451 Let a and b be binary interactive variables. Define

$$loop \;=\; \textbf{if } b \textbf{ then } loop \textbf{ else } ok \textbf{ fi}$$

Add a time variable according to any reasonable measure, and then express

$$b{:=}\,\bot \;\|\; loop$$

as an equivalent program but without using $\|$.

§ The left process owns b. Variable a could belong to either process; let's give it to the right process. Let assignment take time 1. Then the left process is

$$\neg b(t{+}1) \wedge t'{=}t{+}1$$

Add recursive time to $loop$, and the right process is

$$\begin{array}{ll}
 & loop \\
= & \textbf{if } b\ t \textbf{ then } t{:=}\,t{+}1.\ loop \textbf{ else } ok \textbf{ fi} \qquad \text{unroll} \\
= & \textbf{if } b\ t \textbf{ then } t{:=}\,t{+}1.\ \textbf{if } b\ t \textbf{ then } t{:=}\,t{+}1.\ loop \textbf{ else } ok \textbf{ fi} \textbf{ else } ok \textbf{ fi} \qquad \text{Substitution Law} \\
= & \textbf{if } b\ t \textbf{ then if } b(t{+}1) \textbf{ then } t{:=}\,t{+}2.\ loop \textbf{ else } t{:=}\,t{+}1 \textbf{ fi else } ok \textbf{ fi} \\
 & \qquad\qquad \text{The left process gives us } \neg b(t{+}1) \\
= & \textbf{if } b\ t \textbf{ then if } \bot \textbf{ then } t{:=}\,t{+}2.\ loop \textbf{ else } t{:=}\,t{+}1 \textbf{ fi else } ok \textbf{ fi} \\
= & \textbf{if } b\ t \textbf{ then } t{:=}\,t{+}1 \textbf{ else } ok \textbf{ fi} \\
= & \textbf{if } b\ t \textbf{ then } a\ t' = a\ t \ \wedge\ t' = t{+}1 \textbf{ else } t'{=}t \textbf{ fi} \qquad t'{=}t \text{ implies } a\ t' = a\ t \\
= & a\ t' = a\ t \ \wedge\ \textbf{if } b\ t \textbf{ then } t' = t{+}1 \textbf{ else } t'{=}t \textbf{ fi}
\end{array}$$

The independent composition is

$$\exists tL, tR\cdot \quad \neg b(t{+}1) \wedge tL{=}t{+}1 \wedge a\ tR = a\ t \ \wedge\ \textbf{if } b\ t \textbf{ then } tR = t{+}1 \textbf{ else } tR{=}t \textbf{ fi}$$
$$\wedge\ t' = max.tL.tR$$

The left process takes time 1 and the right process takes time 0 or 1, so the maximum is 1

$$= \quad \neg b(t{+}1) \ \wedge\ a(t{+}1) = a\ t \ \wedge\ t' = t{+}1$$

We no longer have an independent composition, so a and b are both variables

$$= \quad b{:=}\,\bot$$