97　　(Russell's barber)　Bertrand Russell stated:　"In a small town there is a male barber who shaves the men in the town who do not shave themselves.".　Then Russell asked:　"Does the barber shave himself?".　If we say yes, then we can conclude from the statement that he does not, and if we say no, then we can conclude from the statement that he does. Formalize this paradox, and thus explain it.

§　　Saying that the barber "shaves the men who do not shave themselves" might mean "shaves all and only the men who do not shave themselves" or it might mean "shaves all the men who do not shave themselves, and possibly also some who do shave themselves".　I suppose it's the former.　It might be argued that the sentence "In a small town there is a male barber who ..." means exactly the same as "In a small town there is a male person who shaves the men in the town who do not shave themselves.".　In other words, "barber" is just a local identifier in the sentence.　And so we might begin our formalization as

$$\exists barber : men \cdot \ ...$$

But Russell's question "Does the barber shave himself?" refers to the barber, so apparently the barber is not a local identifier in the first sentence.　I formalize as follows:

$$barber : men \ \land \ \forall m : men \cdot shaves\ barber\ m = \neg\ shaves\ m\ m$$

$$\text{Now apply specialization, using } barber \text{ for } m \ .$$

$$\implies \quad shaves\ barber\ barber = \neg\ shaves\ barber\ barber \qquad \text{use Completion Rule}$$

$$= \quad \bot$$

So the given information was $\bot$ (inconsistent, self-contradictory), and from it we can conclude anything.　In particular, we can conclude $shaves\ barber\ barber$, and we can also conclude $\neg\ shaves\ barber\ barber$ .　An alternative formalization and proof is

$$barber : men \ \land \ (\S m : men \cdot shaves\ barber\ m) = (\S m : men \cdot \neg\ shaves\ m\ m)$$

$$\implies \quad barber : men \qquad \text{specialize and transparency}$$

$$\land \ (barber : \S m : men \cdot shaves\ barber\ m) = (barber : \S m : men \cdot \neg\ shaves\ m\ m)$$

$$\implies \quad shaves\ barber\ barber = \neg\ shaves\ barber\ barber \qquad \text{solution law and specialization}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{Completion Rule}$$

$$= \quad \bot$$

Bertrand Russell made the statement for the purpose of showing that a reasonable sounding sentence may be inconsistent.　But I think that if anyone were to say that sentence in earnest, they would mean "In a small town there is a male barber who shaves the other men in the town who do not shave themselves.".　This sentence has no inconsistency.

If the barber shaves all the men who do not shave themselves, and possibly also some who do shave themselves, then the given information should be formalized as

$$barber : men \ \land \ \forall m : men \cdot shaves\ barber\ m \Leftarrow \neg\ shaves\ m\ m$$

This interpretation sounds unreasonable:　if someone shaves himself, why does the barber also shave him?　Answer:　if he's the barber, then he shaves himself and the barber shaves him.　There's no inconsistency in this interpretation.