73 (whodunit) Here are ten statements.

(i) Some criminal robbed the Russell mansion.

(ii) Whoever robbed the Russell mansion either had an accomplice among the servants or had to break in.

(iii) To break in one would have to either smash the door or pick the lock.

(iv) Only an expert locksmith could pick the lock.

(v) Anyone smashing the door would have been heard.

(vi) Nobody was heard.

(vii) No one could rob the Russell mansion without fooling the guard.

(viii) To fool the guard one must be a convincing actor.

(ix) No criminal could be both an expert locksmith and a convincing actor.

(x) Some criminal had an accomplice among the servants.

(a) Choosing good abbreviations, translate each of these statements into formal logic.

§ Here are some abbreviations.

Cx = (x is a criminal)
Rx = (x robbed the Russell mansion)
Sx = (x had an accomplice among the servants)
Bx = (x broke in)
Dx = (x smashed the door)
Px = (x picked the lock)
Lx = (x is an expert locksmith)
Hx = (x was heard)
Fx = (x fooled the guard)
Ax = (x is a convincing actor)

Now the statements are formalized as follows.

(i) $\exists x \cdot Cx \wedge Rx$
(ii) $\forall x \cdot Rx \Rightarrow Sx \vee Bx$
(iii) $\forall x \cdot Bx \Rightarrow Dx \vee Px$
(iv) $\forall x \cdot Lx \Leftarrow Px$
(v) $\forall x \cdot Dx \Rightarrow Hx$
(vi) $\neg \exists x \cdot Hx$
(vii) $\neg \exists x \cdot Rx \wedge \neg Fx$
(viii) $\forall x \cdot Fx \Rightarrow Ax$
(ix) $\neg \exists x \cdot Cx \wedge Lx \wedge Ax$
(x) $\exists x \cdot Cx \wedge Sx$

(b) Taking the first nine statements as axioms, prove the tenth.

§ Lemma:

	$\top$	(vii)
$=$	$\neg \exists x \cdot Rx \wedge \neg Fx$	duality (deMorgan)
$=$	$\forall x \cdot \neg(Rx \wedge \neg Fx)$	duality (deMorgan)
$=$	$\forall x \cdot \neg Rx \vee \neg\neg Fx$	double negation
$=$	$\forall x \cdot \neg Rx \vee Fx$	material implication
$=$	$\forall x \cdot Rx \Rightarrow Fx$	

Now the main proof:

	$\top$	(i)
$=$	$\exists x \cdot Cx \wedge Rx$	idempotence
$=$	$\exists x \cdot Cx \wedge Rx \wedge Rx$	lemma and (ii)
$\Rightarrow$	$\exists x \cdot Cx \wedge Fx \wedge (Sx \vee Bx)$	(viii) and (iii)
$\Rightarrow$	$\exists x \cdot Cx \wedge Ax \wedge (Sx \vee Dx \vee Px)$	(v) and (iv)

$\Longrightarrow \quad \exists x \cdot Cx \wedge Ax \wedge (Sx \vee Hx \vee Lx)$ distribute

$= \quad \exists x \cdot Cx \wedge Ax \wedge Sx \;\vee\; Cx \wedge Ax \wedge Hx \;\vee\; Cx \wedge Ax \wedge Lx$ (vi) and (ix)

$= \quad \exists x \cdot Cx \wedge Ax \wedge Sx$ specialize

$\Longrightarrow \quad \exists x \cdot Cx \wedge Sx$