242 (transitive closure) A relation R: $(0,..n)\to(0,..n)\to bin$ can be represented by a square binary array of size n. Given a relation in the form of a square binary array, write a program to find

(a) its transitive closure (the strongest transitive relation that is implied by the given relation).

§ Let $Pijk$ mean “there is a path in R from j to k via zero or more intermediate nodes all of which are less than i”. Formally,

$$P0 = R$$
$$\forall i, j, k\cdot\ P(i+1)jk \ = \ Pijk \ \lor \ Piji \land Piik$$

Then we can say that R' is the transitive closure of R as follows:

$$R' = Pn$$

This is just right for a **for**-loop (Chapter 5).

$$R{=}P0 \Rightarrow R'{=}Pn \ \Leftarrow\ \textbf{for } i{:=}\ 0;..n \textbf{ do } R{=}Pi \Rightarrow R'{=}P(i{+}1) \textbf{ od}$$
$$R{=}Pi \Rightarrow R'{=}P(i{+}1) \ \Leftarrow$$
$$\quad \textbf{for } j{:=}\ 0;..n \textbf{ do for } k{:=}\ 0;..n \textbf{ do } R{:=}\ (j;k) \to Rjk \ \lor\ Rji \land Rik \mid R \textbf{ od od}$$

That's the whole thing. If you want more detail, define A as follows.

$$\begin{aligned} Aijk \ = \ & (\forall r\colon 0,..j\cdot\ \forall c\colon 0,..n\cdot\ Rrc{=}P(i{+}1)rc) \\ \land\ & (\forall c\colon 0,..k\cdot\ Rjc{=}P(i{+}1)jc) \\ \land\ & (\forall c\colon k,..n\cdot\ Rjc{=}Pijc) \\ \land\ & (\forall r\colon j{+}1,..n\cdot\ \forall c\colon k,..n\cdot\ Rrc{=}Pirc) \end{aligned}$$

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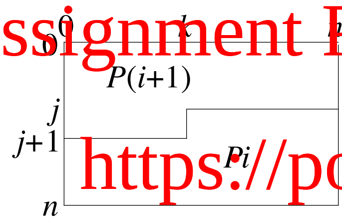

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Now $Ai00 \ = \ R{=}Pi$ and $Aijn \ = \ Ai(j{+}1)0$ and $Ain0 \ = \ A(i{+}1)00$.

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$$A000 \Rightarrow A'n00 \ \Leftarrow\ \textbf{for } i{:=}\ 0;..n \textbf{ do } Ai00 \Rightarrow A'(i{+}1)00 \textbf{ od}$$
$$Ai00 \Rightarrow A'(i{+}1)00 \ \Leftarrow\ \textbf{for } j{:=}\ 0;..n \textbf{ do } Aij0 \Rightarrow A'i(j{+}1)0 \textbf{ od}$$
$$Aij0 \Rightarrow A'i(j{+}1)0 \ \Leftarrow\ \textbf{for } k{:=}\ 0;..n \textbf{ do } Aijk \Rightarrow A'ij(k{+}1) \textbf{ od}$$
$$Aijk \Rightarrow A'ij(k{+}1) \ \Leftarrow\ R{:=}\ (j;k) \to Rjk \ \lor\ Rji \land Rik$$

Of course, **for**-loops are not necessary.

(b) its reflexive transitive closure (the strongest reflexive and transitive relation that is implied by the given relation).

§ This is similar to part (a), but this time we define $P0jk \ = \ j{=}k \lor Rjk$. Since $P0$ is not true initially, we need to start with

$$R'{=}Pn \ \Leftarrow\ \textbf{for } j{:=}\ 0;..n \textbf{ do } R{:=}\ (j;j) \to \top \mid R \textbf{ od}.$$
$$R{=}P0 \Rightarrow R'{=}Pn$$

and then continue as before.