From the second-last line of the first interpretation of part (a), watching tennis. From the second-last line of the second interpretation of part (a), we see that the speaker is either playing tennis or watching tennis, but not both, and we cannot determine which one.