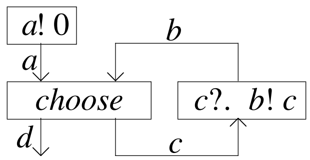
477 (Brock-Ackerman) The following picture shows a network of communicating processes.



The formal description of this network is

$$\textbf{chan } a, b, c\cdot\ a!\,0 \ \|\ choose \ \|\ (c?.\ b!\,c)$$

Formally define *choose*, add transit time, and state the output message and time if

(a) *choose* either reads from a and then outputs a 0 on c and d, or reads from b and then outputs a 1 on c and d. The choice is made freely.

§ We define *choose* as follows:

$$choose \ = \ (a?.\ (c!\,0 \,\|\, d!\,0)) \vee (b?.\ (c!\,1 \,\|\, d!\,1))$$

Now we calculate.

$$\textbf{chan } a, b, c\cdot\ a!\,0 \ \|\ choose \ \|\ (c?.\ b!\,c)$$

$= \ \exists \mathcal{M}a, \mathcal{T}a, \mathbf{r}a, \mathbf{r}a', \mathbf{w}a, \mathbf{w}a', \mathcal{M}b, \mathcal{T}b, \mathbf{r}b, \mathbf{r}b', \mathbf{w}b, \mathbf{w}b', \mathcal{M}c, \mathcal{T}c, \mathbf{r}c, \mathbf{r}c', \mathbf{w}c, \mathbf{w}c'\cdot$
$\quad (\forall i, j\cdot\ i{\le}j \ \Rightarrow\ t \le \mathcal{T}a_i \le \mathcal{T}a_j \le t' \ \wedge\ t \le \mathcal{T}b_i \le \mathcal{T}b_j \le t' \ \wedge\ t \le \mathcal{T}c_i \le \mathcal{T}c_j \le t')$
$\wedge\ \mathbf{r}a{=}\mathbf{w}a{=}\mathbf{r}b{=}\mathbf{w}b{=}\mathbf{r}c{=}\mathbf{w}c{=}0$
$\wedge\ (\mathcal{M}a_{\mathbf{w}a}{=}0 \wedge \mathcal{T}a_{\mathbf{w}a}{=}t \wedge (\mathbf{w}a{:=}\ \mathbf{w}a{+}1)$
$\quad \|\ (\quad t{:=}\ max\ t\ (\mathcal{T}a_{\mathbf{r}a} + 1).\ \ \mathbf{r}a{:=}\ \mathbf{r}a{+}1.$
$\qquad (\ \mathcal{M}c_{\mathbf{w}c}{=}0 \wedge \mathcal{T}c_{\mathbf{w}c}{=}t \wedge (\mathbf{w}c{:=}\ \mathbf{w}c{+}1)$
$\qquad \|\ \mathcal{M}d_{\mathbf{w}d}{=}0 \wedge \mathcal{T}d_{\mathbf{w}d}{=}t \wedge (\mathbf{w}d{:=}\ \mathbf{w}d{+}1)\ )\ )$
$\quad \vee\ (\quad t{:=}\ max\ t\ (\mathcal{T}b_{\mathbf{r}b} + 1).\ \ \mathbf{r}b{:=}\ \mathbf{r}b{+}1.$
$\qquad (\ \mathcal{M}c_{\mathbf{w}c}{=}1 \wedge \mathcal{T}c_{\mathbf{w}c}{=}t \wedge (\mathbf{w}c{:=}\ \mathbf{w}c{+}1)$
$\qquad \|\ \mathcal{M}d_{\mathbf{w}d}{=}1 \wedge \mathcal{T}d_{\mathbf{w}d}{=}t \wedge (\mathbf{w}d{:=}\ \mathbf{w}d{+}1)\ )\ )$
$\|\ (\quad t{:=}\ max\ t\ (\mathcal{T}c_{\mathbf{r}c} + 1).\ \ \mathbf{r}c{:=}\ \mathbf{r}c{+}1.$
$\qquad \mathcal{M}b_{\mathbf{w}b}{=}\mathcal{M}c_{\mathbf{r}c-1} \wedge \mathcal{T}b_{\mathbf{w}b}{=}t \wedge (\mathbf{w}b{:=}\ \mathbf{w}b{+}1)\ )\ )$

Except for time, all processes in independent compositions change different variables, so $\|$ is easily replaced by conjunction. Also, make all substitutions indicated by assignments.

