137 Let a, b, and c be integer variables. Express as simply as possible without using quantifiers, assignments, or dependent compositions

(a)
$b := a-b.\ b := a-b$

§ $b := a-b.\ b := a-b$ expand last assignment

$= b := a-b.\ a'=a \wedge b' = a-b \wedge c'=c$ Substitution Law

$= a'=a \wedge b' = a-(a-b) \wedge c'=c$

$= a'=a \wedge b'=b \wedge c'=c$

$= ok$

(b)
$a := a+b.\ b := a-b.\ a := a-b$

§ $a := a+b.\ b := a-b.\ a := a-b$ expand last assignment

$= a := a+b.\ b := a-b.\ a' = a-b \wedge b'=b \wedge c'=c$ Substitution Law

$= a := a+b.\ a' = a-(a-b) \wedge b' = a-b \wedge c'=c$ Substitution Law

$= a'=b \wedge b' = (a+b)-b \wedge c'=c$ subtract

$= a'=b \wedge b'=a \wedge c'=c$

(c)
$c := a-b-c.\ b := a-b-c.\ a := a-b-c.\ c := a+b+c$

§ $c := a-b-c.\ b := a-b-c.\ a := a-b-c.\ c := a+b+c$ expand last assignment

$= c := a-b-c.\ b := a-b-c.\ a := a-b-c.\ a'=a \wedge b'=b \wedge c'=a+b+c$ Substitution Law

$= c := a-b-c.\ b := a-b-c.\ a'=a-b-c \wedge b'=b \wedge c'=(a-b-c)+b+c$ arithmetic

$= c := a-b-c.\ b := a-b-c.\ a'=a-b-c \wedge b'=b \wedge c'=a$ Substitution Law

$= c := a-b-c.\ a'=a-(a-b-c) \wedge b'=a-b-c \wedge c'=a$ arithmetic

$= c := a-b-c.\ a'=b \wedge b'=a-b-c \wedge c'=a$ Substitution Law

$= a'=b \wedge b'=a-b-(a-b-c) \wedge c'=a$ arithmetic

$= a'=b \wedge b'=c \wedge c'=a$

(d)
$a := a+b.\ b := a+b.\ c := a+b$

§ $a := a+b.\ b := a+b.\ c := a+b$ expand last assignment

$= a := a+b.\ b := a+b.\ a'=a \wedge b'=b \wedge c'=a+b$ substitution law

$= a := a+b.\ a'=a \wedge b' = a+b \wedge c' = a+a+b$ substitution law

$= a' = a+b \wedge b' = a+b+b \wedge c' = a+b+a+b+b$ arithmetic

$= a'=a+b \wedge b' = a + 2\times b \wedge c' = 2\times a + 3\times b$

(e)
$a := a+b.\ b' = a+b.\ c := a+b$

§ $a := a+b.\ b' = a+b.\ c := a+b$ expand last assignment

$= a := a+b.\ b' = a+b.\ a'=a \wedge b'=b \wedge c' = a+b$ dependent composition

$= a := a+b.\ \exists a'', b'', c''\cdot b'' = a+b \wedge a'=a'' \wedge b'=b'' \wedge c' = a''+b''$

one-point for a'' and b'', idempotence for c''

$= a := a+b.\ b' = a+b \wedge c' = a'+b'$ substitution law

$= b' = a+b+b \wedge c' = a'+b'$ arithmetic

$= b' = a + 2\times b \wedge c' = a'+b'$

(f)
$a := a+b+1.\ b := a-b-1.\ a := a-b-1$

§ $a := a+b+1.\ b := a-b-1.\ a := a-b-1$ expand last assignment

$= a := a+b+1.\ b := a-b-1.\ a' = a-b-1 \wedge b'=b \wedge c'=c$ substitution law once

$= a := a+b+1.\ a' = a-(a-b-1)-1 \wedge b'=a-b-1 \wedge c'=c$ simplify

$= a := a+b+1.\ a'=b \wedge b'=a-b-1 \wedge c'=c$ substitution law

$= a'=b \wedge b'=a+b+1-b-1 \wedge c'=c$ simplify

$= a'=b \wedge b'=a \wedge c'=c$

(g)
$a' = a+b+1.\ b' = a-b-1$

§		$a' = a+b+1.\ \ b' = a-b-1$	expand dependent composition
	$=$	$\exists a'', b'', c''\cdot\ \ a'' = a+b+1\ \wedge\ b' = a''-b''-1$	one point for a'' , identity for c''
	$=$	$\exists b''\cdot\ \ b' = a+b+1-b''-1$	simplify, rearrange, and identity
	$=$	$\exists b''\cdot\ \ b'' = a+b-b'\ \wedge\ \mathsf{T}$	one point for b''
	$=$	T	

(h)		$a := a-b.\ \ b := a-b.\ \ a := a+b$	
§		$a := a-b.\ \ b := a-b.\ \ a := a+b$	expand last $:=$
	$=$	$a := a-b.\ \ b := a-b.\ \ a' = a+b\ \wedge\ b'=b\ \wedge\ c'=c$	substitution law
	$=$	$a := a-b.\ \ a' = a+a-b\ \wedge\ b'=a-b\ \wedge\ c'=c$	substitution law
	$=$	$a' = a-b+a-b-b\ \wedge\ b'=a-b-b\ \wedge\ c'=c$	simplify
	$=$	$a' = 2\times a - 3\times b\ \wedge\ b' = a - 2\times b\ \wedge\ c'=c$	