

14 Complete the following laws of Binary Theory

- (a) $\top =$
- (b) $\perp =$
- (c) $\neg a =$
- (d) $a \wedge b =$
- (e) $a \vee b =$
- (f) $a = b =$
- (g) $a \neq b =$
- (h) $a \Rightarrow b =$

by adding a right side using only the following symbols (in any quantity)

- (i) $\neg \wedge a b ()$
- (ii) $\neg \vee a b ()$
- (iii) $\neg \Rightarrow a b ()$
- (iv) $\neq \Rightarrow a b ()$
- (v) \neg **if then else fi** $a b ()$

That's $8 \times 5 = 40$ questions.

§	(i)	(ii)	(iii)	(iv)	(v)
(a)	$\neg(a \wedge \neg a)$	$a \vee \neg a$	$a \Rightarrow a$	$a \Rightarrow a$	if a then a else $\neg a$ fi
(b)	$a \wedge \neg a$	$\neg(a \vee \neg a)$	$\neg(a \Rightarrow a)$	$a \neq a$	if a then $\neg a$ else a fi
(c)	$\neg a$	$\neg a$	$\neg a$	$a \Rightarrow (a \neq a)$	$\neg a$
(d)	$a \wedge b$	$\neg(\neg a \vee \neg b)$	$\neg(a \Rightarrow \neg b)$	$(a \Rightarrow (a \neq b)) \Rightarrow (a \neq a)$	if a then b else a fi
(e)	$\neg(\neg a \wedge \neg b)$	$a \vee b$	$\neg(a \Rightarrow b)$	$(a \Rightarrow (a \neq a)) \Rightarrow b$	if a then a else b fi
(f)	$\neg(a \wedge \neg b)$ $\wedge \neg(\neg a \wedge b)$	$\neg(a \vee b)$ $\vee \neg(\neg a \vee \neg b)$	$\neg((a \Rightarrow b)$ $\Rightarrow \neg(b \Rightarrow a))$	$(a \neq b) \Rightarrow (a \neq a)$	if a then b else $\neg b$ fi
(g)	$\neg(a \wedge b)$ $\wedge \neg(\neg a \wedge \neg b)$	$\neg(a \vee \neg b)$ $\vee \neg(\neg a \vee b)$	$(a \Rightarrow b) \Rightarrow \neg(b \Rightarrow a)$	$a \neq b$	if a then $\neg b$ else b fi
(h)	$\neg(a \wedge \neg b)$	$\neg a \vee b$	$a \Rightarrow b$	$a \Rightarrow b$	if a then b else $\neg a$ fi

Note: using continuing operators, we can write (f)(iii) and (f)(iv) as $a \Rightarrow b \Rightarrow a$.