$= \ \exists \mathcal{M}a, \mathcal{T}a, \mathbf{r}a, \mathbf{r}a', \mathbf{w}a, \mathbf{w}a', \mathcal{M}b, \mathcal{T}b, \mathbf{r}b, \mathbf{r}b', \mathbf{w}b, \mathbf{w}b', \mathcal{M}c, \mathcal{T}c, \mathbf{r}c, \mathbf{r}c', \mathbf{w}c, \mathbf{w}c'\cdot$
$\quad (\forall i, j\cdot\ i{\le}j \ \Rightarrow\ t \le \mathcal{T}a_i \le \mathcal{T}a_j \le t' \ \wedge\ t \le \mathcal{T}b_i \le \mathcal{T}b_j \le t' \ \wedge\ t \le \mathcal{T}c_i \le \mathcal{T}c_j \le t')$
$\wedge\ \mathbf{r}a{=}\mathbf{w}a{=}\mathbf{r}b{=}\mathbf{w}b{=}\mathbf{r}c{=}\mathbf{w}c{=}0$
$\wedge\ \exists ta, tc, tb\cdot$
$\qquad ta{=}\mathcal{T}a_0{=}t \wedge \mathcal{M}a_0{=}0 \wedge \mathbf{w}a'{=}1$
$\quad \wedge\ (\quad tc = \mathcal{T}c_0 = \mathcal{T}d_{\mathbf{w}d} = \mathcal{T}a_0 + 1 \wedge \mathbf{r}a'{=}\mathbf{w}c'{=}1 \wedge \mathcal{M}c_0{=}\mathcal{M}d_{\mathbf{w}d}{=}0 \wedge \mathbf{w}d'{=}\mathbf{w}d{+}1$
$\qquad \vee\ \ tc = \mathcal{T}c_0 = \mathcal{T}d_{\mathbf{w}d} = \mathcal{T}b_0{+}1 \wedge \mathbf{r}b'{=}\mathbf{w}c'{=}1 \wedge \mathcal{M}c_0{=}\mathcal{M}d_{\mathbf{w}d}{=}1 \wedge \mathbf{w}d'{=}\mathbf{w}d{+}1)$
$\quad \wedge\ tb = \mathcal{T}b_0 = \mathcal{T}c_0 + 1 \ \wedge\ \mathbf{r}c'{=}\mathbf{w}b'{=}1 \ \wedge\ \mathcal{M}b_0{=}\mathcal{M}c_0$
$\quad \wedge\ t' = MAX\ [ta;\ tc;\ tb]$ use One-Point laws to eliminate most quantifiers

$= \ \exists tc, tb\cdot$
$\quad (\quad tc = \mathcal{T}d_{\mathbf{w}d} = t{+}1 \ \wedge\ \mathcal{M}d_{\mathbf{w}d}{=}0 \ \wedge\ \mathbf{w}d'{=}\mathbf{w}d{+}1$
$\quad \vee\ \ tc = \mathcal{T}d_{\mathbf{w}d} = tb + 1 \ \wedge\ \mathcal{M}d_{\mathbf{w}d}{=}1 \ \wedge\ \mathbf{w}d'{=}\mathbf{w}d{+}1\ )$
$\wedge\ \ tb = tc{+}1$
$\wedge\ \ t' = MAX\ [t;\ tc;\ tb]$ move the conjunctions into the disjunction

$= \ \exists tc, tb\cdot$
$\quad tc = \mathcal{T}d_{\mathbf{w}d} = t{+}1 \ \wedge\ \mathcal{M}d_{\mathbf{w}d}{=}0 \ \wedge\ \mathbf{w}d'{=}\mathbf{w}d{+}1 \ \wedge\ tb = tc{+}1 \ \wedge\ t' = MAX\ [t;\ tc;\ tb]$
$\vee\ \ tc = \mathcal{T}d_{\mathbf{w}d} = tb{+}1 \ \wedge\ \mathcal{M}d_{\mathbf{w}d}{=}0 \ \wedge\ \mathbf{w}d'{=}\mathbf{w}d{+}1 \ \wedge\ tb = tc{+}1 \ \wedge\ t' = MAX\ [t;tc;tb]$

now we can eliminate tc and tb in each disjunct separately

$= \quad \mathcal{T}d_{\mathbf{w}d} = t{+}1 \ \wedge\ \mathcal{M}d_{\mathbf{w}d}{=}0 \ \wedge\ \mathbf{w}d'{=}\mathbf{w}d{+}1 \ \wedge\ t' = t + 2$
$\vee\ \ \mathcal{T}d_{\mathbf{w}d} = \infty \ \wedge\ \mathcal{M}d_{\mathbf{w}d}{=}1 \ \wedge\ \mathbf{w}d'{=}\mathbf{w}d{+}1 \ \wedge\ t' = \infty$

$= \quad (t{:=}\ t{+}1.\ d!\ 0.\ t{:=}\ t{+}1) \lor (t{:=}\ \infty.\ d!\ 1)$

Either a 0 is output after time 1 or nothing ever happens. There is probably a better way to do this question by using laws of programs and not translating to ordinary logic.

