370 Let all variables be integer. Add recursive time. Using recursive construction, find a fixed-point of

(a) $\quad skip \;=\; \textbf{if } i{\ge}0 \textbf{ then } i{:=}\, i{-}1.\; skip.\; i{:=}\, i{+}1 \textbf{ else } ok \textbf{ fi}$

§ Adding recursive time,

$$skip \;=\; \textbf{if } i{\ge}0 \textbf{ then } i{:=}\, i{-}1.\; t{:=}\, t{+}1.\; skip.\; i{:=}\, i{+}1 \textbf{ else } ok \textbf{ fi}$$

$$skip_0 \;=\; t' \ge t$$

$$skip_{n+1} \;=\; \textbf{if } i{\ge}n \textbf{ then } t' \ge t{+}n{+}1 \textbf{ else if } 0{\le}i{<}n \textbf{ then } t{:=}\, t{+}i{+}1 \textbf{ else } ok \textbf{ fi fi}$$

$$skip_\infty \;=\; \textbf{if } i{\ge}0 \textbf{ then } t{:=}\, t{+}i{+}1 \textbf{ else } ok \textbf{ fi}$$

To show it's a fixed-point, start with the right side of the definition of $skip$, but substitute $skip_\infty$ in place of $skip$,

$$\textbf{if } i{\ge}0 \textbf{ then } i{:=}\, i{-}1.\; t{:=}\, t{+}1.\; \textbf{if } i{\ge}0 \textbf{ then } t{:=}\, t{+}i{+}1 \textbf{ else } ok \textbf{ fi}.\; i{:=}\, i{+}1 \textbf{ else } ok \textbf{ fi}$$

distribute $i{:=}\, i{+}1$ into preceding **if**

$$=\; \textbf{if } i{\ge}0 \textbf{ then } i{:=}\, i{-}1.\; t{:=}\, t{+}1.\; \textbf{if } i{\ge}0 \textbf{ then } t{:=}\, t{+}i{+}1.\; i{:=}\, i{+}1 \textbf{ else } ok.\; i{:=}\, i{+}1 \textbf{ fi else } ok \textbf{ fi}$$

replace first $i{:=}\, i{+}1$ and ok is identity for .

$$=\; \textbf{if } i{\ge}0 \textbf{ then } i{:=}\, i{-}1.\; t{:=}\, t{+}1.\; \textbf{if } i{\ge}0 \textbf{ then } t{:=}t{+}i{+}1.\; i'{=}i{+}1 \wedge t'{=}t \textbf{ else } i{:=}\, i{+}1 \textbf{ fi else } ok \textbf{ fi}$$

substitution law in second **then**-part

$$=\; \textbf{if } i{\ge}0 \textbf{ then } i{:=}\, i{-}1.\; t{:=}\, t{+}1.\; \textbf{if } i{\ge}0 \textbf{ then } i'{=}i{+}1 \wedge t'{=}t{+}i{+}1 \textbf{ else } i{:=}\, i{+}1 \textbf{ fi else } ok \textbf{ fi}$$

replace $i{:=}\, i{+}1$

$$=\; \textbf{if } i{\ge}0 \textbf{ then } i{:=}i{-}1.\; t{:=}\, t{+}1.\; \textbf{if } i{\ge}0 \textbf{ then } i'{=}i{+}1 \wedge t'{=}t{+}i{+}1 \textbf{ else } i'{=}i{+}1 \wedge t'{=}t \textbf{ fi else } ok \textbf{ fi}$$

substitution law twice more

$$=\; \textbf{if } i{\ge}0 \textbf{ then if } i{-}1{\ge}0 \textbf{ then } i'{=}i{-}1{+}1 \wedge t'{=}t{+}1{+}i{-}1{+}1 \textbf{ else } i'{=}i{-}1{+}1 \wedge t'{=}t{+}1 \textbf{ fi else } ok \textbf{ fi}$$

