321 (coin) Repeatedly flip a coin until you get a head. Prove that it takes n flips with probability 2^{-n}. With an appropriate definition of R, the program is

$R \Leftarrow t{:=}\, t{+}1.$ **if** *rand* 2 **then** *ok* **else** R **fi**

§ Starting with the right side, using 1/2 for *rand* 2 and $(t'{>}t) \times 2^{t-t'}$ for R:

$t{:=}\, t{+}1.$ **if** 1/2 **then** $t'{=}t$ **else** $(t'{>}t) \times 2^{t-t'}$ **fi** substitution law

$=$ **if** 1/2 **then** $t'{=}t{+}1$ **else** $(t'{>}t{+}1) \times 2^{t+1-t'}$ **fi** replace **if**

$=$ $(t'{=}t{+}1) / 2 \;+\; (t'{>}t{+}1) \times 2^{t+1-t'} / 2$

$=$ $(t'{=}t{+}1) \times 2^{t-t'} \;+\; (t'{>}t{+}1) \times 2^{t-t'}$

$=$ $(t'{>}t) \times 2^{t-t'}$

$=$ R