(b) as in part (a), *choose* either reads from a and then outputs a 0 on c and d, or reads from b and then outputs a 1 on c and d. But this time the choice is not made freely; *choose* reads from the channel whose input is available first (if there's a tie, then take either one).

§ We define *choose* as follows:

$$\begin{aligned} choose \ = \ & (\surd a \ \lor \ \mathcal{T}a_{ra}{\le}\mathcal{T}b_{rb}) \land (a?.\ (c!\ 0 \parallel d!\ 0)) \\ \lor\ & (\surd b \ \lor \ \mathcal{T}b_{rb}{\le}\mathcal{T}a_{ra}) \land (b?.\ (c!\ 1 \parallel d!\ 1)) \end{aligned}$$

Now we calculate.

$$\textbf{chan } a, b, c\cdot\ a!\ 0 \parallel choose \parallel (c?.\ b!\ c)$$

$$\begin{aligned}
= \quad & \exists \mathcal{M}a, \mathcal{T}a, ra, ra', wa, wa', \mathcal{M}b, \mathcal{T}b, rb, rb', wb, wb', \mathcal{M}c, \mathcal{T}c, rc, rc', wc, wc'\cdot \\
& \quad (\forall i, j\cdot\ i{\le}j \ \Rightarrow\ t \le \mathcal{T}a_i \le \mathcal{T}a_j \le t' \ \land\ t \le \mathcal{T}b_i \le \mathcal{T}b_j \le t' \ \land\ t \le \mathcal{T}c_i \le \mathcal{T}c_j \le t') \\
& \land\ ra{=}wa{=}rb{=}wb{=}rc{=}wc{=}0 \\
& \land\ (\quad \mathcal{M}a_{wa}{=}0 \land \mathcal{T}a_{wa}{=}t \land (wa{:=}\ wa{+}1) \\
& \quad \parallel \ (\ (\quad (\mathcal{T}a_{ra}{\le}t \lor \mathcal{T}a_{ra}{\le}\mathcal{T}b_{rb}) \\
& \qquad\qquad \land\ (t{:=}\ max\ t\ (\mathcal{T}a_{ra} + 1).\ ra{:=}\ ra{+}1. \\
& \qquad\qquad\quad (\ \mathcal{M}c_{wc}{=}0 \land \mathcal{T}c_{wc}{=}t \land (wc{:=}\ wc{+}1) \\
& \qquad\qquad\quad \parallel \mathcal{M}d_{wd}{=}0 \land \mathcal{T}d_{wd}{=}t \land (wd{:=}\ wd{+}1)\ )\ )\ ) \\
& \qquad \lor\ (\quad (\mathcal{T}b_{rb}{\le}t \lor \mathcal{T}b_{rb}{\le}\mathcal{T}a_{ra}) \\
& \qquad\qquad \land\ (t{:=}\ max\ t\ (\mathcal{T}b_{rb} + 1).\ rb{:=}\ rb{+}1. \\
& \qquad\qquad\quad (\ \mathcal{M}c_{wc}{=}1 \land \mathcal{T}c_{wc}{=}t \land (wc{:=}\ wc{+}1) \\
& \qquad\qquad\quad \parallel \mathcal{M}d_{wd}{=}1 \land \mathcal{T}d_{wd}{=}t \land (wd{:=}\ wd{+}1)\ )\ )\ )\ ) \\
& \quad \parallel \ (\ t{:=}\ max\ t\ (\mathcal{T}c_{rc} + 1).\ rc{:=}\ rc{+}1. \\
& \qquad \mathcal{M}b_{wb}{=}\mathcal{M}c_{rc-1} \land \mathcal{T}b_{wb}{=}t \land (wb{:=}\ wb{+}1)\ )\ )
\end{aligned}$$

Except for time, all processes in independent compositions change different variables, so $\parallel$ is easily replaced by conjunction. Also, make all substitutions indicated by assignments.