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simplify

$$=\; \textbf{if } i{\ge}0 \textbf{ then if } i{\ge}1 \textbf{ then } i'{=}i \wedge t'{=}t{+}i{+}1 \textbf{ else } i'{=}i \wedge t'{=}t{+}1 \textbf{ fi else } ok \textbf{ fi} \qquad \text{use } {:=} \text{ twice}$$

$$=\; \textbf{if } i{\ge}0 \textbf{ then if } i{\ge}1 \textbf{ then } t{:=}\, t{+}i{+}1 \textbf{ else } t{:=}\, t{+}1 \textbf{ fi else } ok \textbf{ fi}$$

In the first else-part the context is $i{\ge}0 \wedge \neg(i{\ge}1)$ which is $i{=}0$

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$$=\; \textbf{if } i{\ge}0 \textbf{ then if } i{\ge}1 \textbf{ then } t{:=}\, t{+}i{+}1 \textbf{ else } t{:=}\, t{+}i{+}1 \textbf{ fi else } ok \textbf{ fi} \qquad \text{case idempotent}$$

$$=\; \textbf{if } i{\ge}0 \textbf{ then } t{:=}\, t{+}i{+}1 \textbf{ else } ok \textbf{ fi}$$

and we get $skip_\infty$ again, so it is a fixed-point.

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(b) $\quad inc \;=\; ok \vee (i{:=}\, i{+}1.\; inc)$

§ Adding recursive time,

$$inc \;=\; ok \vee (i{:=}\, i{+}1.\; t{:=}\, t{+}1.\; inc)$$

Now recursive construction. Starting with $\top$,

$$inc_0 \;=\; \top$$

$$inc_1 \;=\; ok \vee (i{:=}\, i{+}1.\; t{:=}\, t{+}1.\; inc_0)$$

$$=\; ok \;\vee\; \top$$

$$=\; \top$$

We have converged, and found that $\top$ is a fixed-point. Perhaps we'll get something more interesting if we start with $t'{\ge}t$.

$$inc_0 \;=\; t'{\ge}t$$

$$inc_1 \;=\; ok \vee (i{:=}\, i{+}1.\; t{:=}\, t{+}1.\; inc_0)$$

$$=\; i'{=}i \wedge t'{=}t \;\;\vee\;\; t'{\ge}t{+}1$$

$$inc_2 \;=\; ok \vee (i{:=}\, i{+}1.\; t{:=}\, t{+}1.\; inc_1)$$

$$=\; i'{=}i \wedge t'{=}t \;\;\vee\;\; i'{=}i{+}1 \wedge t'{=}t{+}1 \;\;\vee\;\; t'{\ge}t{+}2$$

I'm ready to guess

$$inc_n \;=\; (\exists m{:}\; 0,..n\cdot\; i'{=}i{+}m \wedge t'{=}t{+}m) \;\vee\; t'{\ge}t{+}n$$

$$inc_\infty \;=\; (\exists m{:}\; nat\cdot\; i'{=}i{+}m \wedge t'{=}t{+}m) \;\vee\; t'{=}\infty$$

Now I must test inc_∞ to see if it's a fixed-point.

$$ok \vee (i{:=}\, i{+}1.\; t{:=}\, t{+}1.\; inc_\infty)$$

$$=\; i'{=}i \wedge t'{=}t \;\;\vee\;\; (\exists m{:}\; nat\cdot\; i'{=}i{+}1{+}m \wedge t'{=}t{+}1{+}m) \;\vee\; t'{=}\infty$$

$$=\; (\exists m{:}\; nat\cdot\; i'{=}i{+}m \wedge t'{=}t{+}m) \;\vee\; t'{=}\infty$$

$= \quad inc_\infty$

Starting with $\bot$ we get

$$
\begin{aligned}
inc_0 &= \bot\\
inc_1 &= ok \lor (i{:=}\ i+1.\ t{:=}\ t+1.\ inc_0)\\
&= i'=i \land t'=t\\
inc_2 &= ok \lor (i{:=}\ i+1.\ t{:=}\ t+1.\ inc_1)\\
&= i'=i \land t'=t \quad \lor \quad i'=i+1 \land t'=t+1\\
inc_n &= (\exists m\colon 0,..n\cdot\ i'=i+m \land t'=t+m)\\
inc_\infty &= (\exists m\colon nat\cdot\ i'=i+m \land t'=t+m)
\end{aligned}
$$

and it is a fixed-point, and it's implementable too!

