

**CSC465/2104 Final Exam 2021 April 22**

1 page, 7 questions, 100 marks, 2 hours

Aids allowed: one letter-sized page, both sides  
list of laws, 14 pages

Cheating is a serious offense.

The consequence ranges from zero on the test  
to failure on the course to expulsion from the university.

The value of each question is indicated in square brackets.

A blank answer is worth about one-third of the marks;  
to that, marks will be added for readable and relevant and correct information,  
and marks will be subtracted for unreadable or irrelevant or incorrect information.

**Assignment Project Exam Help**

Please position the camera so that your hands and face are visible.

Please write your name and student number on each answer page.

If you are in doubt about the meaning of a question,  
state what meaning you assume as part of your answer.

**Add WeChat powcoder**

start of exam

- 0[9] Let  $n$  be a natural state variable. Is the following specification implementable?  
**if  $n > 0$  then  $n := n - 1$  else ok fi**  
Prove or disprove.
- 1[9] Let  $x$  be an integer variable. What is the exact precondition for  $x' \leq 1 \vee x' \geq 5$  to be refined by  $x := x + 1$ ?
- 2[18] Write a program to find the sum of the digits in the decimal representation of a given natural number. Define and refine all specifications, but you do not need to prove the refinements. Ignore time. Hint: use *div* and *mod*.
- 3[12] Let  $i$  be an integer variable, and let  $t$  be time. Let  $P$  be a specification such that  
 **$P \Leftarrow \text{if } i = 0 \text{ then ok else } i := i - 1. t := t + 1. P \text{ fi}$**   
What fixed-point (solution for  $P$ ) does recursive construction give when we start with  $t' = \infty$ ? (You do not need to prove it is a fixed-point.)
- 4 An old implementer's variable  $c: -1, 0, 1$  is being replaced by two new implementer's variables  $a, b: \text{bin}$  such that  $c = -1$  is replaced by  $a$  and  $b$  both being  $\perp$ ,  $c = 1$  is replaced by  $a$  and  $b$  both being  $\top$ , and  $c = 0$  is replaced by  $a$  and  $b$  being unequal.
- (a)[6] What is the transformer?
- (b)[12] Use your transformer to transform  $c := 0$  and implement (refine as program) the resulting specification.
- 5 Let  $a$  and  $b$  be integer variables. Refine  $a = a + b \wedge b = a - b$  by replacing the question marks in the following. Prove that your answer is a refinement.
- (a)[6]  $a := ?; b := ?$
- (b)[6]  $a := ? \parallel b := ?$
- 6 The equation  
 $\text{time} = \text{screen}! t. t := t + 1. \text{time}$   
outputs the time  $t$  onto the *screen* channel once each time unit forever. Considering *time* as the unknown,
- (a)[13] what is the weakest solution to this equation? (No proof required.)
- (b)[9] what is the strongest solution to this equation? (No proof required.)

end of exam