$$\begin{aligned}
= \quad & \exists \mathcal{M}a, \mathcal{T}a, ra, ra', wa, wa', \mathcal{M}b, \mathcal{T}b, rb, rb', wb, wb', \mathcal{M}c, \mathcal{T}c, rc, rc', wc, wc'\cdot \\
& \quad (\forall i, j\cdot\ i{\le}j \ \Rightarrow\ t \le \mathcal{T}a_i \le \mathcal{T}a_j \le t' \ \land\ t \le \mathcal{T}b_i \le \mathcal{T}b_j \le t' \ \land\ t \le \mathcal{T}c_i \le \mathcal{T}c_j \le t') \\
& \land\ ra{=}wa{=}rb{=}wb{=}rc{=}wc{=}0 \\
& \land\ \exists ta, tc, tb\cdot \\
& \qquad ta{=}\mathcal{T}a_0{=}t \land \mathcal{M}a_0{=}0 \land wa'{=}1 \\
& \quad \land\ (\quad (\mathcal{T}a_0{\le}t \lor \mathcal{T}a_0{\le}\mathcal{T}b_0) \\
& \qquad\quad \land\ tc{=}\mathcal{T}c_0{=}\mathcal{T}d_{wd}{=}\mathcal{T}a_0{+}1 \land ra'{=}wc'{=}1 \land \mathcal{M}c_0{=}\mathcal{M}d_{wd}{=}0 \land wd'{=}wd{+}1 \\
& \qquad \lor \quad (\mathcal{T}b_0{\le}t \lor \mathcal{T}b_0{\le}\mathcal{T}a_0) \\
& \qquad\quad \land\ tc{=}\mathcal{T}c_0{=}\mathcal{T}d_{wd}{=}\mathcal{T}b_0{+}1 \land rb'{=}wc'{=}1 \land \mathcal{M}c_0{=}\mathcal{M}d_{wd}{=}1 \land wd'{=}wd{+}1) \\
& \quad \land\ tb = \mathcal{T}b_0 = \mathcal{T}c_0 + 1 \ \land\ rc'{=}wb'{=}1 \ \land\ \mathcal{M}b_0{=}\mathcal{M}c_0 \\
& \quad \land\ t' = MAX\ [ta;\ tc;\ tb]
\end{aligned}$$

use the One-Point laws to eliminate most quantifiers

$$\begin{aligned}
= \quad & \exists tc, tb\cdot \\
& \quad (\quad (t{\le}t \lor t{\le}tb) \ \land\ tc = \mathcal{T}d_{wd} = t{+}1 \ \land\ \mathcal{M}d_{wd}{=}0 \ \land\ wd'{=}wd{+}1 \\
& \quad \lor \quad (tb{\le}t \lor tb{\le}t) \ \land\ tc = \mathcal{T}d_{wd} = tb{+}1 \ \land\ \mathcal{M}d_{wd}{=}1 \ \land\ wd'{=}wd{+}1\ ) \\
& \land\ tb = tc{+}1 \\
& \land\ t' = MAX\ [t;\ tc;\ tb]
\end{aligned}$$

simplify the two minor disjunctions and move the conjunctions into the major disjunction

$$\begin{aligned}
= \quad & \exists tc, tc\cdot \\
& \quad tc{=}\mathcal{T}d_{wd}{=}t{+}1 \ \land\ \mathcal{M}d_{wd}{=}0 \ \land\ wd' = wd{+}1 \ \land tb = tc{+}1 \land t' = MAX\ [t;\ tc;\ tb] \\
& \lor \ \ tb{\le}t \ \land\ tc{=}\mathcal{T}d_{wd}{=}tb{+}1 \ \land\ \mathcal{M}d_{wd}{=}1 \land wd'{=}wd{+}1 \land tb = tc{+}1 \land t'{=}MAX[t;tc;tb]
\end{aligned}$$

now we can eliminate tc and tb in each disjunct separately

$=\quad \mathcal{T}d_{\boldsymbol{w}d} = t+1 \ \land\ \mathcal{M}d_{\boldsymbol{w}d}=0 \ \land\ \boldsymbol{w}d' = \boldsymbol{w}d+1 \ \land\ t' = t+2$

$\quad \lor\ \infty \le t \ \land\ \mathcal{T}d_{\boldsymbol{w}d}=\infty \ \land\ \mathcal{M}d_{\boldsymbol{w}d}=1 \ \land\ \boldsymbol{w}d'=\boldsymbol{w}d+1 \ \land\ t'=\infty$

$=\quad (t{:=}\ t+1.\ d!\ 0.\ t{:=}\ t+1) \ \lor\ t=t'=\infty \land (d!\ 1)$

If the computation starts before time ∞ the output is definitely 0 after time 1. Again, there is probably a better way to do this question by using laws of programs and not translating to ordinary logic.