(c) $\quad sqr \;=\; \mathbf{if}\ i=0\ \mathbf{then}\ ok\ \mathbf{else}\ s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ sqr\ \mathbf{fi}$

§

$$
\begin{aligned}
sqr_0 &= t'\ge t\\
sqr_1 &= \mathbf{if}\ i=0\ \mathbf{then}\ ok\ \mathbf{else}\ s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ t{:=}\ t+1.\ sqr_0\ \mathbf{fi}\\
&= \mathbf{if}\ i=0\ \mathbf{then}\ ok\ \mathbf{else}\ t'\ge t+1\\
sqr_2 &= \mathbf{if}\ i=0\ \mathbf{then}\ ok\ \mathbf{else}\ s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ t{:=}\ t+1.\ sqr_1\ \mathbf{fi}\\
&= \mathbf{if}\ i=0\ \mathbf{then}\ ok\ \mathbf{else}\quad s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ t{:=}\ t+1.\\
&\qquad\qquad \mathbf{if}\ i=0\ \mathbf{then}\ ok\ \mathbf{else}\ t'\ge t+1\ \mathbf{fi}\ \mathbf{fi}\\
&= \mathbf{if}\ i=0\ \mathbf{then}\ ok\\
&\quad\ \mathbf{else}\ \mathbf{if}\ i-1=0\ \mathbf{then}\ s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ t{:=}\ t+1\\
&\qquad\ \mathbf{else}\ t'\ge t+2\ \mathbf{fi}\ \mathbf{fi}\\
&= \mathbf{if}\ i=0\ \mathbf{then}\ s{:=}\ s+0.\ i{:=}\ 0.\ t{:=}\ t+0\\
&\quad\ \mathbf{else}\ \mathbf{if}\ i=1\ \mathbf{then}\ s{:=}\ s+1.\ i{:=}\ 0.\ t{:=}\ t+1\\
&\qquad\ \mathbf{else}\ t'\ge t+2\ \mathbf{fi}\ \mathbf{fi}\\
sqr_3 &= \mathbf{if}\ i=0\ \mathbf{then}\ ok\\
&\quad\ \mathbf{else}\ s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ t{:=}\ t+1.\\
&\qquad \mathbf{if}\ i=0\ \mathbf{then}\ s{:=}\ s+0.\ i{:=}\ 0.\ t{:=}\ t+0\\
&\qquad \mathbf{else}\ \mathbf{if}\ i=1\ \mathbf{then}\ s{:=}\ s+1.\ i{:=}\ 0.\ t{:=}\ t+1\\
&\qquad\quad \mathbf{else}\ t'\ge t+2\ \mathbf{fi}\ \mathbf{fi}\ \mathbf{fi}\\
&= \mathbf{if}\ i=0\ \mathbf{then}\ ok\\
&\quad\ \mathbf{else}\ \mathbf{if}\ i=1\ \mathbf{then}\quad s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ t{:=}\ t+1.\\
&\qquad\qquad\qquad s{:=}\ s+0.\ i{:=}\ 0.\ t{:=}\ t+0\\
&\qquad\ \mathbf{else}\ \mathbf{if}\ i=1\ \mathbf{then}\ s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ t{:=}\ t+1.\\
&\qquad\qquad\qquad s{:=}\ s+1.\ i{:=}\ 0.\ t{:=}\ t+1\\
&\qquad\quad \mathbf{else}\ s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ t{:=}\ t+1.\ t'\ge t+2\ \mathbf{fi}\ \mathbf{fi}\ \mathbf{fi}\\
&= \mathbf{if}\ i=0\ \mathbf{then}\ s{:=}\ s+0.\ i{:=}\ 0.\ t{:=}\ t+0\\
&\quad\ \mathbf{else}\ \mathbf{if}\ i=1\ \mathbf{then}\ s{:=}\ s+1.\ i{:=}\ 0.\ t{:=}\ t+1\\
&\qquad\ \mathbf{else}\ \mathbf{if}\ i=2\ \mathbf{then}\ s{:=}\ s+4.\ i{:=}\ 0.\ t{:=}\ t+2\\
&\qquad\quad \mathbf{else}\ t'\ge t+3\ \mathbf{fi}\ \mathbf{fi}\ \mathbf{fi}\\
sqr_n &= \mathbf{if}\ 0\le i<n\ \mathbf{then}\ s{:=}\ s+i^2.\ t{:=}\ t+i.\ i{:=}\ 0\ \mathbf{else}\ t'\ge t+n\ \mathbf{fi}\\
sqr_\infty &= \mathbf{if}\ 0\le i\ \mathbf{then}\ s{:=}\ s+i^2.\ t{:=}\ t+i.\ i{:=}\ 0\ \mathbf{else}\ t'=\infty\ \mathbf{fi}
\end{aligned}
$$

