

0 There are four cards on a table showing symbols D, E, 2, and 3 (one per card). Each card has a letter on one side and a digit on the other. Which card(s) do you need to turn over to determine whether every card with a D on one side has a 3 on the other? Why?

§ You may have to turn over both D and 2, or you may have your answer after turning over just one of them. If the D has a 3 on the other side, and the 2 doesn't have a D on the other side, then you answer yes. If the D has anything other than 3 on the other side, you answer no. If the 2 has a D on the other side, you answer no. It doesn't matter what the E has on the other side. As for the 3, if it has a D and you already answered yes, this won't change your answer. If it has a D and you already answered no because you previously saw a D without a 3, again this won't change your answer. And if the 3 has anything other than a D on the other side, then again it doesn't change your answer about what Ds have on their other sides.

Although the question does not ask for it, we can formalize the problem as follows. Let's refer to the top (visible) sides of the cards as  $t0$ ,  $t1$ ,  $t2$ , and  $t3$ , and the bottom (invisible) sides of the same cards as  $b0$ ,  $b1$ ,  $b2$ , and  $b3$  respectively. Part of the given information is

$$t0 = \text{"D"} \wedge t1 = \text{"E"} \wedge t2 = \text{"2"} \wedge t3 = \text{"3"}$$

Another part of the given information is

$$\neg(b0 = \text{"D"}) \wedge \neg(b1 = \text{"D"}) \wedge \neg(b2 = \text{"3"}) \wedge \neg(b3 = \text{"3"})$$

because cards don't have letters on both sides, and they don't have digits on both sides.

The question we are asked is

$$\begin{aligned} & (t0 = \text{"D"} \Rightarrow b0 = \text{"3"}) \wedge (b0 = \text{"D"} \Rightarrow t0 = \text{"3"}) \\ \wedge & (t1 = \text{"D"} \Rightarrow b1 = \text{"3"}) \wedge (b1 = \text{"D"} \Rightarrow t1 = \text{"3"}) \\ \wedge & (t2 = \text{"D"} \Rightarrow b2 = \text{"3"}) \wedge (b2 = \text{"D"} \Rightarrow t2 = \text{"3"}) \\ \wedge & (t3 = \text{"D"} \Rightarrow b3 = \text{"3"}) \wedge (b3 = \text{"D"} \Rightarrow t3 = \text{"3"}) \end{aligned}$$

Now use the given information

$$\begin{aligned} = & (\perp \Rightarrow b0 = \text{"3"}) \wedge (\perp \Rightarrow \perp) \\ \wedge & (\perp \Rightarrow b1 = \text{"3"}) \wedge (\perp \Rightarrow \perp) \\ \wedge & (\perp \Rightarrow \perp) \wedge (b2 = \text{"D"} \Rightarrow \perp) \\ \wedge & (\perp \Rightarrow \perp) \wedge (b3 = \text{"D"} \Rightarrow \top) \end{aligned}$$

$$= b0 = \text{"3"} \wedge \top \wedge \top \wedge \top \wedge \top \wedge \neg(b2 = \text{"D"}) \wedge \top \wedge \top$$

$$= b0 = \text{"3"} \wedge \neg(b2 = \text{"D"})$$

To answer the question with "yes", we need to know that the bottom of card 0 is "3" and that the bottom of card 2 is not "D". To answer the question with "no" we need to know either that the bottom of card 0 is not "3" or that the bottom of card 2 is "D".

Here is a logically equivalent puzzle. Four people are in a bar. The first is drinking beer; the next one is drinking Coke; the next one is 16 years old; the last one is 25 years old. The law says that no-one under 21 years old is allowed to drink beer. To check whether the law is being followed, who do we need to investigate further?