CSC465/2104   Test 2   2021 March 22

1 page, 3 questions, 42 marks, 50 minutes
Aids allowed: one letter-sized page, both sides
list of laws, 14 pages

Cheating is a serious offense.
The consequence ranges from zero on the test
to failure on the course to expulsion from the university.

The value of each question is indicated in square brackets.
A blank answer is worth about one-third of the marks;
to that, marks will be added for readable and relevant and correct information,
and marks will be subtracted for unreadable or irrelevant or incorrect information.

Please position the camera so that your hands and face are visible.
Please write your name and student number on each answer page.
If you are in doubt about the meaning of a question,
state what meaning you assume as part of your answer.

0[9]   We can define bunch *powint* by the equation *powint* = 2^{int} . Define the same bunch by construction and induction, without using exponentiation.

1   Let i be an integer variable, and let P be a specification such that

        $P \;=\;$ **if** $i{=}0$ **then** *ok* **else** $i{:=}\ i{-}1.\ P.\ i{:=}\ i{+}1$ **fi**

(a)[3]   Add recursive time.

(b)[15] Using recursive construction starting with $\top$, find the first three approximations toward the weakest fixed-point (weakest solution for P), including time.

(c)[3]   From these three approximations, guess the weakest fixed-point (weakest solution for P).

2[12]   The axioms of program Queue Theory are

     (0) $isemptyq' \;\Longleftarrow\; mkemptyq$

     (1) $isemptyq \Rightarrow front'{=}x \land \neg isemptyq' \;\Longleftarrow\; join\ x$

     (2) $\neg isemptyq \Rightarrow front'{=}front \land \neg isemptyq' \;\Longleftarrow\; join\ x$

     (3) $isemptyq \Rightarrow (join\ x.\ leave \;=\; mkemptyq)$

     (4) $\neg isemptyq \Rightarrow (join\ x.\ leave \;=\; leave.\ join\ x)$

     Prove

        $front'{=}3 \;\Longleftarrow\; mkempty.\ join\ 3$

end of test