Now we test to see if sqr_∞ is a fixed-point.

$$
\begin{aligned}
&\mathbf{if}\ i=0\ \mathbf{then}\ ok\ \mathbf{else}\quad s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ t{:=}\ t+1.\\
&\qquad\qquad \mathbf{if}\ 0\le i\ \mathbf{then}\ s{:=}\ s+i^2.\ t{:=}\ t+i.\ i{:=}\ 0\ \mathbf{else}\ t'=\infty\ \mathbf{fi}\ \mathbf{fi}\\
=\ &\mathbf{if}\ i=0\ \mathbf{then}\ ok\\
&\mathbf{else}\ \mathbf{if}\ 0\le i-1\ \mathbf{then}\quad s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ t{:=}\ t+1.\\
&\qquad\qquad s{:=}\ s+i^2.\ t{:=}\ t+i.\ i{:=}\ 0\\
&\quad \mathbf{else}\ s{:=}\ s+2\times i-1.\ i{:=}\ i-1.\ t{:=}\ t+1.\ t'=\infty\ \mathbf{fi}\ \mathbf{fi}\\
=\ &\mathbf{if}\ i=0\ \mathbf{then}\ ok\\
&\mathbf{else}\ \mathbf{if}\ 1\le i\ \mathbf{then}\ s{:=}\ s+2\times i-1+(i-1)^2.\ t{:=}\ t+1+i-1.\ i{:=}\ 0\\
&\quad \mathbf{else}\ t'=\infty\ \mathbf{fi}\ \mathbf{fi}\\
=\ &\mathbf{if}\ i=0\ \mathbf{then}\ s{:=}\ s+i^2.\ t{:=}\ t+i.\ i{:=}\ 0
\end{aligned}
$$

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else if $1 \le i$ **then** $s := s+i^2.\ \ t := t+i.\ \ i := 0$
    else $t' = \infty$ **fi fi**

$=$ sqr_∞

(d) $fac \ = \$ **if** $i=0$ **then** $f := 1$ **else** $i := i-1.\ fac.\ i := i+1.\ f := f \times i$ **fi**

§ Adding time,

$fac \ = \$ **if** $i=0$ **then** $f := 1$ **else** $i := i-1.\ t := t+1.\ fac.\ i := i+1.\ f := f \times i$ **fi**

Recursive construction starting with $t' \ge t$ produces

$fac_n \ = \$ **if** $0 \le i < n$ **then** $f' = i! \land i' = i \land t' = t+i$ **else** $t' \ge t+n$ **fi**

where $i!$ is "i factorial". Replacing n with ∞ produces

$fac_\infty \ = \$ **if** $0 \le i$ **then** $f' = i! \land i' = i \land t' = t+i$ **else** $t' = \infty$ **fi**

Now we see if fac_∞ is a fixed-point. Starting with the right side of the fac equation,

if $i=0$ **then** $f := 1$ **else** $i := i-1.\ t := t+1.\ fac.\ i := i+1.\ f := f \times i$ **fi** replace fac with fac_∞

$=$ **if** $i=0$ **then** $f := 1$ expand assignment
else $i := i-1.\ t := t+1.$ **if** $0 \le i$ **then** $f' = i! \land i' = i \land t' = t+i$ **else** $t' = \infty$ **fi**. $i := i+1.\ f := f \times i$ **fi**
combine and expand the final two assignments

$=$ **if** $i=0$ **then** $f' = 1 \land i' = i \land t' = t$ use **if**-context in **then**-part
else $i := i-1.\ t := t+1.$ **if** $0 \le i$ **then** $f' = i! \land i' = i \land t' = t+i$ **else** $t' = \infty$ **fi**.
$i' = i+1 \land f' = f \times (i+1) \land t' = t$ **fi** distribute this line into **then** and **else** parts

$=$ **if** $i=0$ **then** $f' = i! \land i' = i \land t' = t+i$
else $i := i-1.\ t := t+1.$ **if** $0 \le i$ **then** $f' = i! \land i' = i \land t' = t+i.\ i' = i+1 \land f' = f \times (i+1) \land t' = t$
else $t' = \infty.\ i' = i+1 \land f' = f \times (i+1) \land t' = t$ **fi fi** dep't comp.

