

$$\begin{aligned}
(a) \quad & 0 \rightarrow 1 \mid 1 \rightarrow 2 \mid 2 \rightarrow 3 \mid 3 \rightarrow 4 \mid 4 \rightarrow 5 \mid [0;..5] \\
\S \quad & 0 \rightarrow 1 \mid 1 \rightarrow 2 \mid 2 \rightarrow 3 \mid 3 \rightarrow 4 \mid 4 \rightarrow 5 \mid [0; 1; 2; 3; 4] \\
= & 0 \rightarrow 1 \mid 1 \rightarrow 2 \mid 2 \rightarrow 3 \mid 3 \rightarrow 4 \mid [0; 1; 2; 3; 5] \\
= & 0 \rightarrow 1 \mid 1 \rightarrow 2 \mid 2 \rightarrow 3 \mid [0; 1; 2; 4; 5] \\
= & 0 \rightarrow 1 \mid 1 \rightarrow 2 \mid [0; 1; 3; 4; 5] \\
= & 0 \rightarrow 1 \mid [0; 2; 3; 4; 5] \\
= & [1; 2; 3; 4; 5] \\
= & [1;..6]
\end{aligned}$$

$$\begin{aligned}
(b) \quad & (4 \rightarrow 2 \mid [-3;..3]) \ 3 \\
\S \quad & (4 \rightarrow 2 \mid [-3;..3]) \ 3 \\
= & (4 \rightarrow 2 \mid [-3; -2; -1; 0; 1; 2]) \ 3 \\
= & [-3; -2; -1; 0; 2; 2] \ 3 \\
= & 0
\end{aligned}$$

$$\begin{aligned}
(c) \quad & ((3;2) \rightarrow [10;..15] \mid 3 \rightarrow [5;..10] \mid [0;..5]) \ 3 \\
\S \quad & ((3;2) \rightarrow [10;..15] \mid 3 \rightarrow [5;..10] \mid [0;..5]) \ 3 \\
= & ((3;2) \rightarrow [10;..15] \mid 3 \rightarrow [5;..10] \mid [0; 1; 2; 3; 4]) \ 3 \\
= & ((3;2) \rightarrow [10;..15] \mid [0; 1; 2; [5;..10]; 4]) \ 3 \\
= & ((3;2) \rightarrow [10;..15] \mid [0; 1; 2; 5; 6; 7; 8; 9; 4]) \ 3 \\
= & [0; 1; 2; 5; 6; [10;..15]; 8; 9; 4] \ 3 \\
= & [5; 6; [10;..15]; 8; 9]
\end{aligned}$$

$$\begin{aligned}
(d) \quad & ([0;..5] \mid [3; 4]) \\
\S \quad & \text{One way:}
\end{aligned}$$

$$\begin{aligned}
& ([0;..5] \mid [3; 4]) \ 1 \\
= & [[0;..5] \ 3; [0;..5] \ 4] \ 1 \\
= & [0;..5] \ 4 \\
= & 4
\end{aligned}$$

Another way:

$$\begin{aligned}
& ([0;..5] \mid [3; 4]) \ 1 \\
= & [0;..5] \ ([3; 4] \ 1) \\
= & [0;..5] \ 4 \\
= & 4
\end{aligned}$$

$$\begin{aligned}
(e) \quad & (2;2) \rightarrow "j" \mid [["abc"]; ["de"]; ["fghi"]] \\
\S \quad & \text{Item 2 of } [["abc"]; ["de"]; ["fghi"]] \text{ is } ["fghi"] \text{ and its item 2 is } "h" \text{ so replacing} \\
& \text{item 2;2 or } [["abc"]; ["de"]; ["fghi"]] \text{ with } "j" \text{ gives} \\
& [["abc"]; ["de"]; ["fgji"]]
\end{aligned}$$

$$\begin{aligned}
(f) \quad & \#[nat] \\
\S \quad & 1 \quad \text{because "A nonempty bunch of items is also an item." page 17} \\
& \text{or, informally}
\end{aligned}$$

$$\begin{aligned}
& \#[nat] \\
= & \#[0, 1, 2, 3, \dots] \\
= & \#([0], [1], [2], [3], \dots) \\
= & \#[0], \#[1], \#[2], \#[3], \dots \\
= & 1, 1, 1, 1, \dots \\
= & 1
\end{aligned}$$

This is the sort of “proof” that mathematicians accept, but it's not a formal proof because the three dots mean “guess what goes here”. Anyway, the question did not ask for proof.

$$\begin{aligned}
 (g) \quad & \#[*3] \\
 \S \quad & \#[*3] \\
 = & \#[nil, 3, 3;3, 3;3;3, \dots] \\
 = & \#([nil], [3], [3;3], [3;3;3], \dots) \\
 = & \#[nil], \#[3], \#[3;3], \#[3;3;3], \dots \\
 = & 0, 1, 2, 3, \dots \\
 = & nat
 \end{aligned}$$

Again, an informal “proof”, but the question did not ask for proof.

$$\begin{aligned}
 (h) \quad & [3; 4]: [3*4*int] \\
 \S \quad & 4*int = int; int; int; int \\
 & 3*4*int = int; int; int; int; int; int; int; int; int; int; int; int; int; int \\
 & [3*4*int] = [int; int; int; int; int; int; int; int; int; int; int; int; int; int]
 \end{aligned}$$

which is all lists of 12 integers, and  $[3; 4]$  is not a list of 12 integers, so the answer is  $\perp$

$$\begin{aligned}
 (i) \quad & [3; 4]: [3; int] \\
 \S \quad & [3; int] \text{ is all lists of length 2 whose item 0 is 3 and whose item 1 is in } int. \text{ The list } [3; 4] \text{ is one of them, so the answer is } \top
 \end{aligned}$$

$$\begin{aligned}
 (j) \quad & [3, 4; 5]: [2*int] \\
 \S \quad & [2*int] \text{ is all lists of length 2 whose item whose items are both in } int. \\
 & [3, 4; 5] \\
 = & [3, (4; 5)] \\
 = & [3], [4; 5] \\
 \text{and } [3] \text{ is not a list of length 2, so the answer is } \perp
 \end{aligned}$$

$$\begin{aligned}
 (k) \quad & [(3, 4); 5]: [2*int] \\
 \S \quad & [(3, 4); 5] = [3; 5, 4; 5] = [3; 5], [4; 5] \\
 \text{and both these lists are of length 2 and both items of each are in } int \text{ so the answer is } \top
 \end{aligned}$$

$$\begin{aligned}
 (l) \quad & [3; (4, 5); 6; (7, 8, 9)] \cdot [3; 4; (5, 6); (7, 8)] \\
 \S \quad & [3; (4, 5); 6; (7, 8, 9)] \cdot [3; 4; (5, 6); (7, 8)] \\
 = & ([3; 4; 6; 7], [3; 5; 6; 7], [3; 4; 6; 8], [3; 5; 6; 8], [3; 4; 6; 9], [3; 5; 6; 9]) \\
 & \cdot ([3; 4; 5; 7], [3; 4; 6; 7], [3; 4; 5; 8], [3; 4; 6; 8]) \\
 = & [3; 4; 6; 7], [3; 4; 6; 8] \\
 = & [3; 4; 6; (7, 8)]
 \end{aligned}$$