427 Let u be a binary user's variable. Let a and b be old binary implementer's variables. We replace a and b by new integer implementer's variables x and y using the convention (from the C language) that 0 stands for $\bot$ and non-zero integers stand for $\top$.

(a) What is the transformer?

§ $a=(x\ne 0) \land b=(y\ne 0)$

(b) Transform $a:= \neg a$.

§ $\forall a, b\cdot\ a=(x\ne 0) \land b=(y\ne 0) \Rightarrow \exists a', b'\cdot\ a'=(x'\ne 0) \land b'=(y'\ne 0) \land (a:= \neg a)$ replace asmt

$=$ $\forall a, b\cdot\ \ a=(x\ne 0) \land b=(y\ne 0)$

$\Rightarrow \exists a', b'\cdot\ a'=(x'\ne 0) \land b'=(y'\ne 0) \land a'=\neg a \land b'=b \land u'=u$ 1-pt a' and b'

$=$ $\forall a, b\cdot\ a=(x\ne 0) \land b=(y\ne 0) \Rightarrow \neg a=(x'\ne 0) \land b=(y'\ne 0) \land u'=u$ 1-pt a and b

$=$ $\neg(x\ne 0)=(x'\ne 0) \land (y\ne 0)=(y'\ne 0) \land u'=u$

$\Leftarrow$ **if** $x=0$ **then** $x:= 1$ **else** $x:= 0$ **fi**

(c) Transform $u:= a\land b$.

§ $\forall a, b\cdot\ a=(x\ne 0) \land b=(y\ne 0) \Rightarrow \exists a', b'\cdot\ a'=(x'\ne 0) \land b'=(y'\ne 0) \land (u:= a\land b)$ replace asmt

$=$ $\forall a, b\cdot\ \ a=(x\ne 0) \land b=(y\ne 0)$

$\Rightarrow \exists a', b'\cdot\ a'=(x'\ne 0) \land b'=(y'\ne 0) \land a'=a \land b'=b \land u'=a\land b$ 1-pt a' and b'

$=$ $\forall a, b\cdot\ a=(x\ne 0) \land b=(y\ne 0) \Rightarrow a=(x'\ne 0) \land b=(y'\ne 0) \land u'=a\land b$ 1-pt a and b

$=$ $(x\ne 0)=(x'\ne 0) \land (y\ne 0)=(y'\ne 0) \land u'=(x\ne 0)\land(y\ne 0)$

$\Leftarrow$ $u:= (x\ne 0)\land(y\ne 0)$