$=$ **if** $i=0$ **then** $f' = i! \land i' = i \land t' = t+i$
else $i := i-1.\ t := t+1.$ **if** $0 \le i$ **then** $f' = (i+1)! \land i' = i+1 \land t' = t+i$ **else** $t' = \infty$ **fi fi**
substitution law twice

$=$ **if** $i=0$ **then** $f' = i! \land i' = i \land t' = t+i$
else if $1 \le i$ **then** $f' = i! \land i' = i \land t' = t+i$ **else** $t' = \infty$ **fi fi** combine $i=0$ and $1 \le i$ cases

$=$ **if** $0 \le i$ **then** $f' = i! \land i' = i \land t' = t+i$ **else** $t' = \infty$ **fi**

Therefore fac_∞ is a fixed-point.

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(e) $chs \ = \$ **if** $a=b$ **then** $c := 1$ **else** $a := a-1.\ chs.\ a := a+1.\ c := c \times a/(a-b)$ **fi**

§ $chs_0 \ = \ t' \ge t$

$chs_1 \ = \$ **if** $a=b$ **then** $c := 1$ **else** $a := a-1.\ t := t+1.\ chs_0.\ a := a+1.\ c := c \times a/(a-b)$ **fi**

At this point we need to know that $c \times a/(a-b){:}\ int$ and we don't.
But this whole procedure just generates a candidate that needs to be tested.
So we carry on as if $c \times a/(a-b){:}\ int$

$=$ **if** $a=b$ **then** $c := 1$ **else** $t' \ge t+1$ **fi**

$chs_2 \ = \$ **if** $a=b$ **then** $c := 1$ **else** $a := a-1.\ t := t+1.\ chs_1.\ a := a+1.\ c := c \times a/(a-b)$ **fi**

$=$ **if** $a=b$ **then** $c := 1$
else $a := a-1.\ t := t+1.$ **if** $a=b$ **then** $c := 1$ **else** $t' \ge t+1$ **fi**.
$a := a+1.\ c := c \times a/(a-b)$ **fi**

$=$ **if** $a=b$ **then** $c := 1$
else if $a-1=b$ **then** $a := a-1.\ t := t+1.\ c := 1.\ a := a+1.\ c := c \times a/(a-b)$
else $a := a-1.\ t := t+1.\ t' \ge t+1.\ a := a+1.\ c := c \times a/(a-b)$ **fi fi**

$=$ **if** $a=b$ **then** $c := 1$
else if $a-1=b$ **then** $t := t+1.\ c := a$
else $t' \ge t+2$ **fi fi**

$chs_3 \ = \$ **if** $a=b$ **then** $c := 1$
else $a := a-1.\ t := t+1.$
if $a=b$ **then** $c := 1$
else if $a-1=b$ **then** $t := t+1.\ c := a$
else $t' \ge t+2$ **fi fi**.

$a{:=}\ a+1.\ \ c{:=}\ c\times a/(a-b)$ **fi**

$=$ **if** $a{=}b$ **then** $c{:=}\ 1$
else if $a-1{=}b$ **then** $a{:=}\ a-1.\ \ t{:=}\ t+1.\ \ c{:=}\ 1.\ \ a{:=}\ a+1.\ \ c{:=}\ c\times a/(a-b)$
else if $a-2{=}b$ **then** $a{:=}\ a-1.\ t{:=}\ t+1.\ t{:=}\ t+1.\ c{:=}\ a.\ a{:=}\ a+1.\ \ c{:=}\ c\times a/(a-b)$
else $a{:=}\ a-1.\ \ t{:=}\ t+1.\ \ t' \ge t+2.\ \ a{:=}\ a+1.\ \ c{:=}\ c\times a/(a-b)$ **fi fi fi**

