311 The specification **wait** w where w is a length of time, not an instant of time, describes a delay in execution of time w. Formalize and implement it using

(a) the recursive time measure.

§ If t is an extended integer variable and w is an extended natural expression, then define

$$\textbf{wait } w \;=\; t{:=}\, t{+}w$$

and refine it this way:

$$\textbf{wait } w \;\Leftarrow\; \textbf{frame } t\cdot\ \textbf{var } c{:}\ xnat := w\cdot\ \ t' = t{+}c$$
$$t' = t{+}c \;\Leftarrow\; \textbf{if } c{=}0 \textbf{ then } ok \textbf{ else } c{:=}\, c{-}1.\ \ t{:=}\, t{+}1.\ \ t' = t{+}c \textbf{ fi}$$

Proof of first refinement:

$\textbf{frame } t\cdot\ \textbf{var } c{:}\ xnat := w\cdot\ \ t' = t{+}c$

$=\ \ \textbf{frame } t\cdot\ \exists c{:}\ w\cdot\ \exists c'{:}\ xnat\cdot\ \ t' = t{+}c$ — c' is unused; $\exists$ law

$=\ \ \textbf{frame } t\cdot\ \ t' = t{+}w$ — **frame** law

$=\ \ t{:=}\, t{+}w$

$=\ \ \textbf{wait } w$

Proof of last refinement, first case, assuming the nonlocal variables are x:

$c{=}0 \wedge ok$ — expand ok

$=\ \ c{=}0 \wedge x'{=}x \wedge c'{=}c \wedge t'{=}t$ — context and specialization

$\Rightarrow\ \ t' = t{+}c$

Proof of last refinement, last case:

$c{>}0 \wedge (c{:=}\, c{-}1.\ \ t{:=}\, t{+}1.\ \ t' = t{+}c)$ — substitution law twice; specialization

$\Rightarrow\ \ t' = t{+}c$

(b) the real time measure (assume any positive operation times you need).

§ This time t is an extended real variable and w is a nonnegative extended real expression. The solution can be like part (a), but in the real time measure, we have to account for the time to make the test (which was $c{=}0$ in part (a)) and to make a conditional branch, and the time for the assignment (which was $c{:=}\, c{-}1$ in part (a)), and the time for the recursive call. I'll use time 1 for all three. As in part (a), we can introduce a counter c initialized to w and count down. But w here is real, not necessarily an integer, so either the test must be $c{\le}0$, or the initial value of c must be rounded up. I'll do the latter. Define

$$\textbf{wait } w \;=\; t{:=}\, t + 3{\times}(ceil\ w) + 1$$

and refine it this way:

$$\textbf{wait } w \;\Leftarrow\; \textbf{frame } t\cdot\ \textbf{var } c{:}\ xnat := ceil\ w\cdot\ \ t' = t + 3{\times}c + 1$$
$$t' = t + 3{\times}c + 1 \;\Leftarrow$$
$$t{:=}\, t{+}1.\ \ \textbf{if } c{=}0 \textbf{ then } ok \textbf{ else } t{:=}\, t{+}1.\ \ c{:=}\, c{-}1.\ \ t{:=}\, t{+}1.\ \ t' = t + 3{\times}c + 1 \textbf{ fi}$$

Proof of first refinement:

$\textbf{frame } t\cdot\ \textbf{var } c{:}\ xnat := ceil\ w\cdot\ \ t' = t + 3{\times}c + 1$

$=\ \ \textbf{frame } t\cdot\ \exists c{:}\ ceil\ w\cdot\ \exists c'{:}\ xnat\cdot\ \ t' = t + 3{\times}c + 1$ — c' is unused; $\exists$ law

$=\ \ \textbf{frame } t\cdot\ \ t' = t + 3{\times}(ceil\ w) + 1$ — **frame** law

$=\ \ t{:=}\, t + 3{\times}(ceil\ w) + 1$

$=\ \ \textbf{wait } w$

Proof of last refinement, assuming the nonlocal variables are x:

$t{:=}\, t{+}1.\ \ \textbf{if } c{=}0 \textbf{ then } ok \textbf{ else } t{:=}\, t{+}1.\ \ c{:=}\, c{-}1.\ \ t{:=}\, t{+}1.\ \ t' = t + 3{\times}c + 1 \textbf{ fi}$ — substitution law 3 times

$=\ \ t{:=}\, t{+}1.\ \ \textbf{if } c{=}0 \textbf{ then } ok \textbf{ else } t' = t + 3{\times}c \textbf{ fi}$ — expand ok

$=\ \ t{:=}\, t{+}1.\ \ \textbf{if } c{=}0 \textbf{ then } c'{=}c \wedge x'{=}x \wedge t'{=}t \textbf{ else } t' = t + 3{\times}c \textbf{ fi}$ — substitution law

$=\ \ \textbf{if } c{=}0 \textbf{ then } c'{=}c \wedge x'{=}x \wedge t'{=}t{+}1 \textbf{ else } t' = t + 3{\times}c + 1 \textbf{ fi}$ — use context $c{=}0$

$=\ \ \textbf{if } c{=}0 \textbf{ then } c'{=}c \wedge x'{=}x \wedge t' = t + 3{\times}c + 1 \textbf{ else } t' = t + 3{\times}c + 1 \textbf{ fi}$ — specialize

$\Rightarrow\ \ t' = t + 3{\times}c + 1$