$=$ **if** $a{=}b$ **then** $c{:=}\ 1$
else if $a-1{=}b$ **then** $t{:=}\ t+1.\ \ c{:=}\ a$
else if $a-2{=}b$ **then** $t{:=}\ t+2.\ \ c{:=}\ a\times(a-1)/2$
else $t' \ge t+3$ **fi fi fi**

$chs_4 =$ **if** $a{=}b$ **then** $c{:=}\ 1$
else $a{:=}\ a-1.\ \ t{:=}\ t+1.$
if $a{=}b$ **then** $c{:=}\ 1$
else if $a-1{=}b$ **then** $t{:=}\ t+1.\ \ c{:=}\ a$
else if $a-2{=}b$ **then** $t{:=}\ t+2.\ \ c{:=}\ a\times(a-1)/2$
else $t' \ge t+3$ **fi fi fi**.
$a{:=}\ a+1.\ \ c{:=}\ c\times a/(a-b)$ **fi**

$=$ **if** $a{=}b$ **then** $c{:=}\ 1$
else if $a-1{=}b$ **then** $t{:=}\ t+1.\ \ c{:=}\ a$
else if $a-2{=}b$ **then** $t{:=}\ t+2.\ \ c{:=}\ a\times(a-1)/2$
else if $a-3{=}b$ **then** $t{:=}\ t+3.\ \ c{:=}\ a\times(a-1)\times(a-2)/(2\times3)$
else $t' \ge t+4$ **fi fi fi fi**

$chs_n =$ **if** $b \le a < b+n$ **then** $t{:=}\ t+a-b.\ \ c{:=}\ \Pi[b+1;..a+1]/\Pi[1;..a-b+1]$ **else** $t' \ge t+n$ **fi**

$chs_\infty =$ **if** $a{\ge}b$ **then** $t{:=}\ t+a-b.\ \ c{:=}\ \Pi[b+1;..a+1]/\Pi[1;..a-b+1]$ **else** $t'{=}\infty$ **fi**

Now I test to see if chs_∞ is a fixed-point.

if $a{=}b$ **then** $c{:=}\ 1$ **else** $a{:=}\ a-1.\ \ t{:=}\ t+1.\ \ chs_\infty.\ \ a{:=}\ a+1.\ \ c{:=}\ c\times a/(a-b)$ **fi**

$=$ **if** $a{=}b$ **then** $c{:=}\ 1$
else $a{:=}\ a-1.\ \ t{:=}\ t+1.$
if $a{\ge}b$ **then** $t{:=}\ t+a-b.\ c{:=}\ \Pi[b+1;..a+1]/\Pi[1;..a-b+1]$ **else** $t'{=}\infty$ **fi**.
$a{:=}\ a+1.\ \ c{:=}\ c\times a/(a-b)$ **fi**

$=$ **if** $a{=}b$ **then** $c{:=}\ 1$
else if $a-1{\ge}b$ **then** $a{:=}\ a-1.\ \ t{:=}\ t+1.$
$t{:=}\ t+a-b.\ \ c{:=}\ \Pi[b+1;..a+1]/\Pi[1;..a-b+1].$
$a{:=}\ a+1.\ c{:=}\ c\times a/(a-b)$
else $a{:=}\ a-1.\ \ t{:=}\ t+1.\ \ t'{=}\infty.\ \ a{:=}\ a+1.\ \ c{:=}\ c\times a/(a-b)$ **fi fi**

$=$ **if** $a{=}b$ **then** $c{:=}\ 1$
else if $a{>}b$ **then** $t{:=}\ t+a-b.\ \ c{:=}\ \Pi[b+1;..a+1]/\Pi[1;..a-b+1]$
else $t'{=}\infty$ **fi fi**

$=$ **if** $a{\ge}b$ **then** $t{:=}\ t+a-b.\ \ c{:=}\ \Pi[b+1;..a+1]/\Pi[1;..a-b+1]$ **else** $t'{=}\infty$ **fi**

$=$ chs_∞

So chs_∞ is a fixed-point. Note that for $1{\le}b{\le}a$, c' is the number of ways of choosing b things from a things.

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